FONDAZIONE BRUNO KESSLER

ECT*
EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

QUANTUM FRACTALS

Bira van Kolck

cnrs

THE UNIVERSITY OF ARIZONA

Outline

- Emergence of structure
- Discrete scale invariance
- Bosonic clusters
- Nucleons
- Conclusion

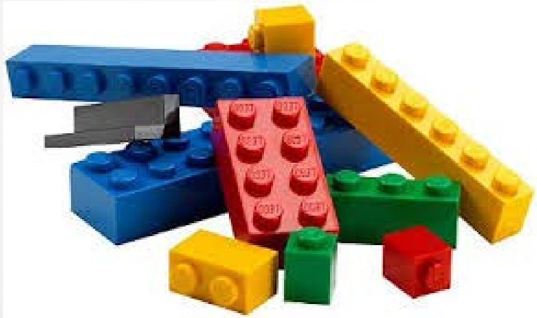
IN MEMORIAM

Manuel Malheiro

1960-2024

Nuclear physics and the emergence of quantum structures

"reduction":
what are the
building blocks?



p			r
	atoms, molecules		
1 keV		$2\,\mathring{A}$	
	hadrons/ nuclei		
1 GeV		$0.2\,\text{fm}$	
	"elementary" particles		

$$pr \sim 1$$

uncertainty principle

Heisenberg '27

"emergence":
how do the building
blocks fit together?

Here: $\hbar = 1, c = 1$

$$[m] = [E] = [p] = [r]^{-1} = [t]^{-1}$$

Structures and scales

"… were the world to be made between now and tomorrow 100 or 1,000 times larger or smaller than it is at present, all its parts being enlarged or diminished proportionally, everything would appear tomorrow exactly as now, just as though nothing had been changed."

Nicolas Oresme
a commentary on Aristotle's *De caelo et mundo*, 1377

stock

scale transformation

$$r \to \alpha r \qquad \alpha \geq 0$$

(quantum)

$$p \to \alpha^{-1} p$$

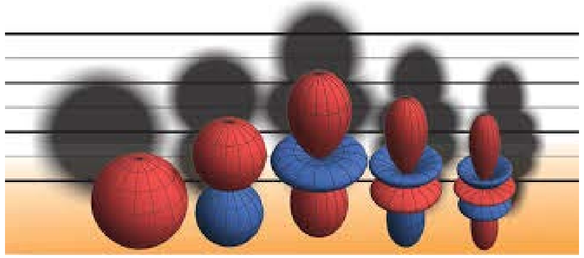
(nonrelativistic)

$$t/m \to \alpha^{2}\, t/m$$

$$mE \to \alpha^{-2} mE$$

≠

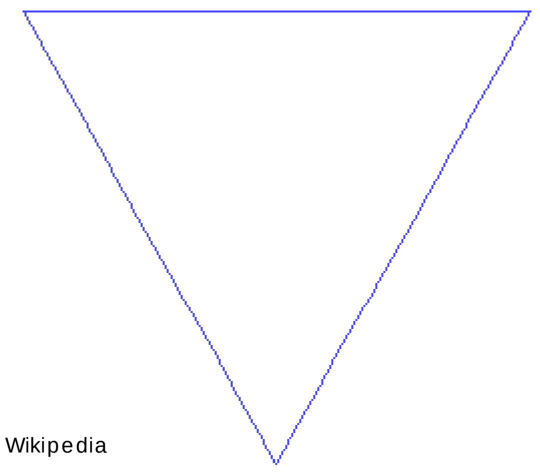
≠



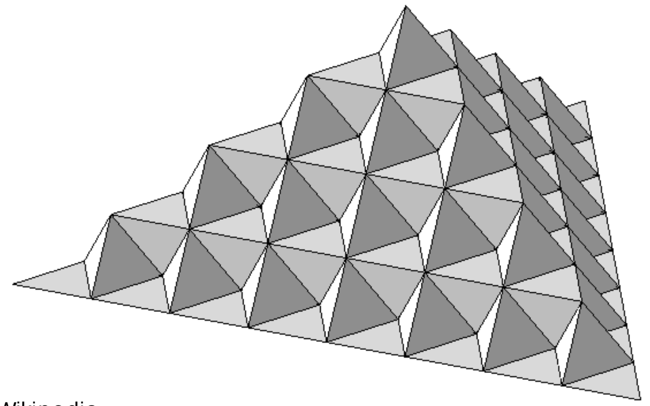
universe
not
scale
invariant
...

Simplest "complex" structures: one scale

von Koch 1904



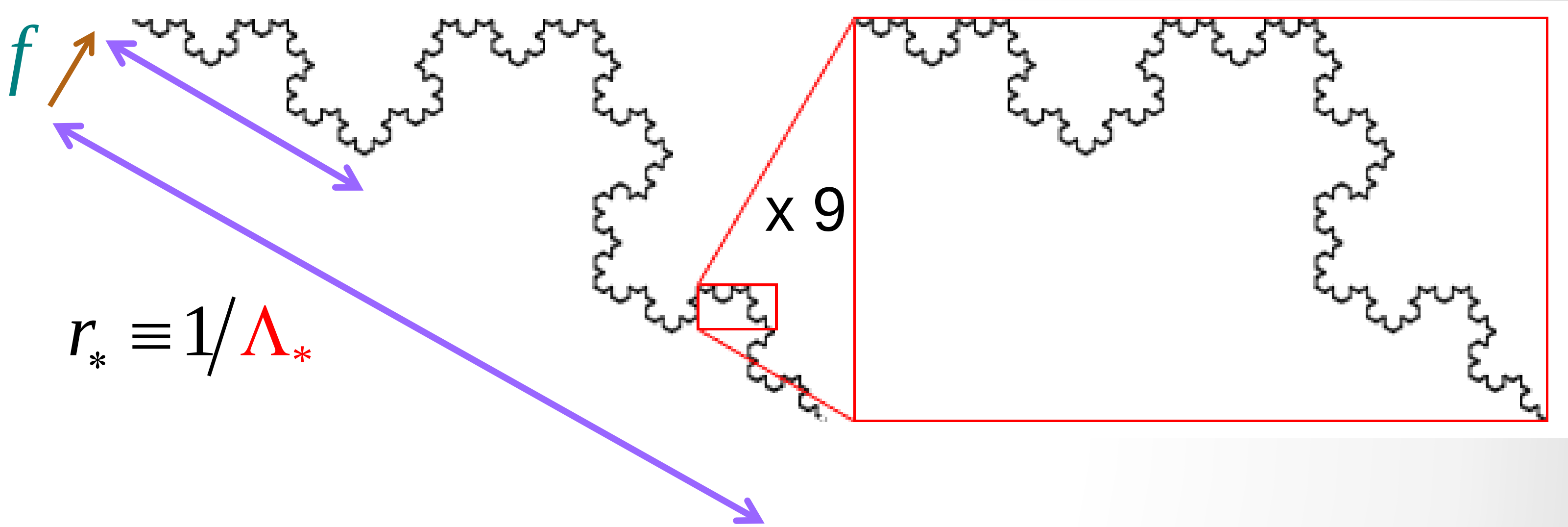
Wikipedia



Wikipedia

(Scaling) Fractal

yozh.org/2010/10/21/mset001/



$$r_* \equiv 1/\Lambda_*$$

$$r \rightarrow \alpha_n\, r = f^{\,n}\, r$$

$$\begin{cases} f \text{ real} \\ n \text{ integer} \end{cases}$$

Discrete scale invariance

romanesco or "fractal broccoli"

Nuclear physics: nucleons (proton or neutron) with spin S=½, nearly the same mass

four-component fermions $m_N \simeq 940$ MeV

lightest exchanged particles: pions

$m_\pi \simeq 140$ MeV

$$V(r) = -\frac{g_{\pi N}^2}{m_N}\frac{e^{-m_\pi r}}{r^3} S_{12} + \ldots$$

range $R \sim m_\pi^{-1} \simeq 1.4$ fm

Yukawa '35

proton + neutron, S=1: deuteron

$B_2 \simeq 2.2$ MeV ⇒ $\frac{R}{a_2} \simeq \frac{\sqrt{m_N B_2}}{m_\pi} \approx \frac{1}{3}$

binding energy

Atomic physics: neutral atoms, mass $m_{at} \approx A m_N$

fermions or bosons

lightest exchanged particles: two photons

$m_\gamma = 0$

$$V(r) = -\frac{l_{\rm vdW}^4}{4\pi m_{\rm at} r^6} + \ldots$$

"range" $R \sim l_{\rm vdW}$

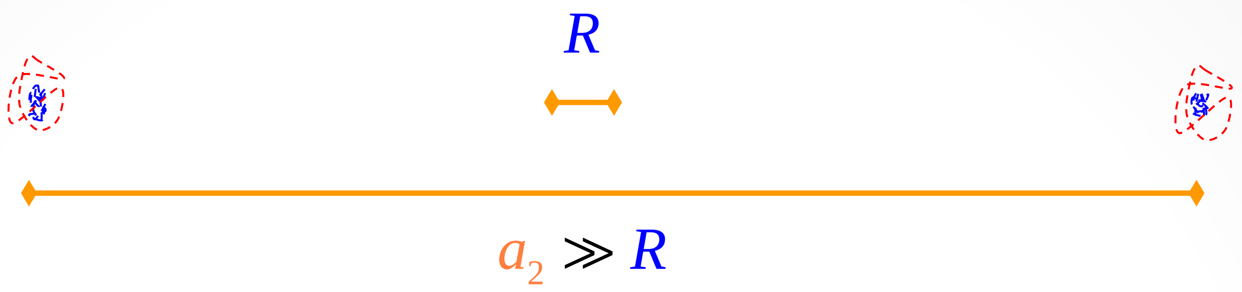
v.d. Waals 1873

two bosonic ^{4}He atoms, S=0: ^{4}He dimer

$B_2 \simeq 1.3$ mK

$l_{\rm vdW} \simeq 5.4$ Å

⇒ $\frac{R}{a_2} \simeq \sqrt{m_N B_2}\, l_{\rm vdW} \approx \frac{1}{20}$



multipole expansion of interactions

leading order — next-to-leading order

$$V_2(\vec{r};\Lambda) = \frac{4\pi}{m}\left\{ C_0(\Lambda)\ \delta_\Lambda^{(3)}(\vec{r}) + C_2(\Lambda)\ \nabla^2\delta_\Lambda^{(3)}(\vec{r}) + \ldots \right\}$$

cutoff-dependent parameters — smeared delta function

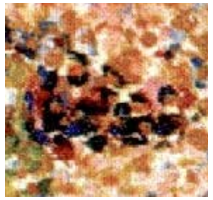
"Short-Range Effective Field Theory"

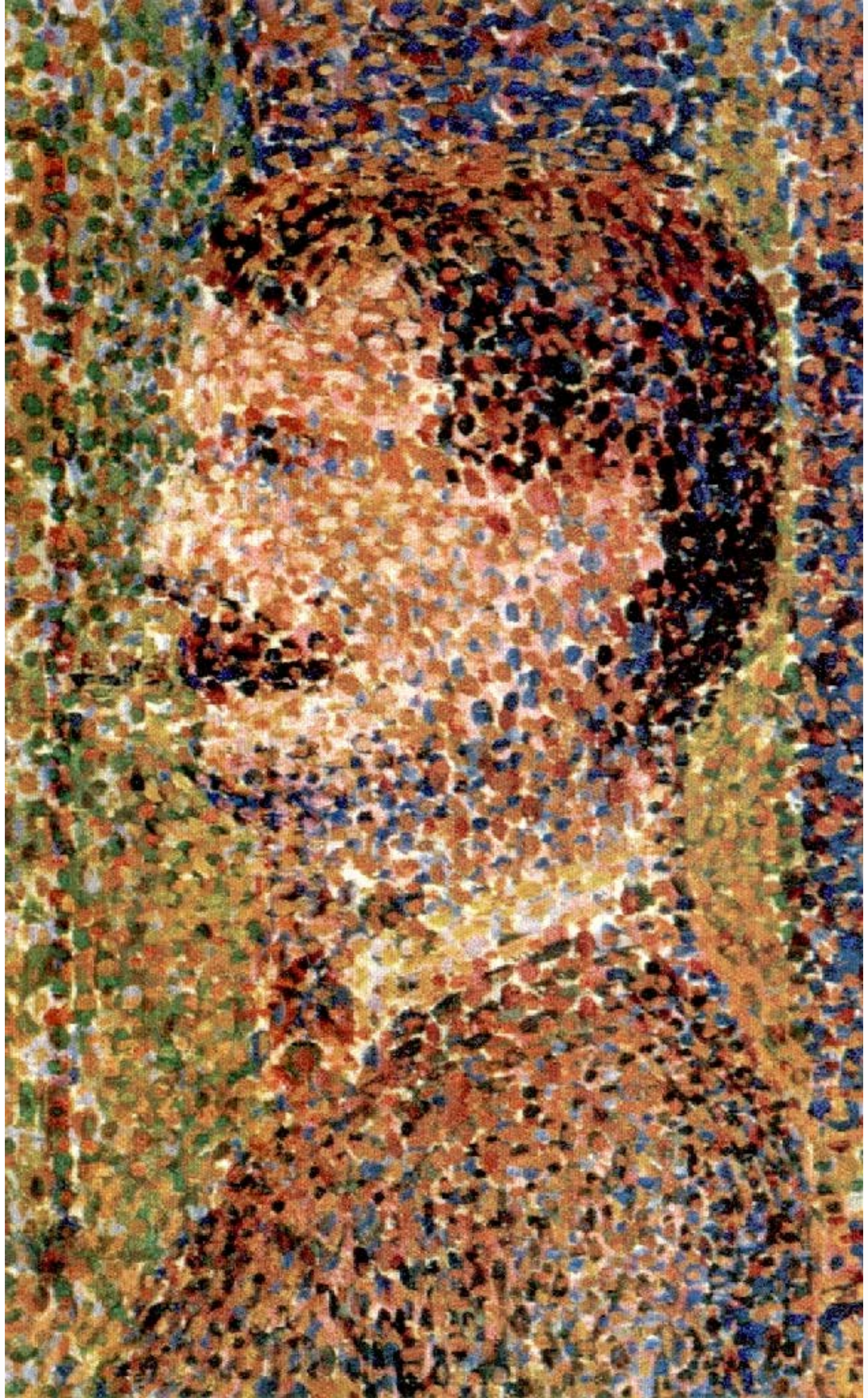
vK '97'98
Bedaque + vK '97
Kaplan, Savage, Wise '98
…

cf. Bethe, Peierls '35

⬇ renormalization

S matrix in expansion in powers of *R*/*r*, insensitive to *arbitrary* choice of regularization as long as $\Lambda^{-1} \ll R$





Seurat,

La Parade

(detail)

LO point limit $R \to 0$

scale invariance

unitarity limit $a_2 \to \infty$ ⇒ $mE_2 = 0 \Leftrightarrow \alpha^{-2}\, mE_2 = 0$

no scale!

no two-body bound states...

cf. Oresme

Two-component fermions

no *N*-body bound states either
unless
scale invariance broken by external interaction/trap

e.g. $$\left.\frac{E_N^{(0)}}{N}\right|_{N\to\infty} = \xi\, \frac{3(3\pi^2\rho)^{2/3}}{10m}$$

universal number Bertsch '99

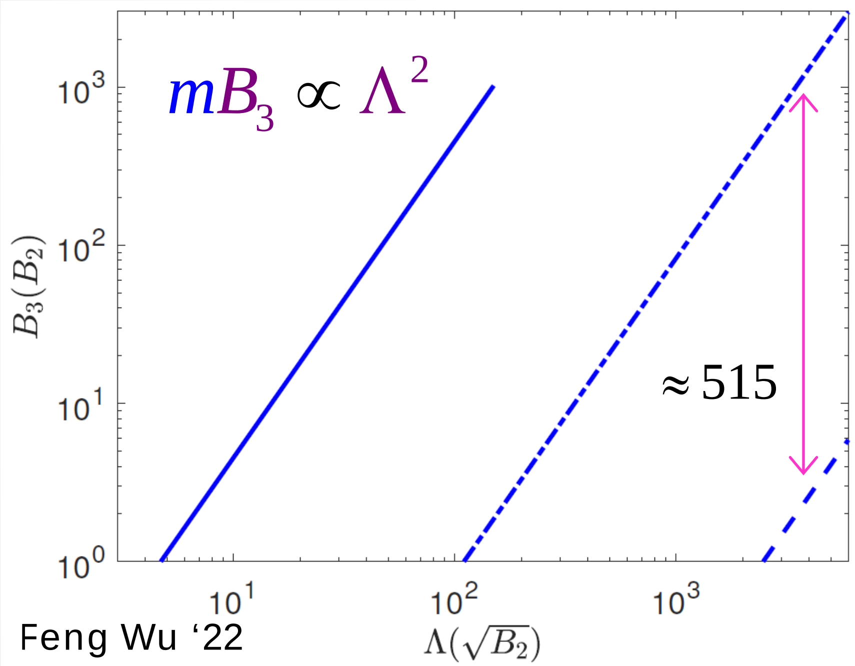
free-gas energy, uniform density

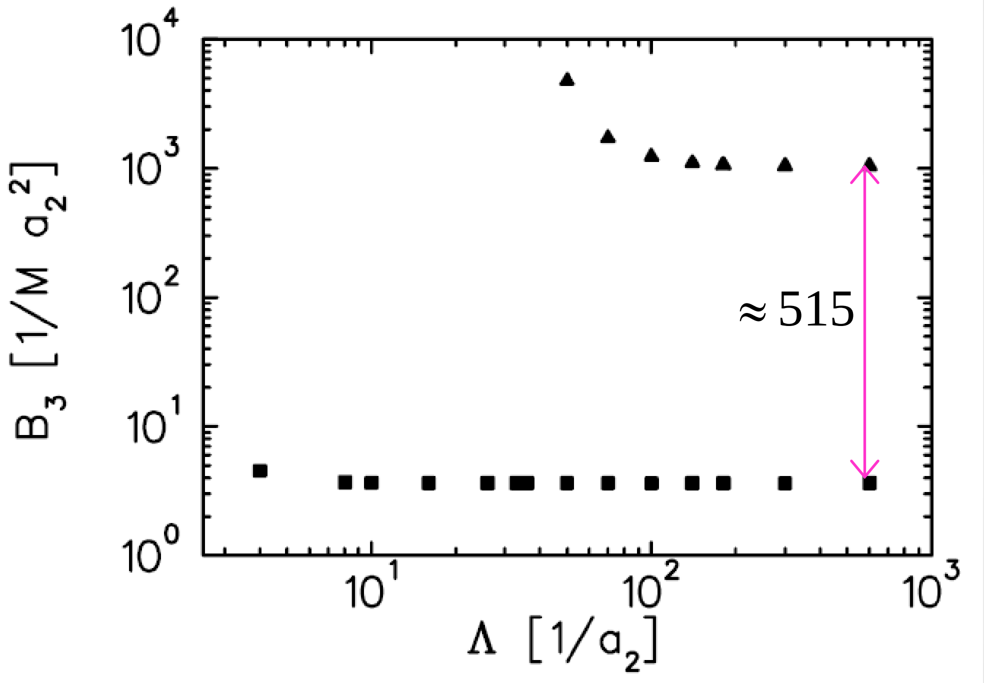
Other cases?

Bosons, more-component fermions

Three-body system

Bedaque, Hammer + v.K. '99 '00 …





"Thomas collapse"

Thomas '35

$$V_3(\vec{r}_1-\vec{r}_2,\vec{r}_2-\vec{r}_3)=\frac{(4\pi)^2}{m}D_0(\Lambda)\,\delta^{(3)}_{\Lambda}(\vec{r}_1-\vec{r}_2)\,\delta^{(3)}_{\Lambda}(\vec{r}_2-\vec{r}_3)$$

one parameter

determined by one three-body datum

Bedaque, Hammer + vK '99 '00
...

$$H(\Lambda) \equiv \frac{\Lambda^2 D_0(\Lambda)}{m C_0^2(\Lambda)} \simeq \frac{\sin\left(s_0 \ln(\Lambda_*/\Lambda) - \arctan(s_0^{-1})\right)}{\sin\left(s_0 \ln(\Lambda_*/\Lambda) + \arctan(s_0^{-1})\right)}$$

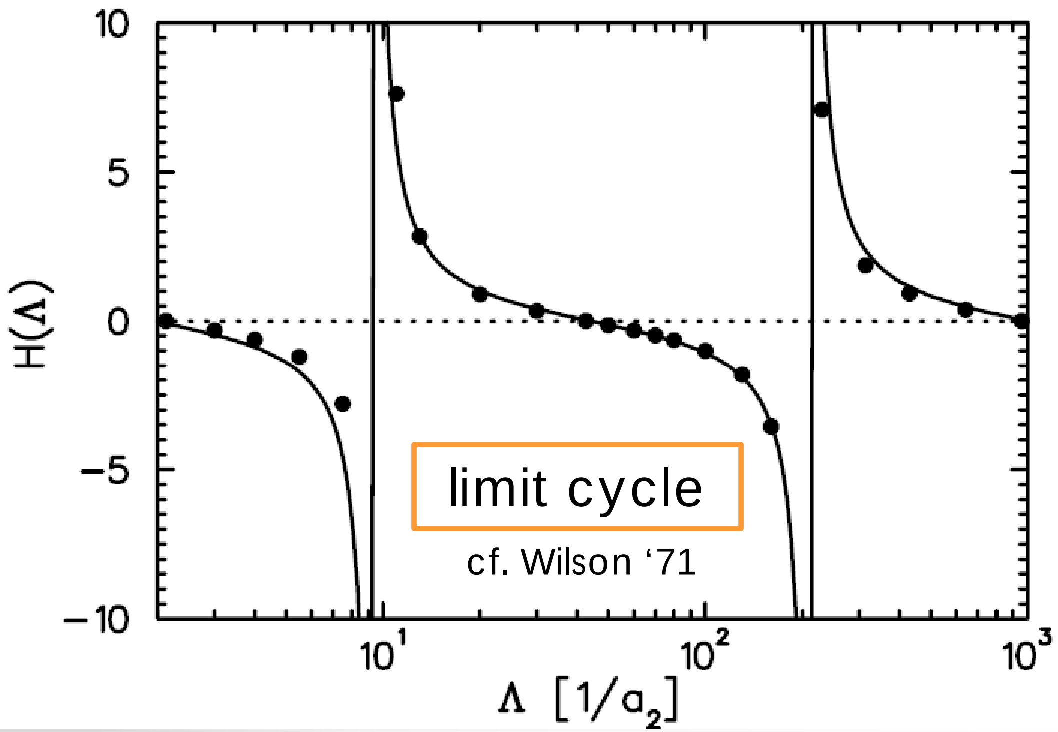
$s_0 \simeq 1.00624$

dimensionful parameter

anomalous breaking of
(continuous) scale invariance

quantum phenomenon!

f^{-1}



$$\Lambda \to \alpha_n^{-1}\Lambda = f^{-n}\Lambda$$

$$f^{-1} = \exp\left(\pi/s_0\right) \simeq 22.7$$

Discrete scale invariance

Templo Mayor

Two consequences

1) Towers of excited states

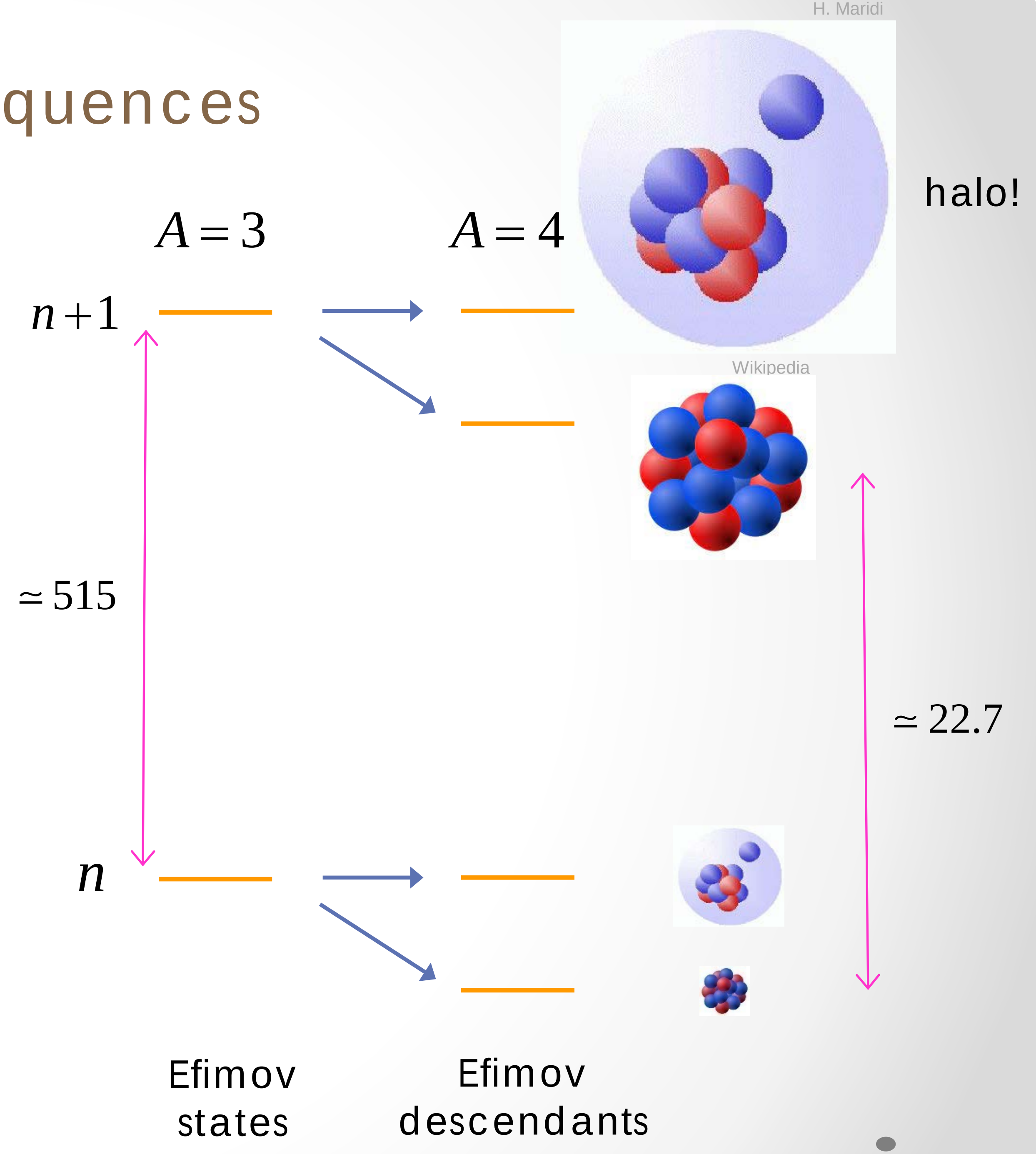
$$mB^{(0)}_{A,n} \to \alpha_l^{-2}\, mB^{(0)}_{A,n} = mB^{(0)}_{A,n+l}$$

$$\Rightarrow \quad mB^{(0)}_{A,n}(\Lambda_*) = mB^{(0)}_{A,0}(\Lambda_*)\exp\left(-2n\pi/s_0\right)$$

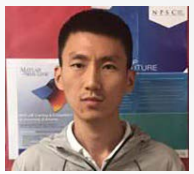
ground state — fixes tower position

$A=3$ Efimov ’70

$A=4$ Hammer, Platter ’07



Wu, Frederico, Higa + vK ‘24



“Halo EFT”

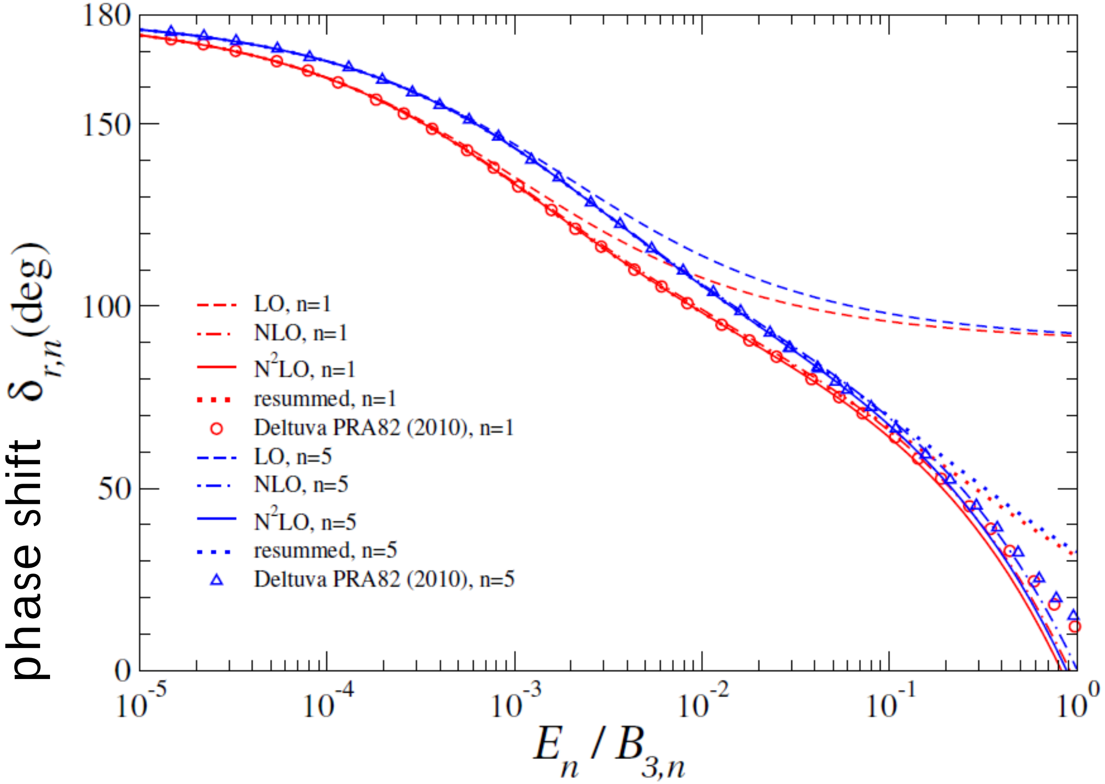
Bertulani, Hammer + vK ’03 ...

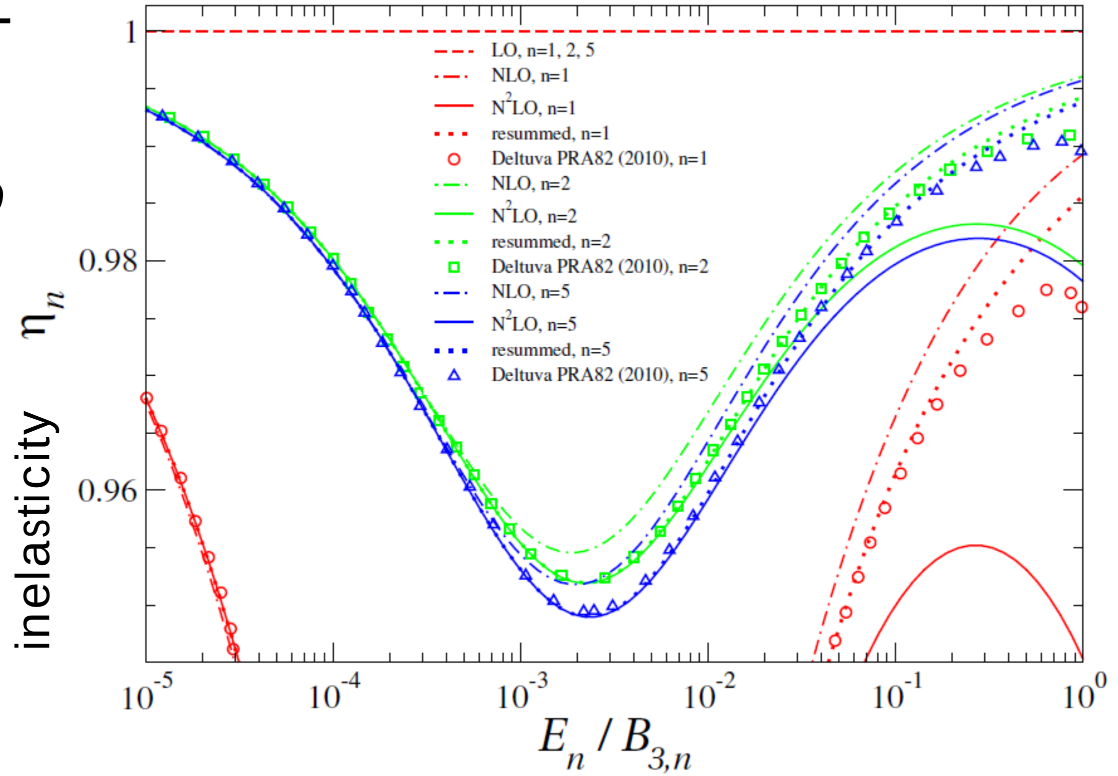
$$r_h \sim \left(3m\left(B^*_{4,n} - B_{3,n}\right)/2\right)^{-1/2} \gg \left(2mB_{3,n}/3\right)^{-1/2} \sim R_h$$

- integrate out all states except neighboring trimer
- treat trimer as point in “halo” expansion in R_h/r_h

$\Rightarrow$ $C_{2n}(\Lambda) = C_{2n;R}(\Lambda) + i\,C_{2n;I}(\Lambda)$

1+3 scattering: Halo EFT vs *ab initio*





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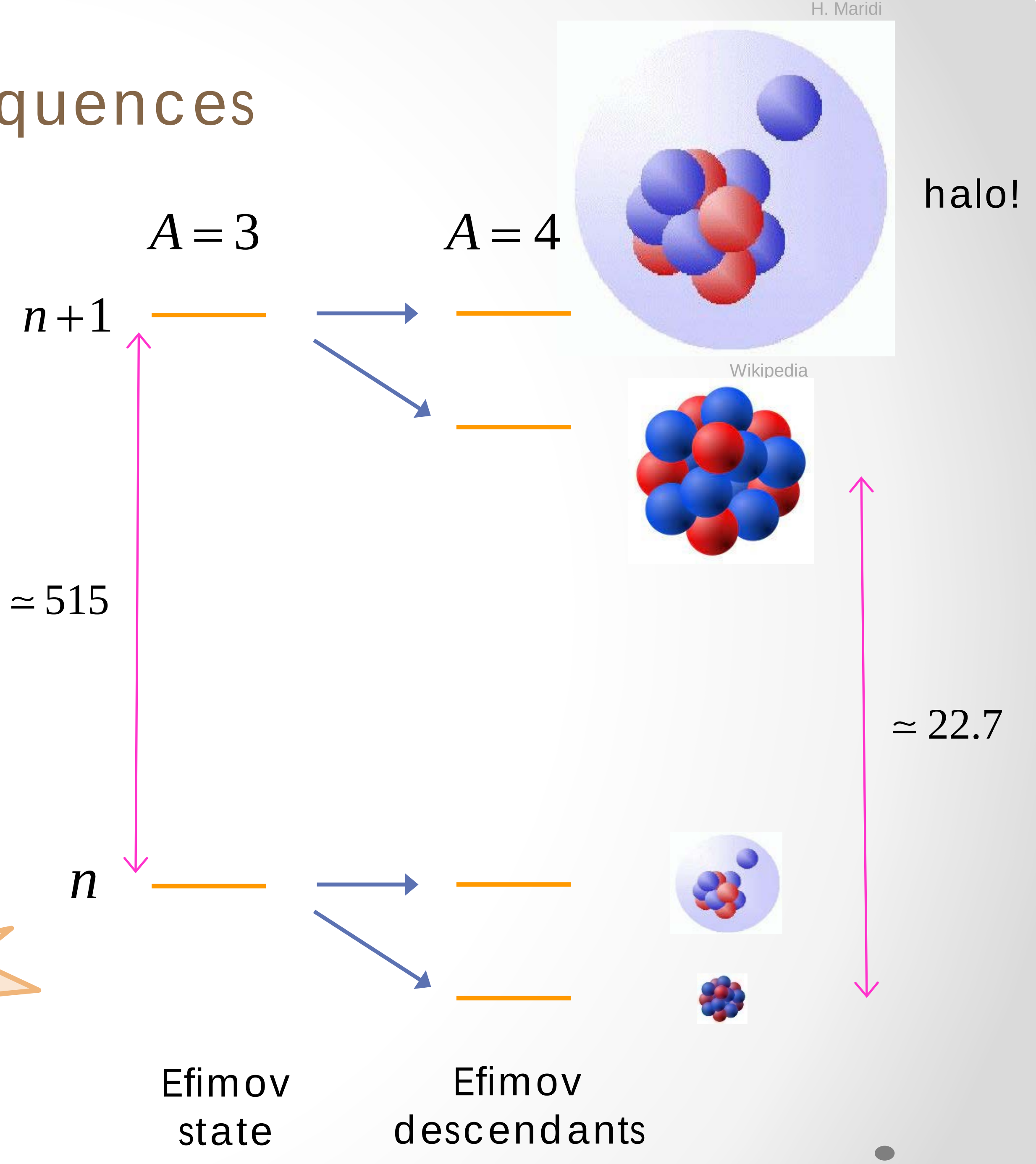
ground state — fixes tower position

$A=3$ Efimov '70

$A=4$ Hammer, Platter '07

$A=5,6$ bosons — von Stecher '10'11, Gattobigio, Kievsky, Viviani '11'12

Towers upon towers!



2) Ground-state correlations

single scale → $$\frac{B^{(0)}_{A,0}(\Lambda_*)}{A} = \kappa_A \frac{B^{(0)}_{3,0}(\Lambda_*)}{3}$$

universal numbers

$\kappa_2 \equiv 0$

$\kappa_3 \equiv 1$

$\kappa_4 \simeq 3.5$ Hammer, Platter '07

$\kappa_{A\geq 5} \simeq ?$ von Stecher '10

...

Carlson, Gandolfi, Vitiello + vK '17

varying Λ_*

$A = 4$ Tjon line Platter, Hammer, Meißner '05

Tjon '75

Nakaichi, Akaishi, Tanaka, Lim '78

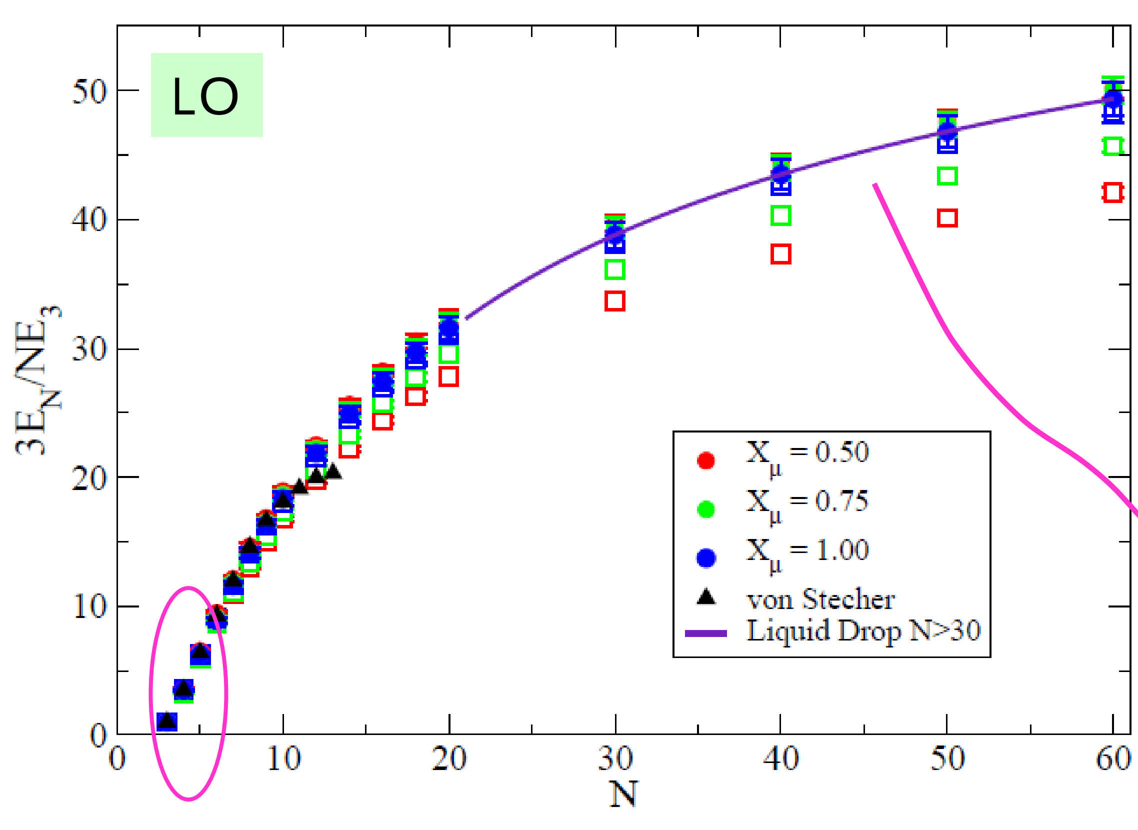
$A = 5, 6$ bosons Generalized Tjon lines Bazak, Eliyahu + vK '16

Nakaichi, Akaishi, Tanaka, Lim '79'80

N unitary bosons

Gandolfi, Carlson, Vitiello + vK '17





↑ increasing Λ

saturation!

cf. Piatecki + Krauth '14
Comparin + Krauth '16

$$\kappa_N \approx \frac{3}{N}(N-2)^2$$

Bazak, Eliyahu + vK '16

$$\kappa_N = \kappa_\infty \left[1 - \eta_S N^{-1/3} + \mathcal{O}\left(N^{-2/3}\right)\right]$$

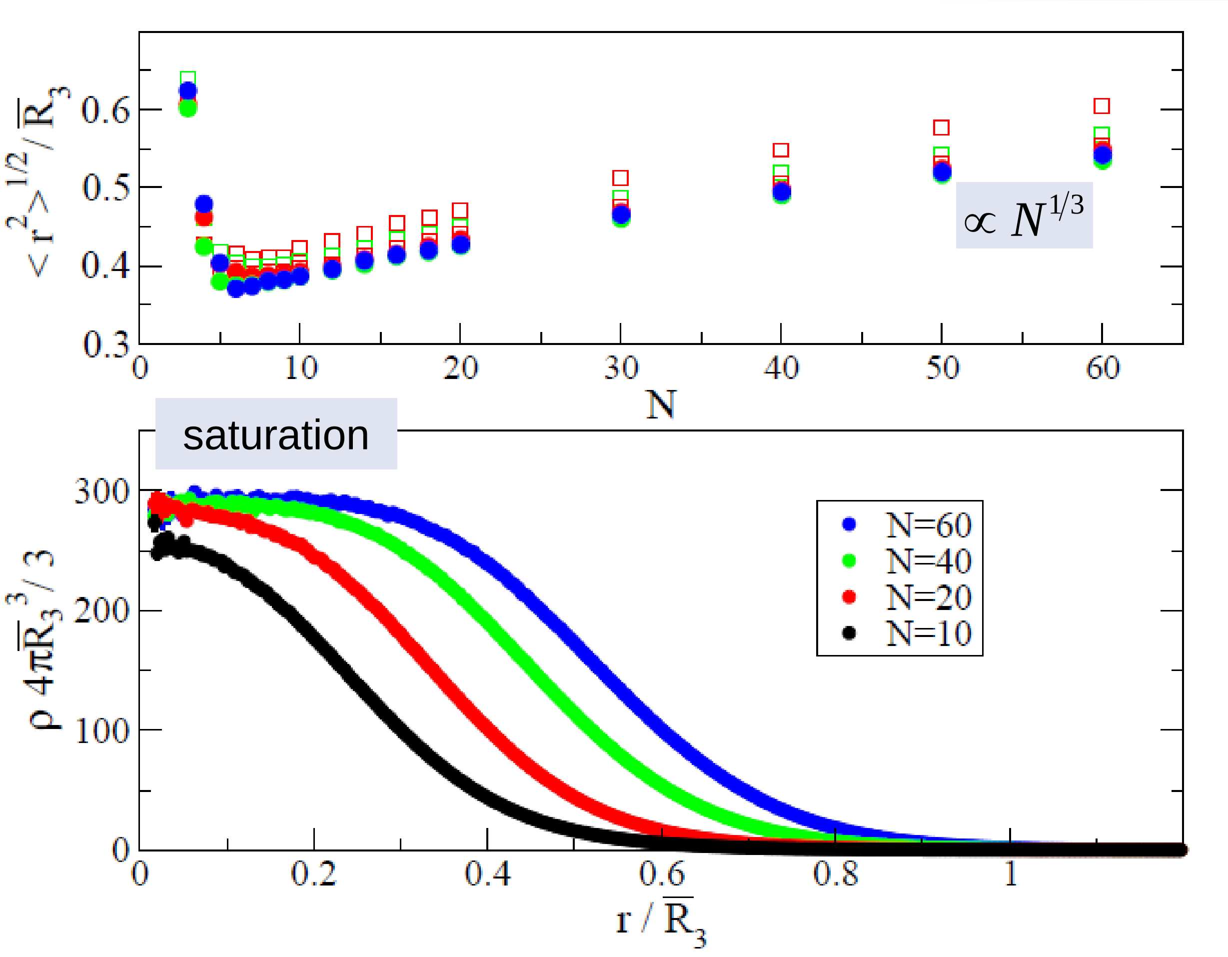
$$\kappa_\infty = 90 \pm 10 \qquad \eta_S = 1.7 \pm 0.3$$

cf. ^{4}He $\quad \kappa_\infty \simeq 182 \qquad \eta_S \simeq 2.7$

Pandharipande *et al.* '83

Gandolfi, Carlson, Vitiello + vK ’17

A liquid indeed…

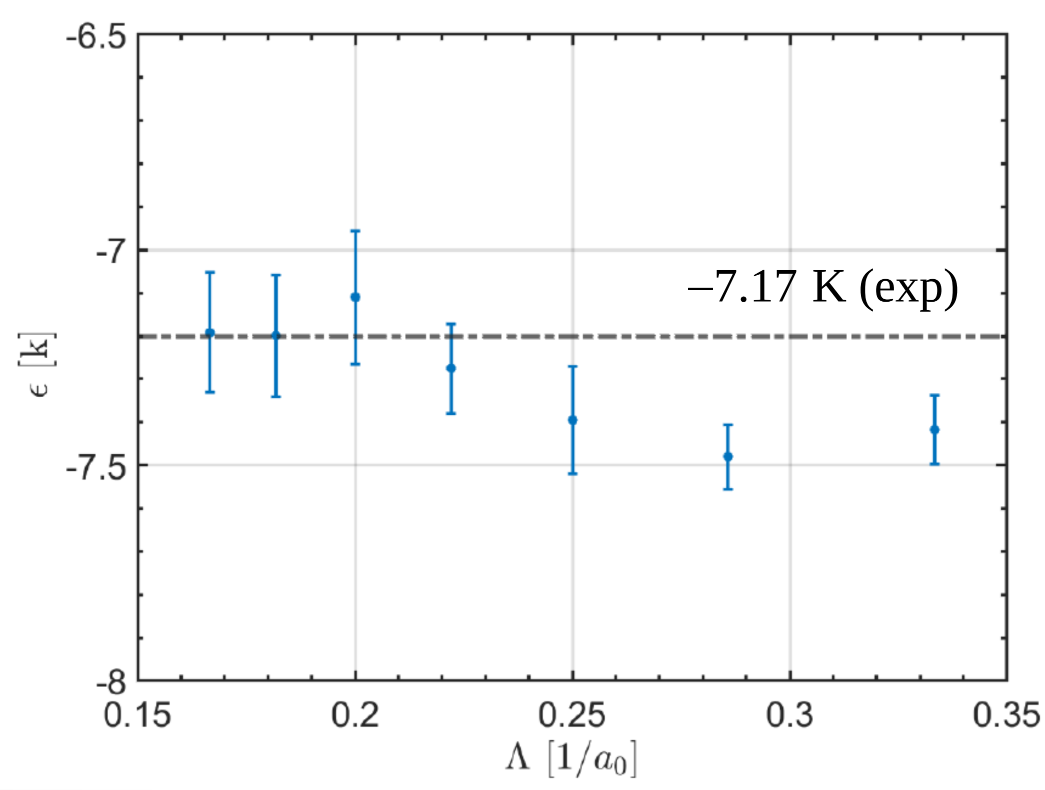


$$\bar{R}_3 \equiv \left(2mB_3\right)^{-1/2}$$

^{4}He atoms

De-Leon, Pederiva ’22

energy per particle
at saturation density



$\Rightarrow$ $\kappa_\infty \simeq 190$

vs.

$\kappa_\infty \simeq 182$ Pandharipande *et al.* ’83

$\kappa_\infty \simeq 184$ (exp)

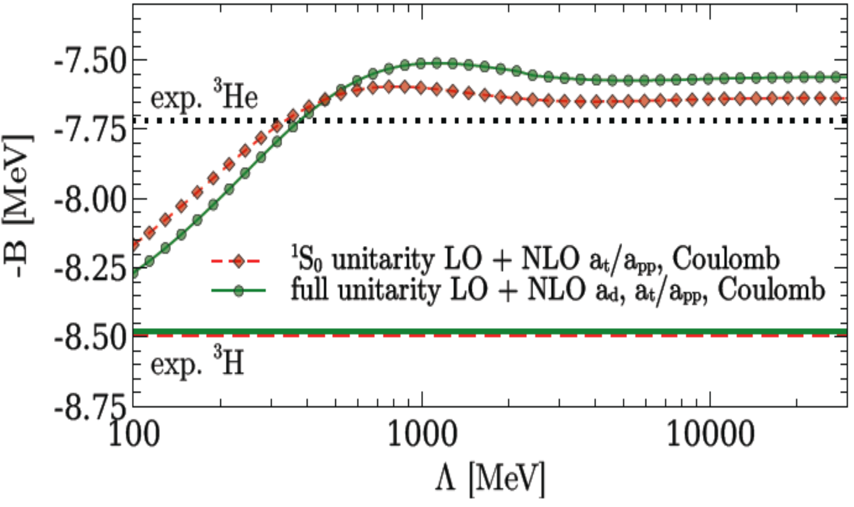
a_2^{-1} corrections nonperturbative!?

a_2^{-1} perturbative corrections

Contessi, Gandolfi, Carlson + vK, in progress

Nucleons around unitarity

Helion-triton splitting



$A=3$

NLO

$$B_h^{(1)} - B_t \simeq -(0.92 \pm 0.18)\ \text{MeV}$$

vs.

$$-0.764\ \text{MeV (exp)}$$

LO

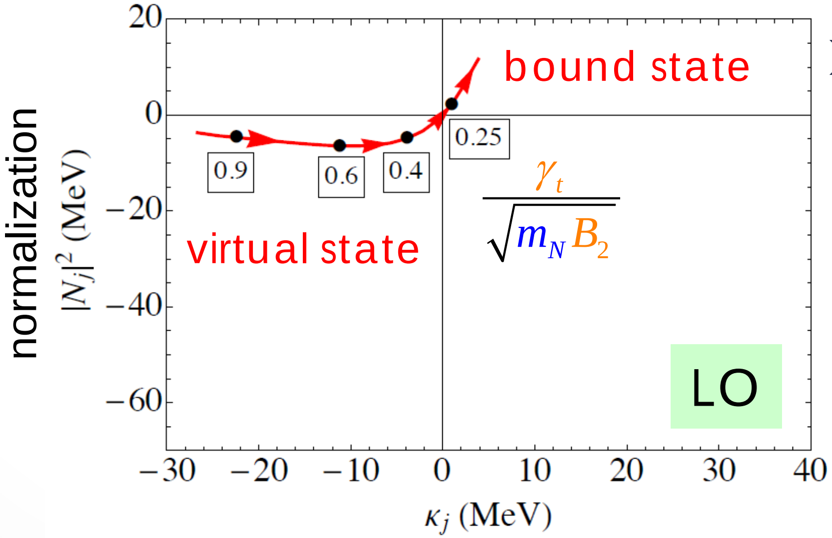
König, Grießhammer, Hammer + vK '16



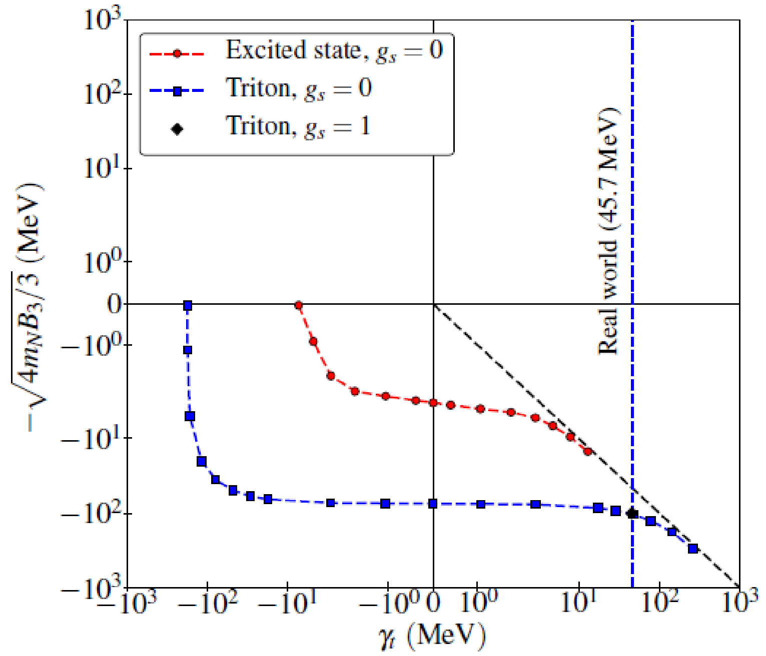
First-excited state of triton

Rupak, Vaghani, Higa + vK '18





binding momentum

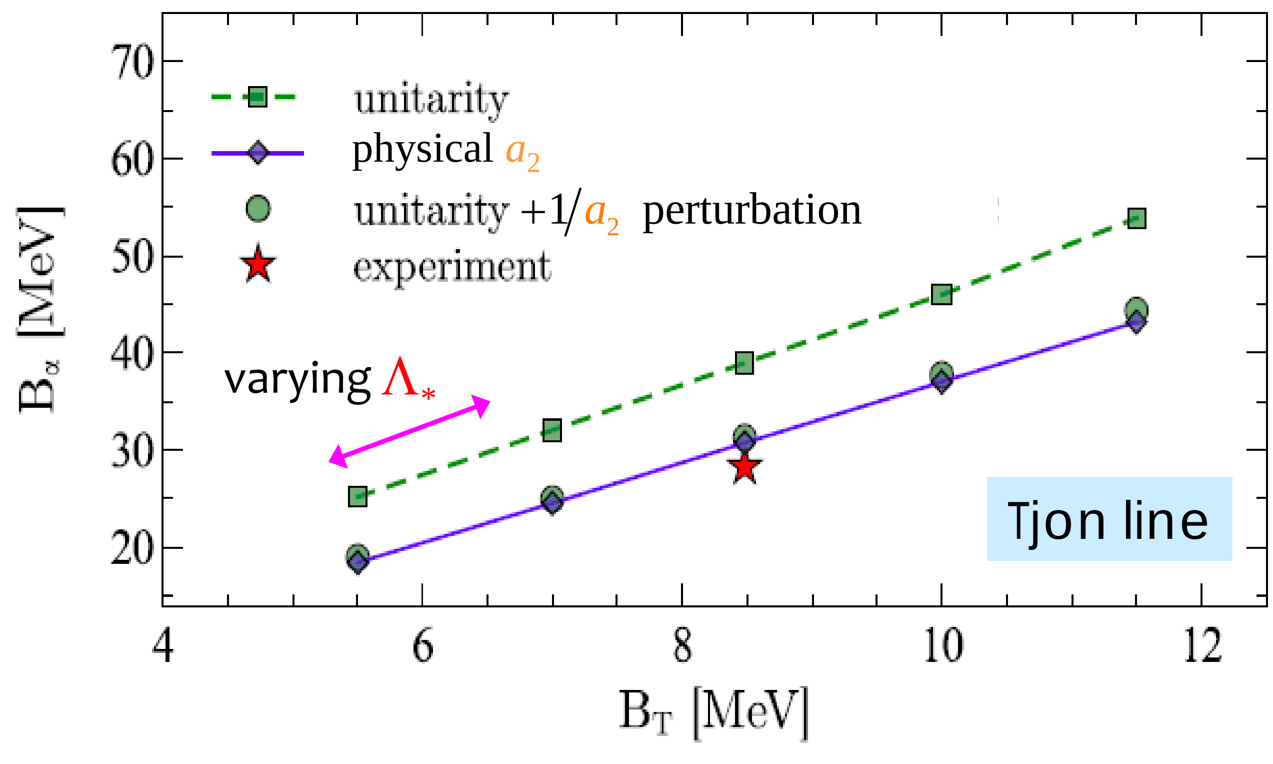


deuteron binding momentum

Alpha particle

$A=4$

König, Grießhammer, Hammer + vK '17



Ground state

LO

incomplete NLO

perturbatively close to unitarity limit!

First-excited state

$$\frac{B^{(0)}_{\alpha*}}{B_t} = \frac{B^{(0)}_{\alpha*}}{B^{(0)}_h} \simeq 1.0023$$

Hammer, Platter '07
...
Deltuva '10

vs.

$$\frac{B_{\alpha*}}{B_h} = 1.05 \text{ MeV (exp)}$$

$$\frac{B_{\alpha*}}{B_t} = 0.95 \text{ MeV (exp)}$$

but needs full NLO

Similar for [4]He atoms Wu, König + vK, in progress

More nucleons around unitarity

discrete scale invariance

Multiple towers of excited states?

Ground states

single scale → $$\frac{B_A^{(\mathrm{LO})}(\Lambda_*)}{A} = \kappa_A \frac{B_3(\Lambda_*)}{3}$$

$$\begin{cases} \kappa_2 \equiv 0 \\ \kappa_3 \equiv 1 \\ \kappa_4 \simeq 3.5 \\ \kappa_{A\geq 5} \simeq ? \end{cases}$$

= bosons

grows slower than bosons?

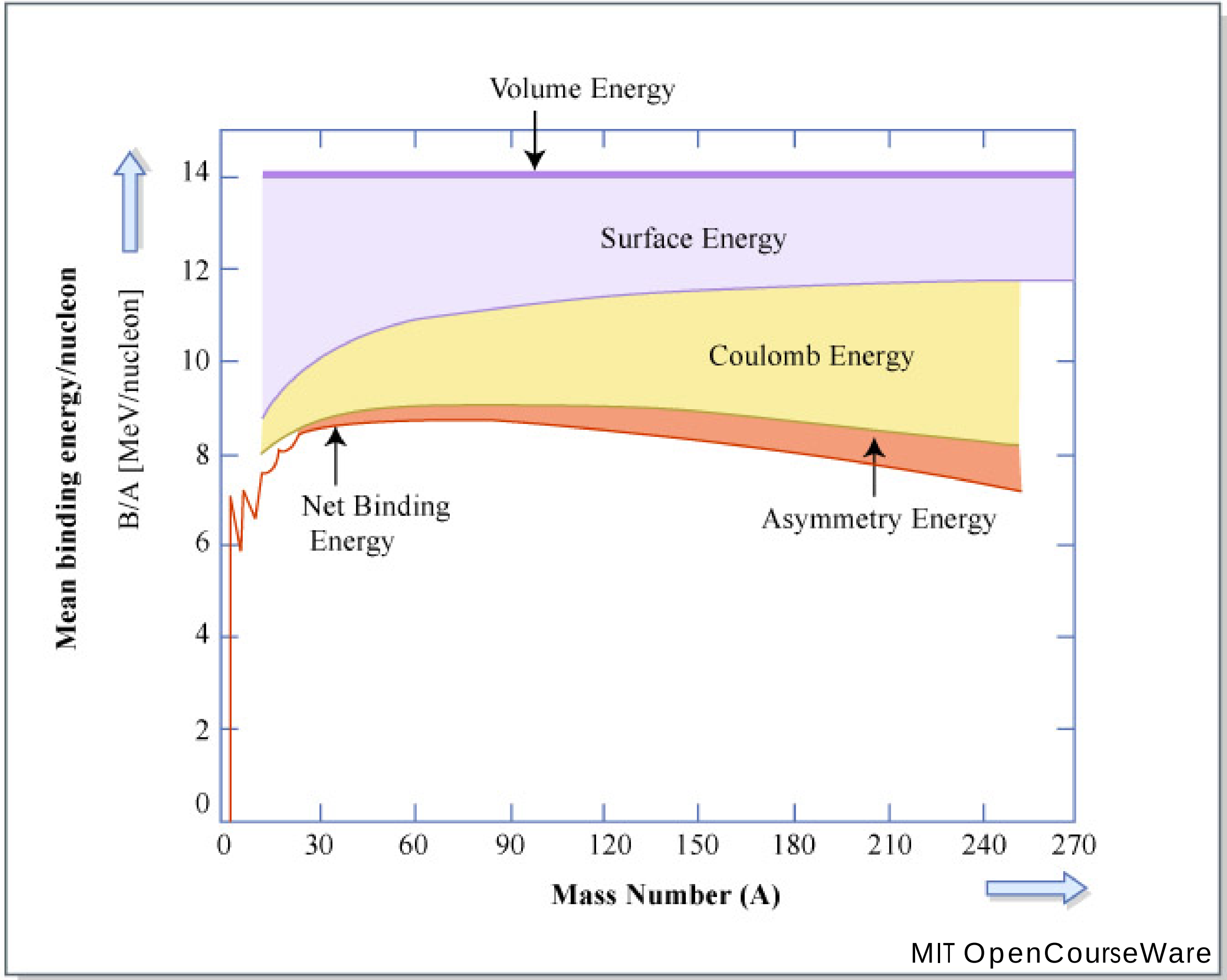
nuclear structure from a single parameter?

Semi-empirical mass formula

Von Weizsäcker ‘35

$$\kappa_A = \kappa_\infty \left[1 - \eta_S A^{-1/3} - \eta_C A^{-4/3} Z(Z-1) - \eta_A (1-2Z/A)^2 + \ldots \right]$$

$$\frac{B_3(\Lambda_*)}{3} \simeq 2.8$$



$\kappa_\infty \approx 5.6$

$\eta_S \approx 1.2$

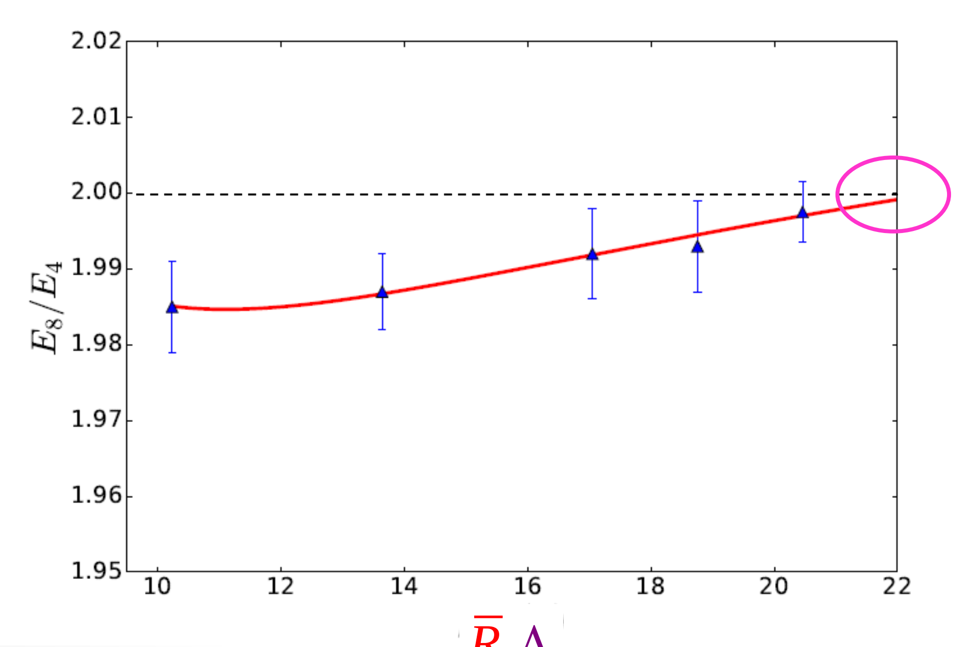
$\eta_C \approx 0.05$

$\eta_A \approx 1.5$

Dawkins, Carlson, vK + Gezerlis '20

$A=8$

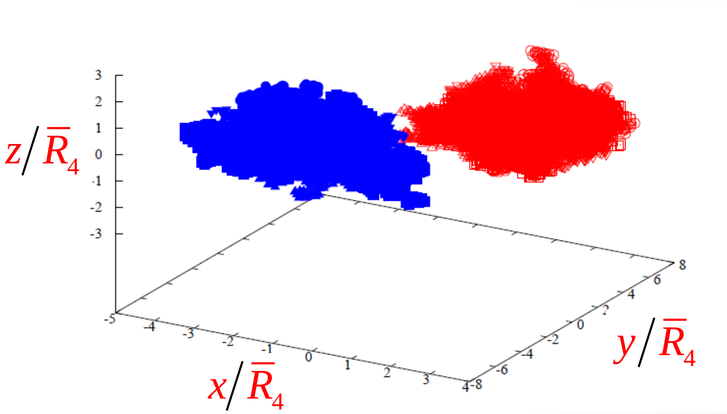
LO



$\bar{R}_4\Lambda$

$\bar{R}_4 \equiv (2mB_4)^{-1/2}$

$\kappa_8 \simeq \kappa_4$?



consistent with ^{8}Be

Clustering a universal property of multi-component unitary fermions?

See also Schäfer, Contessi, Kirscher, Mareš '20

connection to Bijker+Iachello?

¿ how to get stable states at higher orders ?

Contessi, Schäfer, Kirscher, Lazauskas, Carbonell '23

- evidence for nearby poles at LO
- most nuclear ground states shallow $\left|\frac{B_A}{A}-\frac{B_4}{4}\right| \ll \frac{B_A}{A}$

⇓

improve LO with some subLO corrections
while maintaining renormalization and power counting

- no new physical parameter at LO
- effect no larger than NLO → removed at NLO (within N^{2}LO uncertainty)

Contessi, Schäfer + vK '23
Contessi, Pavón Valderrama + vK '24

Conclusion

Quantum systems near unitarity can be described by essentially *one* parameter Λ_*

Fractal-like structure emerges from discrete scale invariance

Bosons saturate and form a quantum liquid

Multi-component fermions tend to cluster

Perturbative expansion around unitarity works for light nuclei. How far can we go?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2024 PROGRAMME OF ACTIVITIES

FEBRUARY

5-9.2 alpha_S(2024): Workshop on Precision Measurements of the Strong Coupling Constant
D. D'ENTERRIA (CERN), S. KLUTH (MPP), G. ZANDERIGHI (MPP)

12-16.2 New Jet Quenching Tools to Explore Equilibrium and Non-Equilibrium Dynamics in Heavy-Ion Collisions
A. SADOFYEV (LIP), C. ANDRES (LIP), J. BARATA (BNL), C. SALGADO (IGFAE)

26.2-1.2 Inaugural Workshop on Nuclear Astrochemistry
N. MASON (University of Kent), D. BEMMERER (HZDR), E. MASHA (HZDR), D. MIFSUD (Atomki)

MARCH

04-08.2 EDMs: Complementary Experiments and Theory Connections
S. DEGENKOLB (Heidelberg University), P. SCHMIDT-WELLENBURG (Paul Scherrer Institute), G. Pignol (LPSC), J. DE VRIES (University of Amsterdam), R. BERGER (Philipps-Universität Marburg)

APRIL

15-19.4 Bridging Scales: At the Crossroads among Renormalisation Group, Multi-Scale Modelling, and Deep Learning
R. MENICHETTI (University of Trento), F. PEDERIVA (University of Trento), R. POTESTIO (University of Trento), A. ROGGERO (University of Trento)

22-26.4 The Physics of Strongly Interacting Matter: Neutron Stars, Cold Atomic Gases and Related Systems
A. SCHWENK (TU Darmstadt), F. FERLAINO (University of Innsbruck), C. PETHICK (Niels Bohr Institute), A. WATTS (University of Amsterdam)

MAY

7-10.5 Quantum Science Generation 2024
D. DE BERNARDIS (INO-CNR), V. PANIZZA (University of Trento), L. VESPUCCI (University of Trento), A. BALDAZZI (University of Trento), V. AMITRANO (University of Trento), C. BENAVIDES-RIVEROS (INO-CNR), A. BERTI (INO-CNR), A. NARDIN (University of Trento)

13-17.5 SPICE: Strange Hadrons as a Precision Tool for Strongly InteraCting SystEms
J. POCHODZALLA (University of Mainz), C. CURCEANU (INFN-LNF), B. DOENIGUS (University of Frankfurt), L. FABBIETTI (TU Munich), S. NAKAMURA (University of Tokyo), F. SAKUMA (RIKEN), I. VIDANA (INFN Catania)

20-24.5 Beyond-Eikonal Methods in High-Energy Scattering
J. JALILIAN-MARIAN (Baruch College), A. CZAJKA (NCBJ), Y. KOVCHEGOV (Ohio State University)

27-31.5 Machine Learning and the Renormalization Group
J. URBAN (MIT), D. HACKETT (Fermilab), A. HASENFRATZ (University of Colorado Boulder), J. PAWLOWSKI (Heidelberg University), B. LUCINI (Swansea University)

JUNE

3-7.6 A Modern Odyssey: Quantum Gravity meets Quantum Collapse at Atomic and Nuclear Physics Energy Scales in the Cosmic Silence
C. CURCEANU (INFN-LNF), A. BASSI (University and INFN Trieste), L. BAUDIS (University of Zurich), A. MARCIANO (Fudan University China), K. PISCICCHIA (CREF & Centro Ricerche Enrico Fermi), L. DIOSI (Wigner, University of Budapest)

10-14.6 Diffraction and Gluon Saturation at the LHC and the EIC
C. ROYON (University of Kansas), M. HENTSCHINSKI (Universidad de las Americas Puebla), A. SABIO VERA (Universidad Autonoma de Madrid), S. SCHLICHTING (University of Bielefeld), A. DESHPANDE (Stony Brook University)

17-21.6 Towards a Consistent Approach for Nuclear Structure and Reaction: Microscopic Optical Potentials
C. BARBIERI (University of Milan), C. ELSTER (Ohio University), C. HEBBORN (FRIB), A. OBERTELLI (TU Darmstadt)

JUY

1-5.7 New Opportunities and Challenges in Nuclear Physics with High Power Lasers
C.J. YANG (ELI-NP), K. SPOHR (ELI-NP), P. TOMOSSINI (ELI-NP), Y. FUKADA (Kansai Photon Science Institute), V. HORNY (ELI-NP), L. GIZZI (INO), D. DOMENICO (ELI-NP)

8-12.7 Synergies between LHC and EIC for Quarkonium Physics
F. CELIBERTO (Universidad de Alcalà), C. VAN HULSE (Universidad de Alcalà), J.P. LANSBERG (CNRS), D. KIKOLA (Warsaw University of Technology), D. BOER (University of Groningen), E. GONZALES-FERREIRO (IGFAE), C. FLORE (University of Turin)

15.7-2.8 DTP/TALENT: Training in Advanced Low Energy Nuclear Theory: Nuclear Theory for Astrophysics
A. ARCONES (TU Darmstadt & GSI), B. GIACOMAZZO (University of Milano-Bicocca), J. PIEKAREWICZ (Florida State University)

AUGUST

5-9.8 Towards Improved Hadron Tomography with Hard Exclusive Reactions
M. BOER (Virgina Tech), A. CAMSONNE (Jlab), J. WAGNER (NCBJ)

19-23.8 The Nuclear Interaction: Post-Modern Developments
R. TIMMERMANS (University of Groningen), J. McGOVERN (University of Manchester), M. PIARULLI (Washington University), U. VAN KOLCK (IJClab Orsay)

SEPTEMBER

9-13.9 New Developments in Studies of the QCD Phase Diagram
H. DING (Central China Normal University), F. KARSCH (University of Bielefeld), M.P.LOMBARDO (INFN Florence), P. PETRECZKY (BNL)

16-20.9 Spin and Quantum Features of QCD Plasma
F. BECATTINI (University and INFN Florence), X. HUANG (Fudan University), D. RISCHKE (Goethe University Frankfurt), Y. YIN (CAS)

30.9-4.10 KAMPAI - Kaonic, Antiprotonic, Muonic, Pionic and "onia" exotic Atoms: Interchanging Knowledge
A. SCORDO (INFN Frascati), P. INDELICATO (Laboratoire Kastler Brossel), J. OBERTOVA (Czech Technical University, Prague), C. CURCEANU (INFN-LNF), A. KNECHT (PSI), M. SKURZOK (Jagiellonian University of Krakow), T. HASHIMOTO (JAEA)

OCTOBER

14-25.10 Measuring Neutrino Interactions for Next-Generation Oscillation Experiments
S. DOLAN (CERN), C. WILKINSON (LBNL), C. WRET (University of Oxford), L. PICKERING (Rutherford Appleton Laboratory)

NOVEMBER

4-8.11 Universal Themes in Bose-Einstein Condensation
J. CARUSOTTO (INO-CNR BEC Center), T. GIAMARCHI (University of Geneva), G. FERRARI (University of Trento), D. SNOKE (University of Pittsburgh), P. LITTLEWOOD (University of Chicago), F. M. MARCHETTI (UAM), N. PROUKAKIS (University of Newcastle)

DECEMBER

02-06.12 Penetrating Probes of Hot High-mu_B matter: Theory Meets Experiment
E. SCOMPARIN (INFN Turin), T. GALATYUK (TU Darmstadt), M.P. Lombardo (INFN Florence), R. RAPP (Texas A&M University), G. USAI (University Cagliari)

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For information: staff@ectstaff.eu | www.ectstar.eu

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