Laboratory for Nuclear Science At Avery Point

aka Laboratory for Astrophysics http://astro.uconn.edu

The Algebraic Cluster Model (ACM) and Cluster Shell Model (CSM) *

Moshe Gai, University of Connecticut, http://astro.uconn.edu, moshe.gai@uconn.edu

Theory (ACM, CSM):

1. ACM: ^{8}Be,^{12}C,^{16}O,^{20}Ne
   The $\mathcal{D}3h$ Symmetry: ^{12}C,^{20}Ne
2. CSM: ^{9}Be, ^{13}C, ^{21}Ne
3. The $\mathcal{D}3h$' Symmetry: ^{13}C, ^{21}Ne

Phenomenology (CSM):

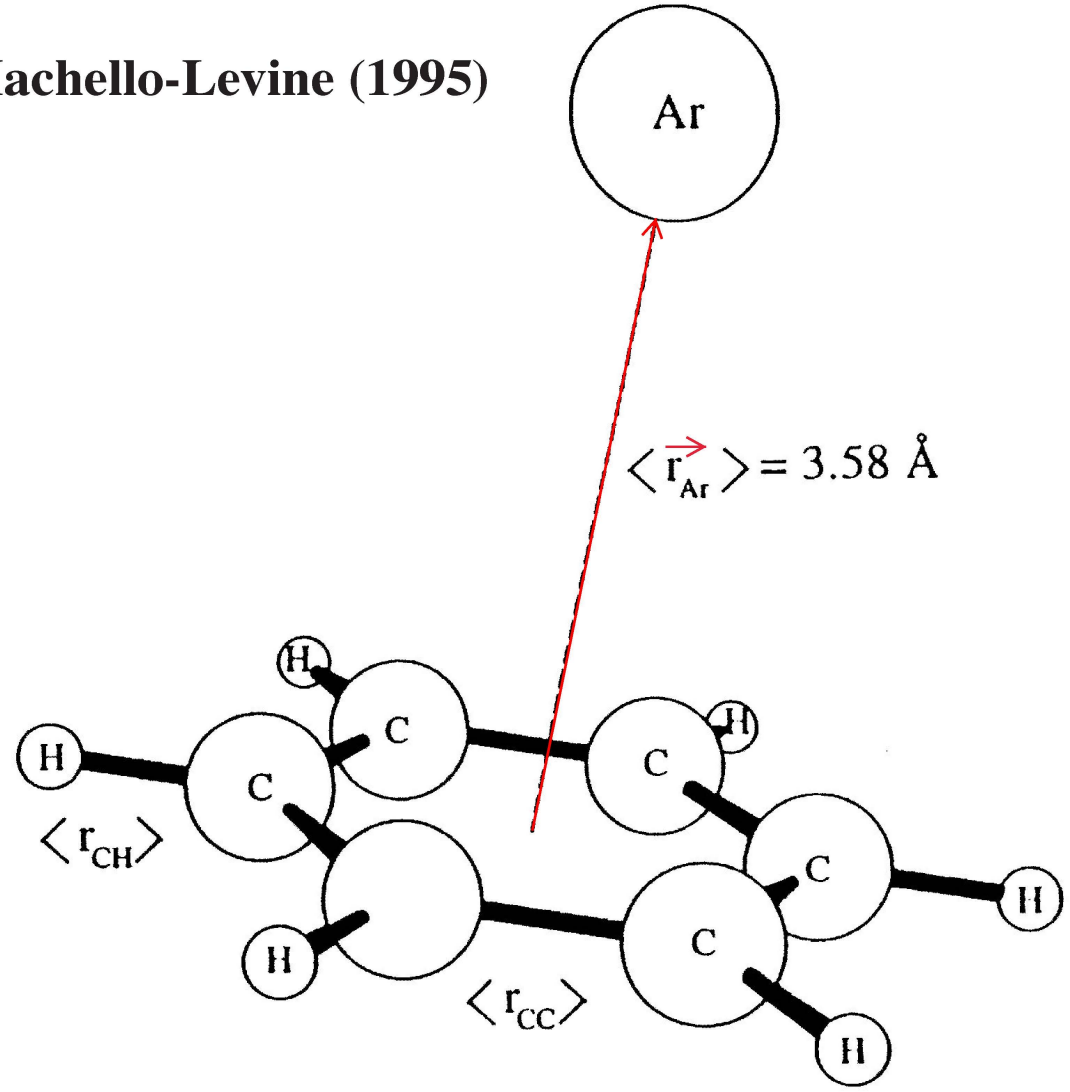
4. Conjectured Hole States: ^{7}Be, ^{19}F
5. Conjectured Particle-Hole States: ^{8}Be

Experiments:

1. The Hoyle Rotational Band (UConn-OTPC)
2. The $\mathcal{D}3h$ Symmetry: ^{12}C (UConn-Birmingham)
3. Search for 2^{+}, 3^{-} states in ^{12}C (Warsaw TPC)
4. Search for Rotational Bands in ^{8}Be (SHU/ISS)

* Supported by the U.S. Department of Energy,
Office of Science, Nuclear Physics
Grants Number DE-FG02-94ER40870.

Iachello-Levine (1995)



Algebraic Models

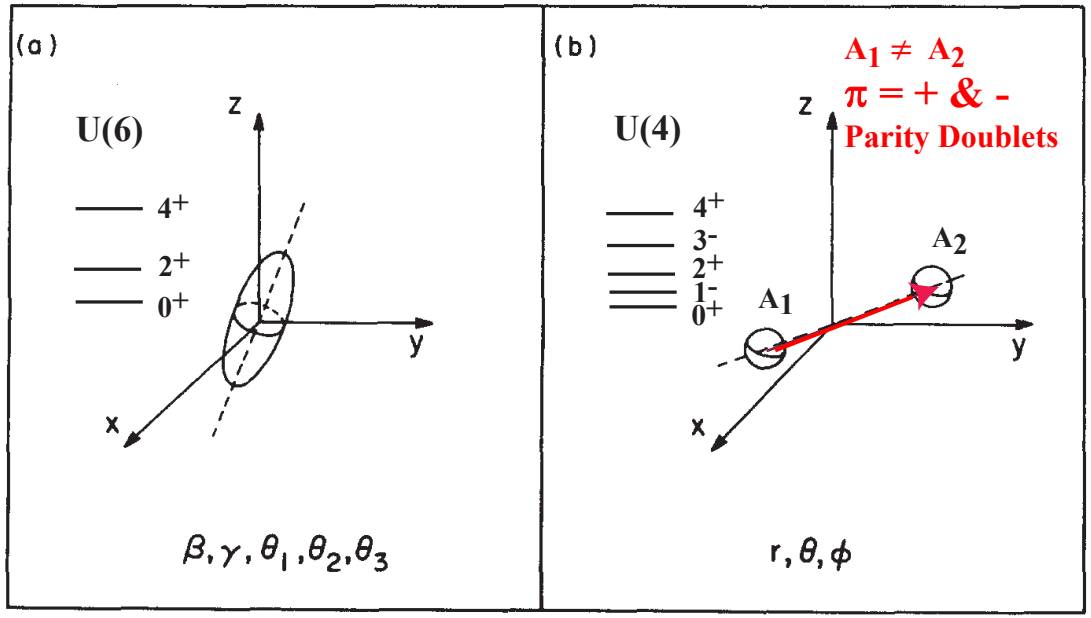
Deformed State:

$(\theta_1, \theta_2, \theta_3, \beta, \gamma) \rightarrow U(6)$

Molecular State:

$\mathcal{R}$: **Separation vector**

$(r, \theta, \phi) \rightarrow U(4)$



J. Chem. Phys., Vol. 77, No. 6, 15 September 1982

VOLUME 49, NUMBER 20 PHYSICAL REVIEW LETTERS 15 NOVEMBER 1982

Radiative Width of Molecular-Cluster States

Yoram Alhassid and Moshe Gai
A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06511

and

George F. Bertsch
Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48823

(Received 5 August 1982)

Molecular states are characterized by enhanced electromagnetic deexcitations of many different multipolarities. The expected enhancement of $E1$, $E2$, and $E3$ transitions is examined by deriving molecular sum rules for radiative deexcitation widths and via a dimensionality approach. The enhancement of the $E1$ transitions is the most striking.

PACS numbers: 21.60.Gx, 23.20.Ck, 25.40.Lw

"Molecular shape, when expanded in spherical harmonics, require substantial higher order terms": (Enhanced E1, E2, E3, E4...)

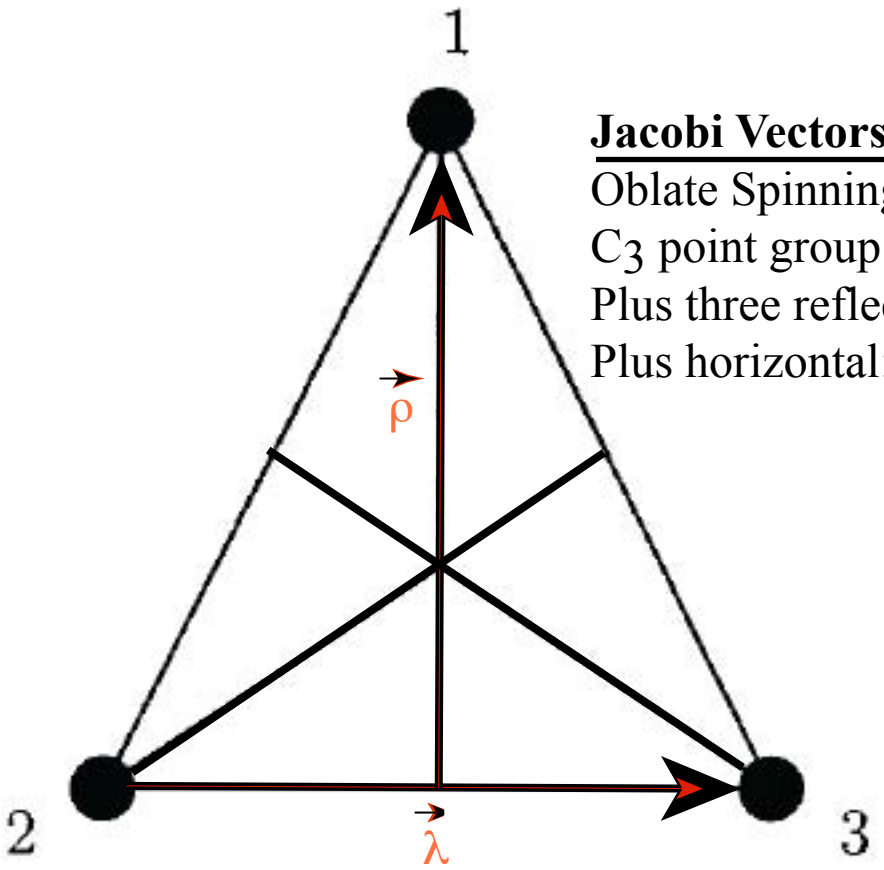
$\mathcal{D}_{3h}$ Symmetry in ^{12}C:

Spectrum of the (Symmetric) Triangular Spinning Top:

Molecular Physics: H_3^+ Molecule

Hadron Structure: Three Quark Model

Nuclear Structure: ^{12}C Three Alpha-Particles



Jacobi Vectors: $\vec{\lambda}$, $\vec{\rho}$

Oblate Spinning Top: U(6+1)

C_3 point group symmetry

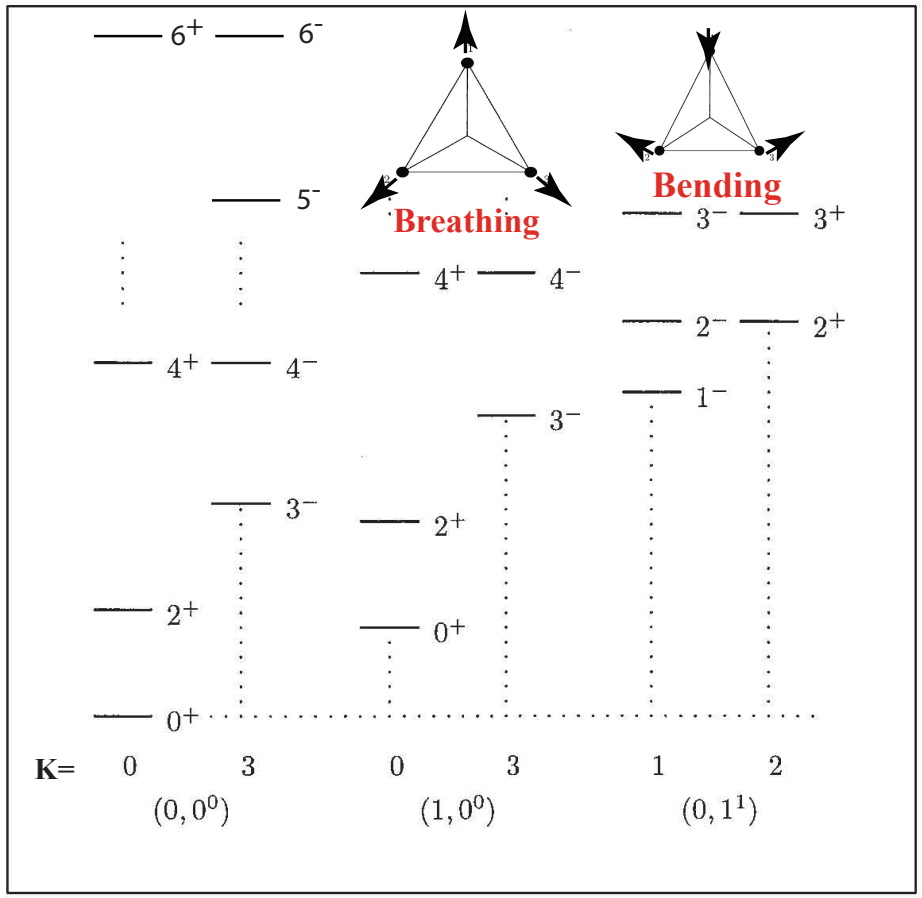
Plus three reflections

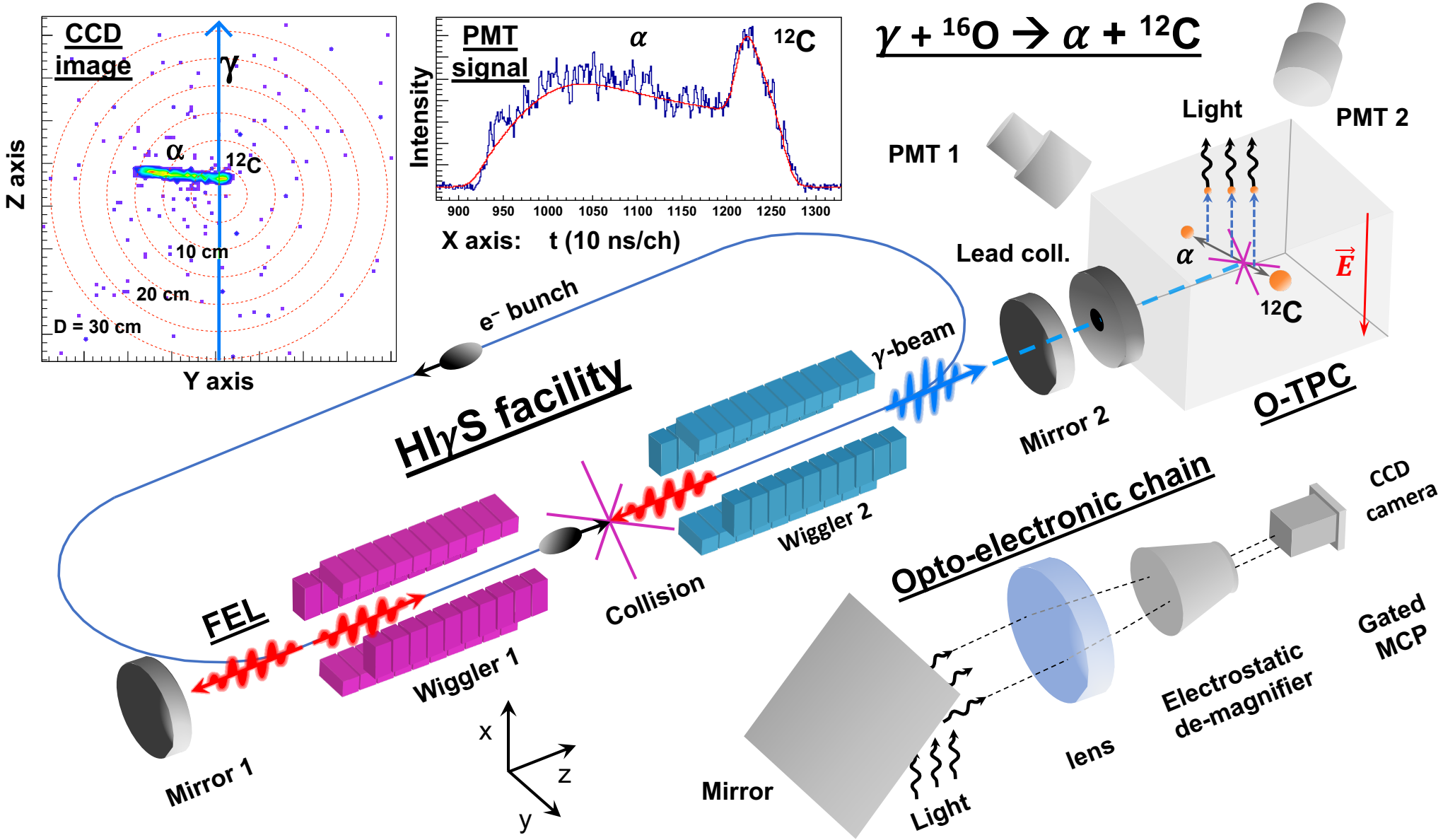
Plus horizontal: D_{3h} Symmetry

Rotation-Vibration Spectrum of the Three Alpha Triangular Spinning Top

U(7) Model/ $\mathcal{D}_{3h}$ Symmetry

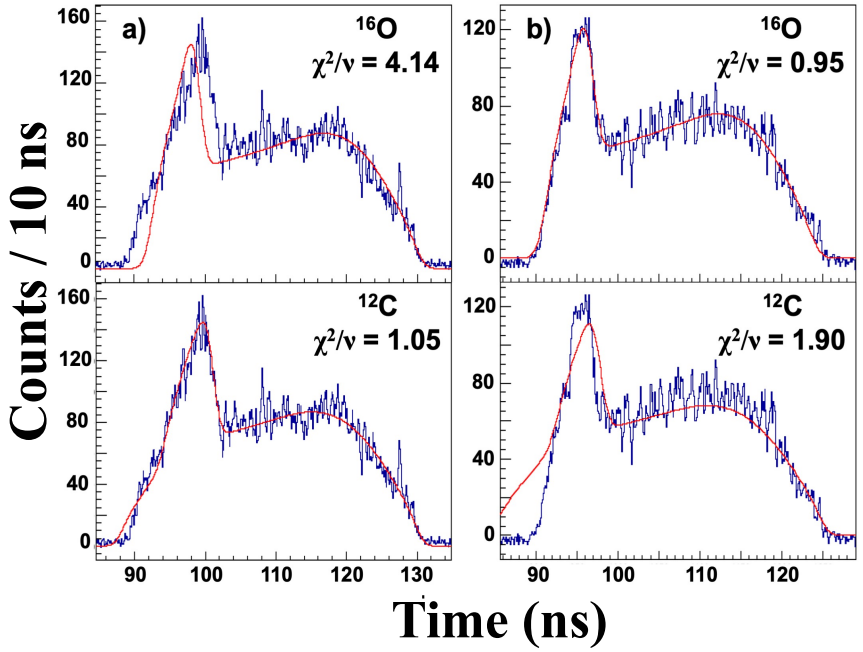
R. Bijker and F. Iachello; Ann. Phys. **298**(2002)334

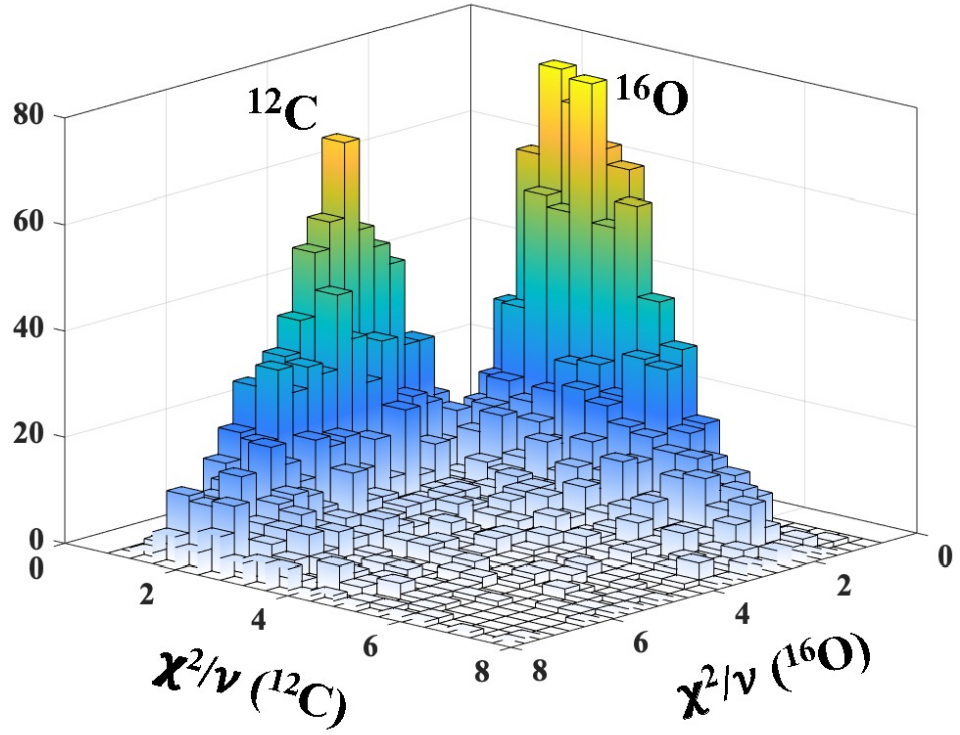


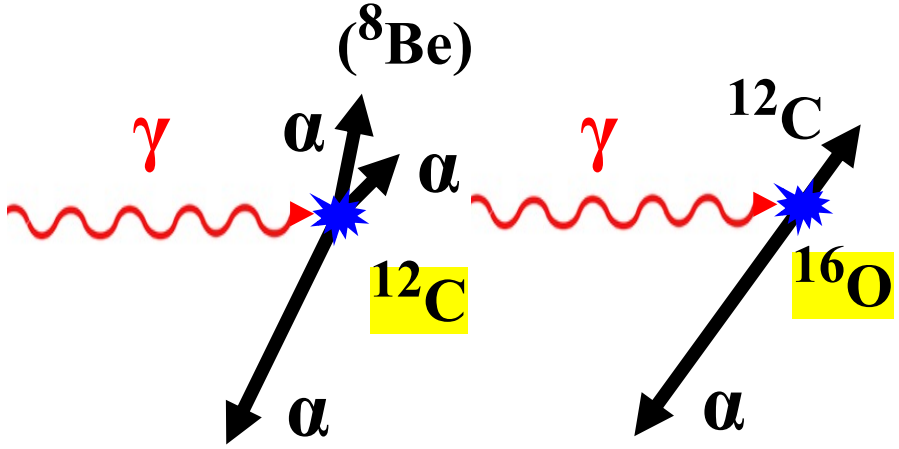


R. Smith, M. Gai, D.K. Schweitzer, S.R. Stern and M.W. Ahmed,
Nature Communications, 12, 5920 (2021).
https://www.nature.com/articles/s41467-021-26179-x

Line Shape Analysis (CO_2 Gas)





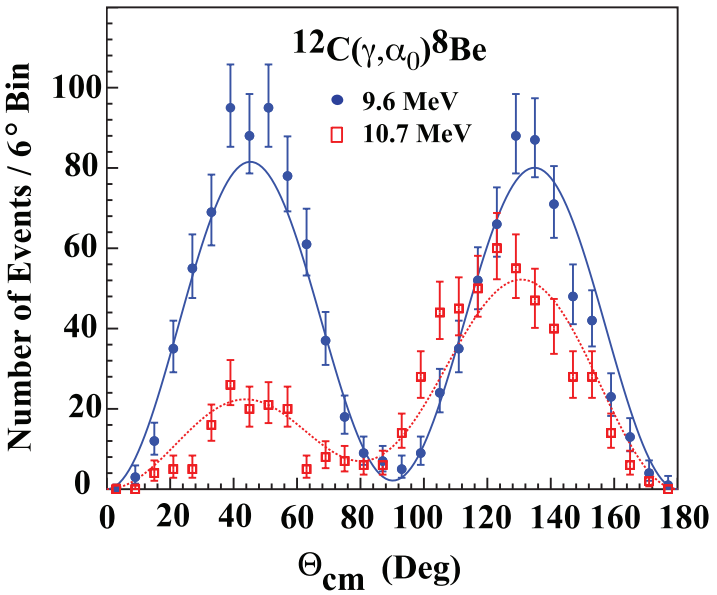


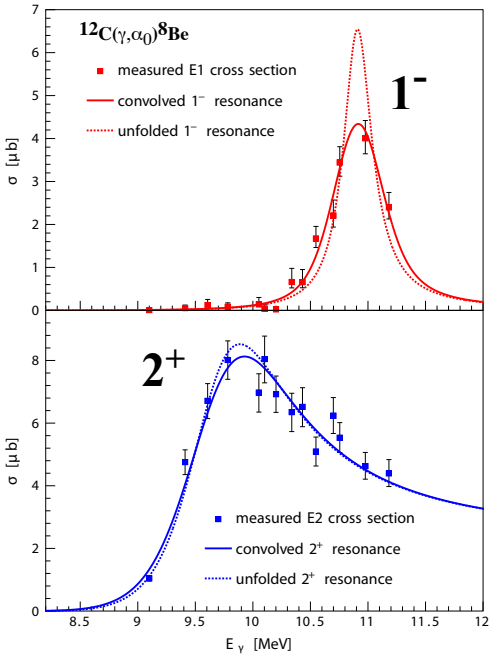
Machine Learning

UConn-TUNL Optical Readout TPC (O-TPC)

$^{12}C(\gamma,3\alpha)$

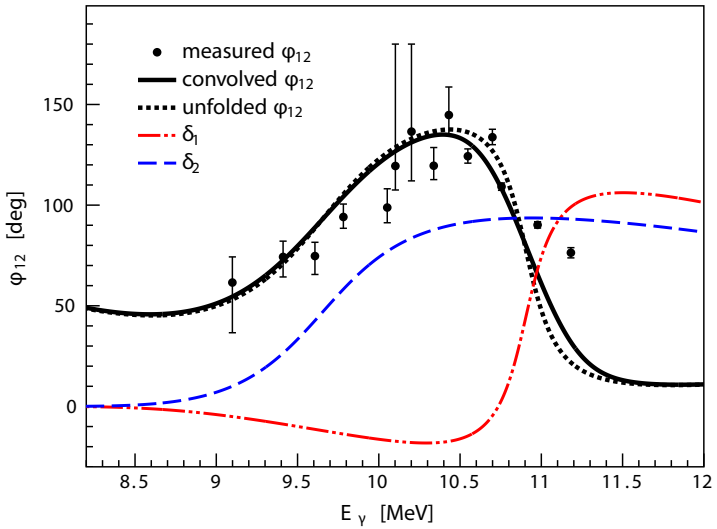
	9.6 MeV	10.7 MeV
E2/(E1+E2) =	0.97 ± 0.02	0.71 ± 0.05
ϕ_{12} =	80 ± 6°	132 ± 5°





W.R. Zimmerman *et al.*; Phys. Rev. Lett. 110(2013)152502

$\phi_{12} = \delta_2 - \delta_1 + \arctan(\eta/2)$



B(EL): ACM/ $\mathcal{D}3h$ Symmetry

$$B(EL: 0^+ \rightarrow L) = (\frac{Ze\beta L}{3})^2 \frac{2L+1}{4\pi} [3 + 6P_L(-\frac{1}{2})]$$

$$B(E2: 0^+ \rightarrow 2^+) = (Ze)^2 \frac{1}{4\pi} \frac{5}{4}\beta^4$$

$$B(E3: 0^+ \rightarrow 3^-) = (Ze)^2 \frac{1}{4\pi} \frac{35}{8}\beta^6$$

$$B(E4: 0^+ \rightarrow 4^+) = (Ze)^2 \frac{1}{4\pi} \frac{81}{64}\beta^8$$

(β Determined from g.s. radius)

R-Matrix Fit: $\Gamma\gamma(1^-) = 29 \pm 2.1$ meV B(E1) = 6.5×10^{-5} W.u.

(O-TPC Data) $\Gamma\gamma(2^+) = 182\ ^{+43}_{-53}$ meV B(E2) = 1.2 W.u.

At 12.5 MeV Broad ($\Gamma \approx 1$ MeV): 1^-, 2^+ and 3^- ?

B(E1) = 10^{-4} W.u., B(E2) = 1 W.u. B(E3) = 1 W.u.

Ratio of Γγ (eV) x (2J+1): E1/E2/E3 = 1/8/0.003

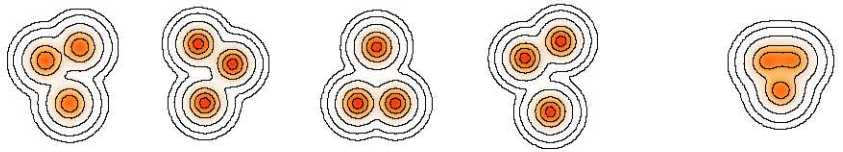
A1/A2/A3 = 1/3/0.05 → A3: 2-5% sensitivity (at our limit)

A1/A2 ≈ $\mathcal{O}(1)$

M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, and A. Richter
Fermionic Molecular Dynamic Model (FMD), Phys. Rev. lett. 98, 032501 (2007)

PRL 98, 032501 (2007) PHYSICAL REVIEW LETTERS week ending 19 JANUARY 2007

^{12}C

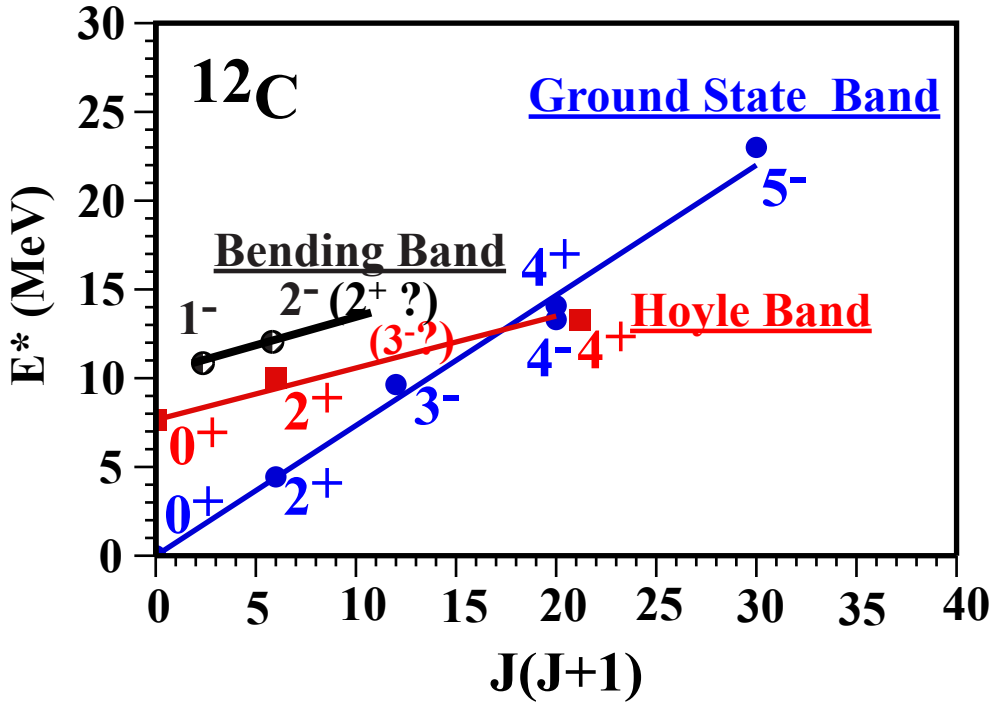


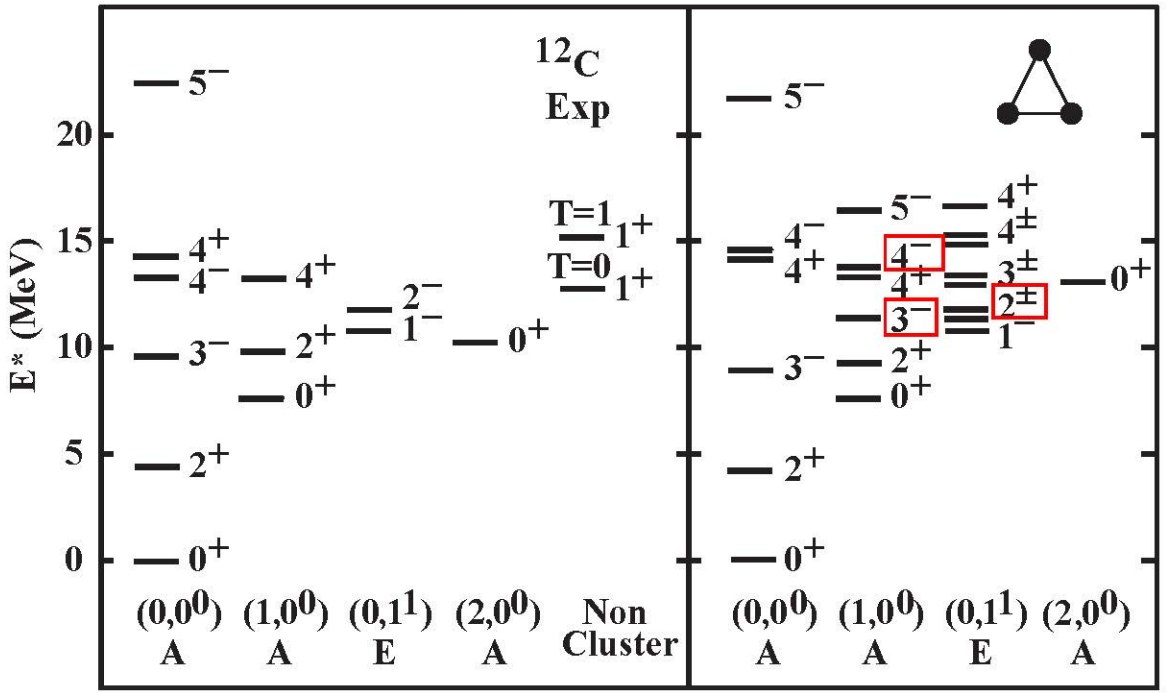
ground state: $|\langle 1|0_1^+\rangle| = 0.30$ $|\langle 2|0_1^+\rangle| = 0.25$ $|\langle 3|0_1^+\rangle| = 0.15$ $|\langle 4|0_1^+\rangle| = 0.08$ $|\langle 5|0_1^+\rangle| = 0.94$

Hoyle state: $|\langle 1|0_2^+\rangle| = 0.72$ $|\langle 2|0_2^+\rangle| = 0.71$ $|\langle 3|0_2^+\rangle| = 0.61$ $|\langle 4|0_2^+\rangle| = 0.61$ $|\langle 5|0_2^+\rangle| = 0.04$

UConn-Birmingham

D.J. Marin-Lambarri, R. Bijker, M. Freer, M. Gai, Tz. Kokalova,
D.J. Parker, C. Wheldon, Phys. Rev. Lett. 113, 012502 (2014)





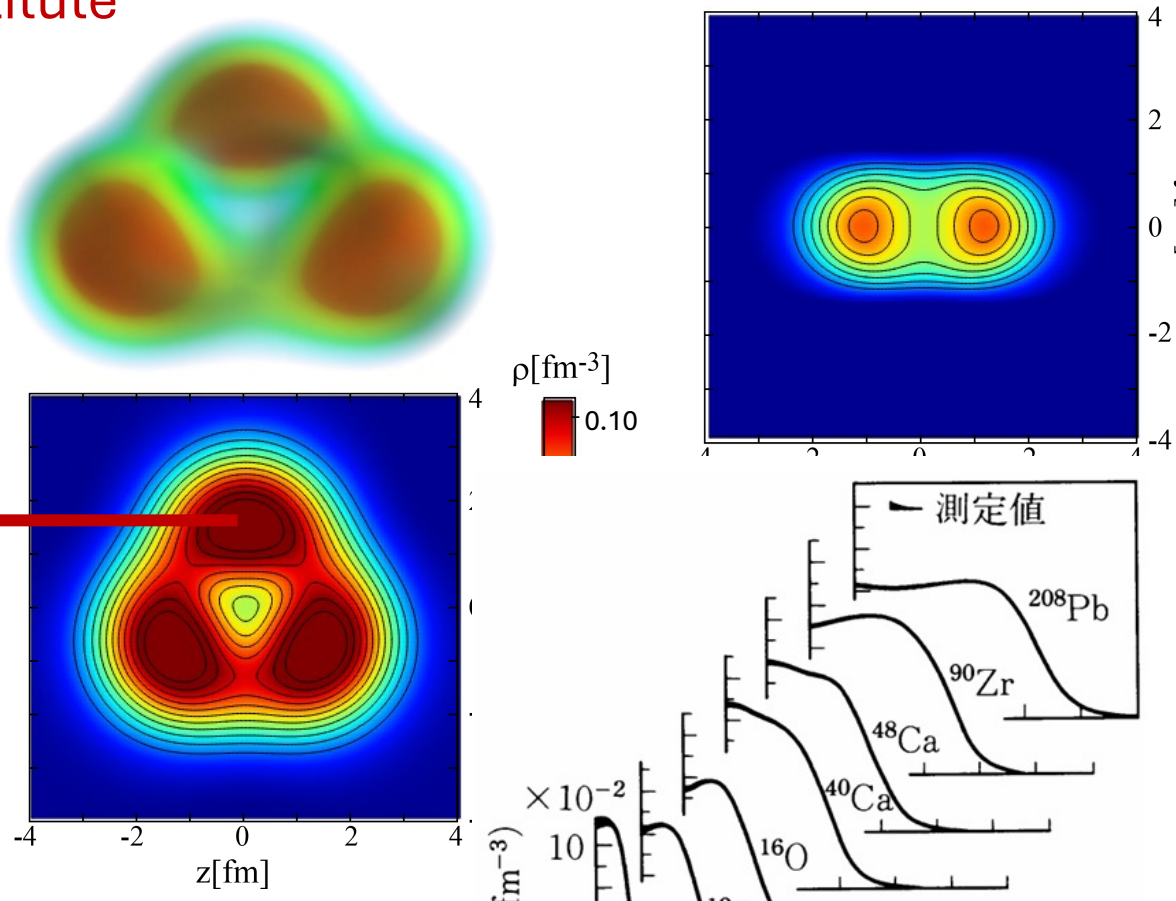
Summary for the shape of ^{12}C/ Courtesy of Masaaki Kimura, RIKEN

I only assumed that the 0^+, 2^+, 3^- and 4^+ states constitute "the ground band" sharing the same intrinsic state
(This is the only assumption I made)

A beautiful triangular shape of ^{12}C showed up! (3 alphas?)

Density at the vertex is as high as 0.11 fm^{-3} which is much larger than typical density ($\sim$ 0.08 fm^{-3}).
Do you think this is unrealistic (no saturation)?

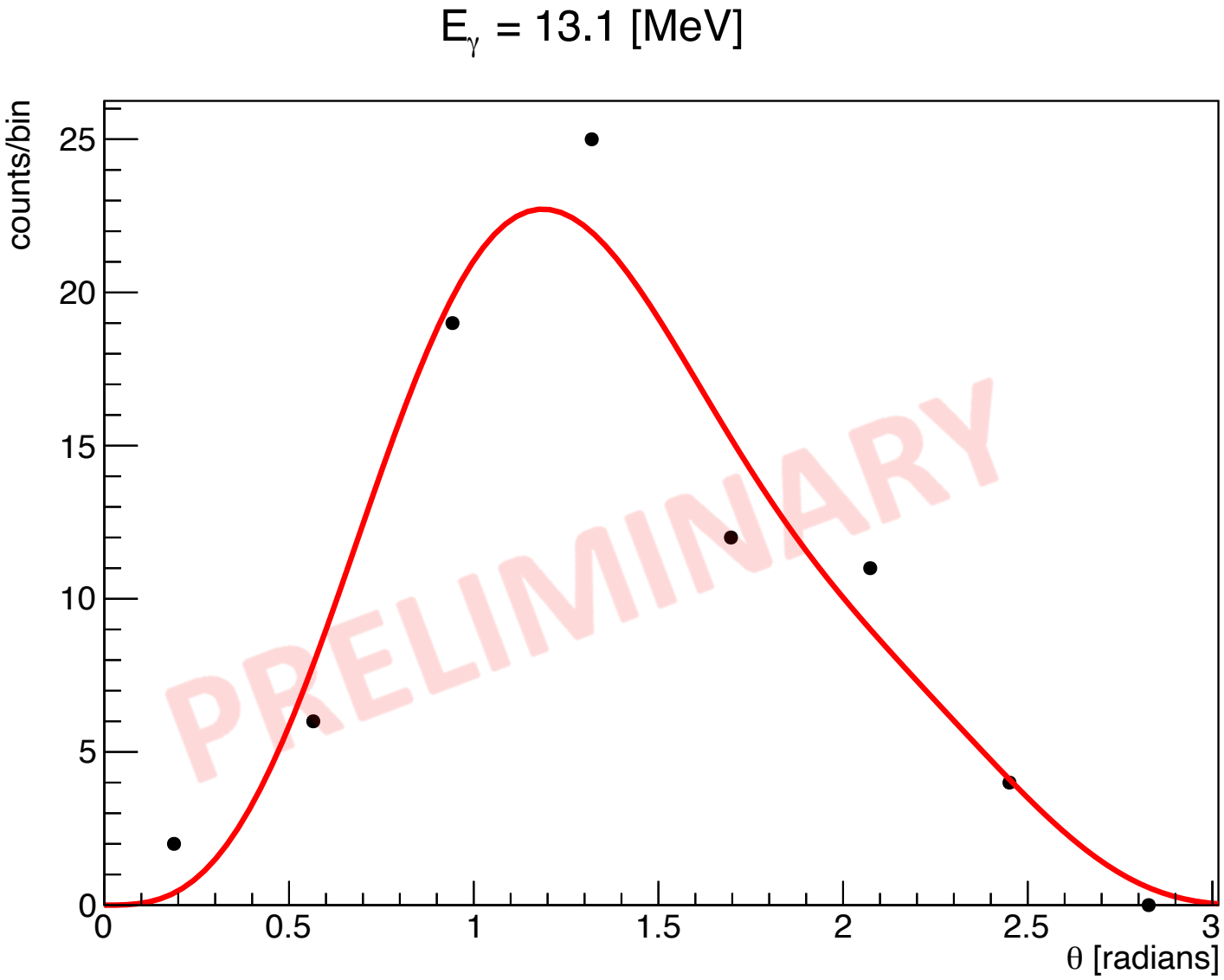
Among all nuclei, there is only one nucleus which has such high central density.
That's alpha particle (0.11 fm^{-3}).
Do you think that this is an accidental coincidence?



M.K. and Y. Taniguchi, EPJA 60, (2024)

$^{12}C(\gamma,\alpha_0)^8Be(g.s.)$ E1 + E2 (1^- & 2^+ ?)

Kristian C.Z. Haverson, SHU, 2023



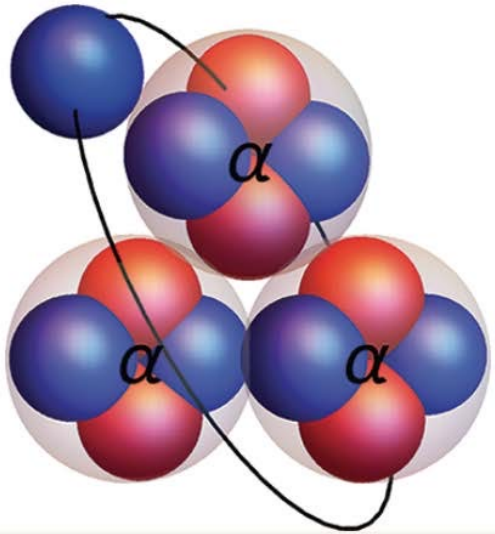
$\mathcal{D}_{3h}$' Symmetry in ^{13}C:

R. Bijker and F. Icahello, Prog. Part. Nucl. Phys. 110, 103735 (2020)

PHYSICAL REVIEW LETTERS

Volume 122, Issue 16

26 April 2019



On the Cover

A ^{13}C nucleus modeled as a triangle of three α particles plus an additional neutron.

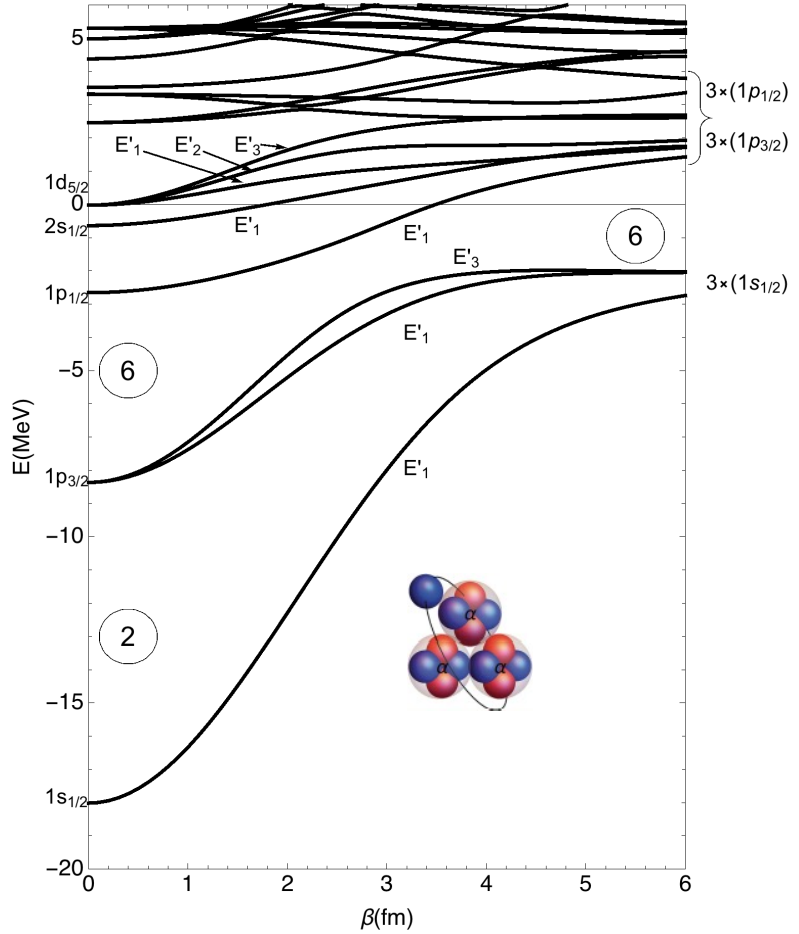
From the article

Evidence for Triangular D_{3h}' Symmetry in ^{13}C

R. Bijker and F. Iachello

Phys. Rev. Lett. **122**, 162501 (2019)

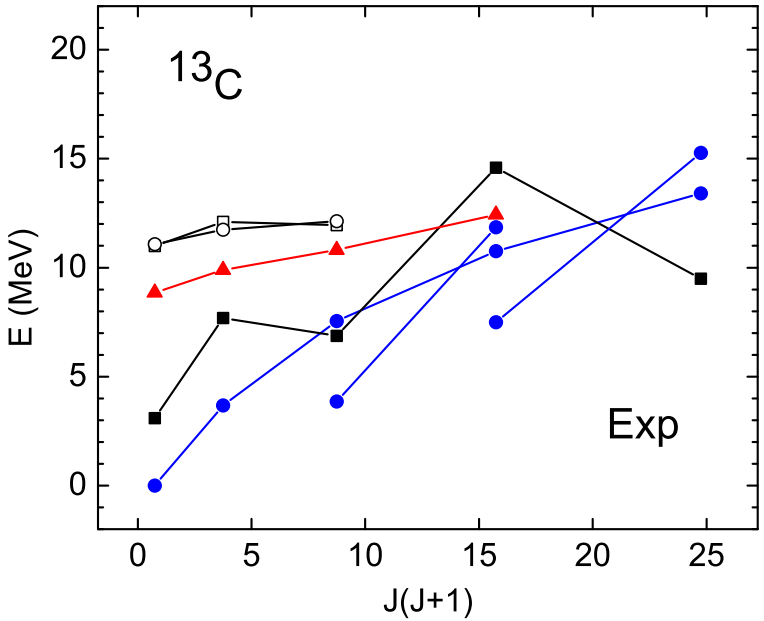
View Issue



β = distance of α to 3 body c.m.

Figure 26: Single-particle energies in a cluster potential with D_{3h} symmetry calculated with $V_0 = 13.3$ MeV, $V_{0,\mathrm{so}} = 16.9$ MeV fm^2, $\alpha = 0.0872$ fm^{-2}. Reproduced from [61] with permission.

Evidence for Triangular $\mathcal{D}_{3h}$' Symmetry in ^{13}C, R. Bijker, F. Iachello, Phys. Rev. Lett. 122, 162501 (2019)



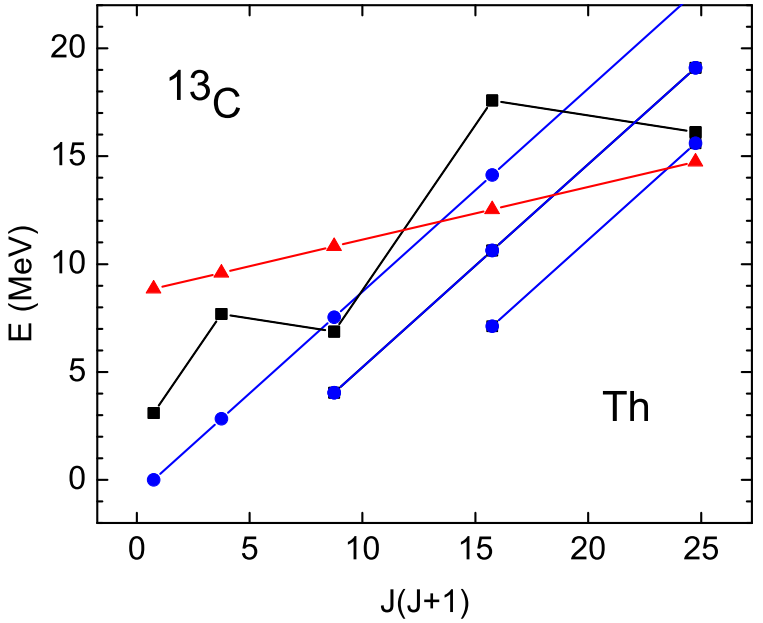


TABLE I. B(EL) values in ^{12}C and ^{13}C in W.U. [15].

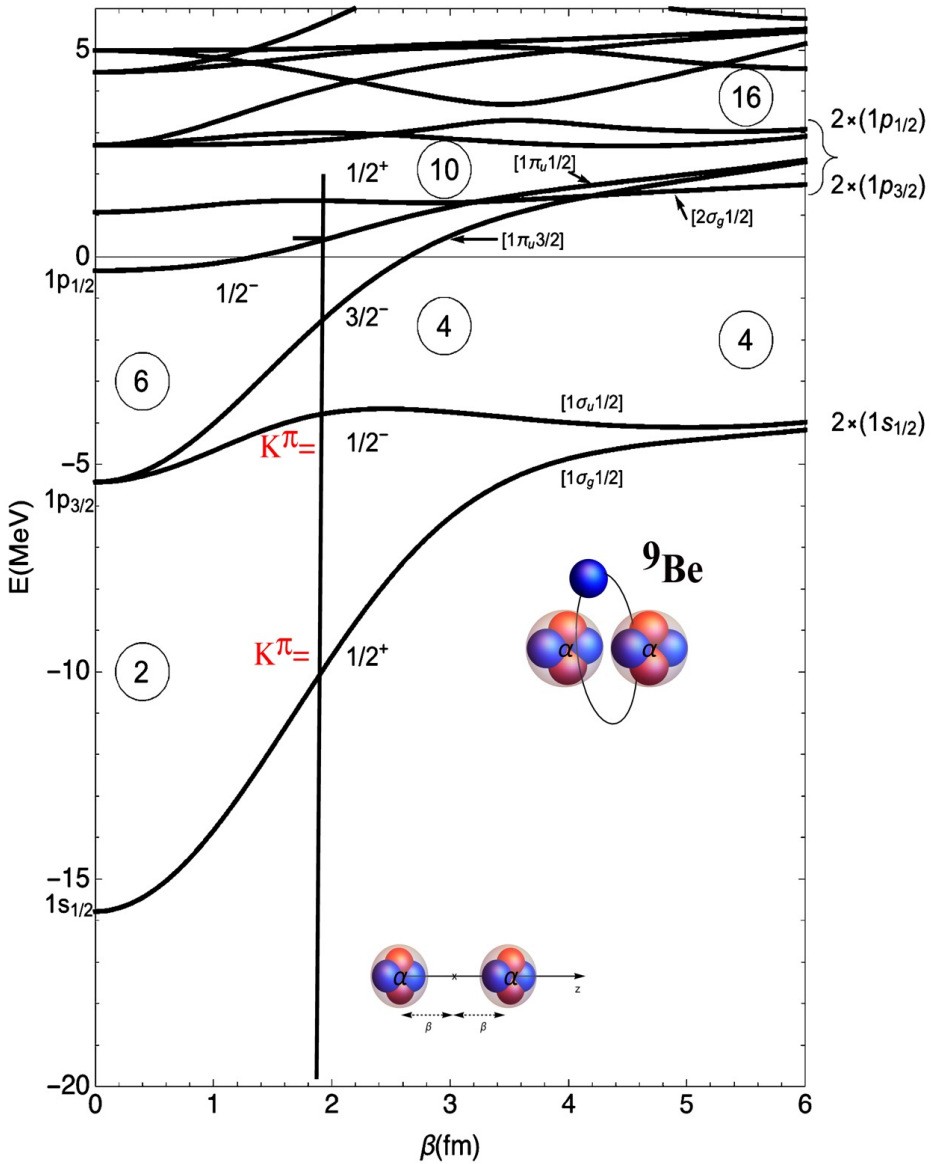
	B(EL)	Exp	Th
^{12}C	$B(E2; 2_1^+ \rightarrow 0_1^+)$	4.65 ± 0.26	4.8
	$B(E3; 3_1^- \rightarrow 0_1^+)$	12 ± 2	7.6
^{13}C	$B(E2; 3/2_1^- \rightarrow 1/2_1^-)$	3.5 ± 0.8	4.8
	$B(E2; 5/2_1^- \rightarrow 1/2_1^-)$	3.1 ± 0.2	3.2
	$B(E3; 5/2_1^+ \rightarrow 1/2_1^-)$	10 ± 4	4.3

$$B(E2{:}\ 0^+ \rightarrow\ 2^+) = (Ze)^2\,\frac{5}{4\pi}\,\frac{1}{4}\,\beta^4$$

$$B(E2{:}J', K \rightarrow J, K) = (Ze)^2\,\frac{5}{4\pi}\,\beta^4 (J', K, 2, 0 \mid J, K)^2$$

C_2' Symmetry in ^{9}Be:

V. Della Rocca, R. Bijker, F. Iachell0, Nucl. Phys. A 966(2017)156
V. Della Rocca, F. Iachello, Nucl. Phys. A 973(2018)1



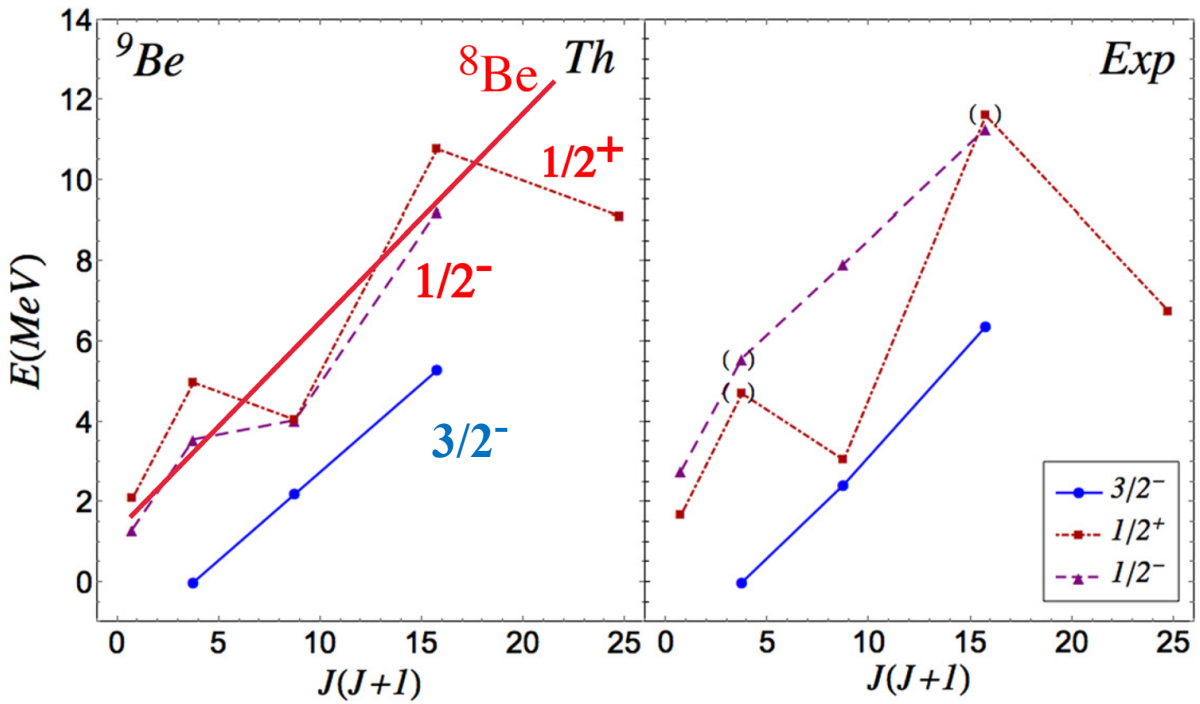
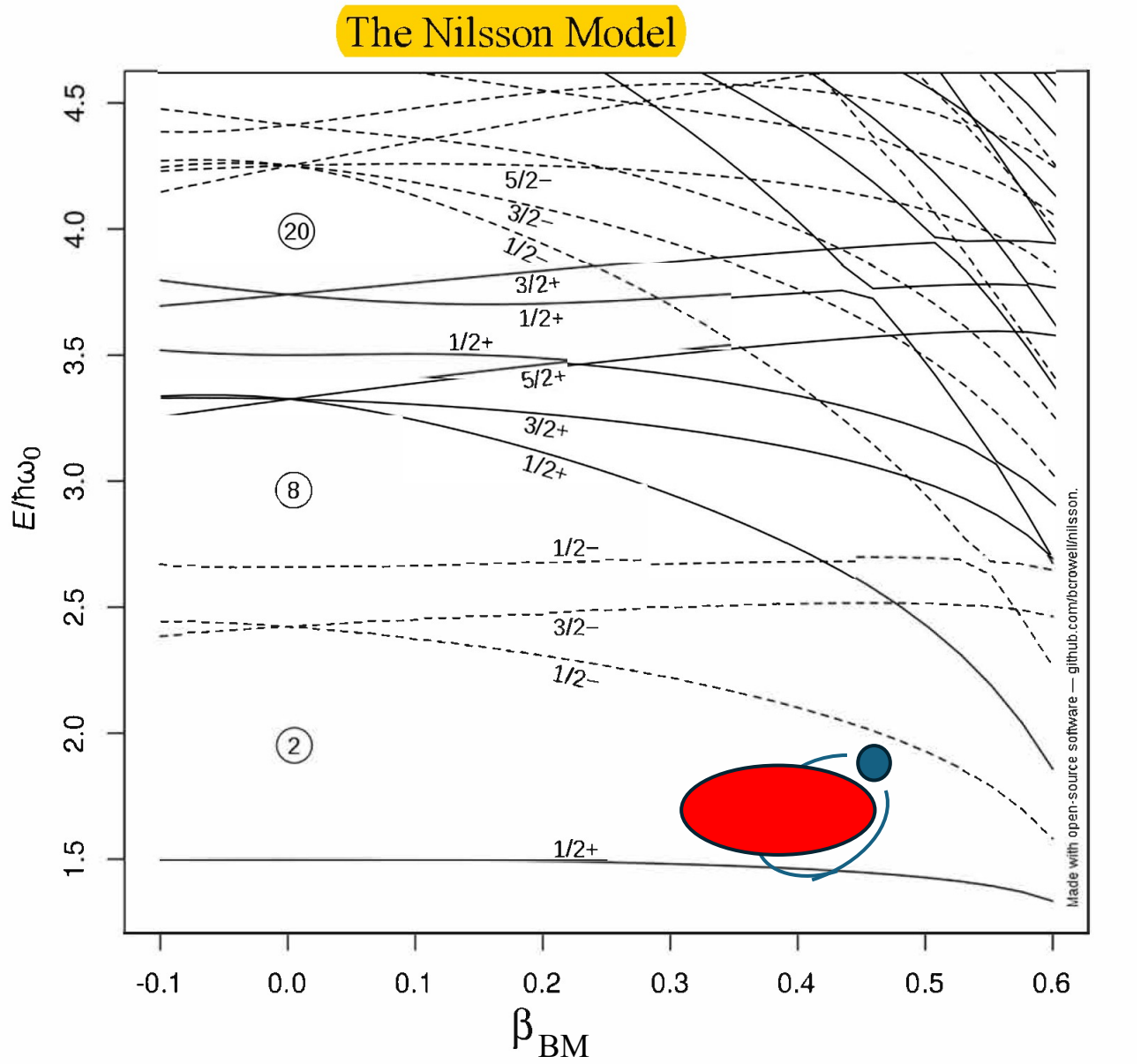


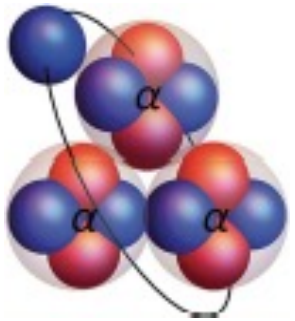
Fig. 8. Observed rotational bands in ^{9}Be.



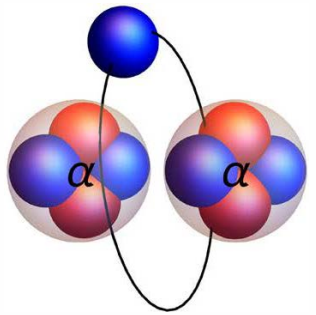
Only Enhanced B(E2)

Molecular orbits (Many Center Shell Model)

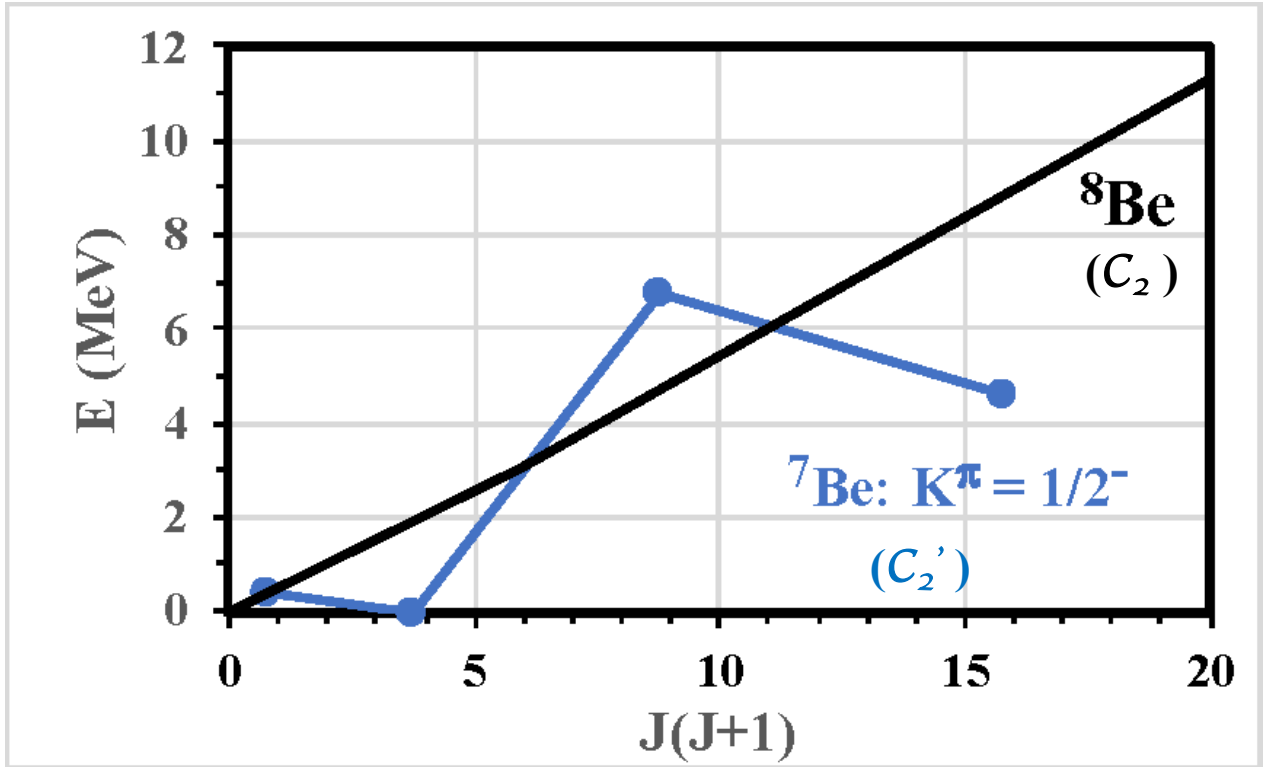
^{13}C

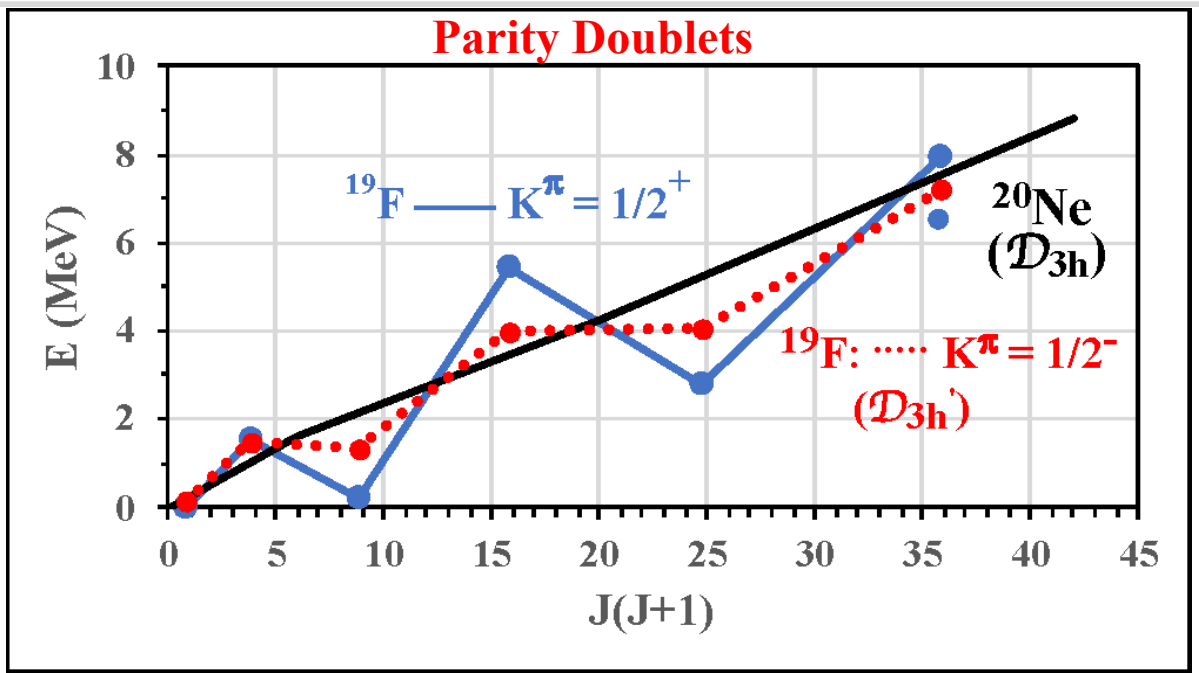


^{9}Be



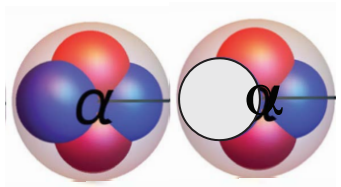
^{13}C: Enhanced B(E1), B(E2), B(E3)… and Parity doublets





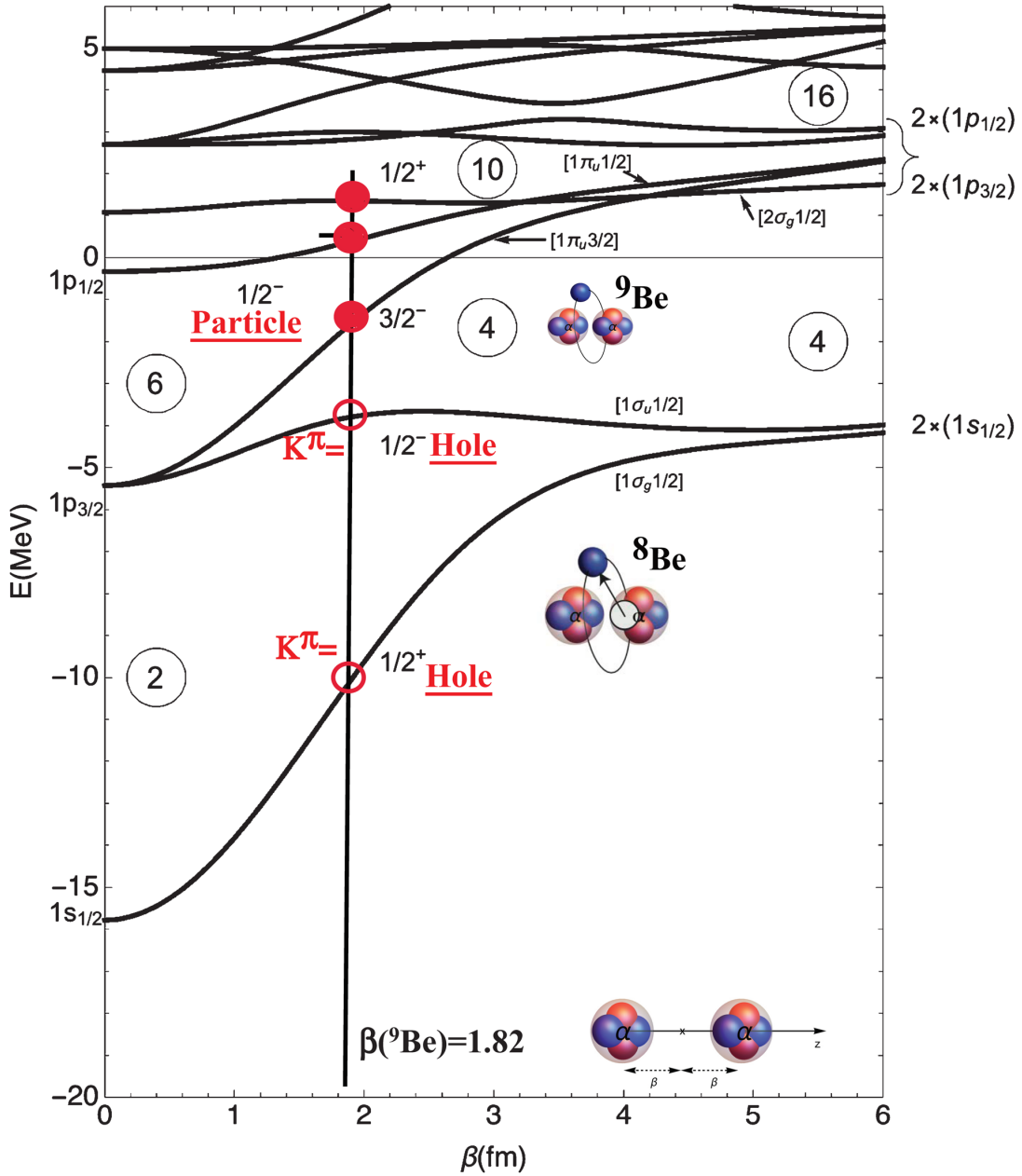
Hole States in Clustering?
A Phenomenological Study

^{7}Be?

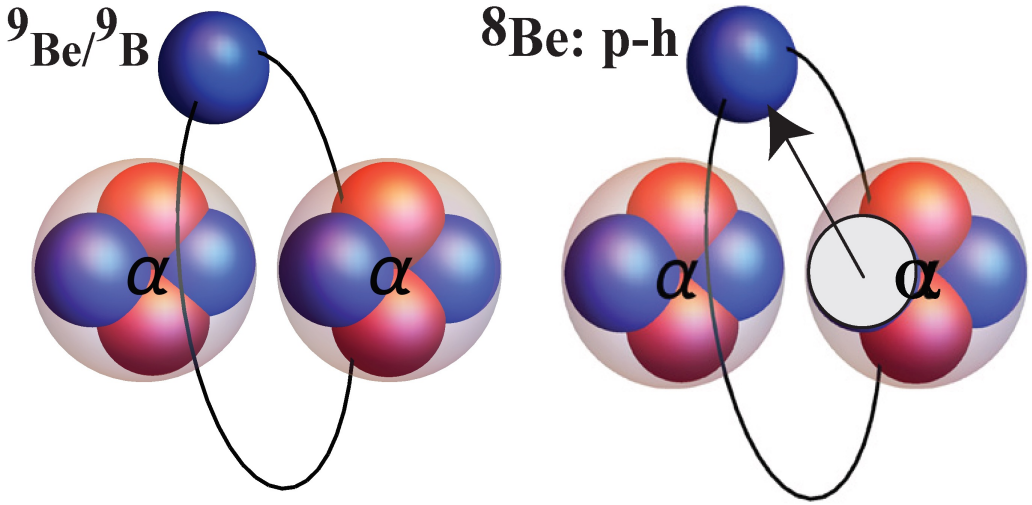


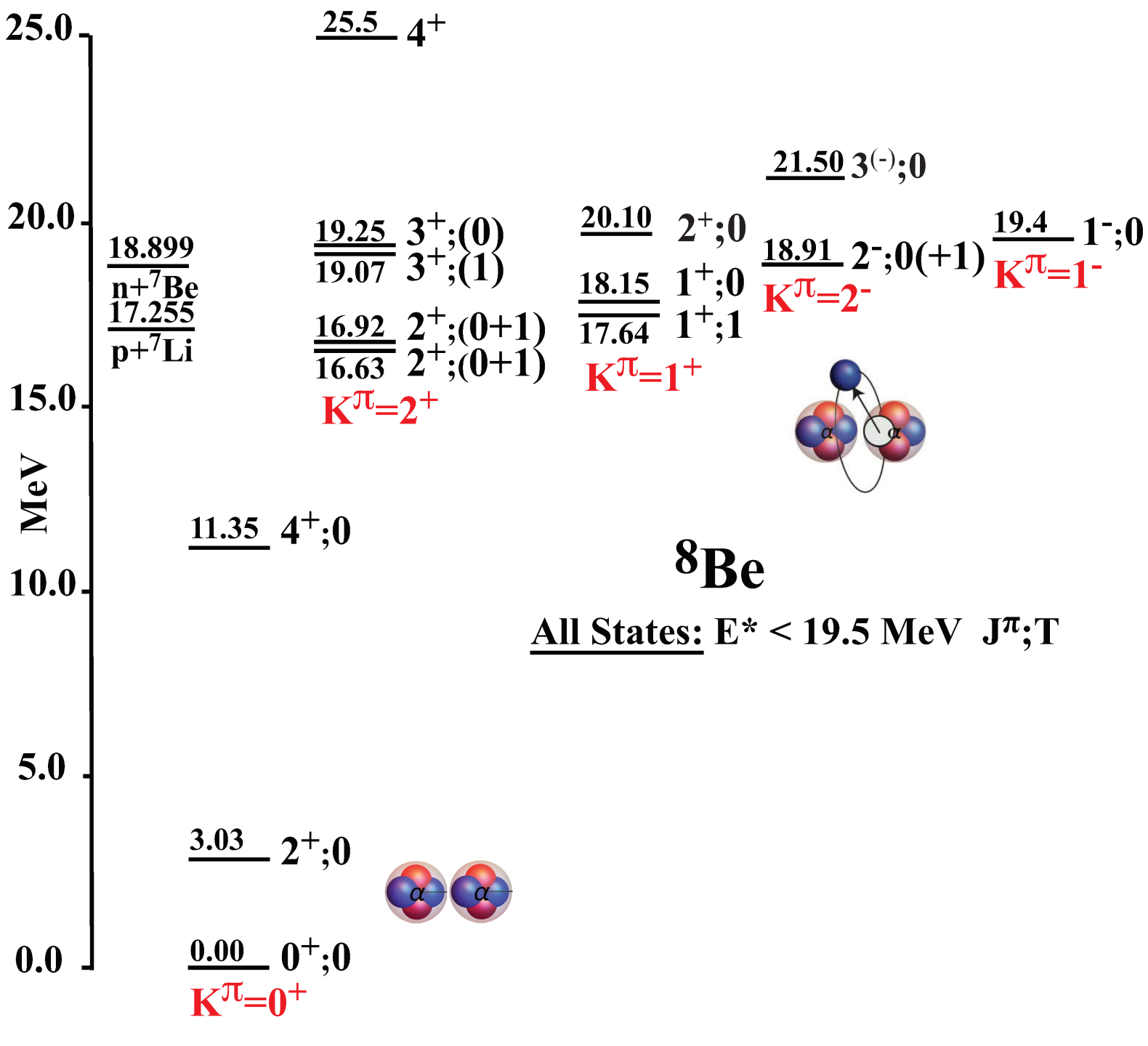
V. Della Rocca, F. Iachello / Nuclear Physics A 973 (2018) 1–32

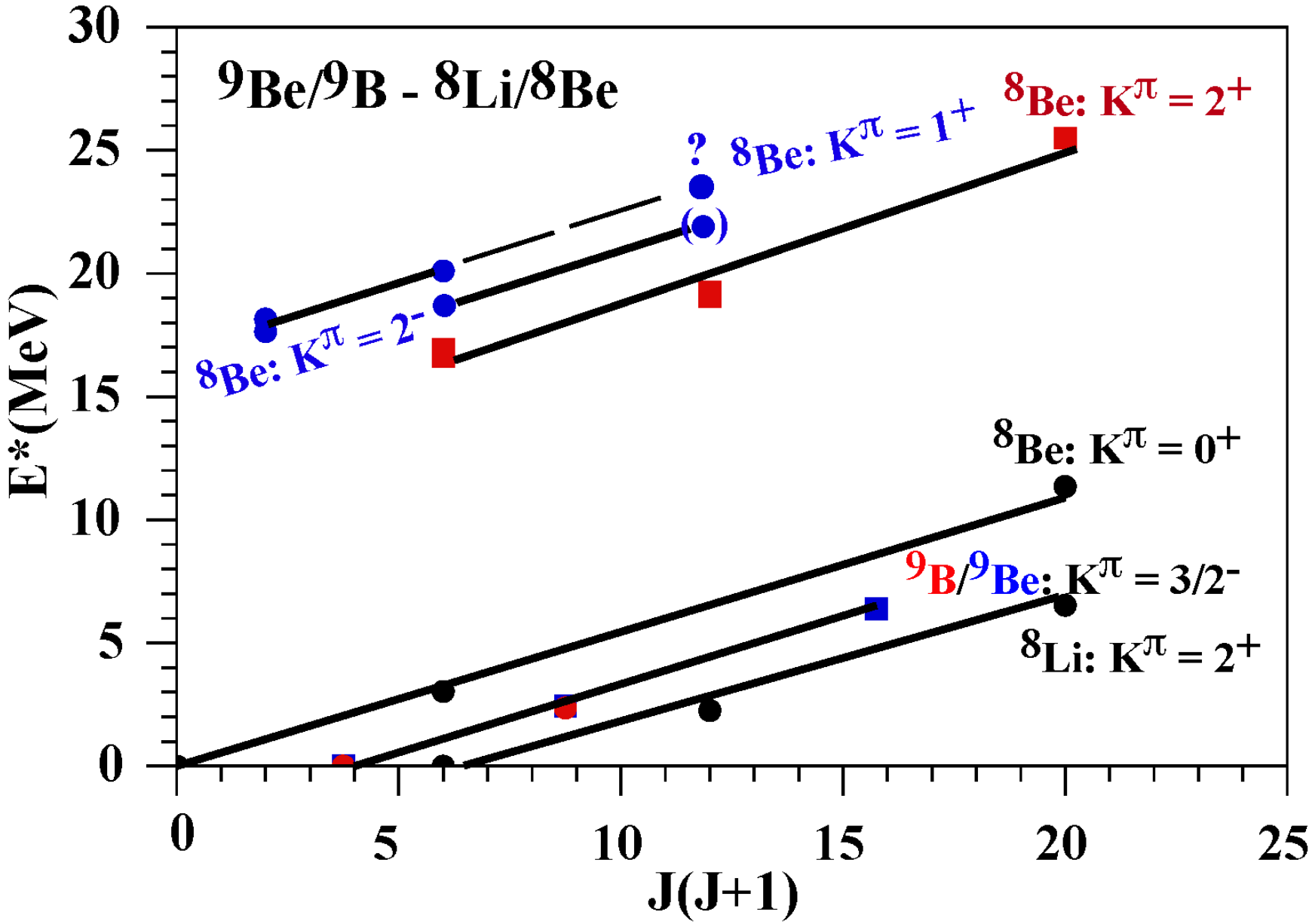
(The Cluster Shell Model)



Particle-Hole States in Clustering?



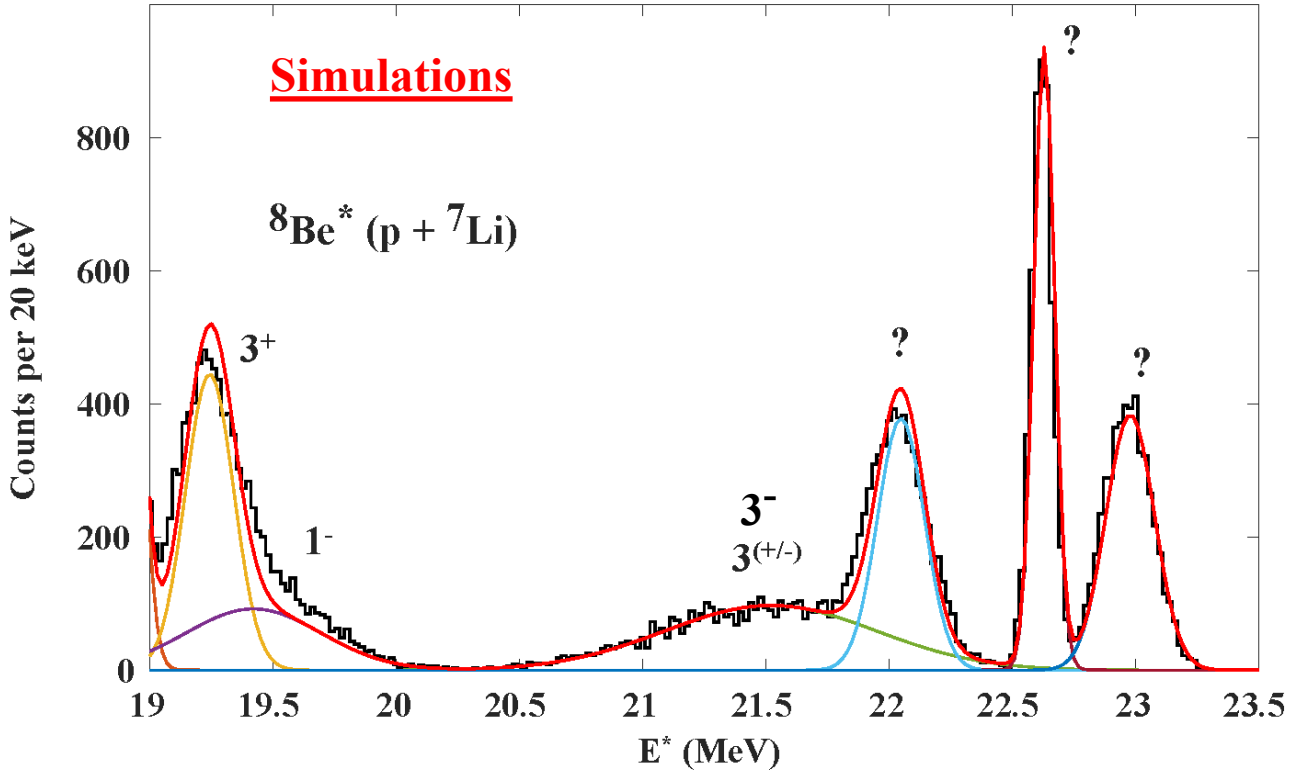

25.0
20.0
15.0
10.0
5.0
0.0
MeV
25.5 4+
21.50 3(-);0
19.25 3+;(0)
19.07 3+;(1)
20.10 2+;0
18.15 1+;0
17.64 1+;1
18.91 2-;0(+1)
19.4 1-;0
Kπ=1-
Kπ=2-
Kπ=1+
18.899
n+7Be
17.255
p+7Li
16.92 2+;(0+1)
16.63 2+;(0+1)
Kπ=2+
11.35 4+;0
8Be
All States: E* < 19.5 MeV Jπ;T
3.03 2+;0
0.00 0+;0
Kπ=0+
α
α
α
α



Philip R. Page, R-Matrix Analysis: ^{8}Be*(21.5 MeV) is a 3^{-}
Phys. Rev. C 72, 054312 (2005).

ISOLDE Solenoidal Spectrometer (ISS) Collaboration

Robin Smith, Liam Gafney, September 21, 2020



Most intense 11 MeV/u ^{7}Be beam: ~6 x 10^{6} /sec

HIE-ISOLDE: Experiment IS692 (15 Shifts + 13 Days irradiation: ^{7}Be production), November 2023

Conclusions: **Molecular Nuclear Physics**

1. **The Algebraic Cluster Model, A new chapter in Cluster Physics**
2. **The Cluster Shell Model, An extension of the Shell Model To molecular orbits**
3. **Tantalizing Phenomenology of conjectured CSM Hole and CSM Particle-Hole States (Awaiting theoretical formulation)**