Multiple shape coexistence in nuclei

Paul Garrett
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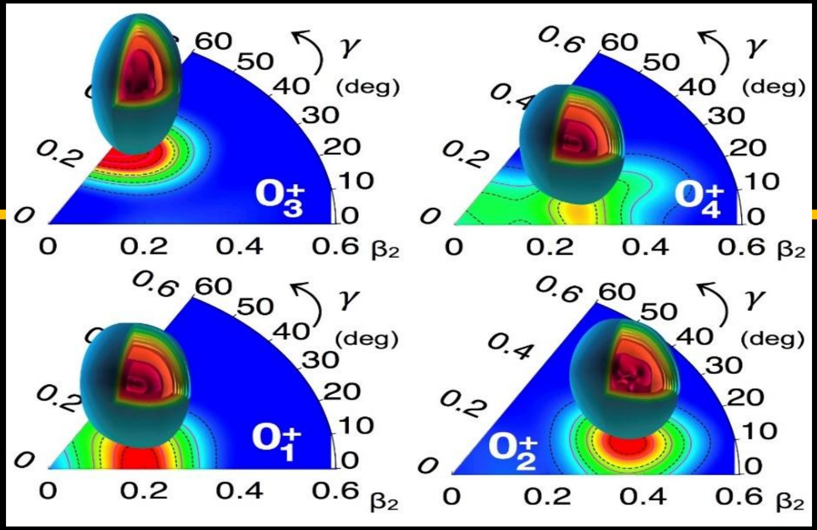
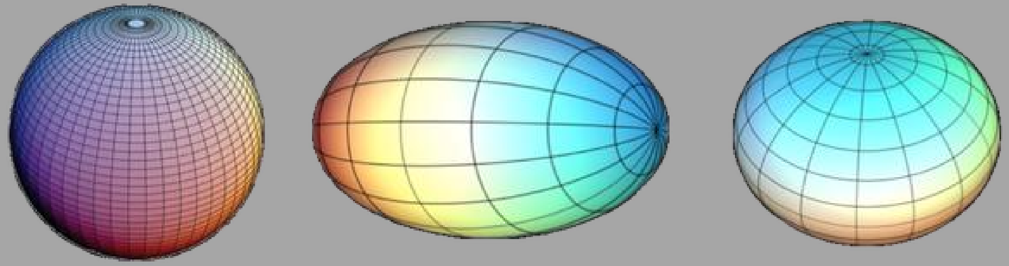


Why are shape coexistence studies interesting?

- **What do we mean by shape coexistence?**
  - **presence of states in the same nucleus within a narrow energy range of two (or more) states that have well defined and distinct properties that can be interpreted in terms of different intrinsic shapes**

- **The shapes we observe, e.g., spherical, prolate, oblate, triaxial**



emerge as a consequence of the nucleon-nucleon interaction manifest in a many-body system

- **The presence of competing shapes due to effects of correlations, especially pairing and quadrupole-quadrupole correlations amongst nucleons, overcoming cost of energy promoting particles into different orbits – even across major shell gaps**

- **Studies of shape coexistence enables us to study effects of correlation energies on deformation (or *vice versa*) within a system having the same number of protons and neutrons**

Shape coexistence in the nuclear landscape

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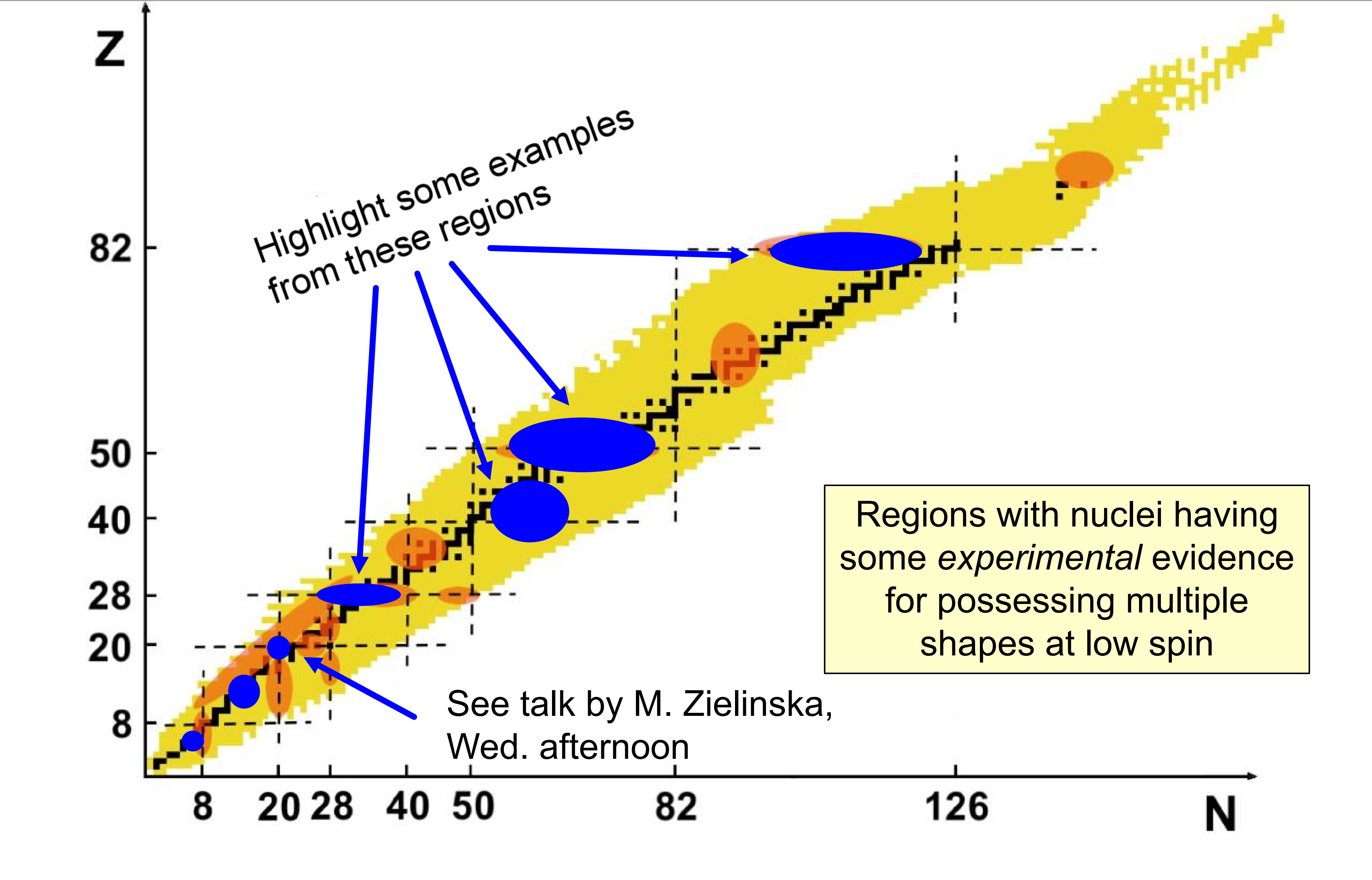
- **Regions of shape coexistence are now known to be located throughout the nuclear chart, although still mainly concentrated in the vicinity of closed shell or subshells**
  - There has been much activity recently, including areas suggested to possess multiple shape coexistence – C, Si/Mg, Ni, Sr/Zr, Cd, Pb/Hg

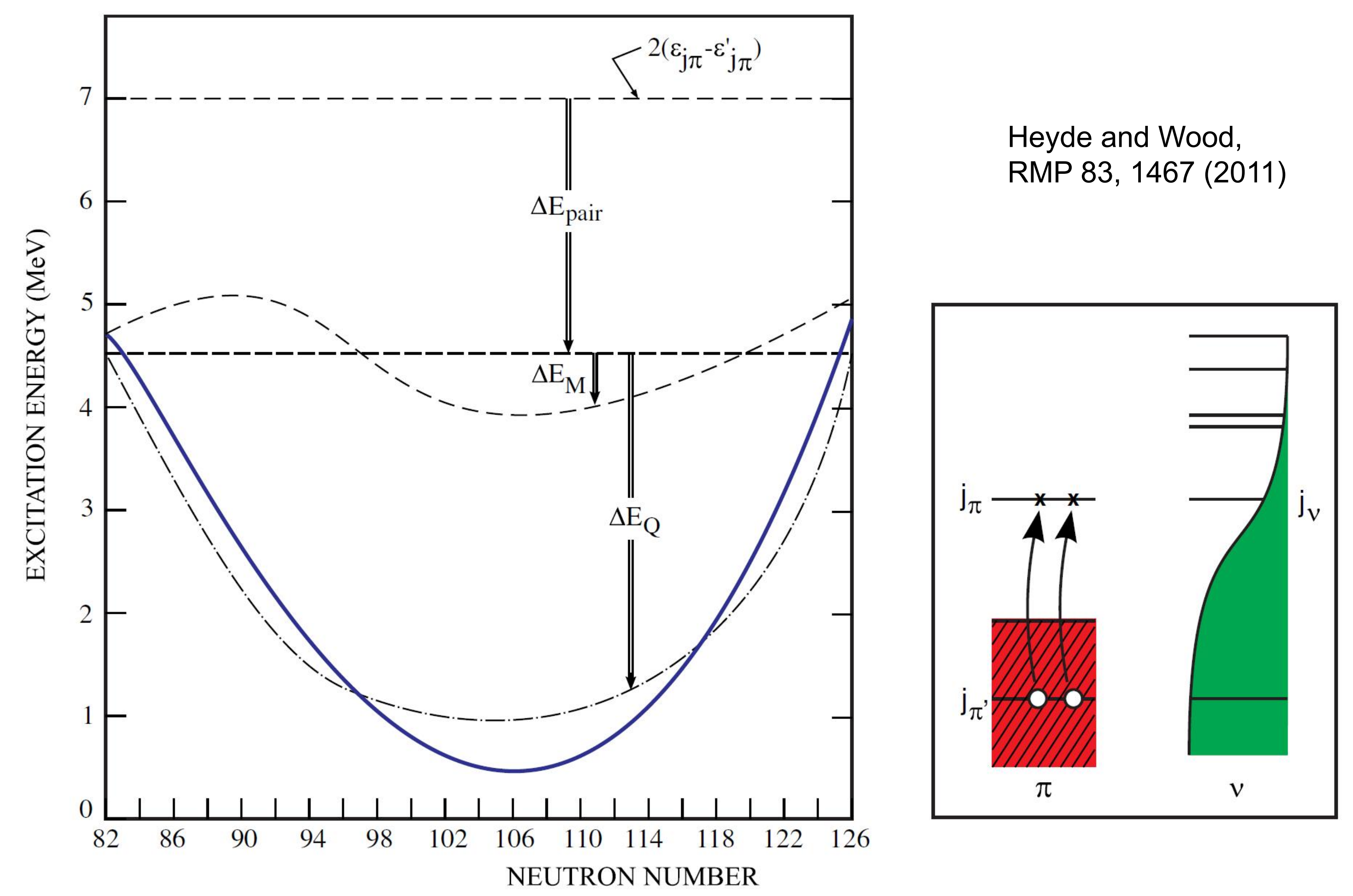


Multiparticle-multihole states and shape coexistence

- **Deformation is driven by the quadrupole-quadrupole interaction between protons and neutrons – the more valence particles of both types, the greater the deformation (up to a point)**

- **There can be a substantial gain in energy due to correlations between the particles such that the energy cost for promoting particle across shells can be offset**

- **The most common mechanism to generate states with different shapes involves the creation of multiparticle-multihole states**



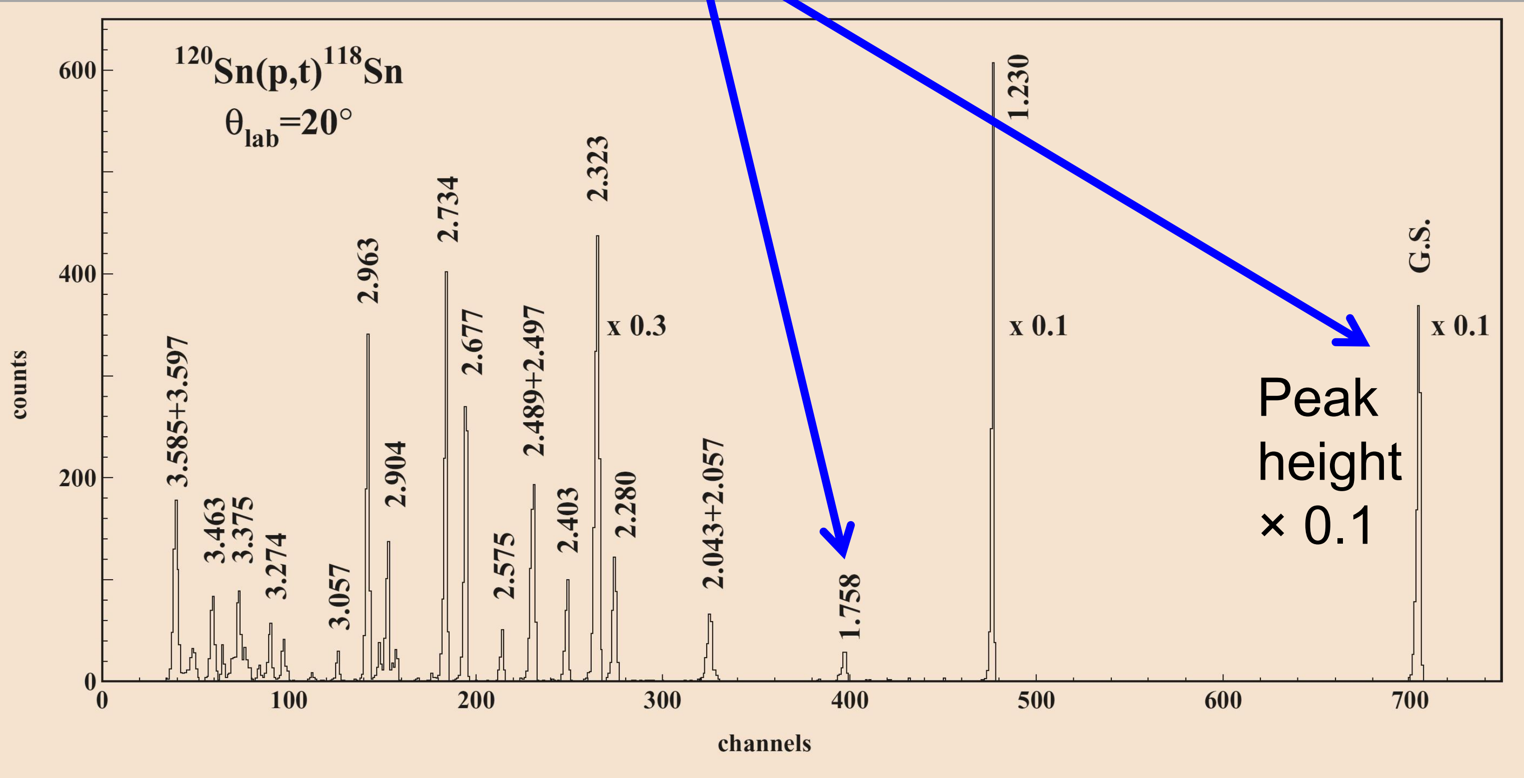
Heyde and Wood, RMP 83, 1467 (2011)

0^+ states in Sn populated very weakly in two-neutron transfer, but strongly in two-proton transfer

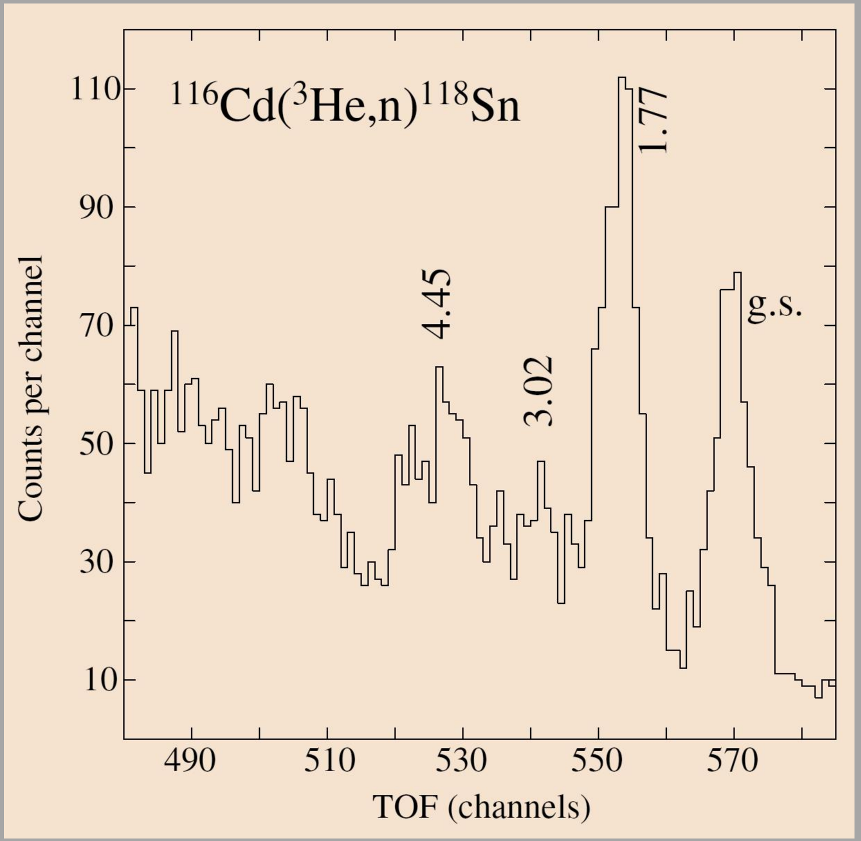
- In normal or superfluid nuclei, the two-nucleon-transfer should be dominated by ground state–to ground state transitions – typically >95% of L=0 strength to the ground state

- Near Z=50, two-proton transfer strongly populates excited 0^+ state – reminiscent of proton pairing vibration – 2p-2h excitation across Z=50 closed shell

Excited 0^+ @ 1758 keV very weakly populated



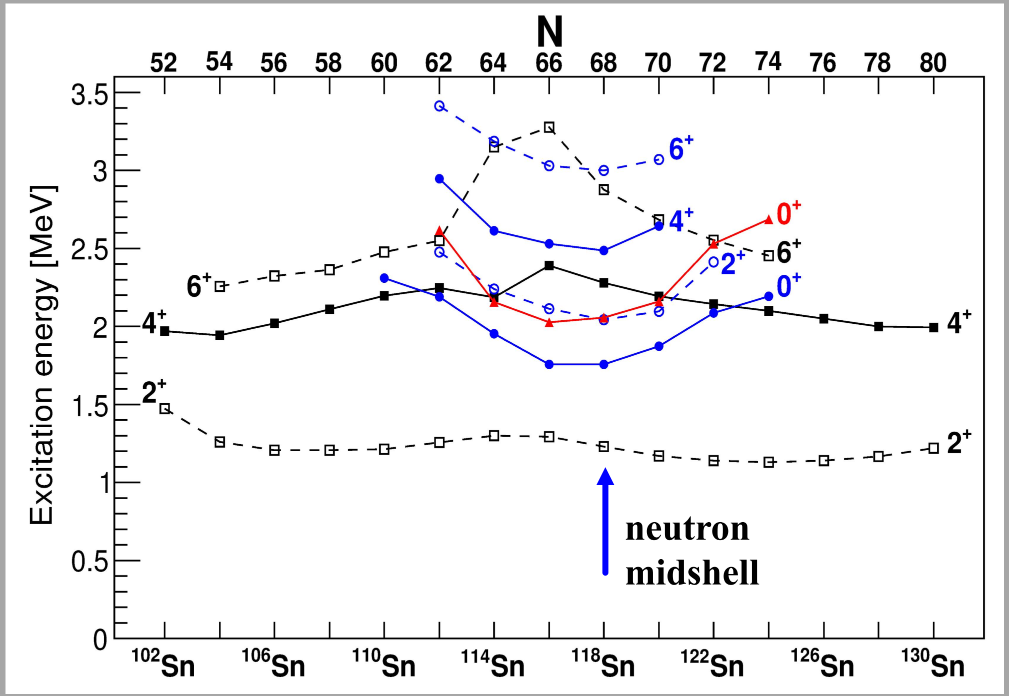
Guazzoni et al., PRC **78**, 064608 (2008)



Fielding et al., Nucl. Phys. **A281**, 392 (1977)

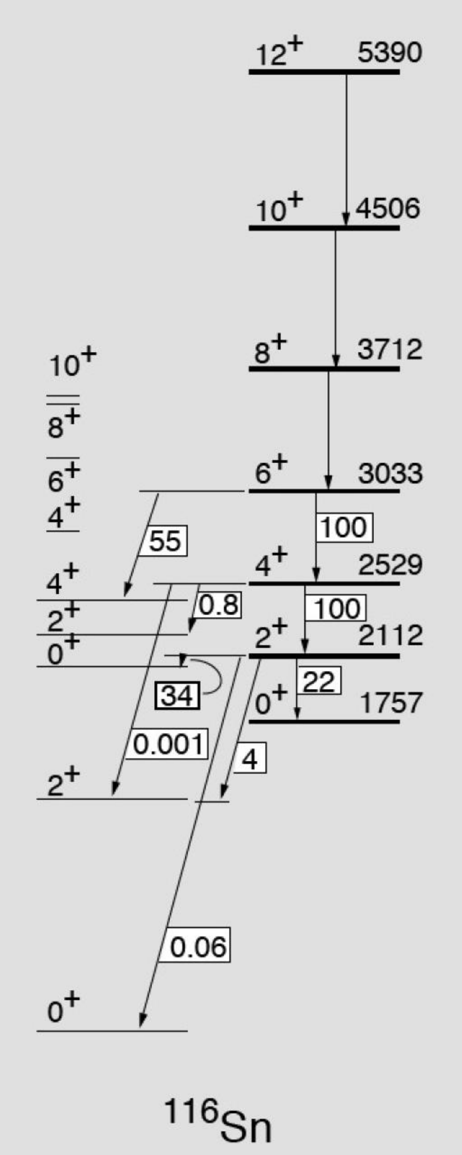
Example of the data for deformed 2*p*-2*h* "intruder" bands at closed shells – ^{116}Sn

Sn isotopes have characteristic parabolic pattern to excitation energies



Garrett, Zielińska, & Clement, PPNP **124**, 103931 (2022)

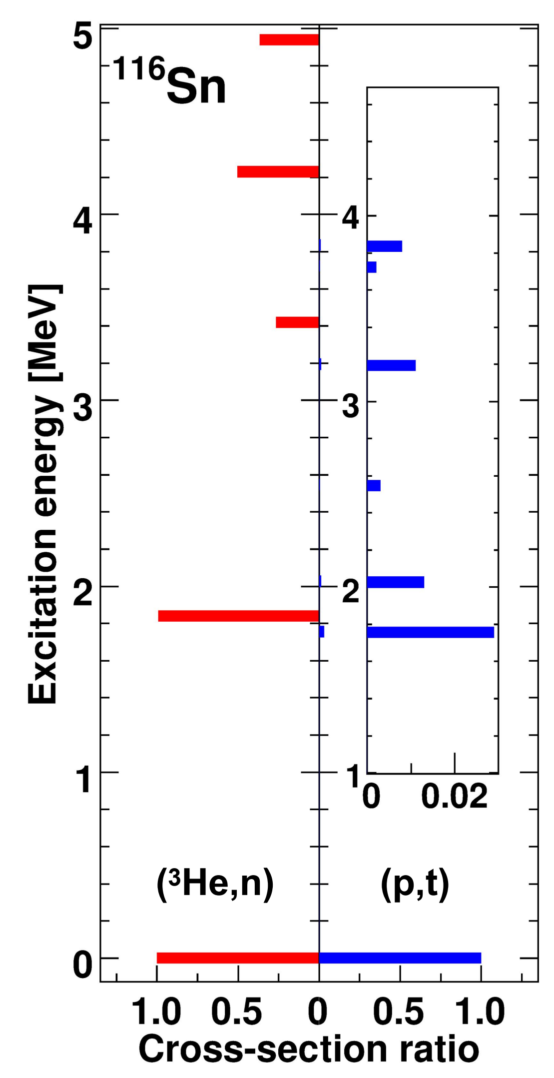
Appearance of rotational-like bands, with enhanced in-band B(E2) values, that stand out amongst spherical "shell-model" states



Normally, the two-nucleon-transfer is dominated by gs-to-gs transitions – typically > 95% of *L*=0 strength

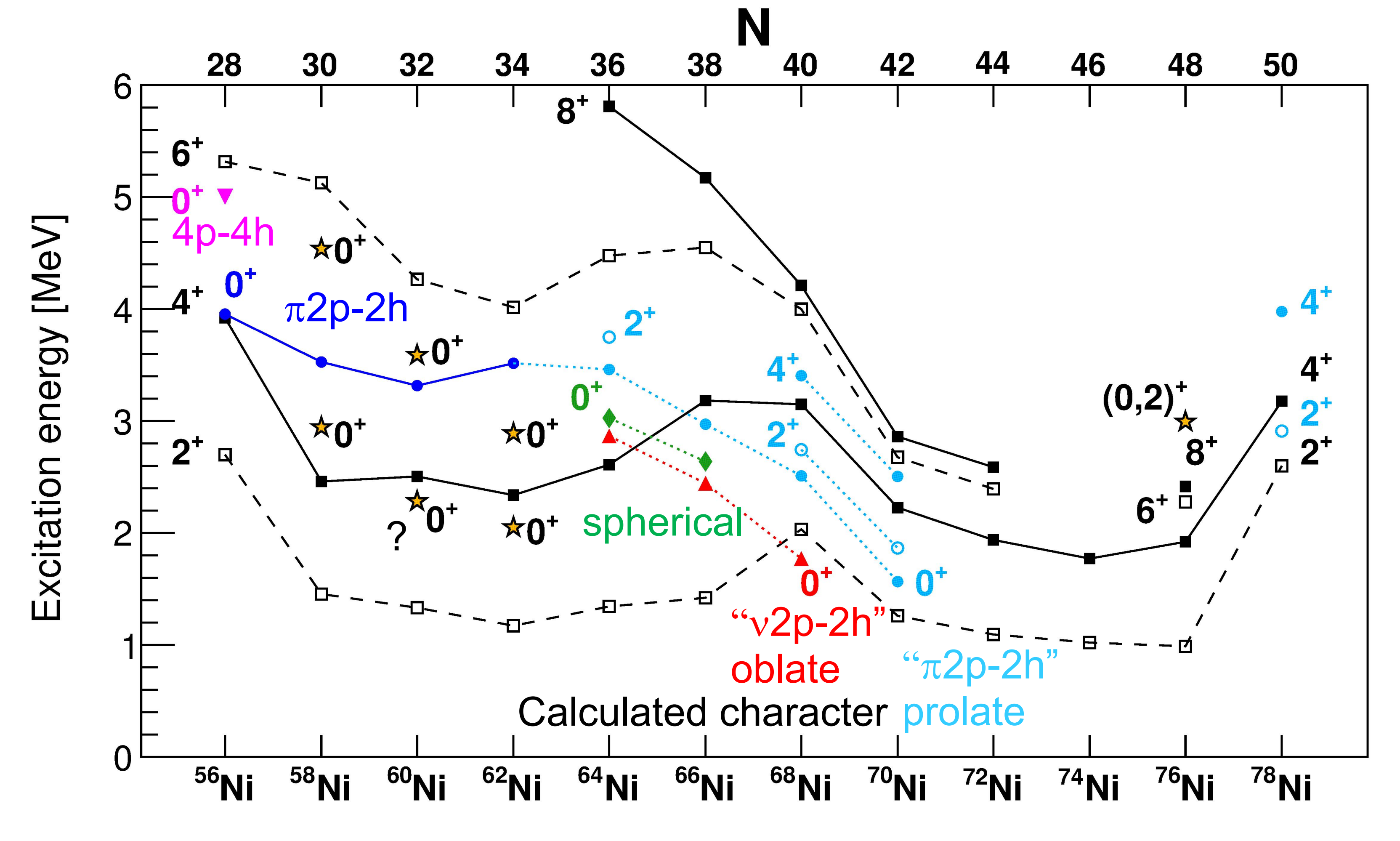
Near *Z* = 50, two-proton transfer strongly populates the 0_2^+ state

Garrett, Zielińska, & Clement, PPNP **124**, 103931 (2022)



The Ni isotopes

- Variety of highly deformed structures observed in vicinity of ^{56}Ni by Lund group (e.g., ^{56}Ni by D. Rudolf et al., PRL 82, 3763 (1999))

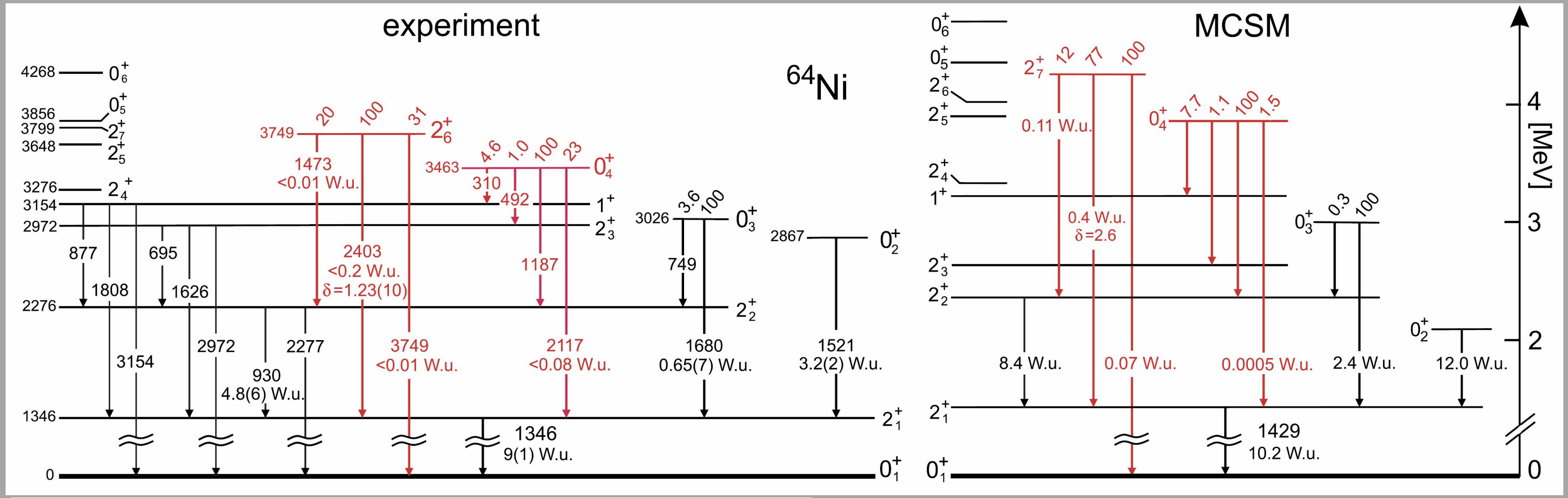


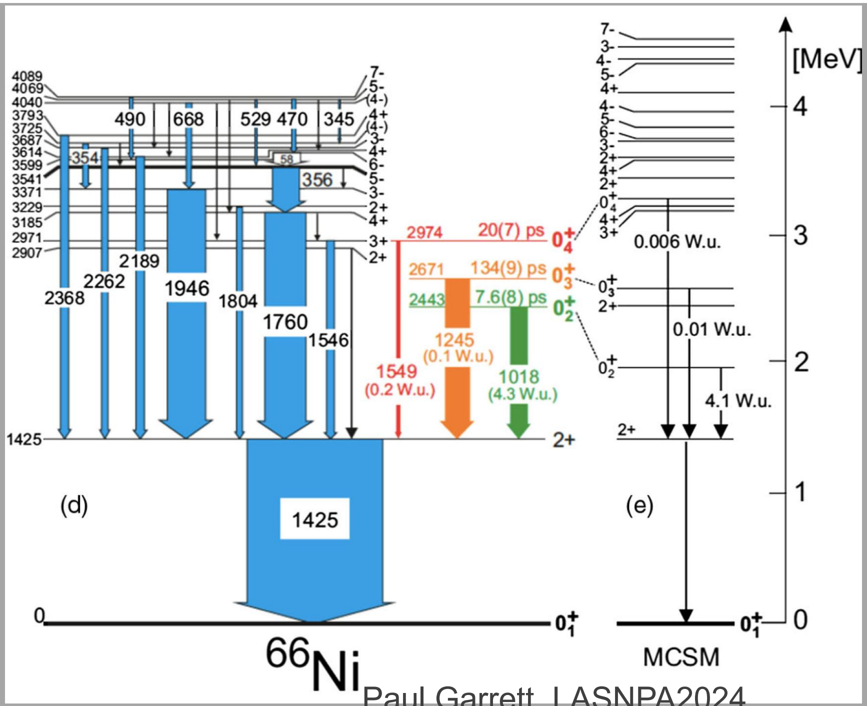
PG, M. Zielinska, and E. Clement, PPNP 124, 123931 (2022)

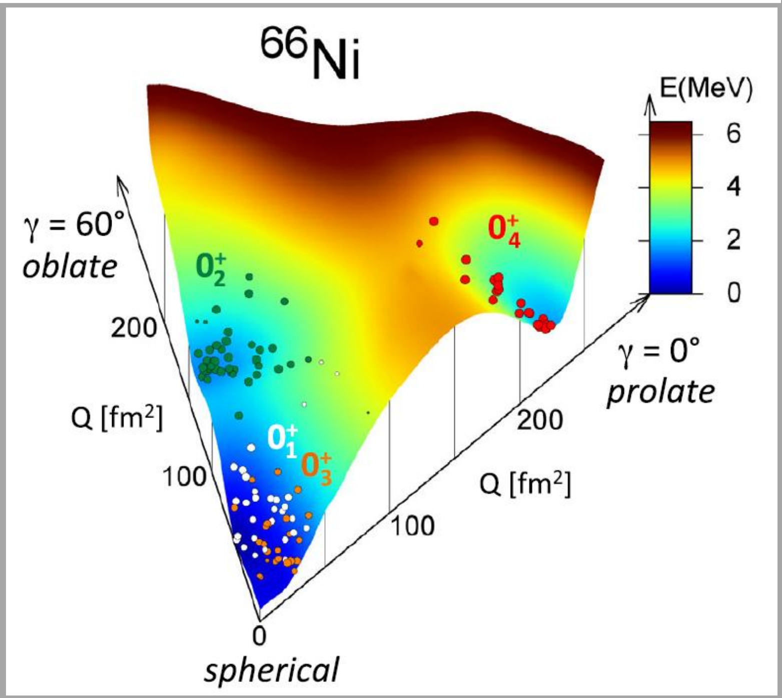
Suggestions for multiple shapes in $^{64-68}$Ni through comparisons with MCSM

- **Suggested shapes need to be confirmed through additional measurements**

S. Leoni et al., PRL 118, 162502 (2017)
N. Mărginean et al., PRL 125, 102502 (2020)

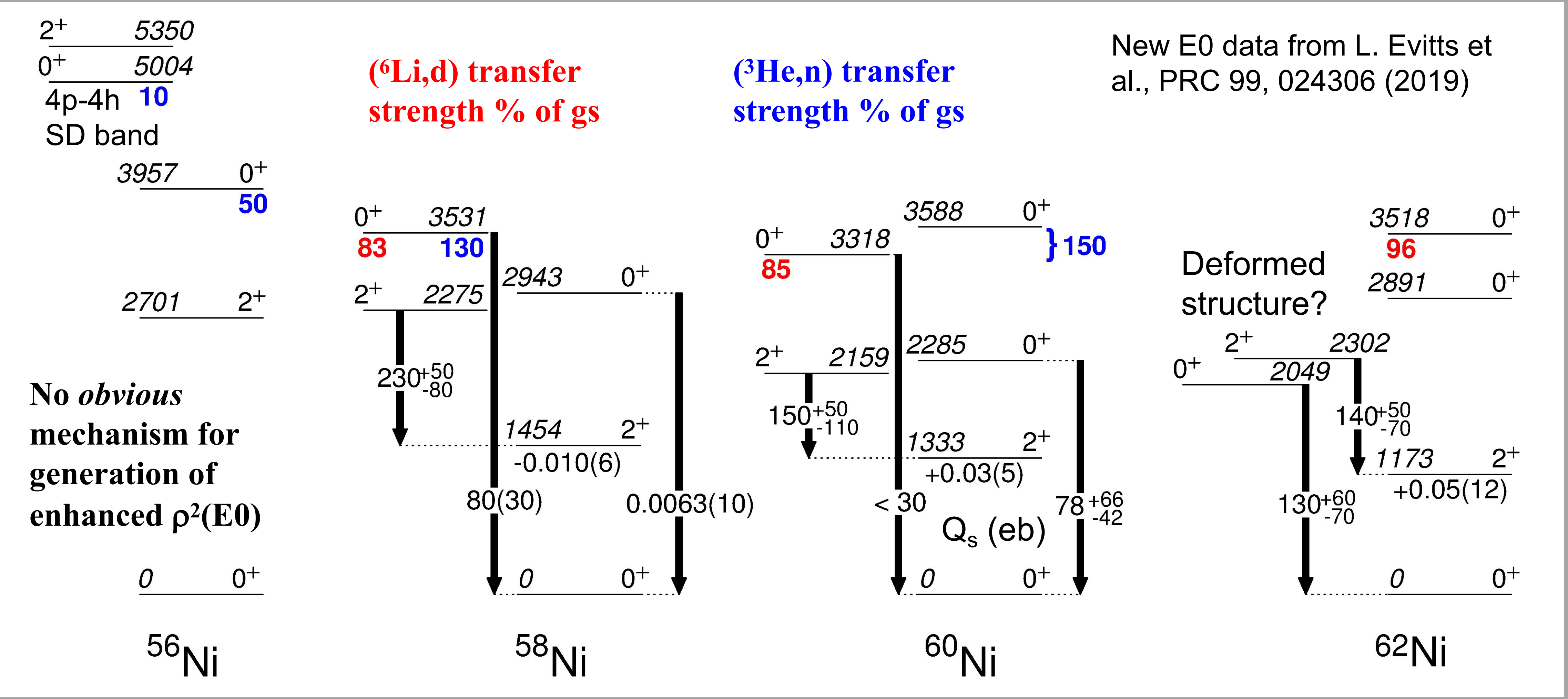






Trouble in paradise for *E*0s – the mysterious case of the Ni isotopes

Transitions labelled with $10^3\rho^2(E0)$

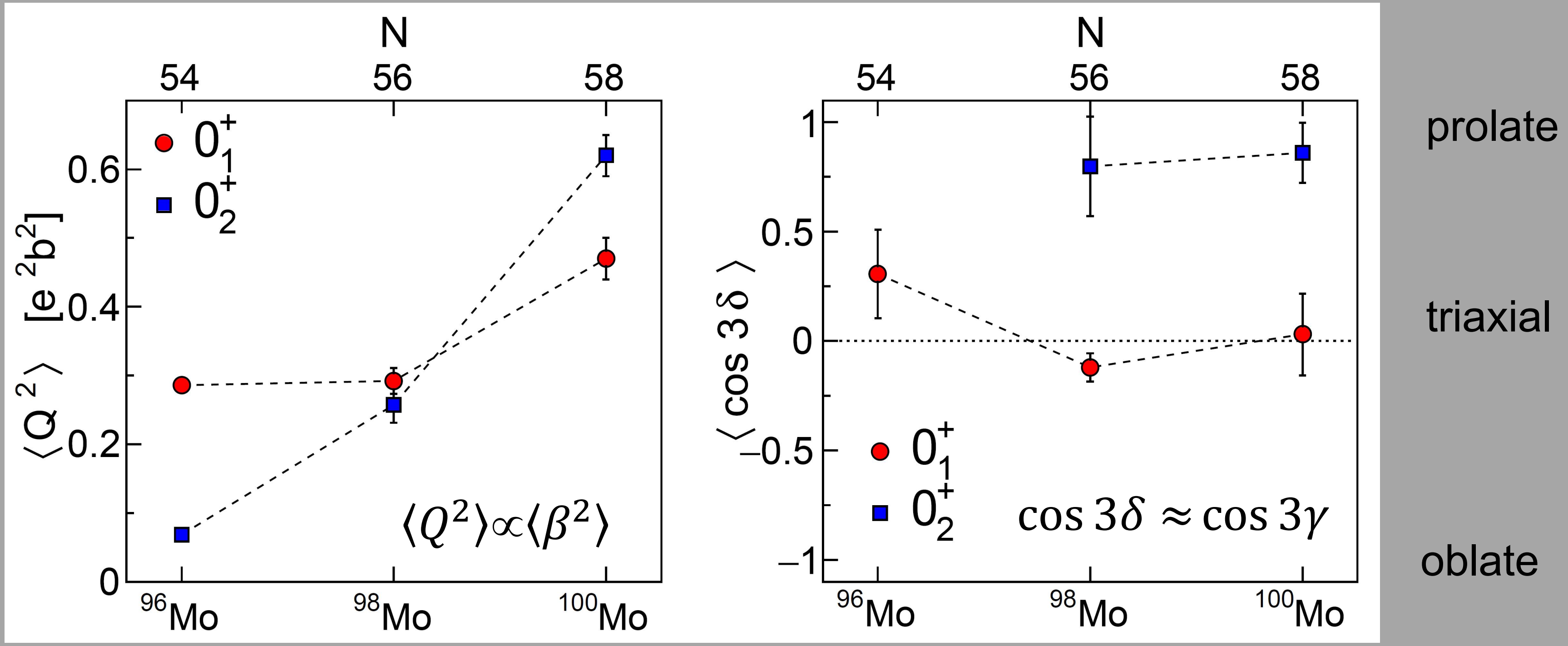


In 2-state mixing model $\rho^2(E0) = \alpha^2\beta^2(\Delta\langle r^2\rangle)\frac{Z^2}{R^4}$

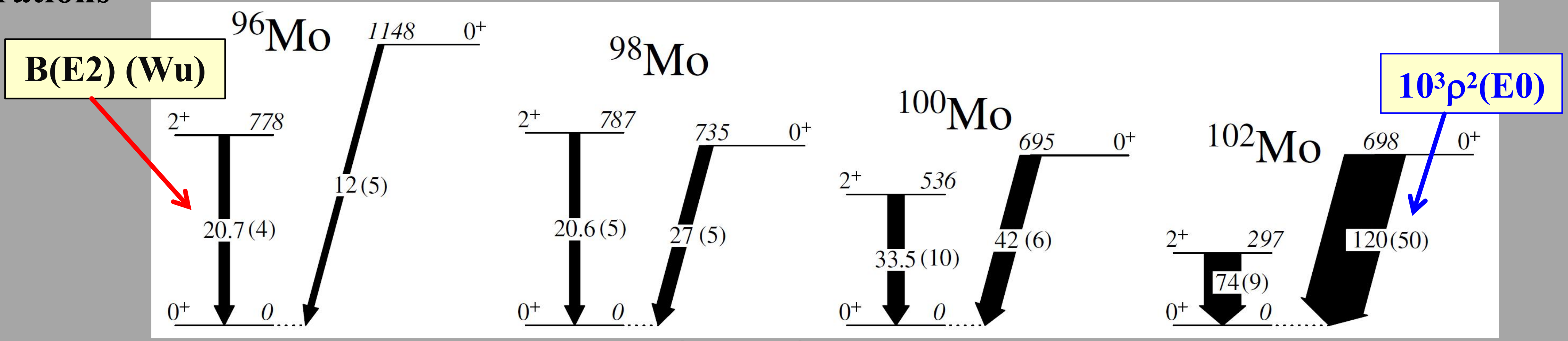
PG, M. Zielinska, and E. Clement, PPNP 124, 123931 (2022)

Shape coexistence in the Z = 40 – 50 region: $^{96-102}$Mo show clear evidence, and anchor the region

- **Detailed Coulomb-excitation studies enable extraction of shape invariants indicating *clearly different* shapes for 0_1^+ and 0_2^+ states**
- **Strong *E*0 transitions indicate differences in deformation and mixing of configurations**

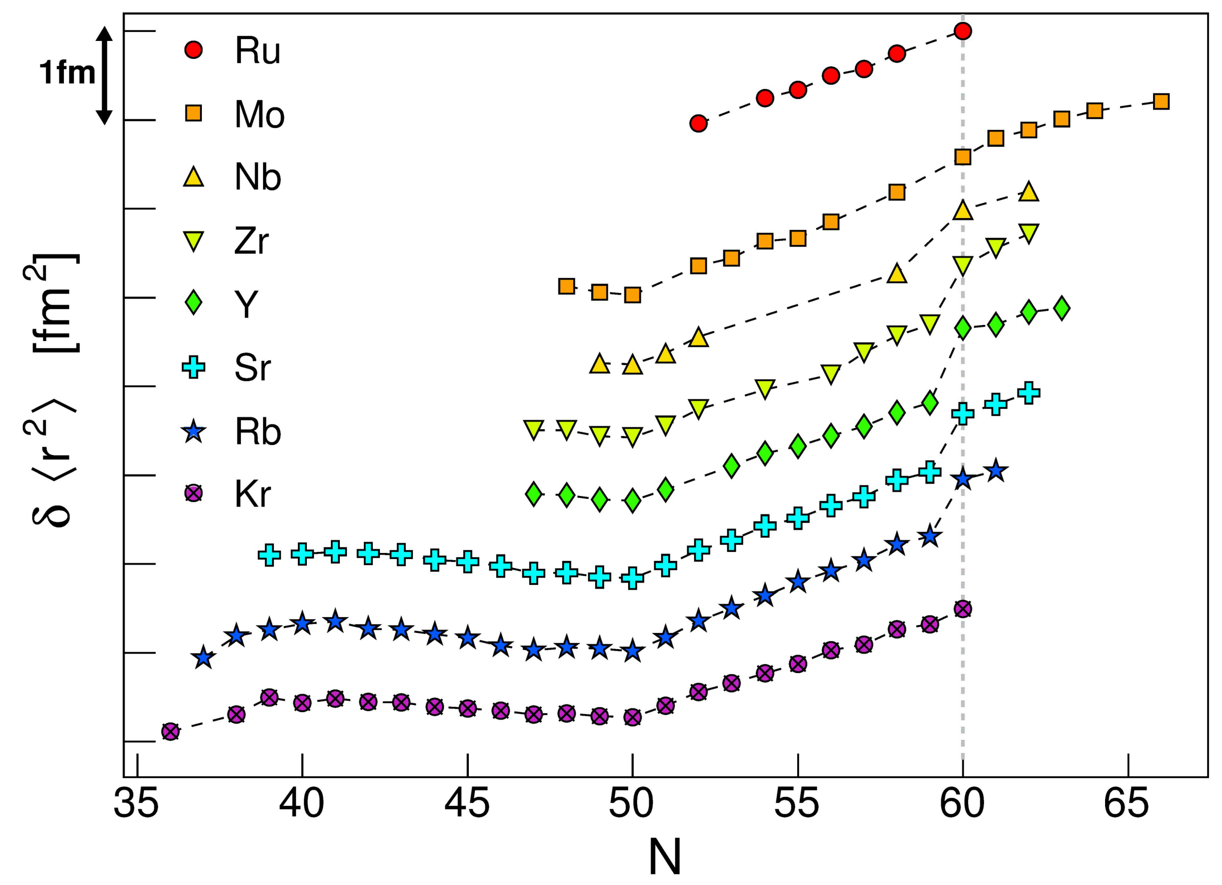


M. Zielińska et al., NPA 712, 3 (2002), K. Wrzosek-Lipska et al., PRC 86 064305 (2012)

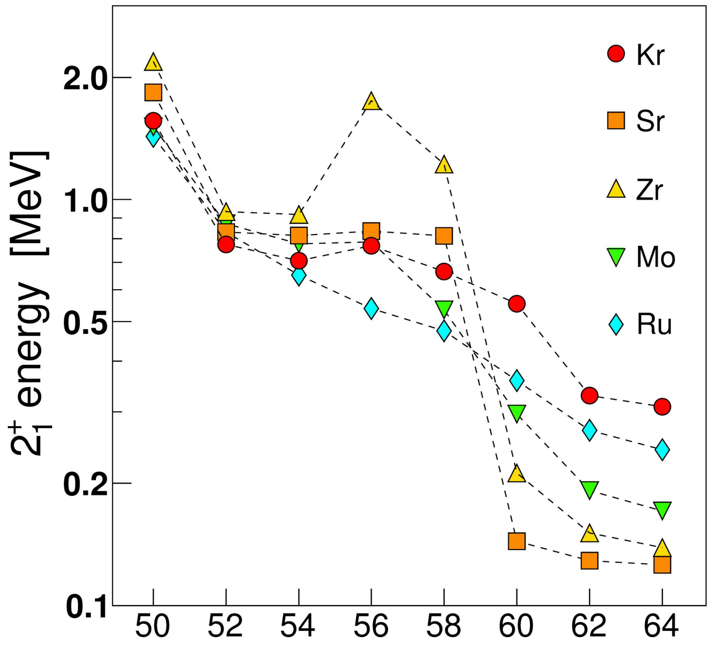


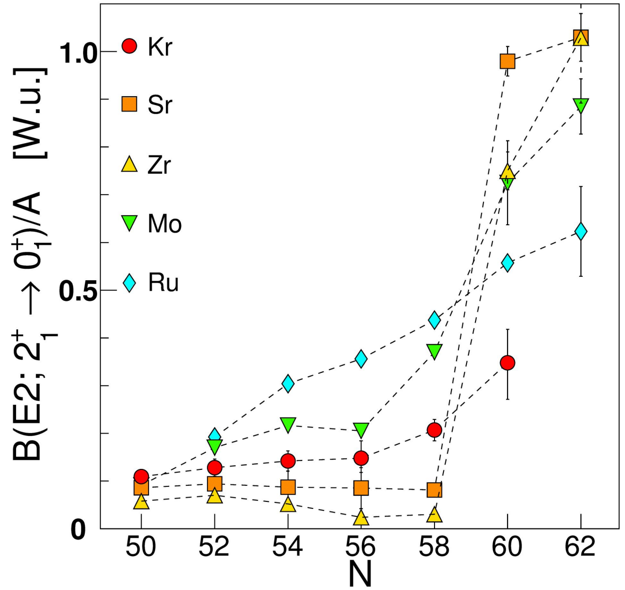
Zr isotopes undergo the most rapid change of ground state structure across the nuclear chart

- **There have been numerous experimental investigations, but firm evidence for shape coexistence has been lacking, and only recently *B(E2)*s determined for deformed states**



Measured isotope shifts





MCSM calculations – multiple shape coexistence predicted in the Zr isotopes

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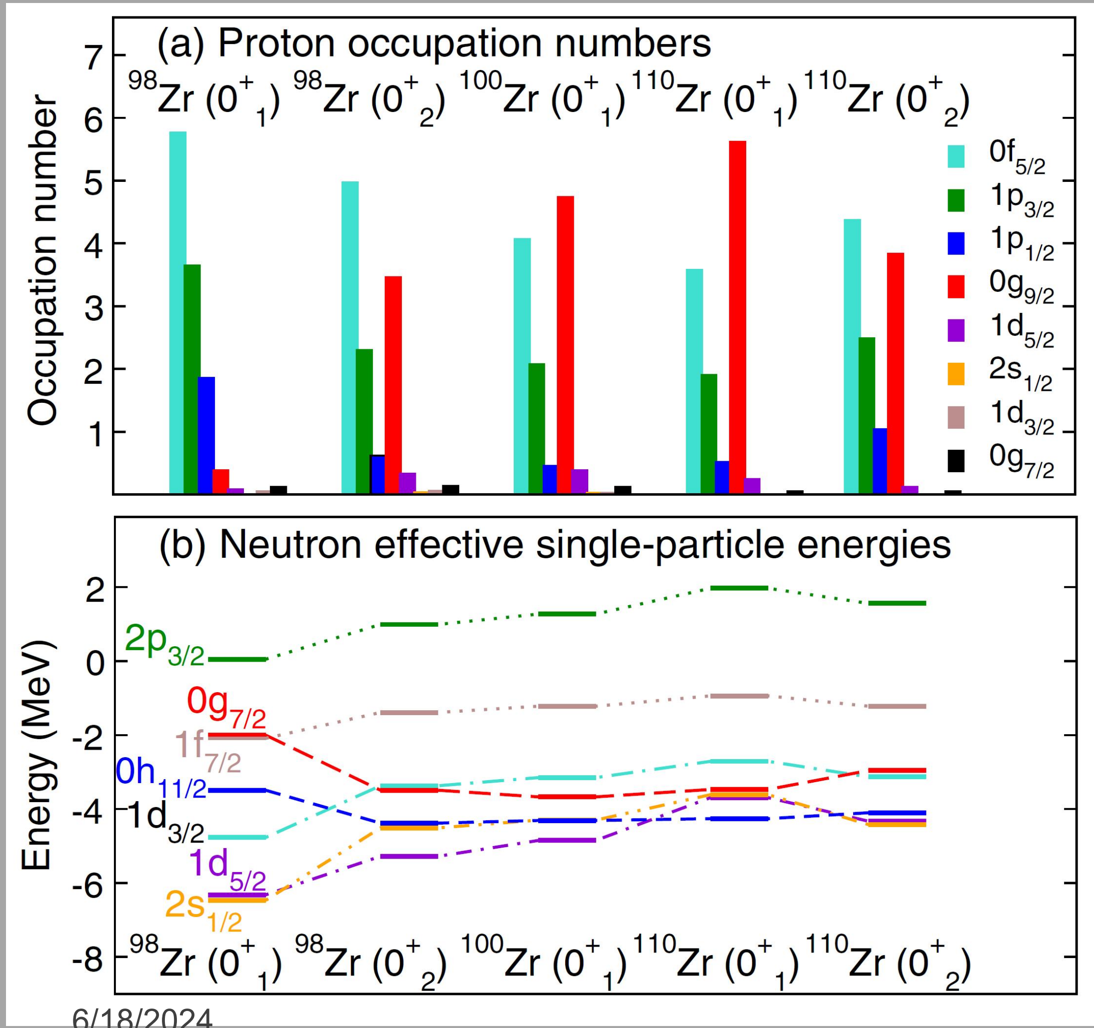
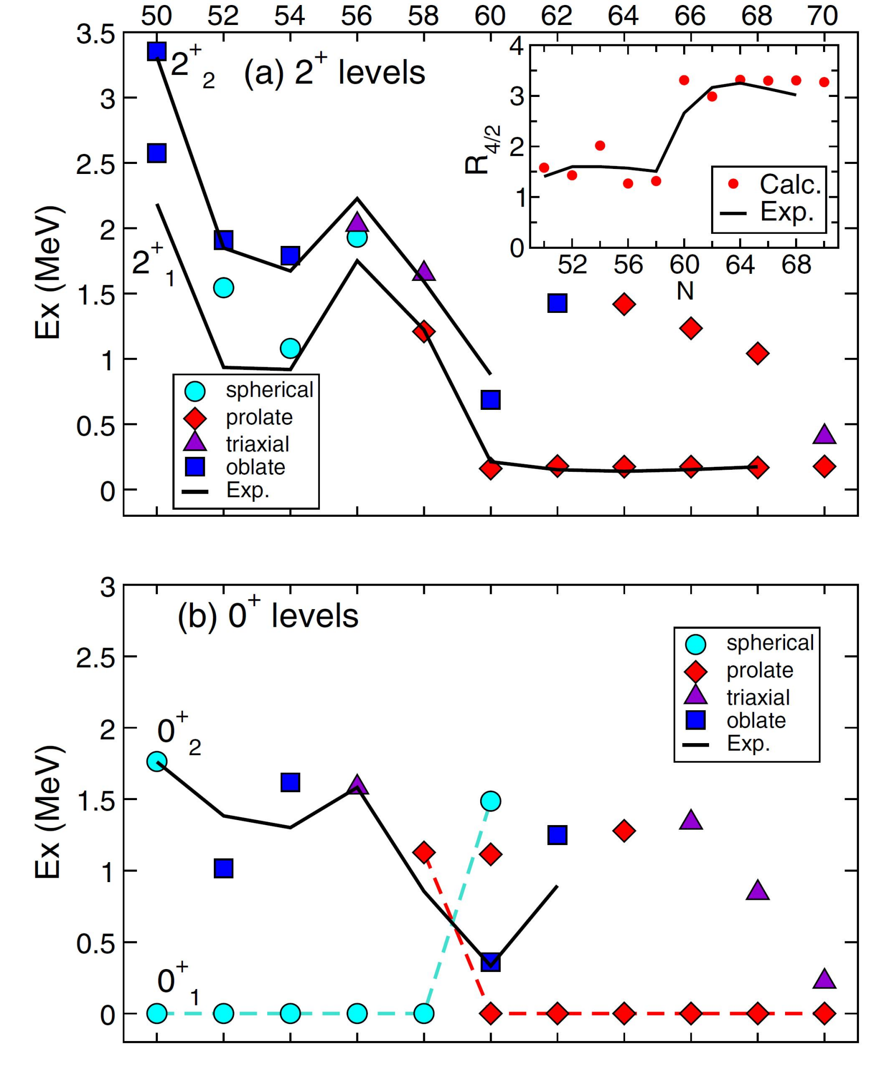
- ***p-n*** **tensor interaction reduces *Z*=40 gap when $\nu g_{7/2}$ shell is filled**
- **Lower gap enables additional proton excitations, creating a self-reinforcing mechanism – type-II shell evolution**



Deformed 0^+ state configuration includes 2*p*-2*h* (+4*p*-4*h*, …) excitations across *Z*=40 gap

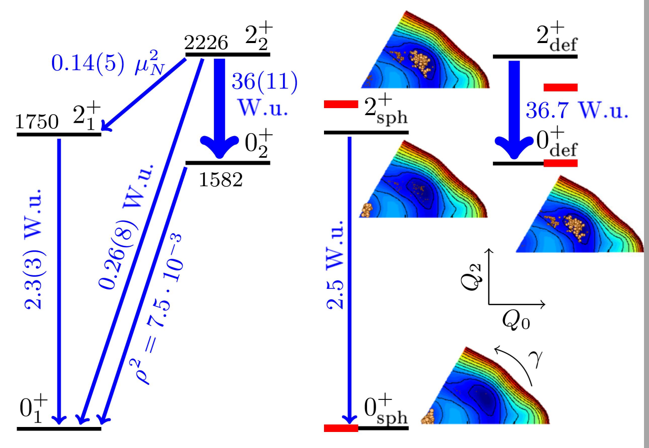
Very different configurations and (generally) weak mixing between 0_1^+ (spherical) and 0_2^+ (deformed) until *N*=60 is reached



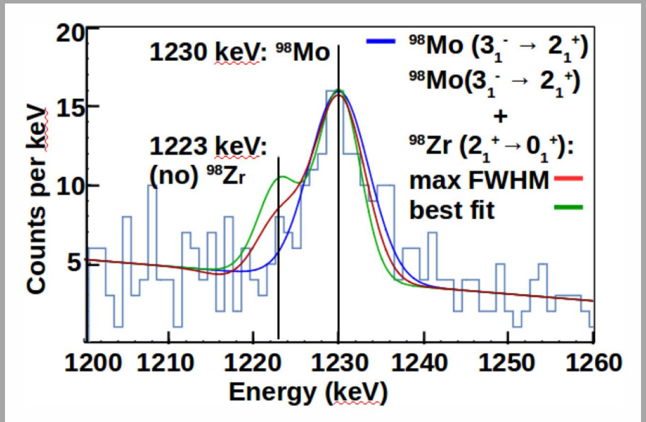
Togashi et al., PRL 117 172502 (2016)

Some recent work on the Zr isotopes

^{96}Zr – high resolution e^- scattering yielding B(E2;$2_2^+ \rightarrow 0_1^+$) and comparisons with MCSM

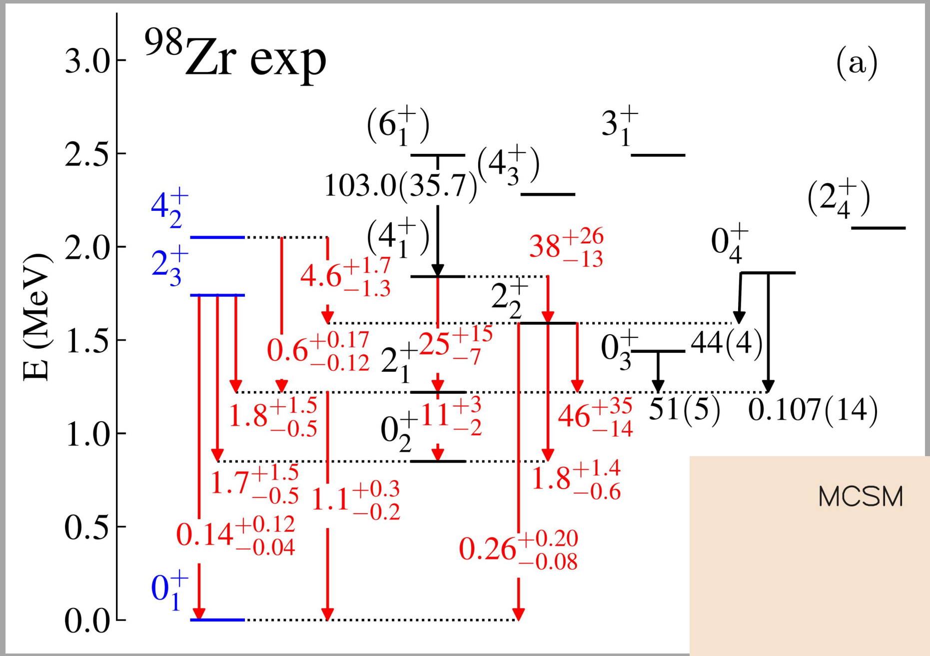


C. Kremer *et al.*, PRL 117, 172503 (2016)



W. Witt *et al.*, PRC 98, 041302(R) (2018)

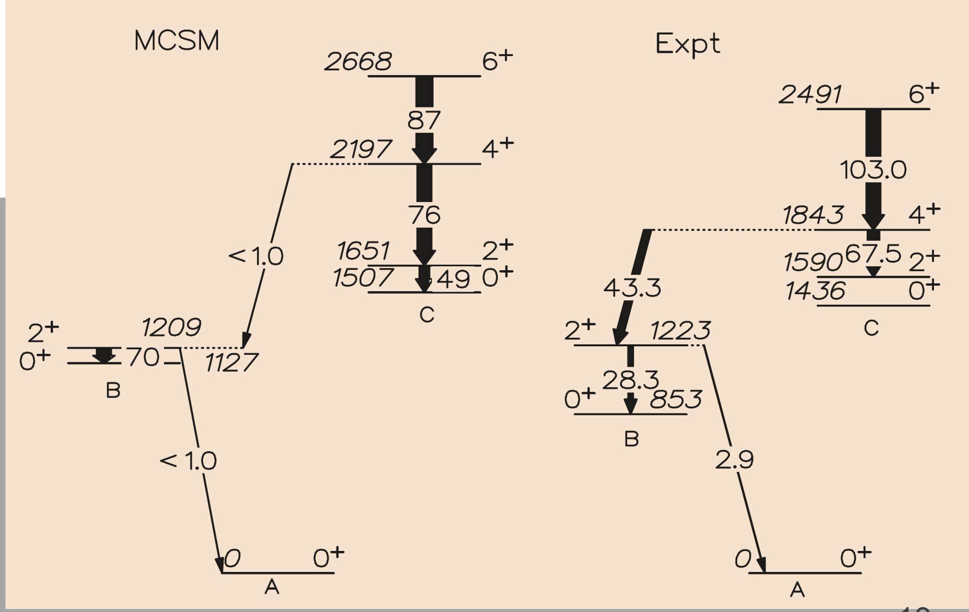
^{98}Zr – lifetimes measured in ^{9}Be induced fission of ^{238}U, and ^{96}Zr+^{18}O *2p* transfer reaction



V. Karayonchev et al., PRC 102, 064314 (2020)

Substantial differences in both measured lifetimes, and assignments/interpretations

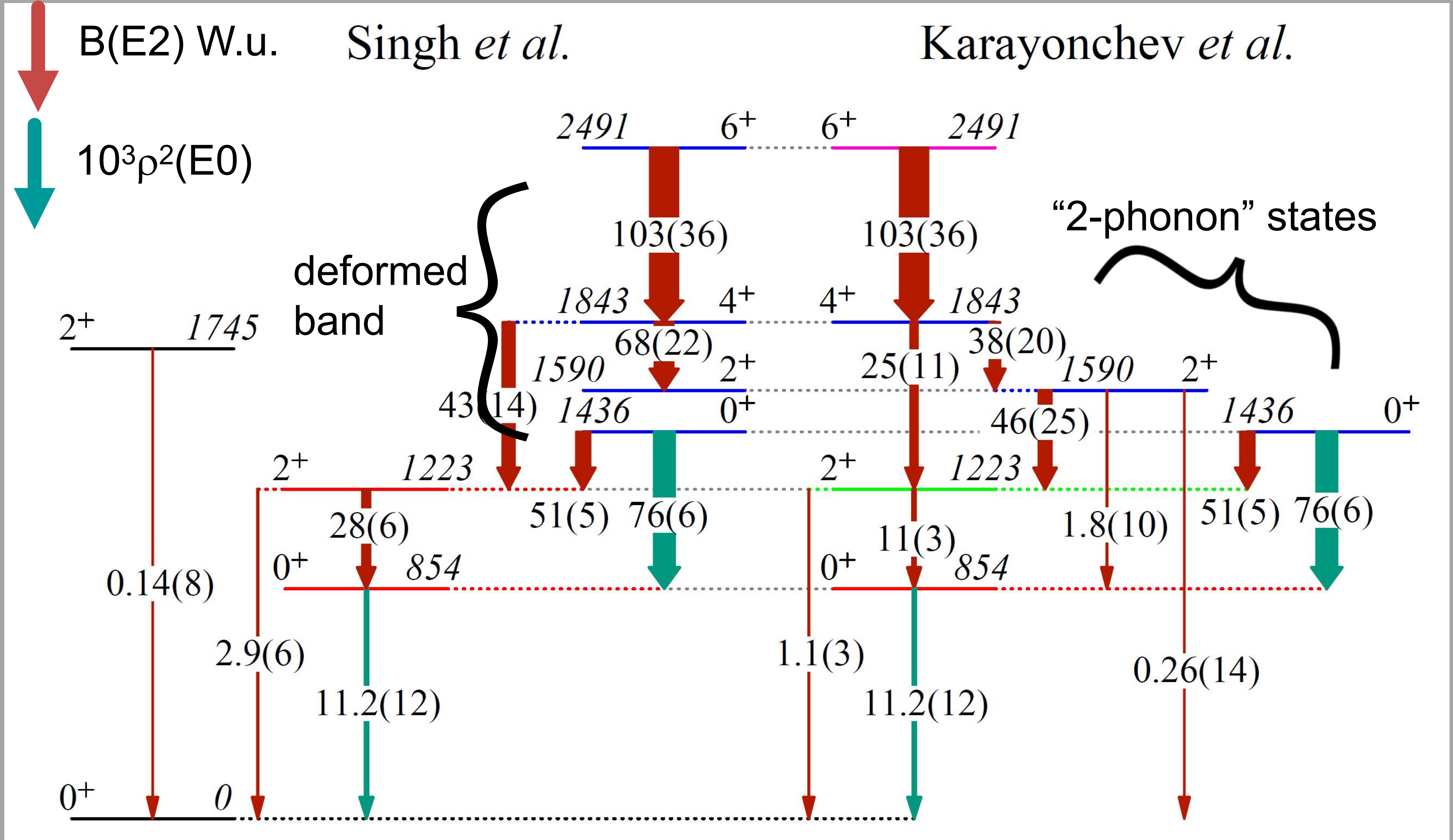
P. Singh et al., PRL 121, 192501 (2018)

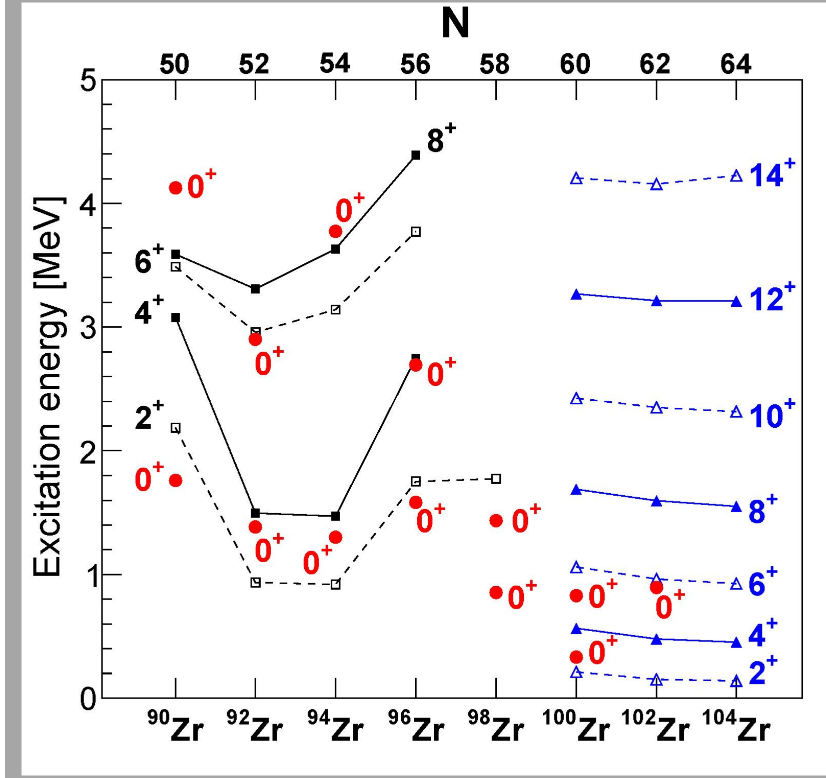


^{98}Zr – Coulomb excitation of mass 98 beam placed limit on B(E2;$2_1^+ \rightarrow 0_1^+$) value

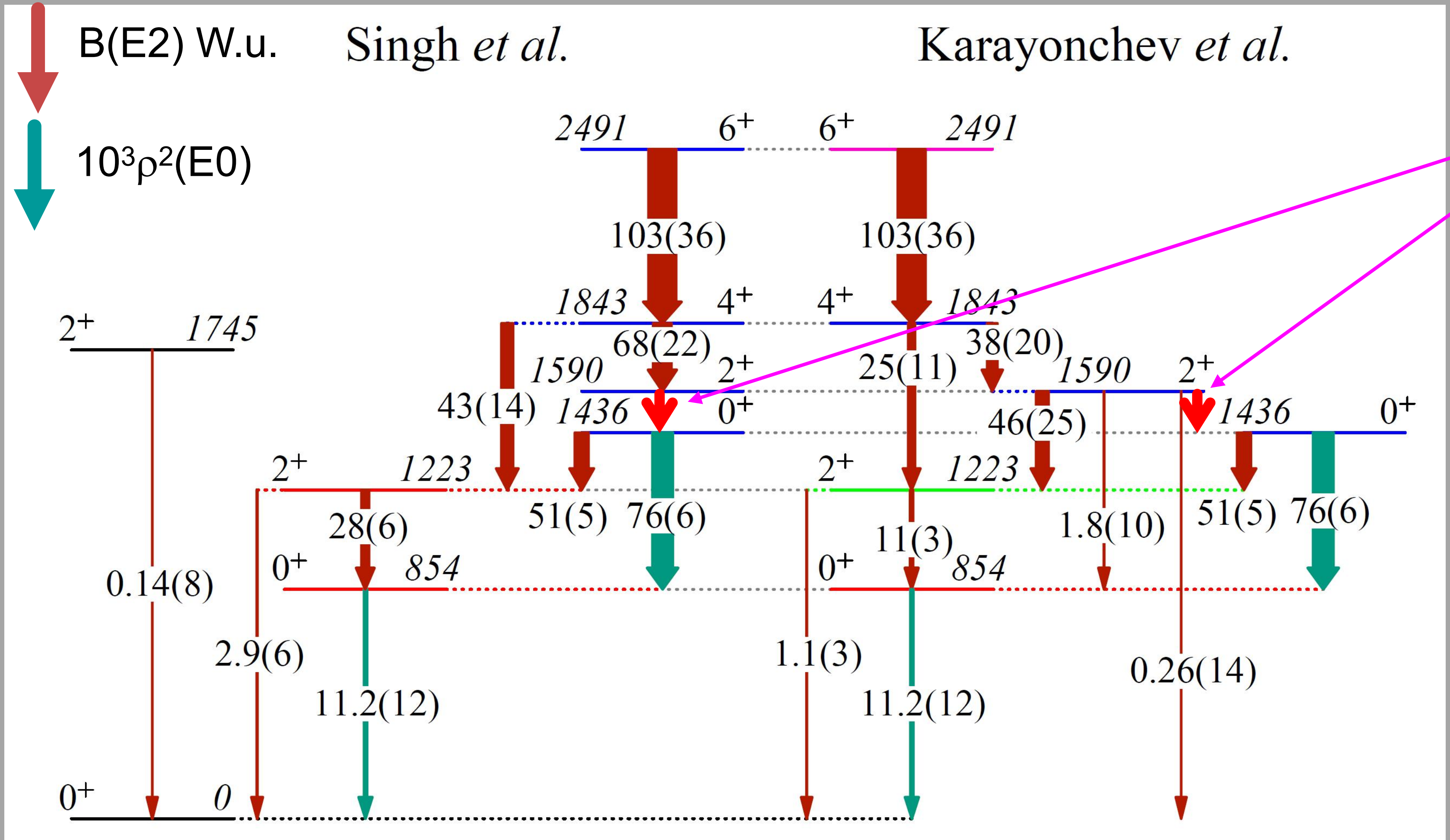
“Tension” in ^{98}Zr between recent measurements and interpretations

- Lifetimes from Singh *et al*. generally shorter than those of Karayonchev *et al*.
- Singh *et al*. favoured multiple shape coexistence with deformed band structures, Karayonchev *et al*. favoured a multiphonon-like structure with configuration mixing





“Tension” between recent measurements and interpretations in ^{98}Zr



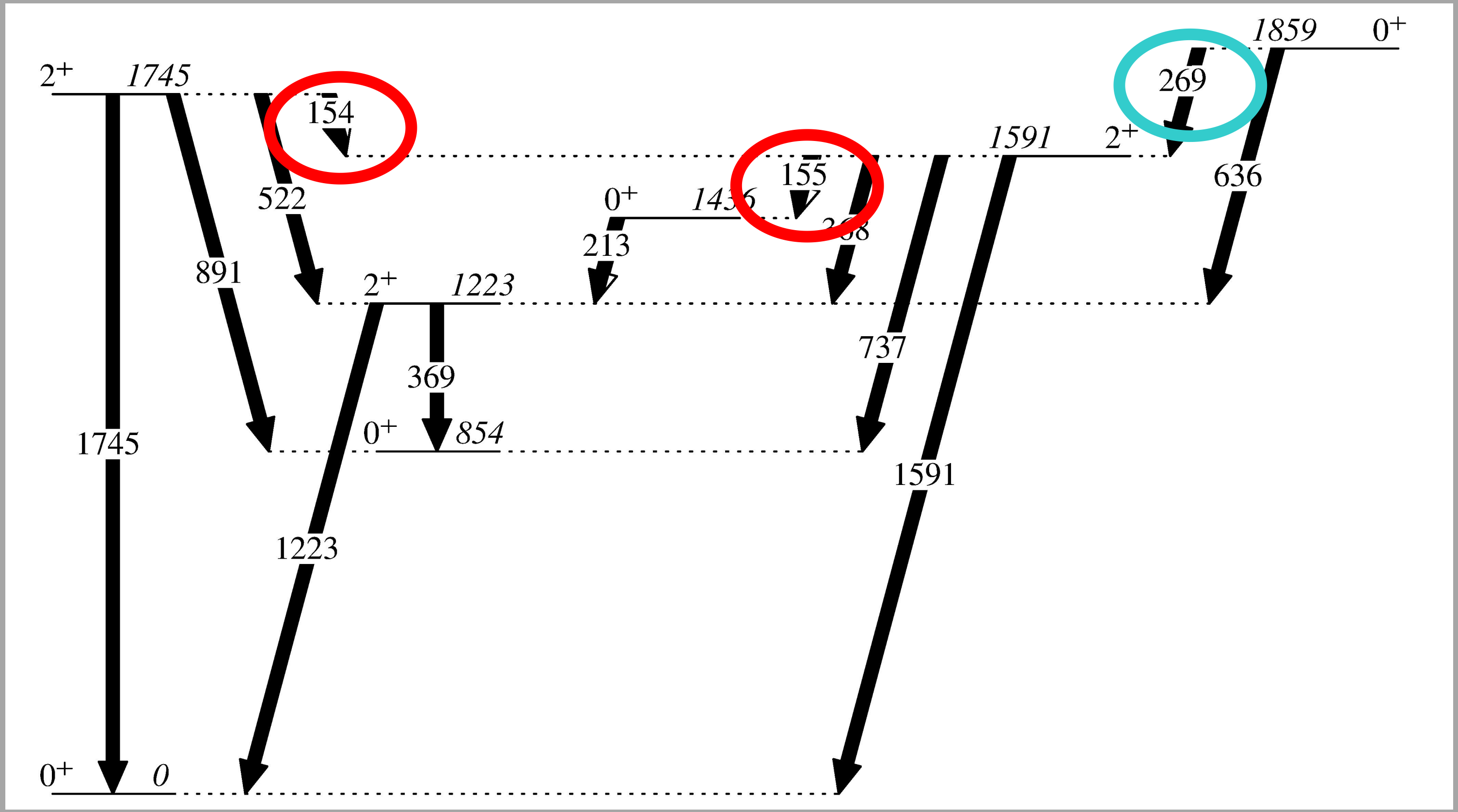
Key is the 155-keV $2_2^+ \rightarrow 0_3^+$ transition:

- **in deformed picture, MCSM predicts 49 W.u.**
- **in multiphonon picture with configuration mixing, 3 – 6 W.u.**

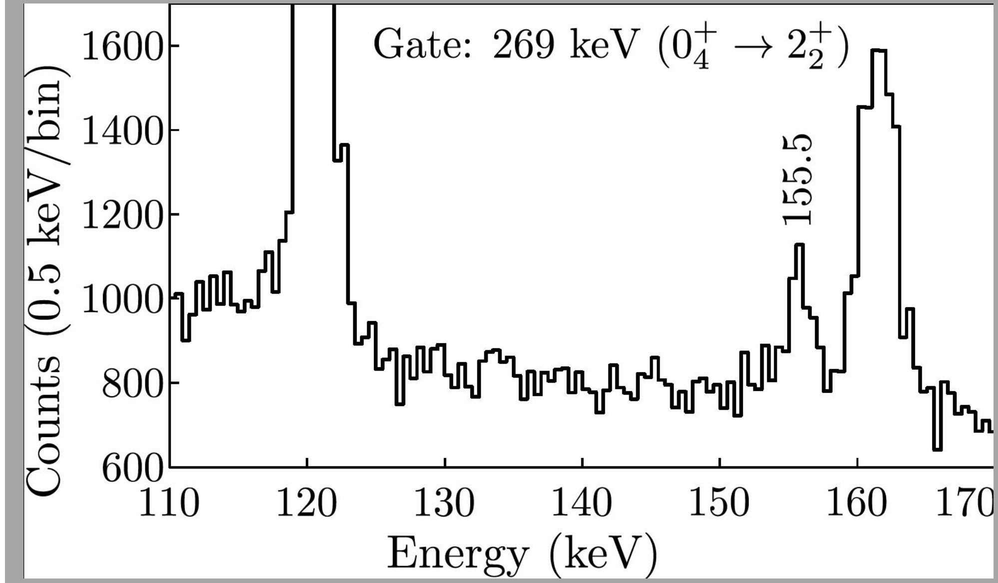
Using measured lifetime and current branching ratio leads to ~ 500 W.u., recognized as unrealistically large

Measurements using the 8π@TRIUMF-ISAC and β-decay to populate ^{98}Zr

- Issue: 155-keV $2_2^+ \rightarrow 0_3^+$ is part of a doublet.
- High statistical level achieved in the experiment enables us to cleanly separate lines by coincidence gating on 269-keV transition

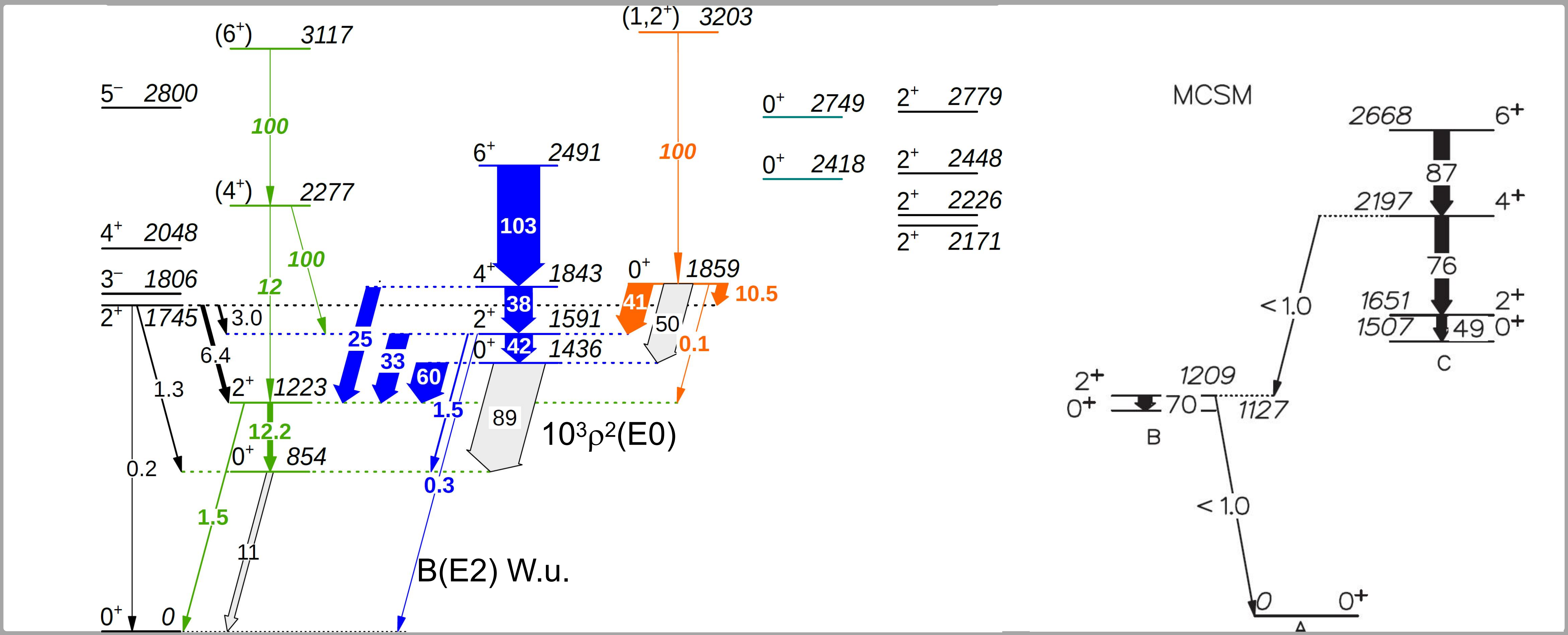


Previous branch: ~1.5%
New result: =0.109(8)%
$B(E2;2_2^+ \rightarrow 0_3^+)$ = 41(18) W.u.



New results consistent with band structure

- **Following the strong $J \rightarrow J\text{–}2$ transition $B(E2)$ values would appear to affirm the band structure for the 0_3^+ state suggested by Singh et al.**



Expanded level scheme for ^{98}Sr from our mass 98 β decay data

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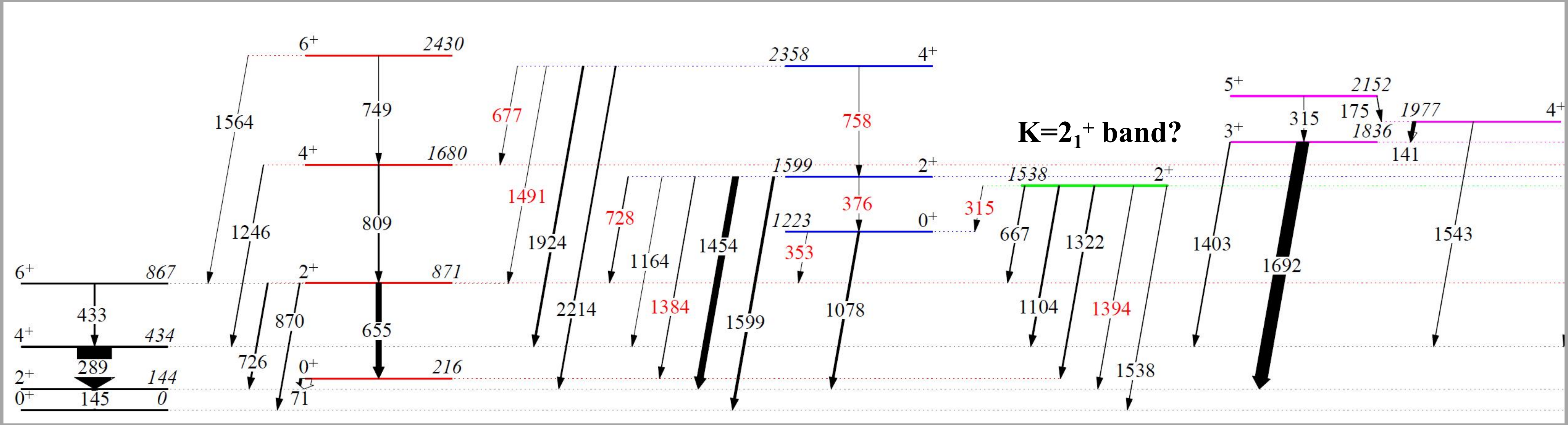
- Transitions labelled in red are new compared with NNDC
- *Candidate* 0_3^+ band with in-band transitions and firm spin assignments

Previously known $K=0_2^+$ band

New $K=0_3^+$ band

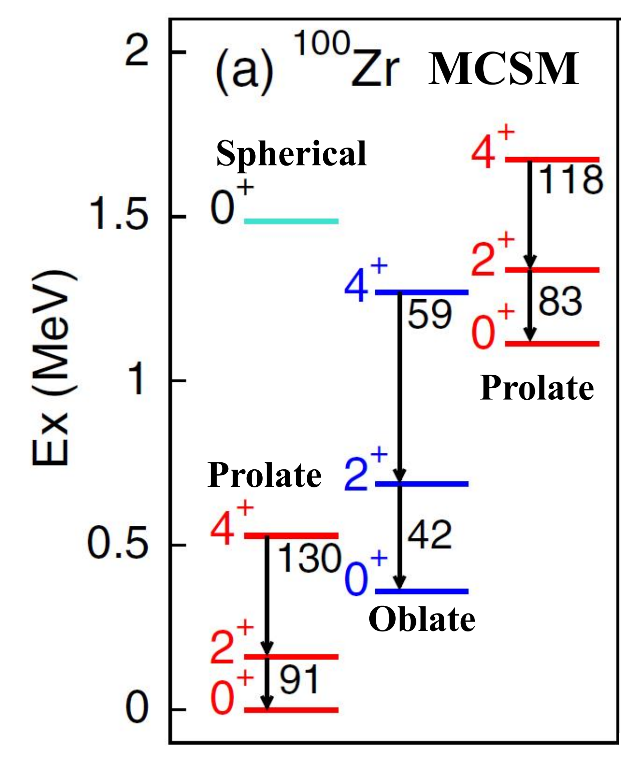
Previously known $K=3^+$ two-qp band

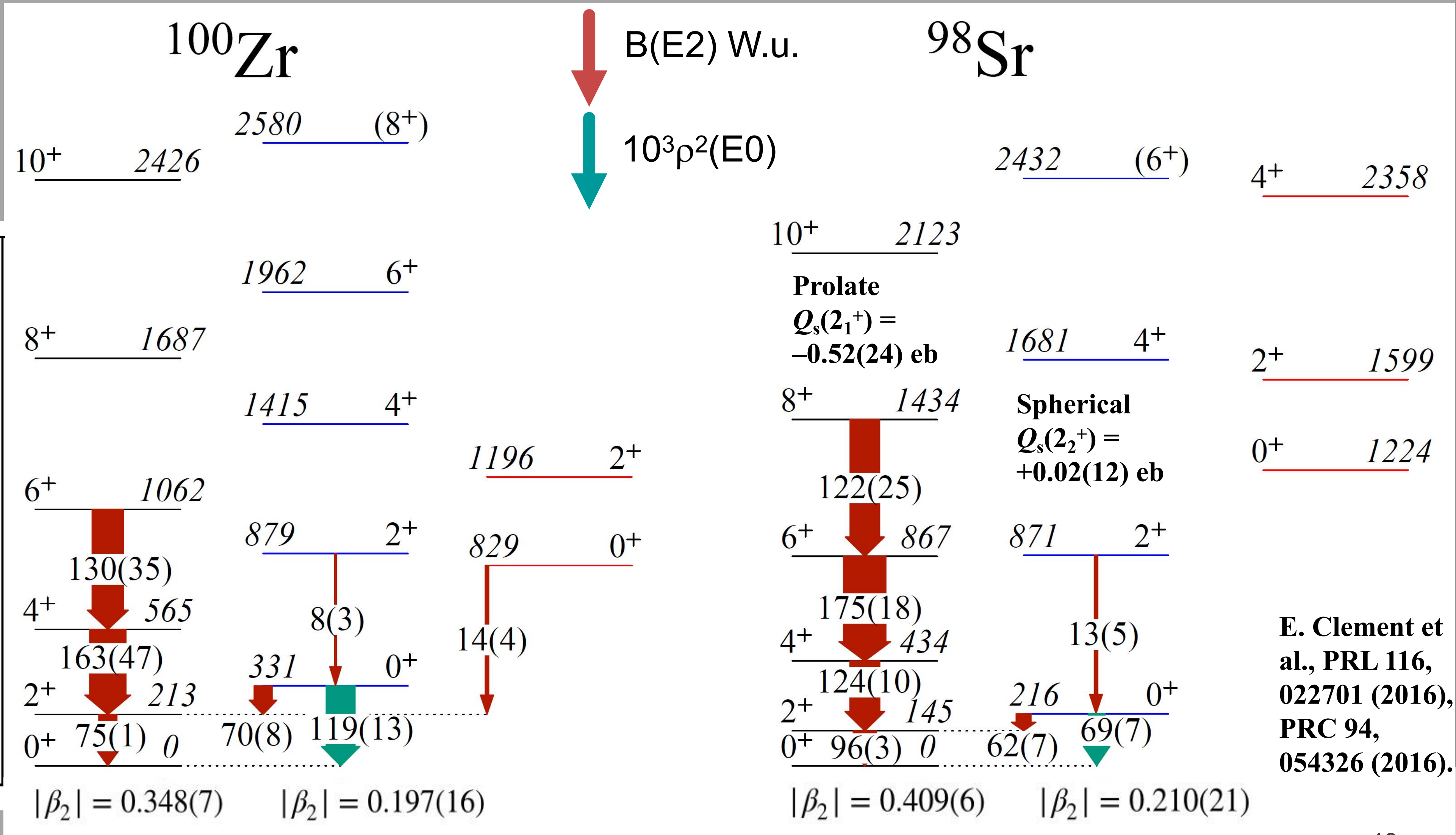


Similarity of ^{98}Sr, ^{100}Zr structure

- gsb appears to be well reproduced in MCSM
- Less collectivity in 0_2^+ band than predicted – likely the "spherical" configuration

^{100}Zr studied with β-decay at TRIUMF-ISAC; level scheme constructed (D. Kalaydjieva, M. Zielinska, Saclay), 0_3^+, 2_2^+ lifetimes measured (H. Bidaman, Guelph)





Evolution of $<Q^2>$

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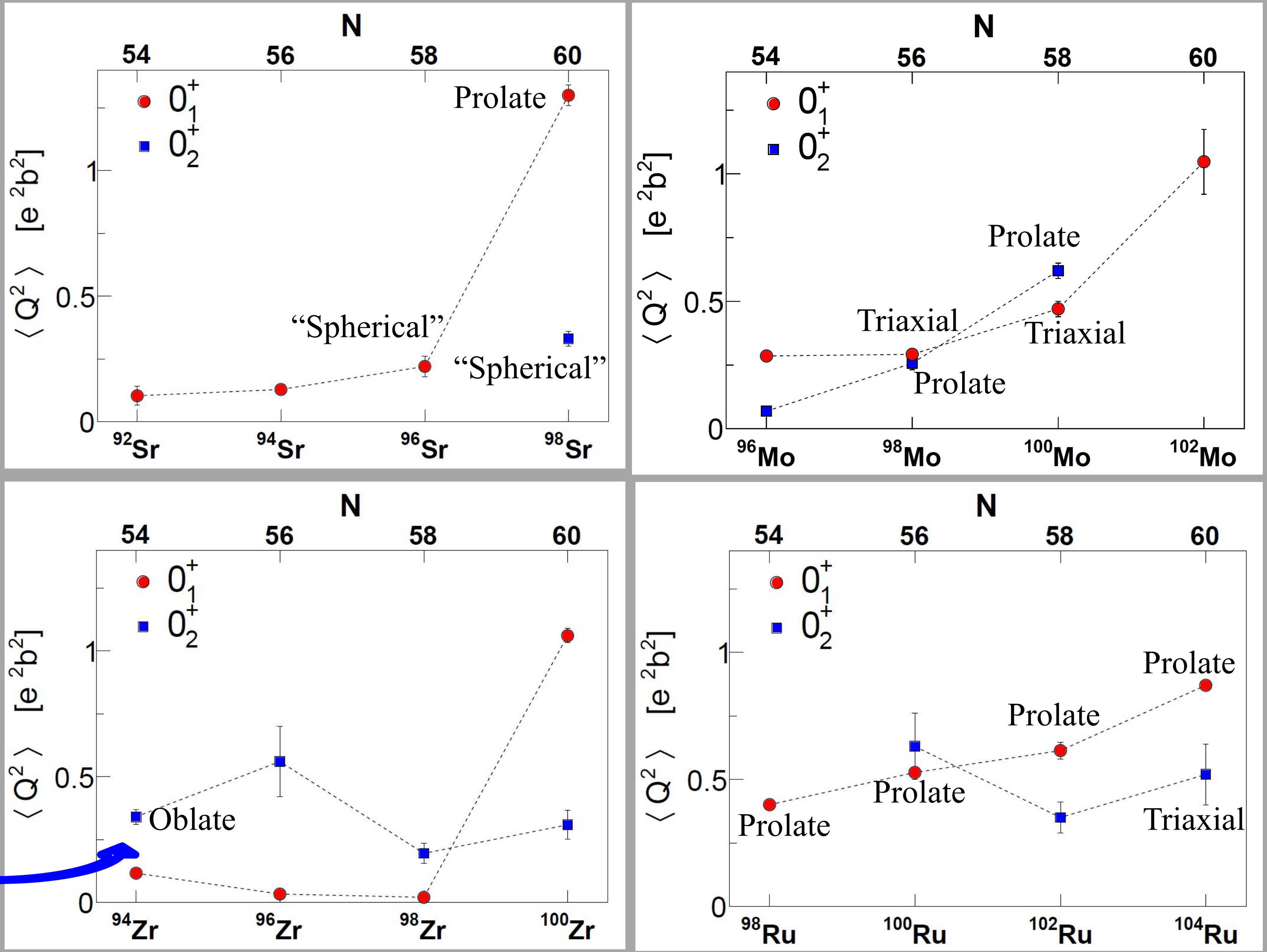
- **Ground state Q^2**
  - **Values increasing with N except for Z=40**
  - **Still some effect of N=60 observed in Ru, but not nearly as dramatic**
- **0_2^+ Q^2**
  - **Pattern in Zr and Ru remarkably similar**



N. Marchini et al., to be published

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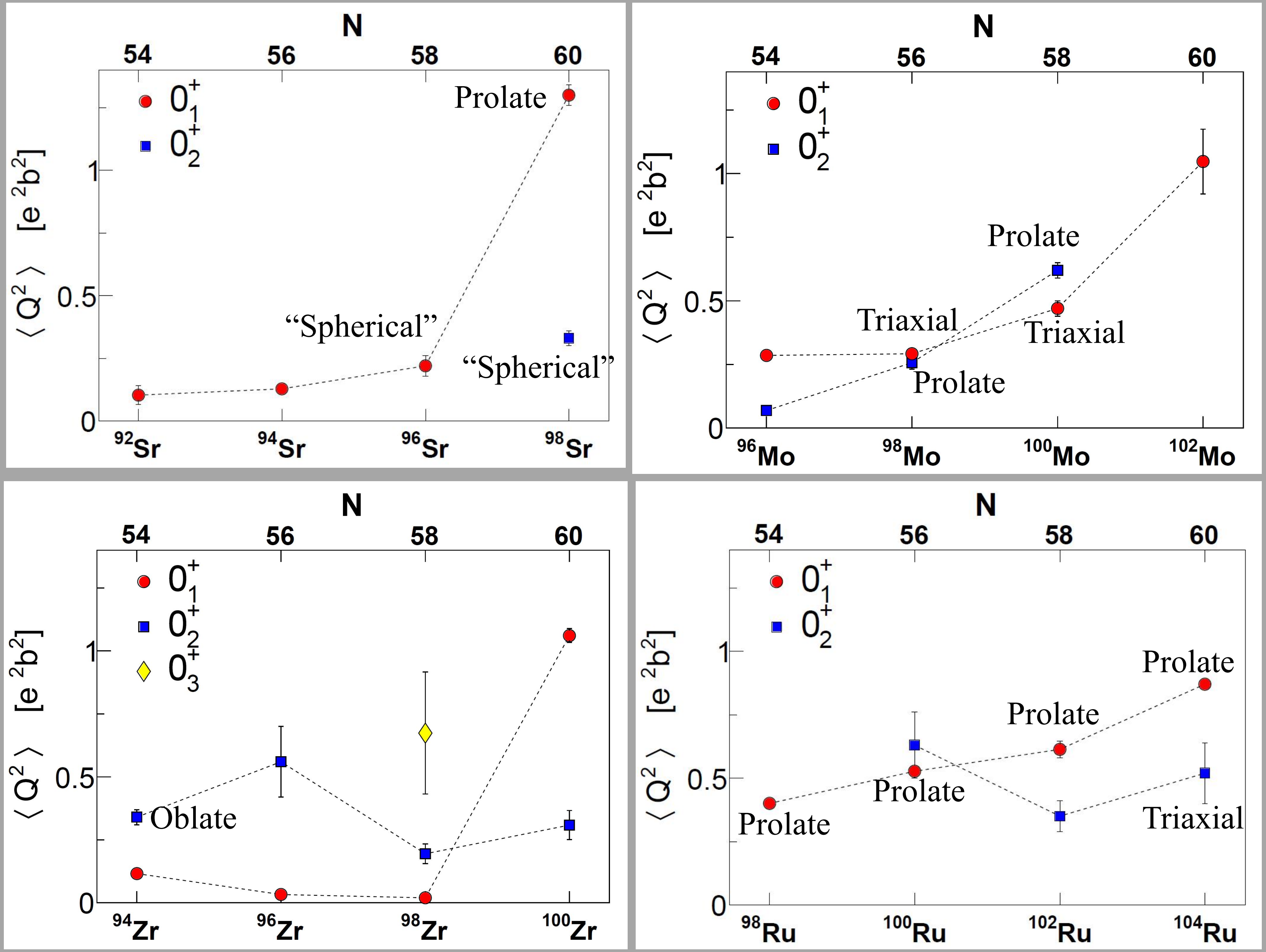


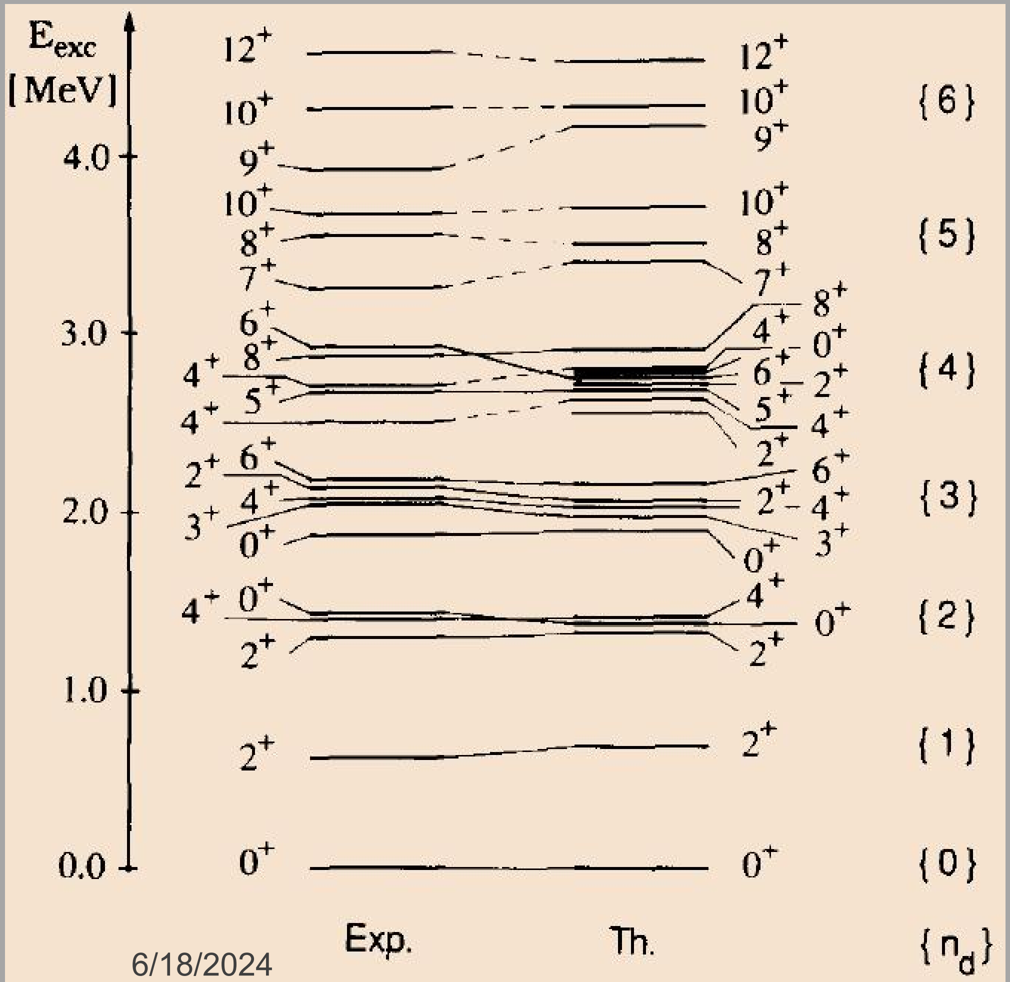
The “evolution” of the structure of the Cd isotopes

- From spherical vibrators….

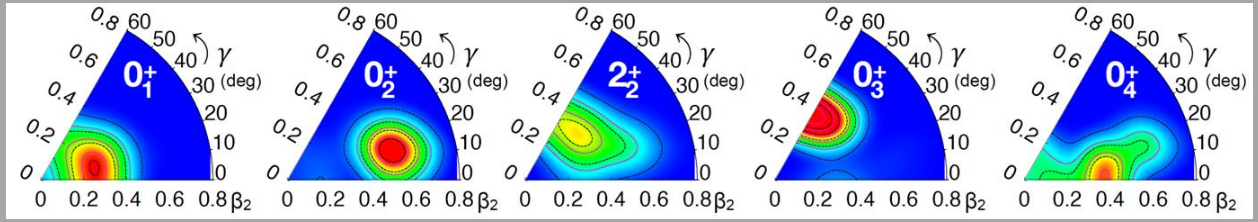
…to deformed with multiple shapes

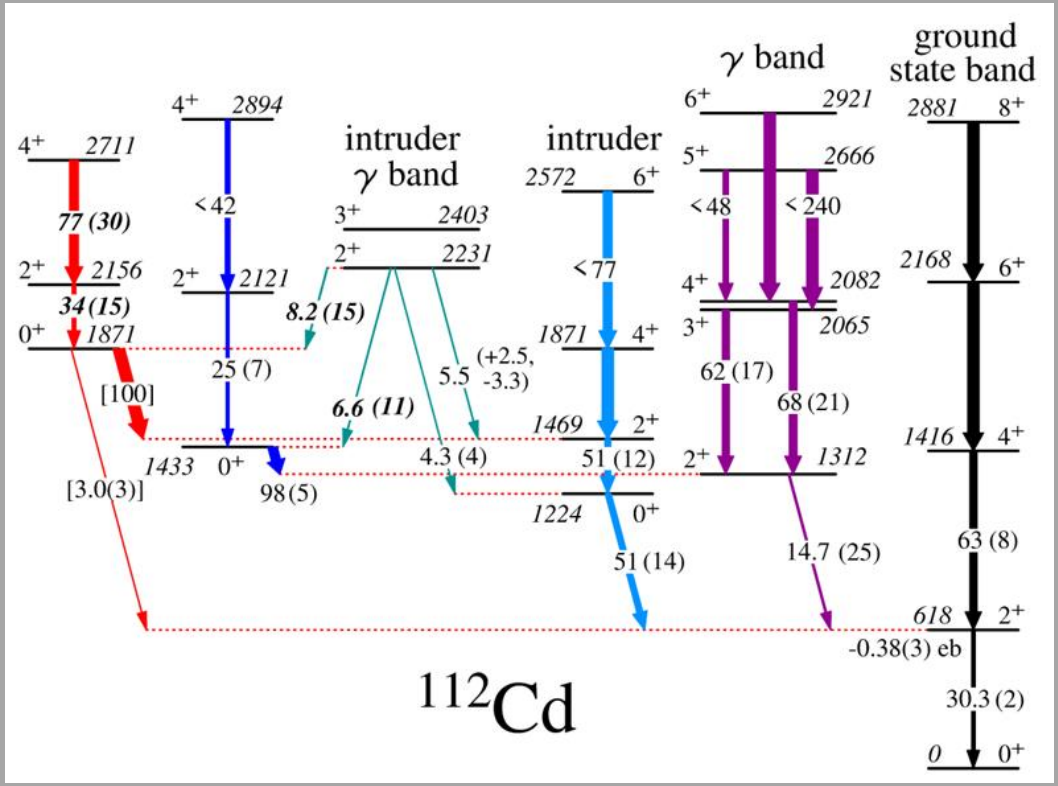
PG et al., PRL 123, 142502 (2019)

Deleze et al., NPA 544, 1 (1993)



Proposed states with phonon number up to N=6



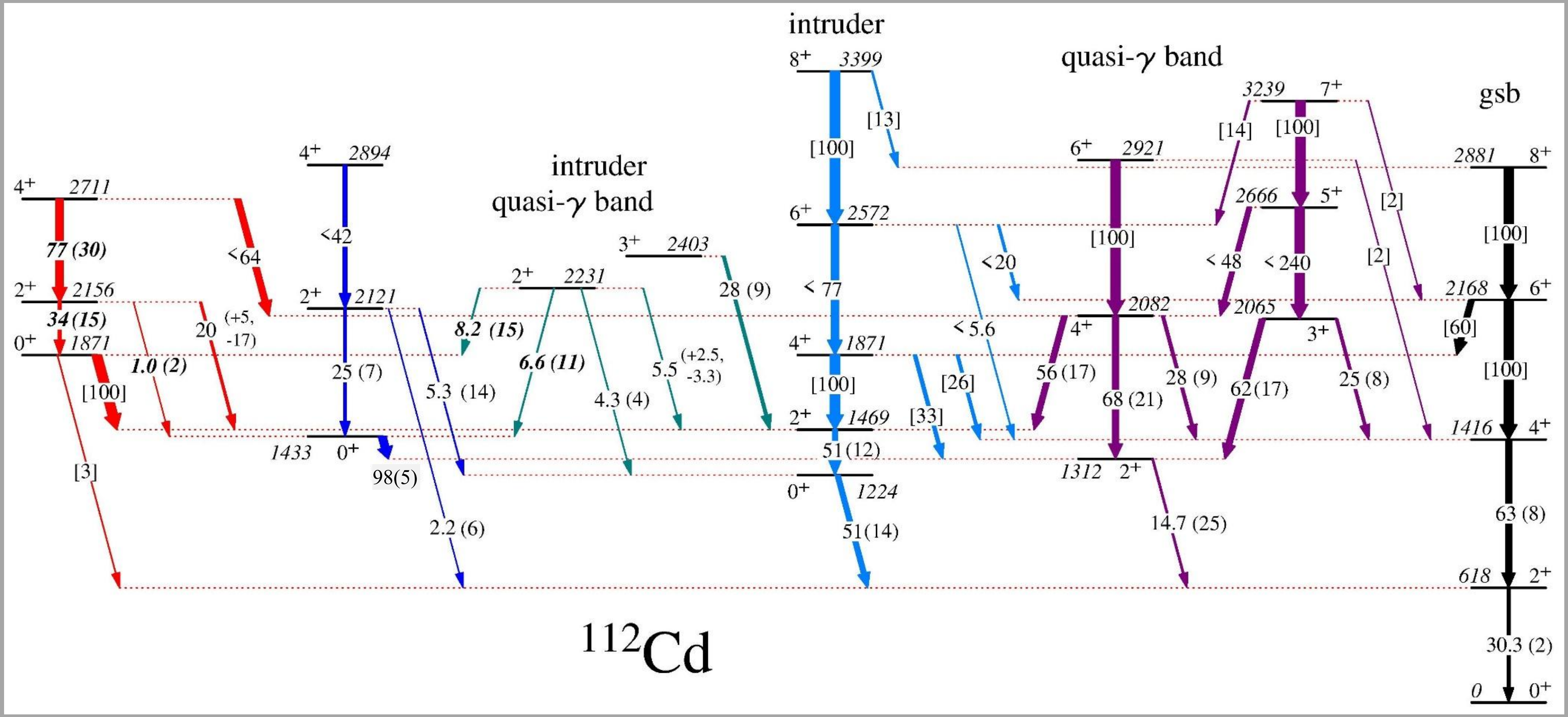


Proposed four distinct shapes for the lowest four 0^+ states

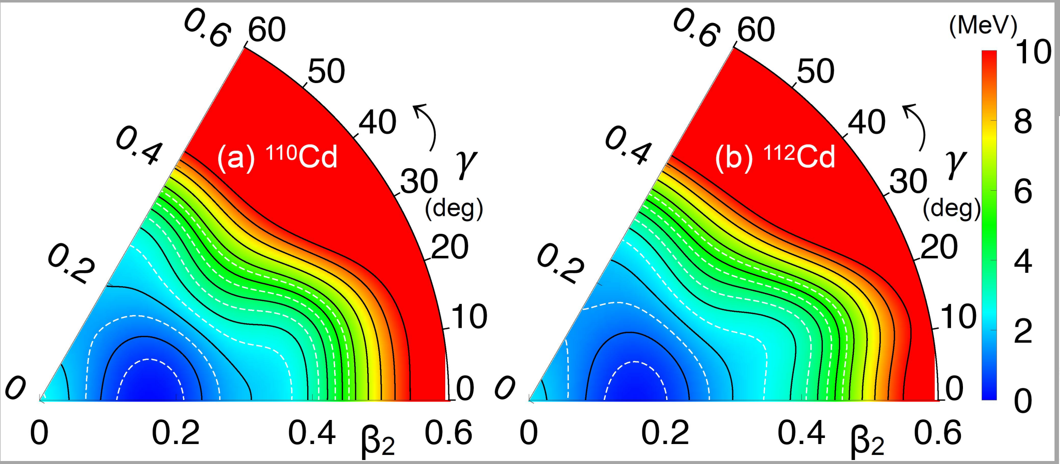
Detailed spectroscopy of Cd isotopes performed following β decay and (n,n′γ) reactions

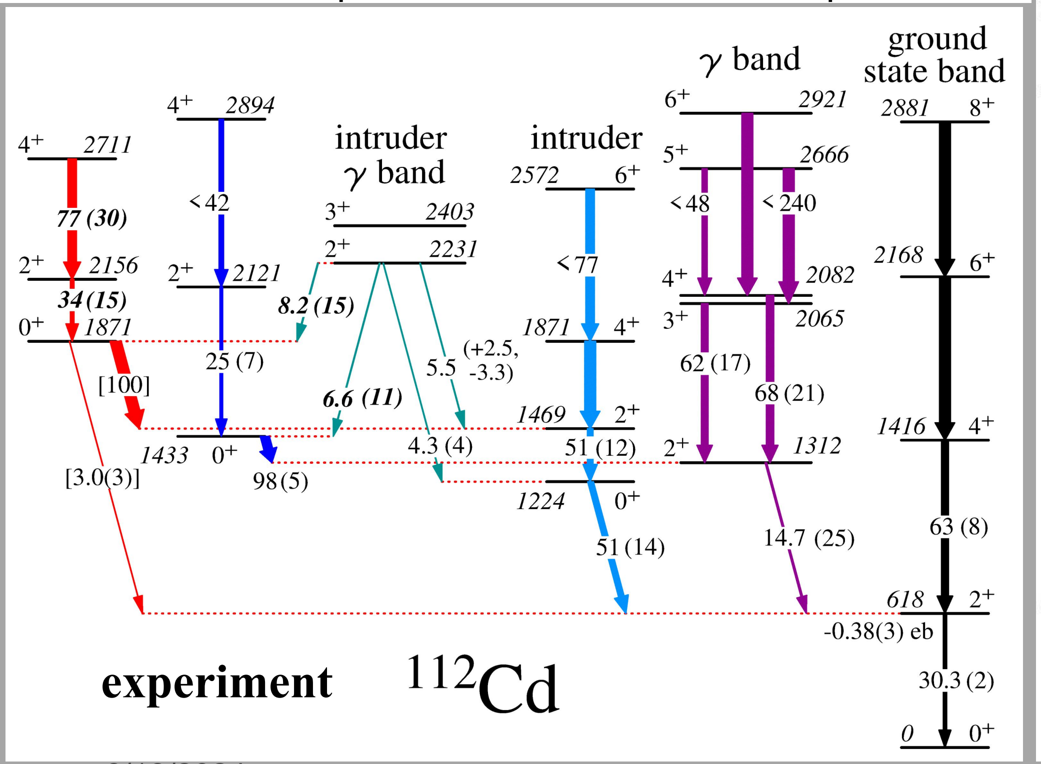
- Extensive expansion of decay scheme of these well-known and well-studied nuclei

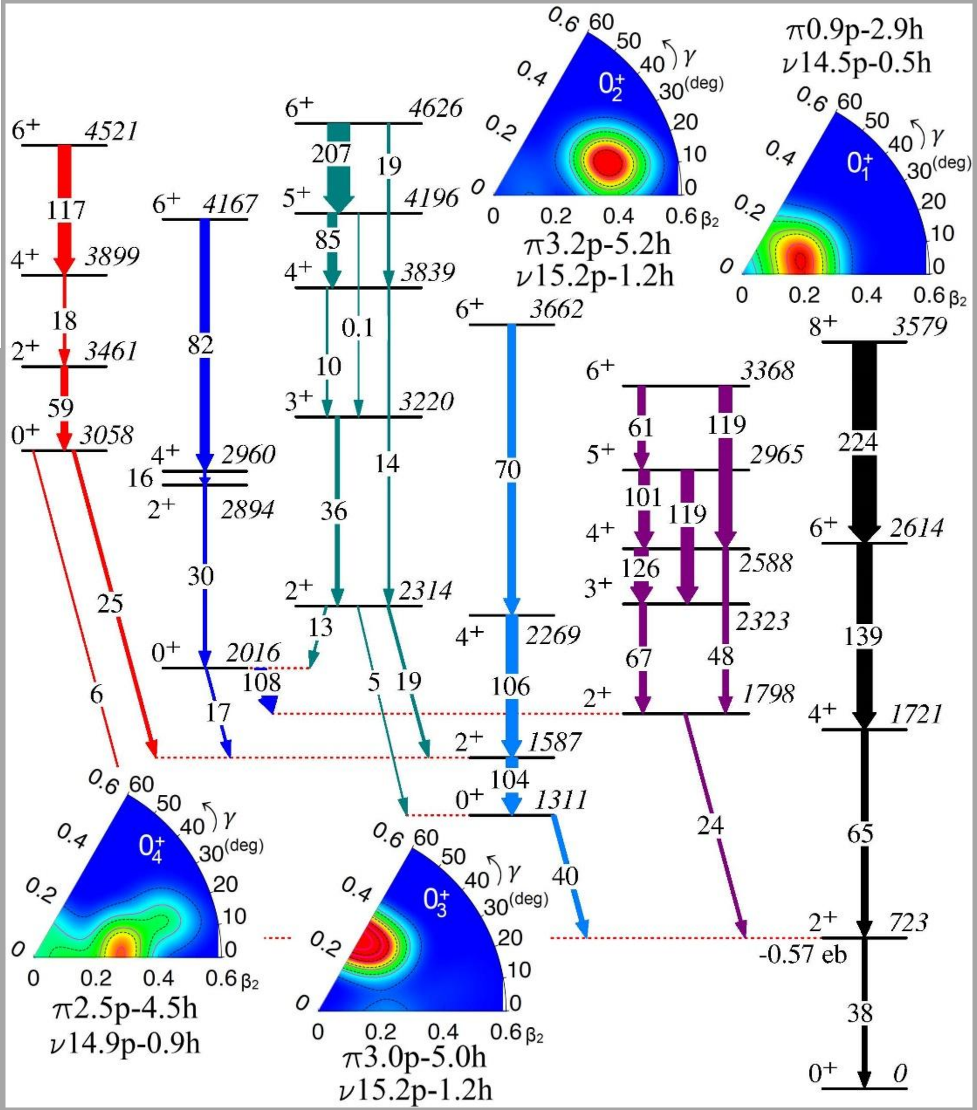
Transitions labelled with *B(E2)* in W.u.
Square brackets indicate relative *B(E2)* values
Very weak transitions removed



Four distinct shapes predicted for 0^+ bands in $^{110,112}Cd$





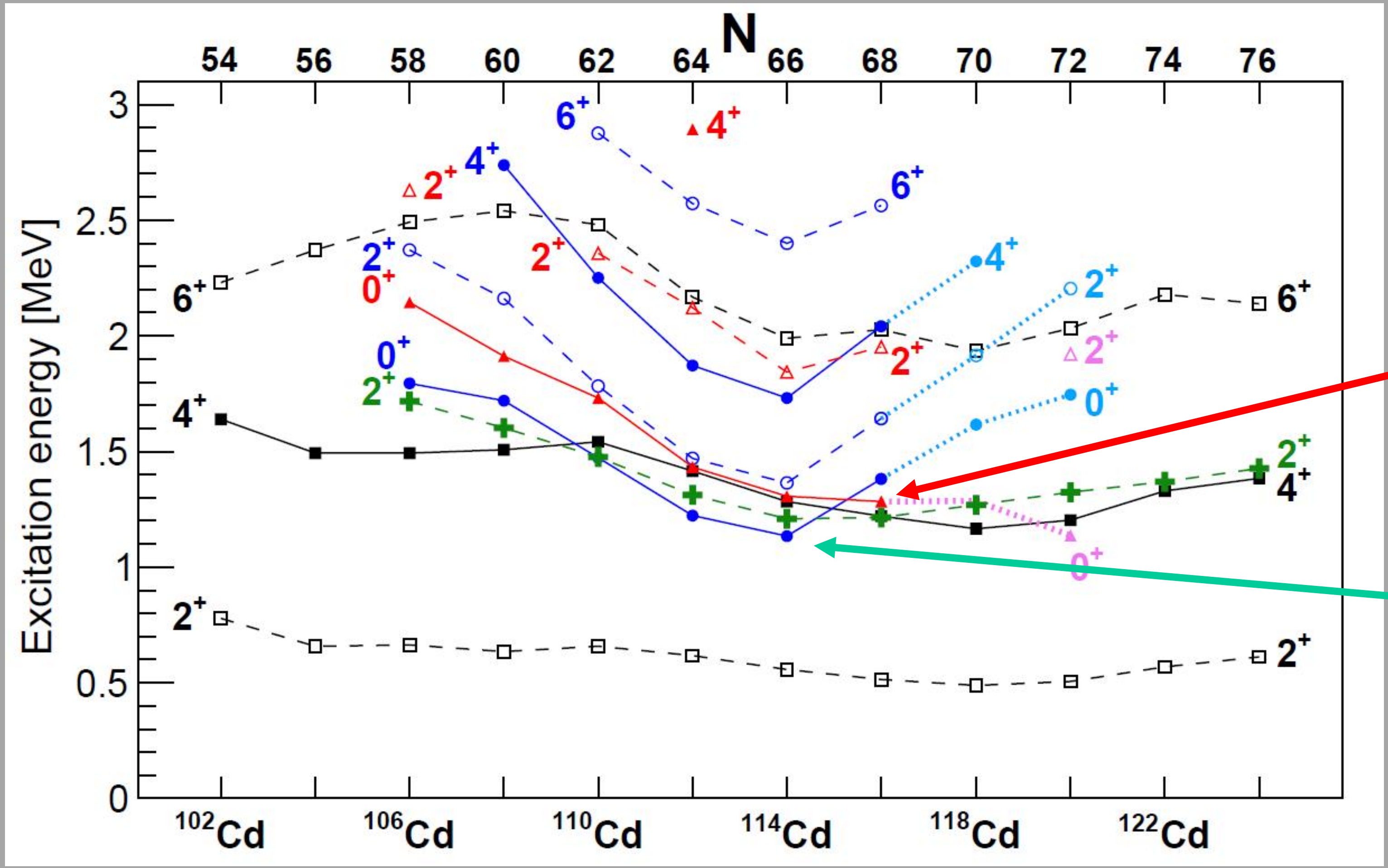


0_1^+ $\beta \approx 0.20$ prolate
0_2^+ $\beta \approx 0.4, \gamma \approx 20°$
0_3^+ $\beta \approx 0.30$ oblate
0_4^+ $\beta \approx 0.25$ prolate

Favoured decays of $0_2^+ \rightarrow 2_1^+$, $0_3^+ \rightarrow 2_2^+$, $0_4^+ \rightarrow 2_3^+$ reproduced

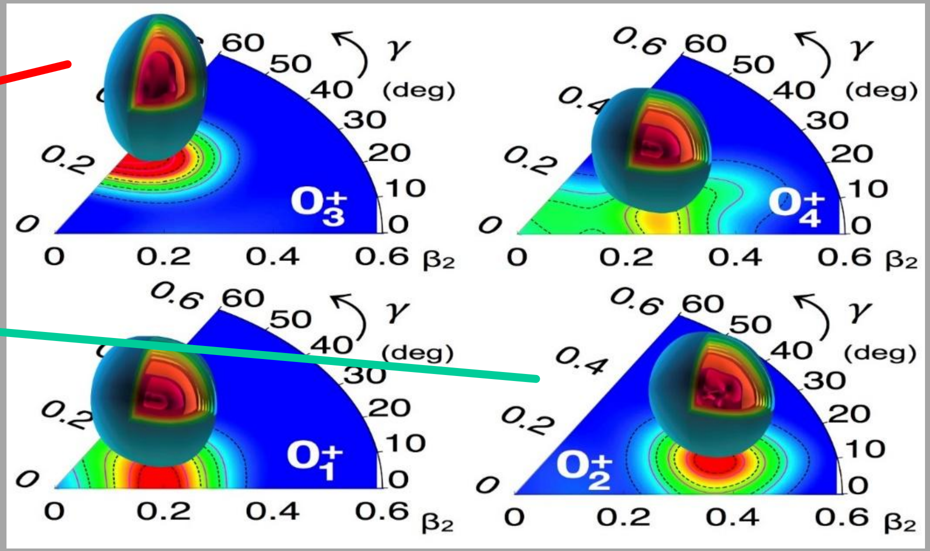
P. Garrett, T. Rodriguez, et al., PRL 123 (2019) 142502, PRC 101, 044302 (2020)

Energy systematics Cd isotopes



"Blue" states – the original "intruder" band, triaxial shape in ^{110,112}Cd from BMF calculations

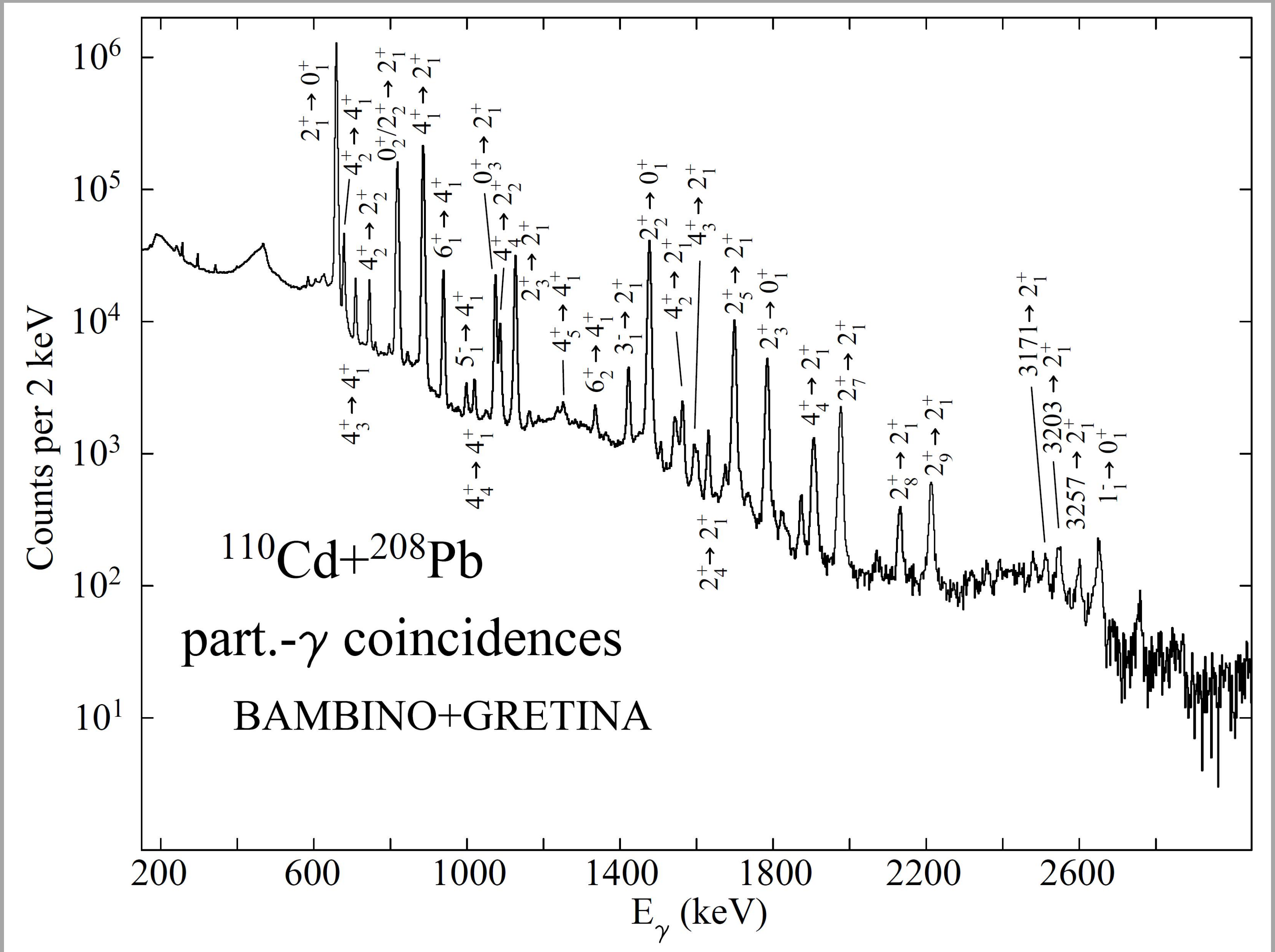
"Red" states – oblate band in ^{110,112}Cd from BMF calculations



The presumed shapes are based on systematics and similarities of decay properties – but become increasingly uncertain towards the neutron rich isotopes

Coulomb excitation studies of ^{110}Cd

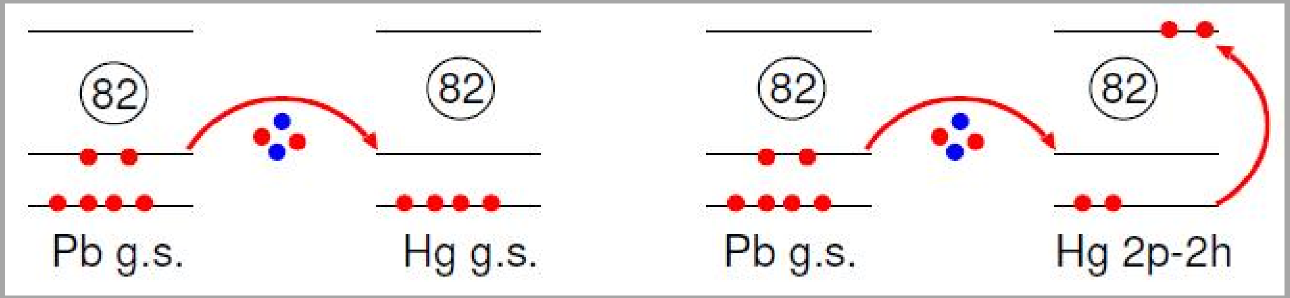
- **Aim to provide definitive results regarding shapes of 0^+ states**
  - **^{14}N/^{32}S + ^{110}Cd using EAGLE array at HIL Warsaw**
  - **^{60}Ni + ^{110}Cd using AGATA at Legnaro**
  - **^{110}Cd + ^{208}Pb using GRETINA at Argonne**
- **New spectroscopic studies following β decay using GRIFFIN array at TRIUMF-ISAC**



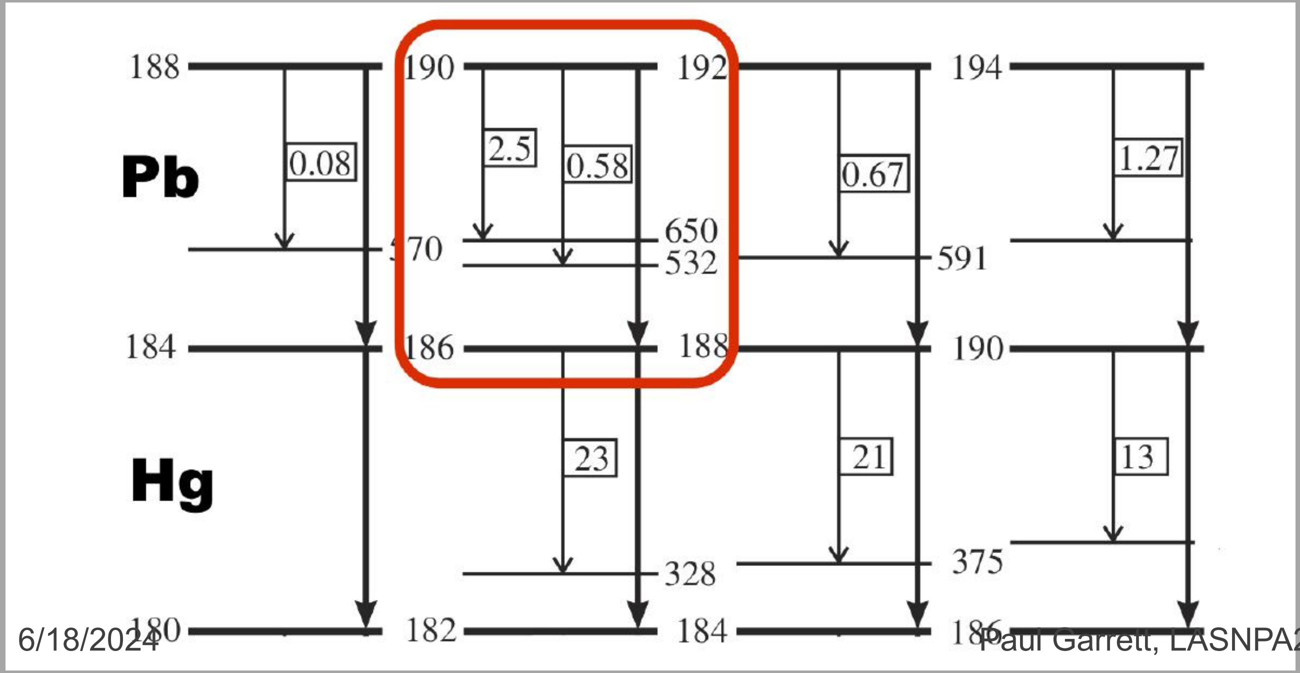
S. Pannu, Guelph

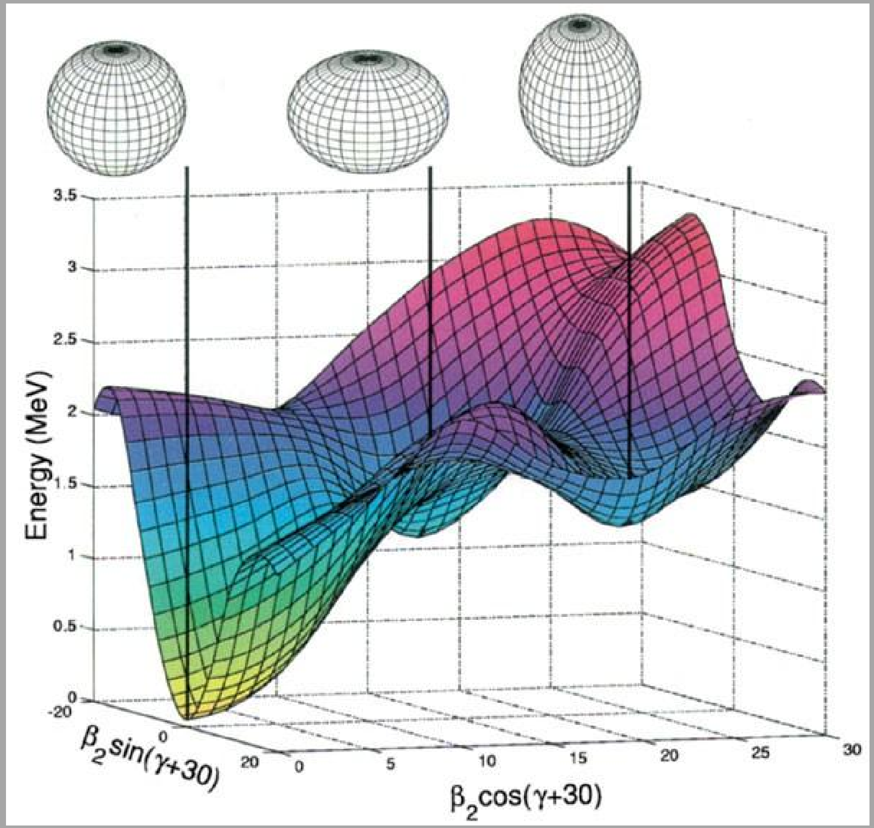
Triple shape coexistence in ^{186}Pb

- ^{186}Pb *the* famous example of multiple-shape coexistence



- α-decay similar to α-transfer to gain information on 2*p*-2*h* enhancements

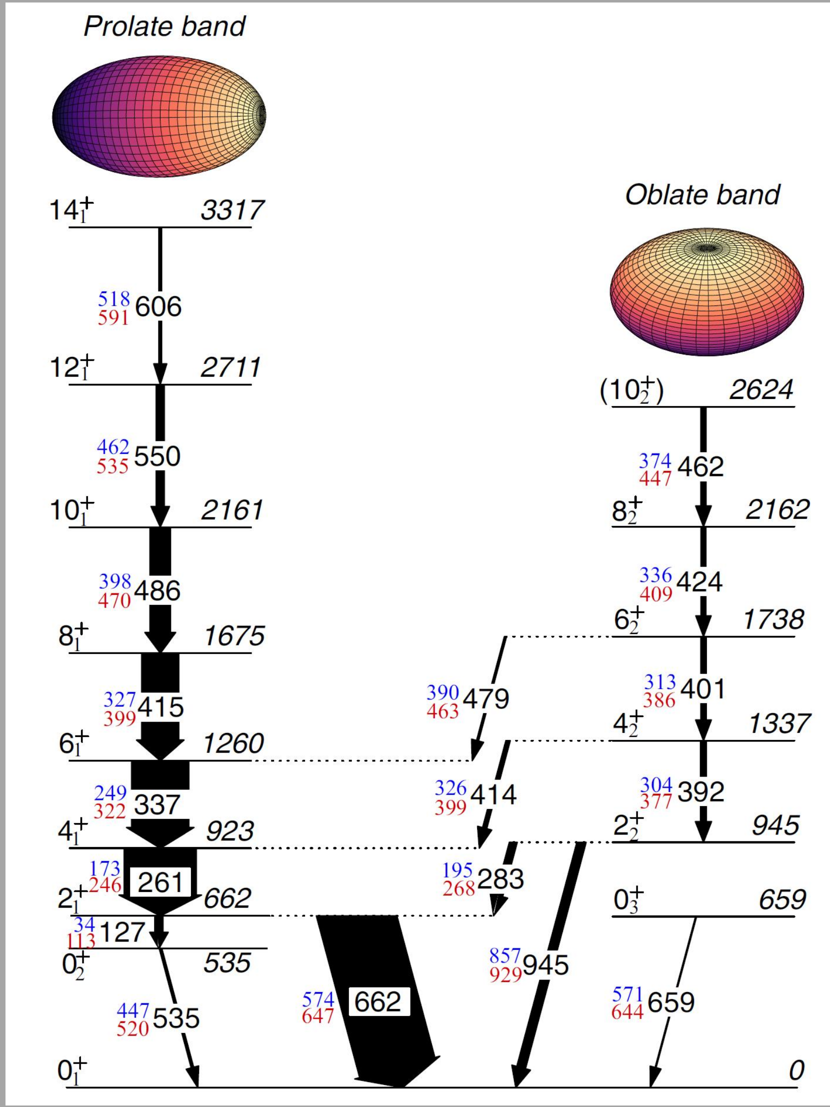




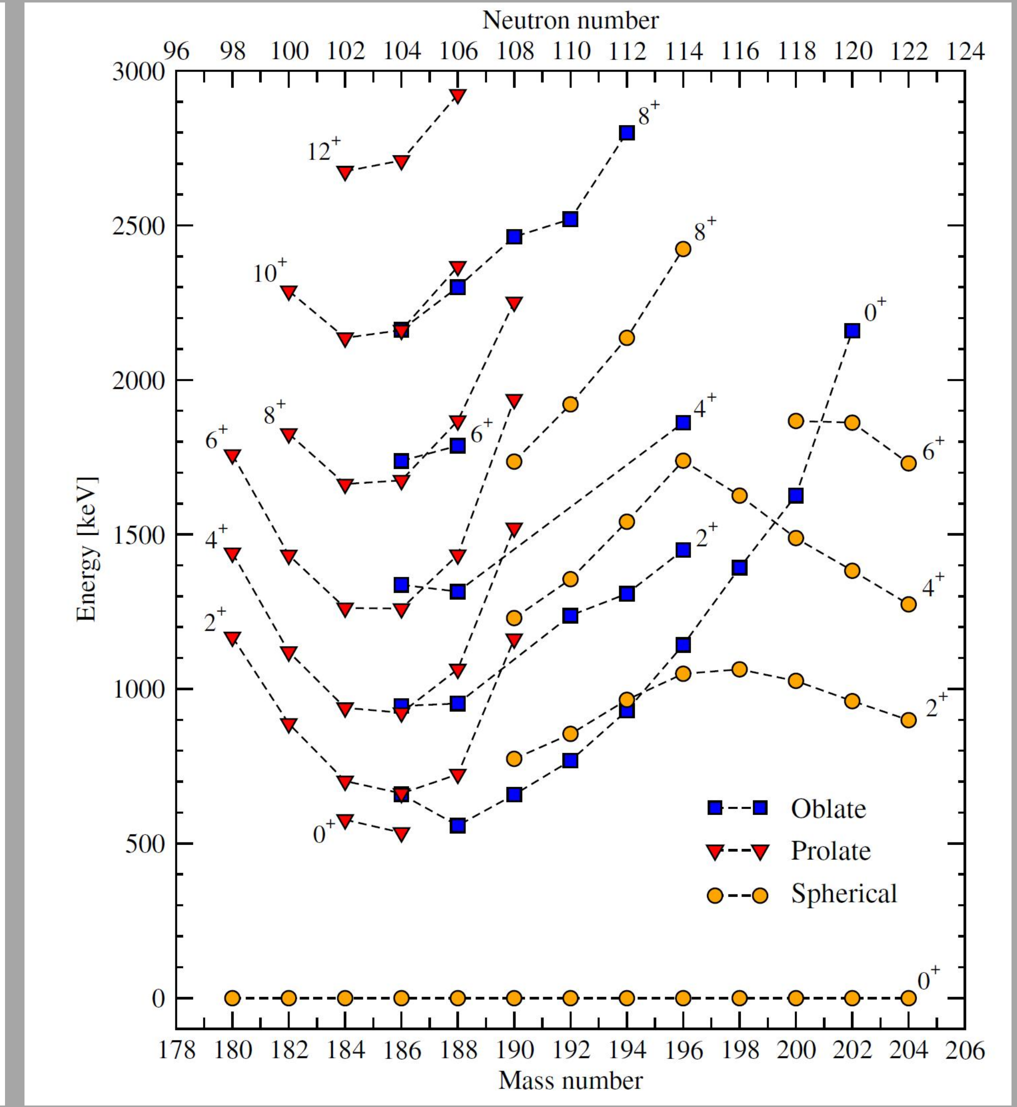
Andreyev et al., Nature **405**, 430 (2000).

- **A spectroscopic *tour de force*, using γγ and γe⁻ spectroscopy with recoil decay tagging at Jyvaskyla, observed the weak, in-band $2_1^+ \rightarrow 0_2^+$ transition establishing the 0_2^+ state as the head of the prolate band with $B(E2; 2_1^+ \rightarrow 0_2^+)$ = 190(80) W.u.**

- **Results indicated small mixing of 0_2^+ and 0_3^+ states**



Ojala et al, Nature Communications **5**, 213 (2022)



Summary

- **Shape coexistence, once thought to be a rare and unique phenomena, is now believed to be pervasive throughout the nuclear chart**
- **Evidence beginning to emerge for multiple-shapes coexisting in nuclei**
  - **in light mass nuclei**
  - **in nuclei near closed shells**
  - **in regions where 2 shapes that coexist have been well established**
- **Some of the suggested examples of nuclei possessing multiple shapes are stable or near stability and offer the possibility of detailed studies by a variety of probes**
- **Much work needs to be done to firmly establish these candidates**

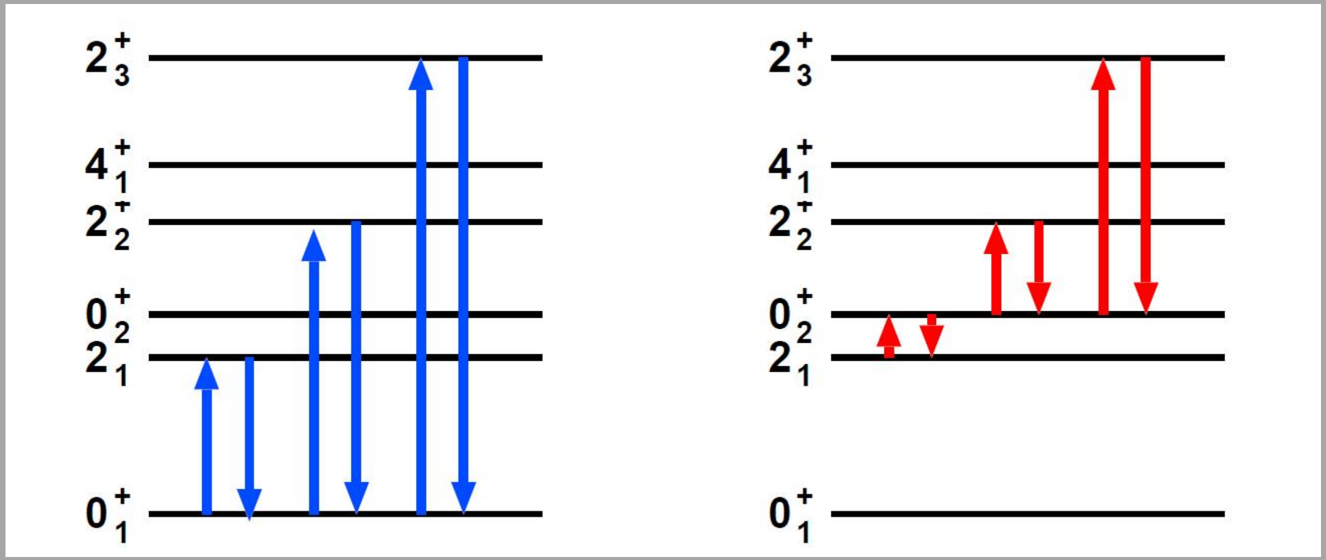
Collaborators (^{98,100}Y β-decay projects)

- Guelph: K. Mastakov (^{98}Zr), B. Olaizola (^{98}Zr), H. Bidaman (^{100}Zr), V. Bildstein, Z.Ahmed, S. Buck, C. Burbadge, H. Dawkins, G. Demand, G. Deng, A. Diaz Varela, R. Coleman, B. Greaves, B. Hadinia, S.Pannu, A. Radich, E. Rand, C. Svensson, T. Zidar
- CEA Saclay: D. Kalaydjieva (^{100}Zr analysis), M. Zielińska (^{100}Zr analysis), W. Korten
- TRIUMF: V. Vedia, A.Garnsworthy, D. Annen, G. Ball, S. Devinyak, I. Dillmann, R. Caballero-Folch, E. Fuakye, F. Garcia, C. Griffin, G. Hackman, M. Moukaddam, J. Park, M. Rajabali, D. Torres, R. Umashankar, Z. Wang
- Simon Fraser University: A. Chester, D. Cross, P. Spagnoletti, C. Andreoiu, K. Ortner, U. Rizwan, P. Voss
- University of Regina: G. Grinyer, K. Kapoor, N. Saei
- INFN Florence: M. Rocchini, N. Marchini, A. Nannini
- Georgia Tech: J. Wood
- University of Kentucky: E. Peters, S. Yates

Deformation in terms of rotational invariant quantities – Kumar-Cline sum rules for <Q^2> invariant

- $E2$ operator is a rank-2 spherical tensor – products coupled to zero are rotationally invariant
- Coupling scheme for $< Q^2 >$ for, e.g., for 0^+ states

$$\frac{1}{\sqrt{5}}\langle Q^2\rangle = \sum_i \langle 0\|M(E2)\|2_i\rangle\langle 2_i\|M(E2)\|0\rangle \begin{Bmatrix} 2 & 2 & 0 \\ 0 & 0 & 2 \end{Bmatrix}$$



- For 0^+ states, <Q^2> reduces to $\Sigma_i B(E2;0^+ \rightarrow 2_i^+)$

$$\langle Q^2\rangle = q_0^2\langle\beta^2\rangle \qquad q_0 = \frac{3}{4\pi}ZR_0^2$$

- Products of three MEs extract $< \cos 3\delta >$ – axiality of shape distribution

What are the experimental signatures of shape coexistence?

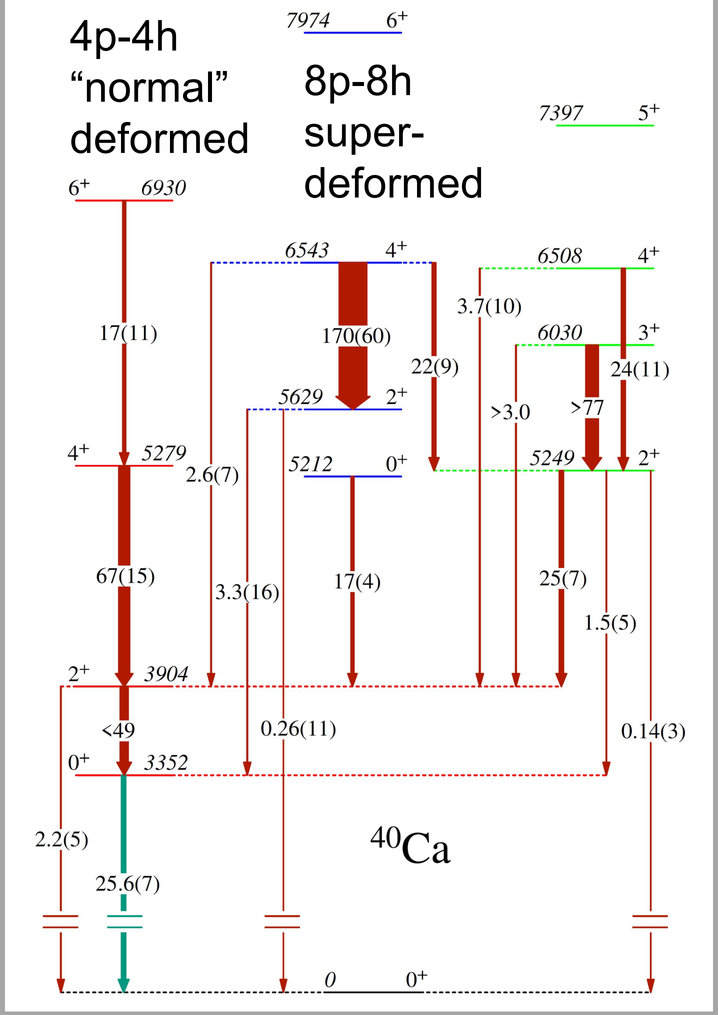
- Level energies
  - Appearance of "unexpected" levels at low energies, e.g., low-lying 0+ states
  - Appearance of rotational bands, especially in a spherical nucleus
  - Inferred moments of inertia
- Transition rates – vastly different $B(E2)$ values within bands
- Transfer reaction cross sections – large enhancements of cross sections, especially to excited 0^+ states
- Quadrupole moments – measure of charge distribution revealing deformation
- Charge radii – directly measuring size of nuclear state
- Sets of EM transition matrix elements to form "invariant" quantities
- $E0$ transition strengths – enhancements to $E0$ transition rates require mixing of states with different deformations

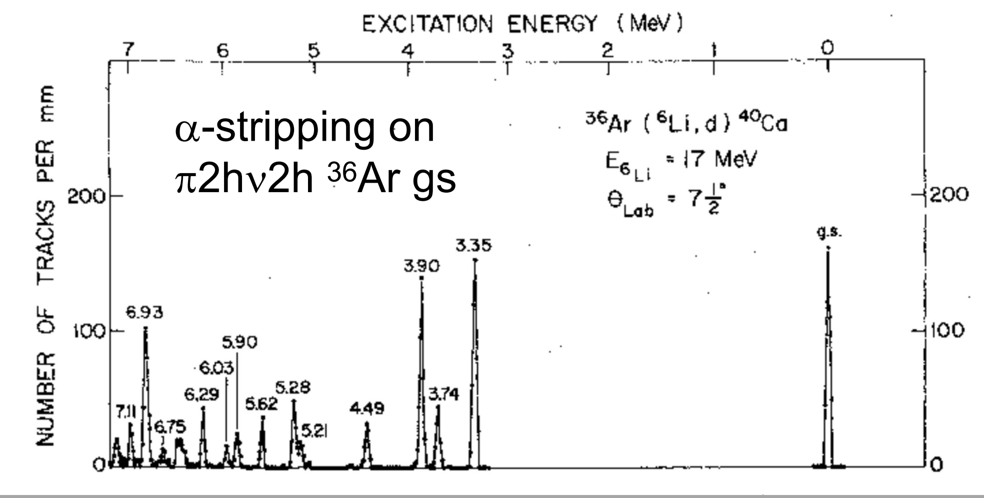
The case of ^{40}Ca – spherical, normal, and superdeformed bands

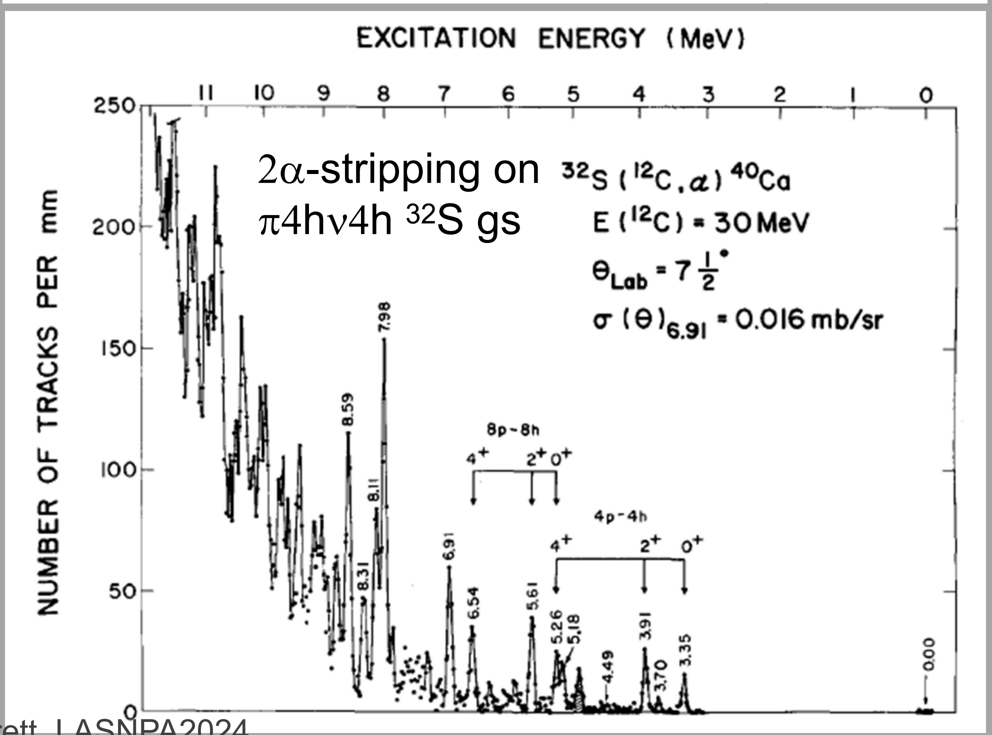
- **Microscopic nature of bands in ^{40}Ca revealed through multi-nucleon transfer**

B(E2) W.u.

$10^3\rho^2$(E0)







The case of ^{40}Ca – spherical, normal, and superdeformed bands

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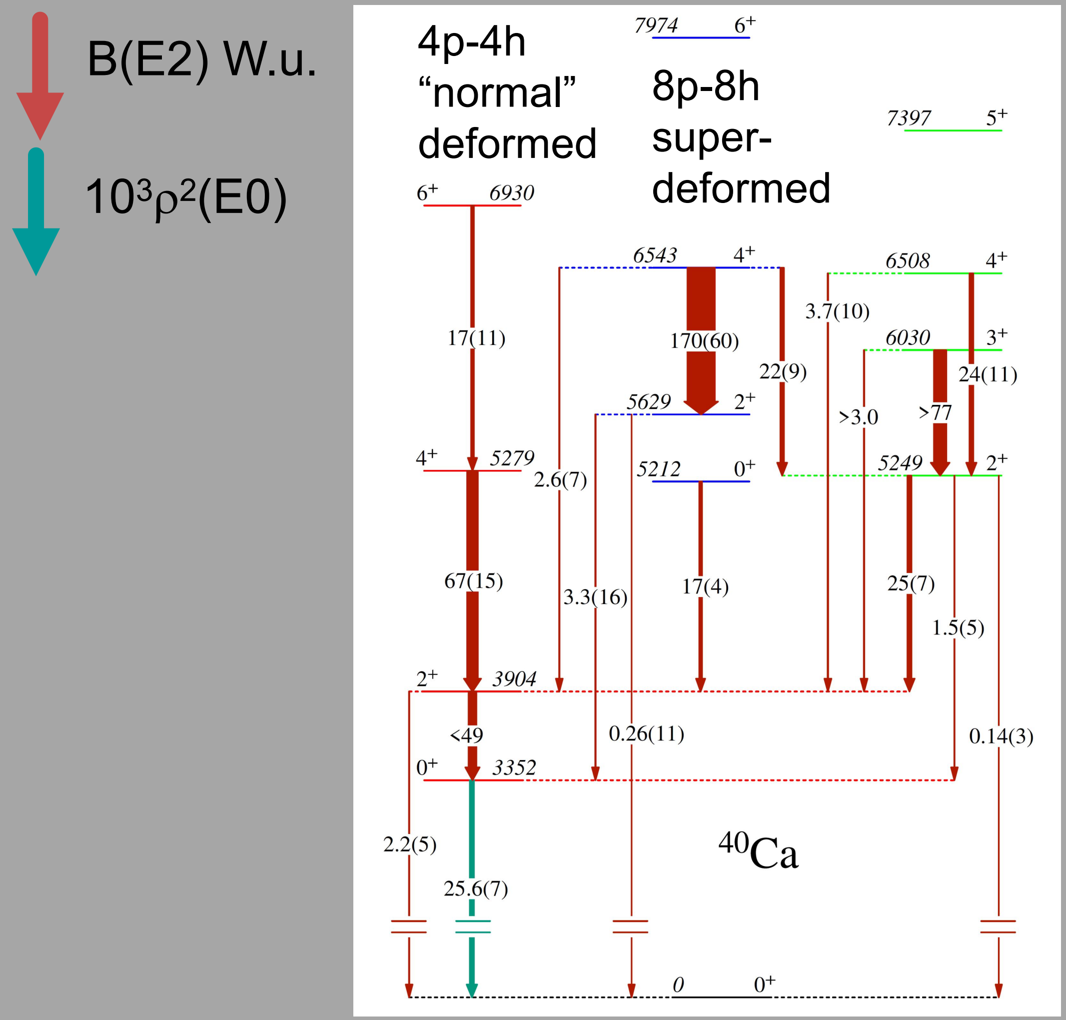

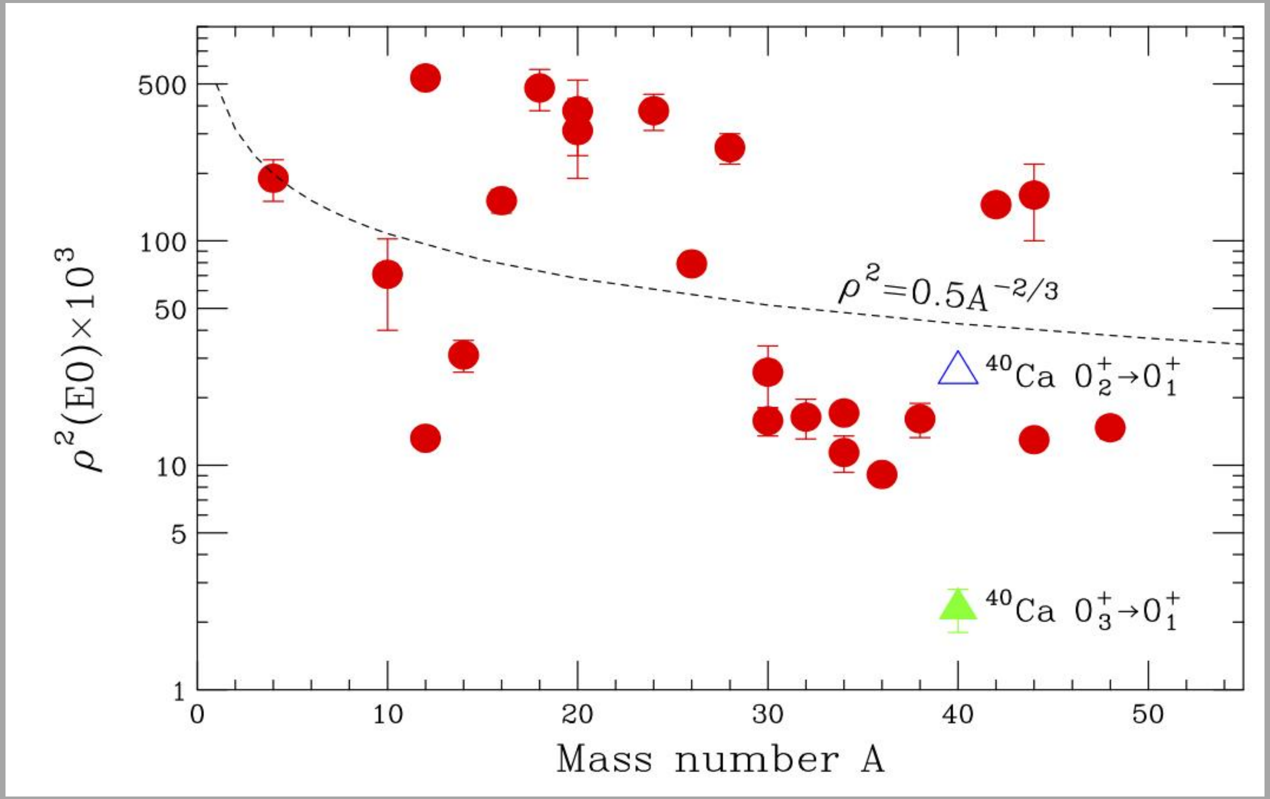

Recent work measured E0 transitions, and used 3-state mixing to explain the small $0_3^+\to0_2^+$ ρ^2(E0)

E. Ideguchi et al., PRL 128, 252501 (2022).

Our first measurement in the region – ^{94}Zr – performed with the 8π γ-ray spectrometer

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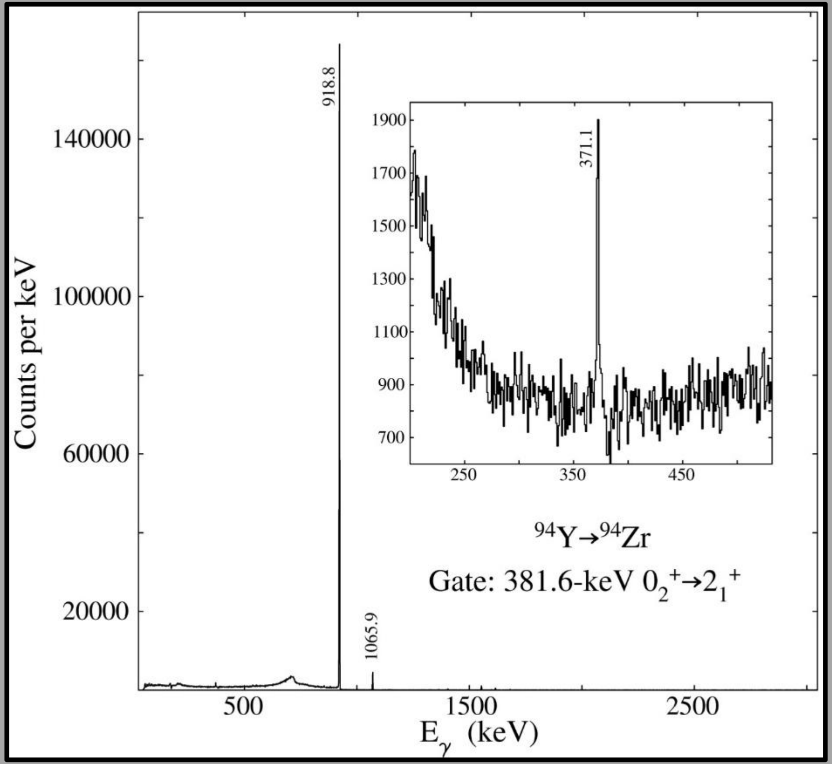
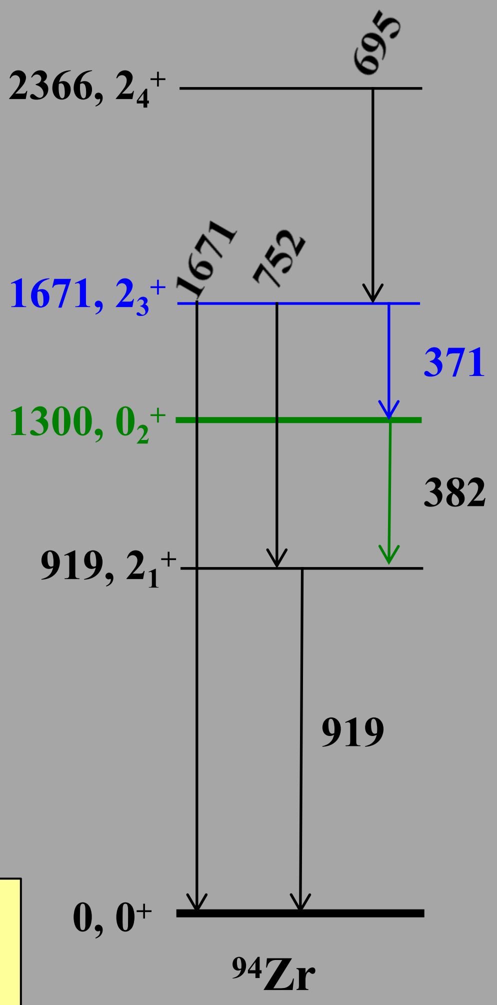
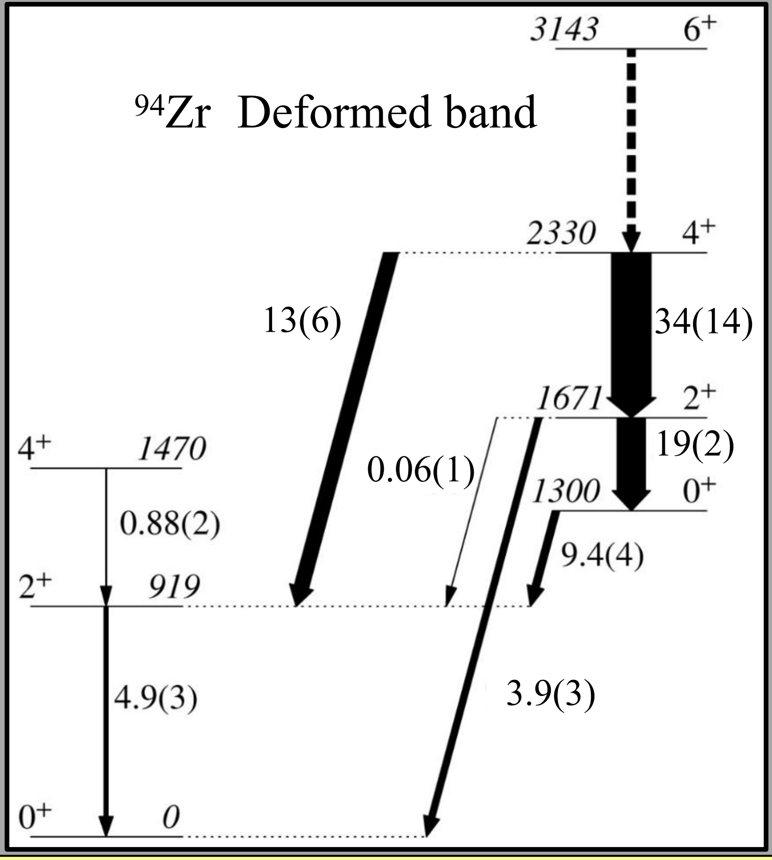
- Goal was to seek critical $2_3^+ \rightarrow 0_2^+$ transition to resolve conflicting interpretations for ^{94}Zr structure



Gate from below revealed new 371-keV transition – a 0.15% branch.





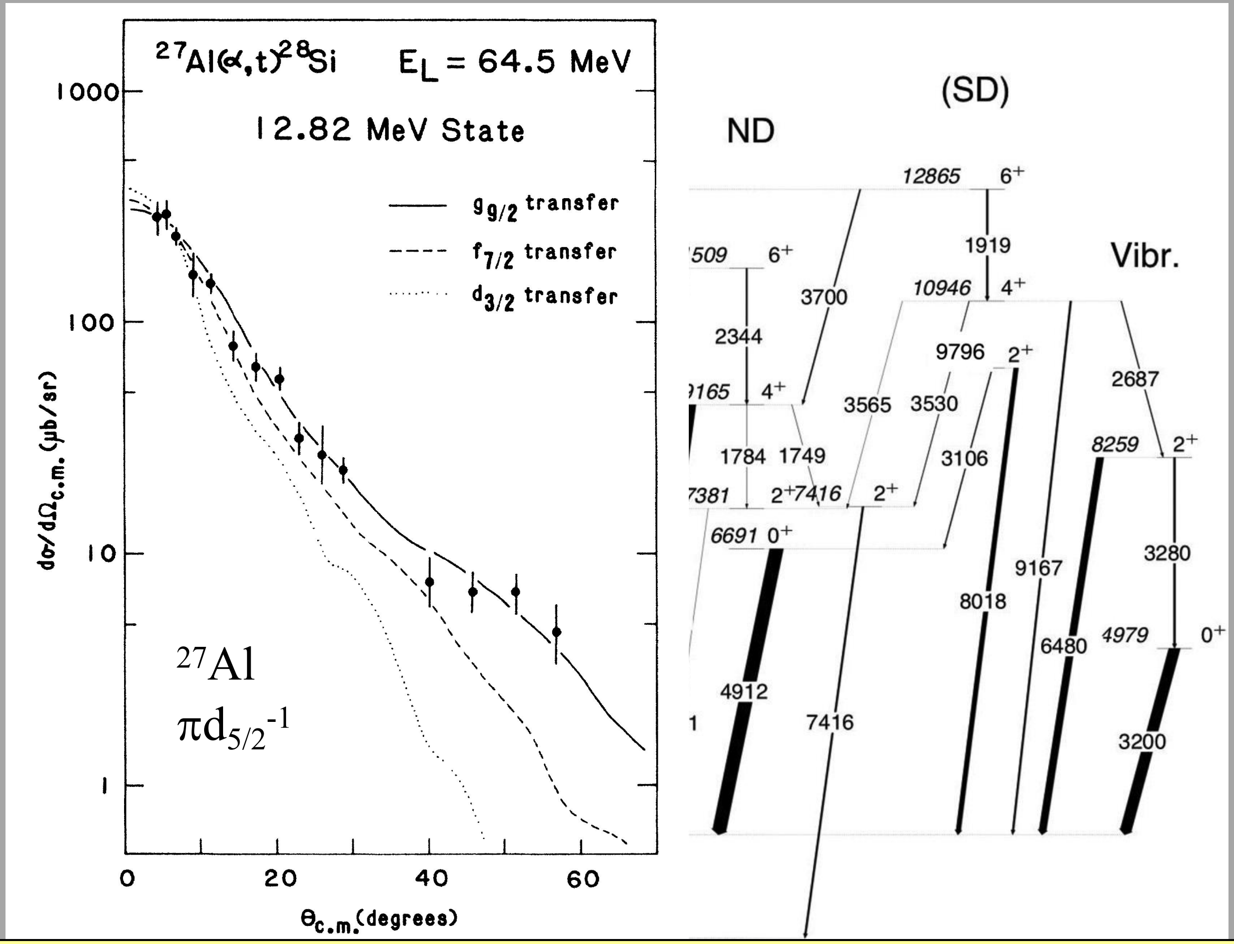
B(*E*2) values in W.u.

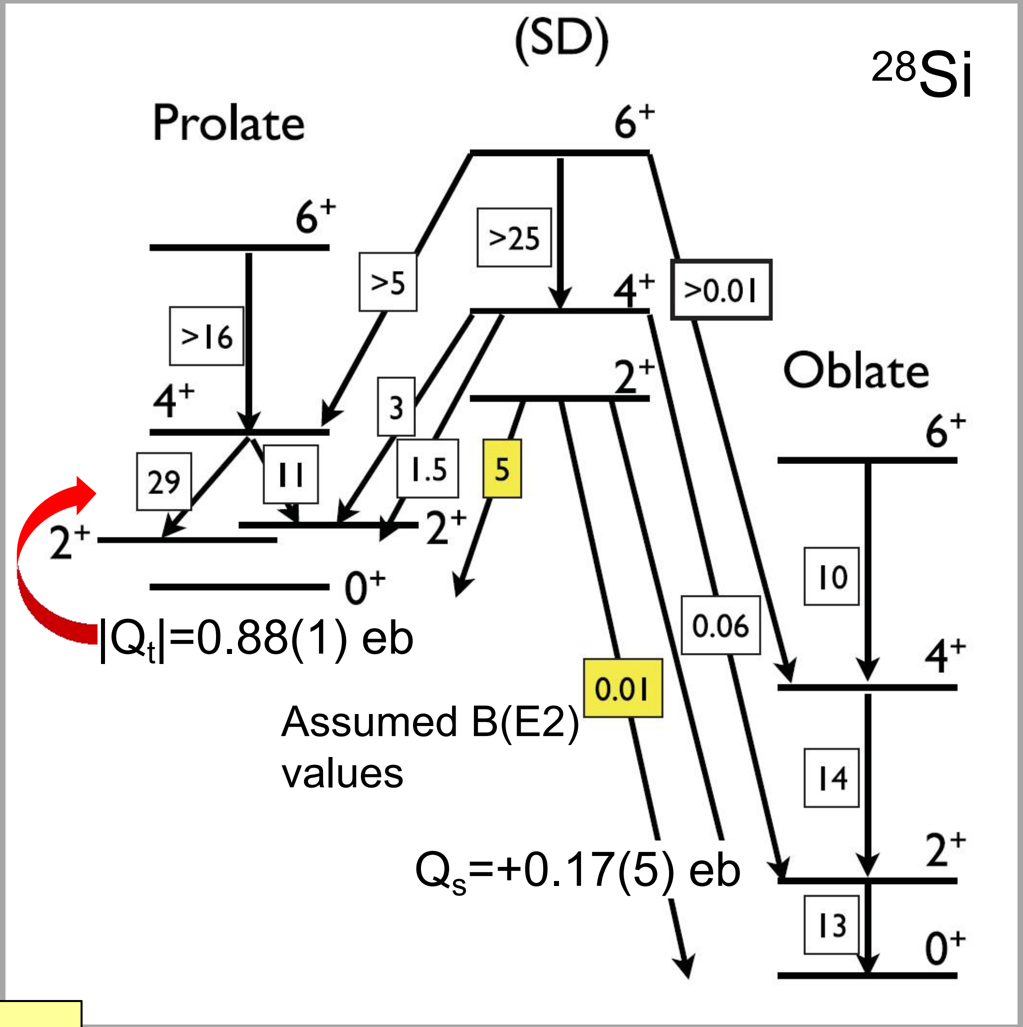
Branching ratio combined with lifetimes from (n,n′γ) reaction provided absolute *B*(*E*2) values, establishing the shape-coexisting band

A. Chakraborty, et al. PRL **110**, 022504 (2013)

Light nuclei with N~Z, e.g. ^{28}Si

- ^{28}Si studied by γ-ray spectroscopy following ^{12}C(^{20}Ne,α) reaction at ANL using GAMMASPHERE (D. Jenkins et al., PRC **86**, 064308 (2012))





Evidence for $\pi g_{9/2}$ component in 6$^+$ SD state (S. Kubono et al., PRC **33**, 1524 (1986))

Our approach: use β decay with the 8π (decommissioned) and GRIFFIN γ-ray spectrometers

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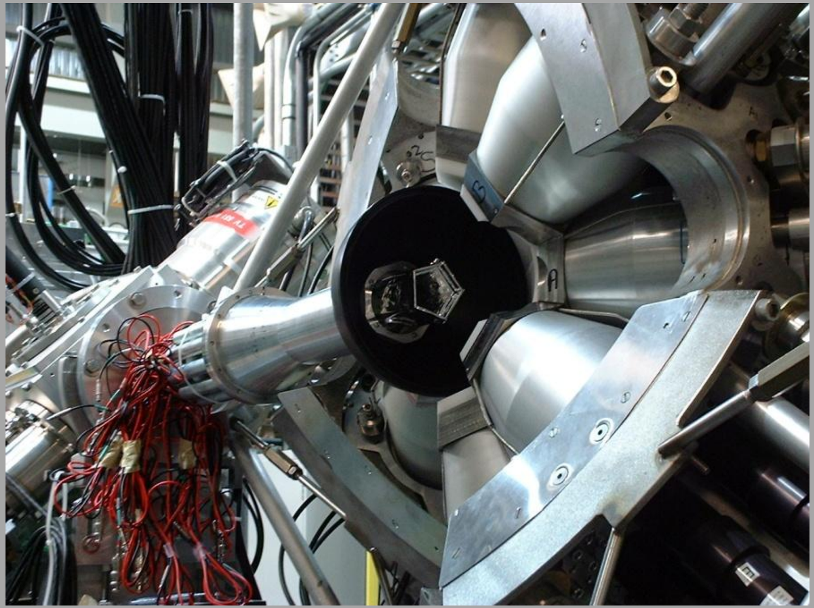
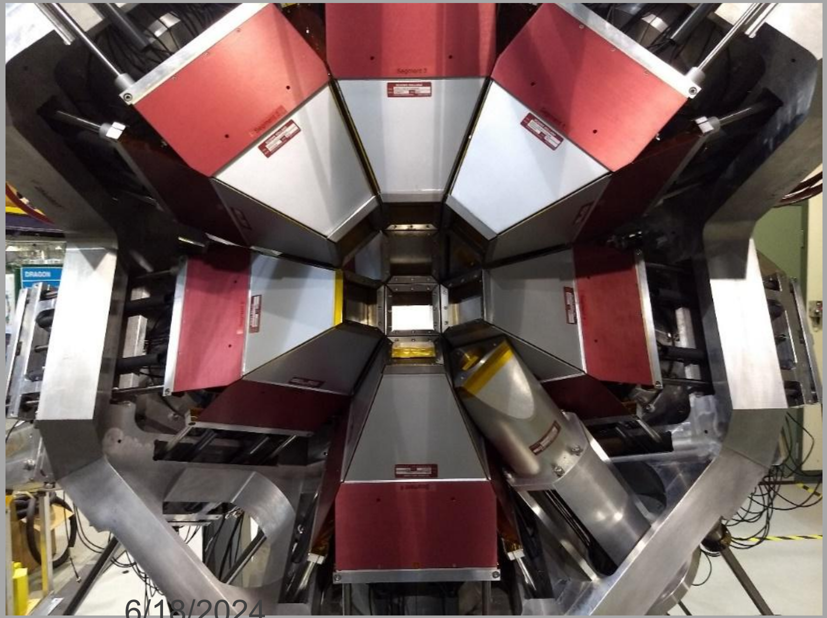
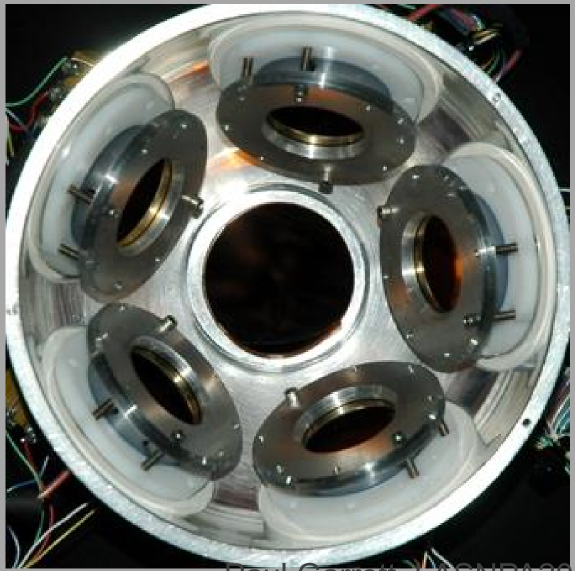
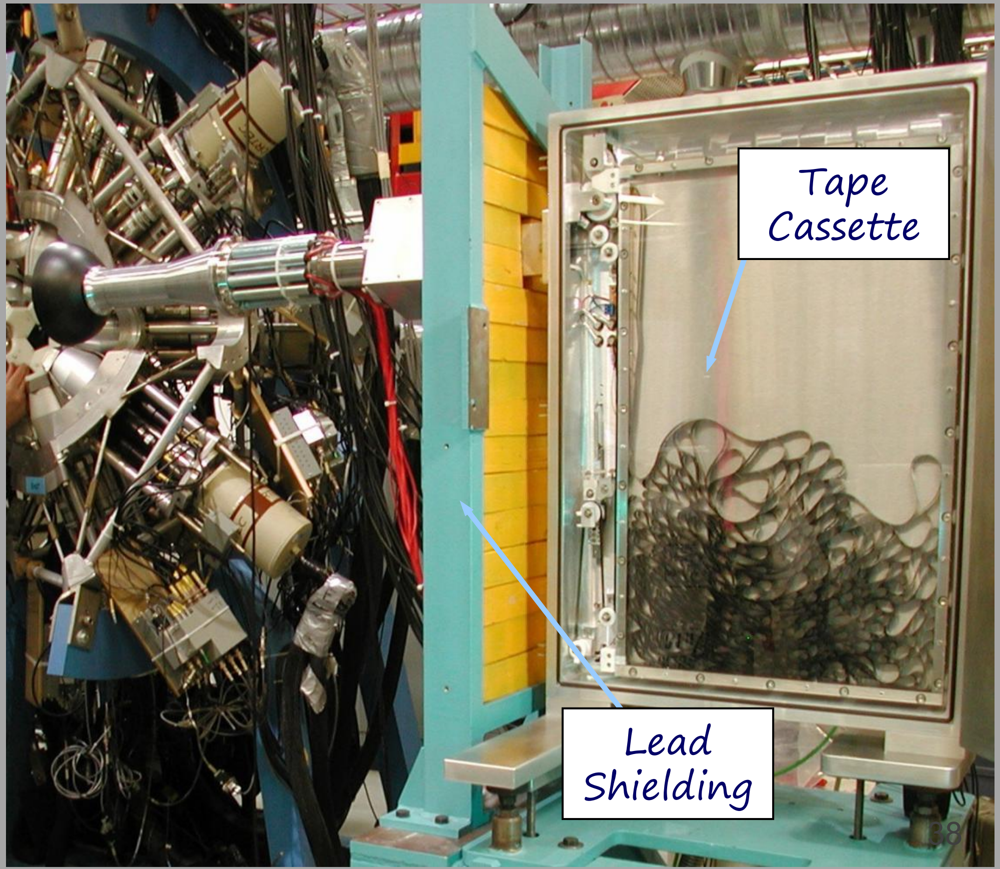


- 8π had 20 HPGe detectors with ~ 1% photopeak efficiency @ 1.3 MeV
- GRIFFIN has 16 large volume clover detectors with ~10% efficiency @1.3 MeV
- Radioactive beam implanted onto a moving tape at center of arrays
  - Allows for movement of long-lived activity out of focus of spectrometer
- 5 Si(Li) detectors for conversion electrons
- 8 $LaBr_3$ detectors, with BGO shields, for fast timing measurements

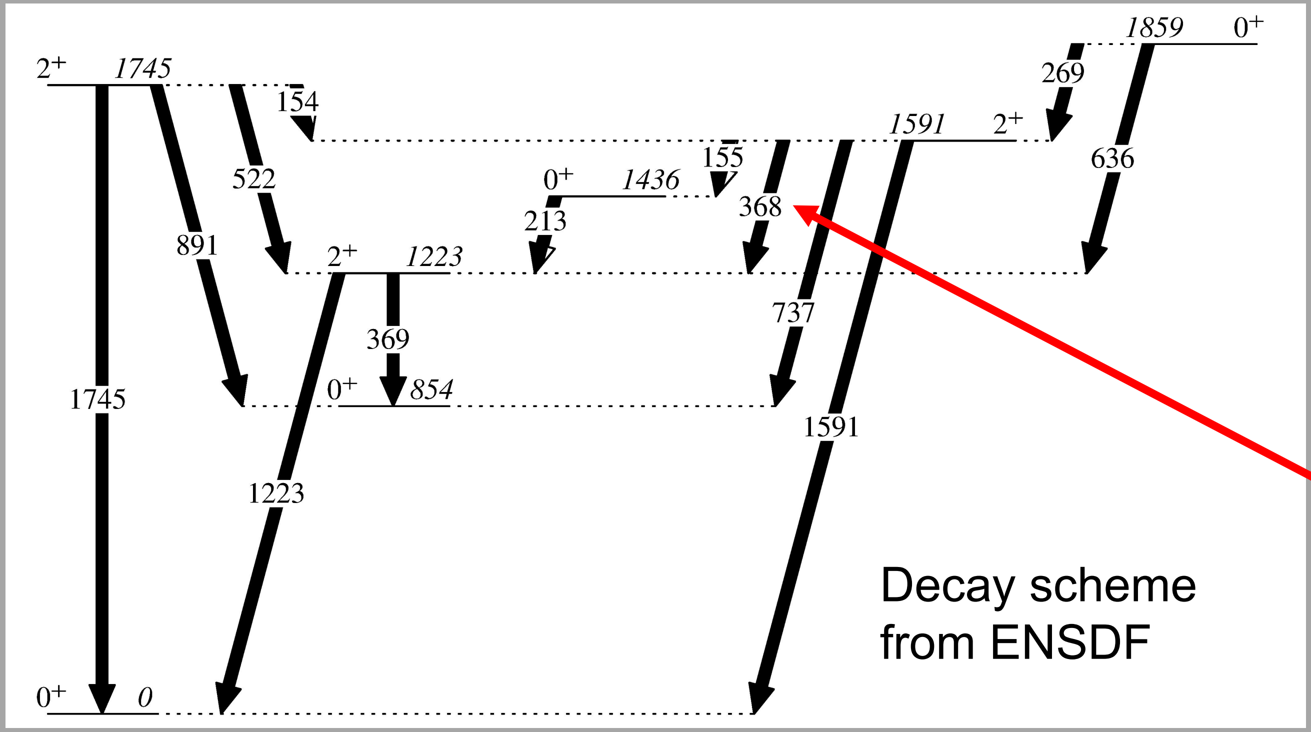


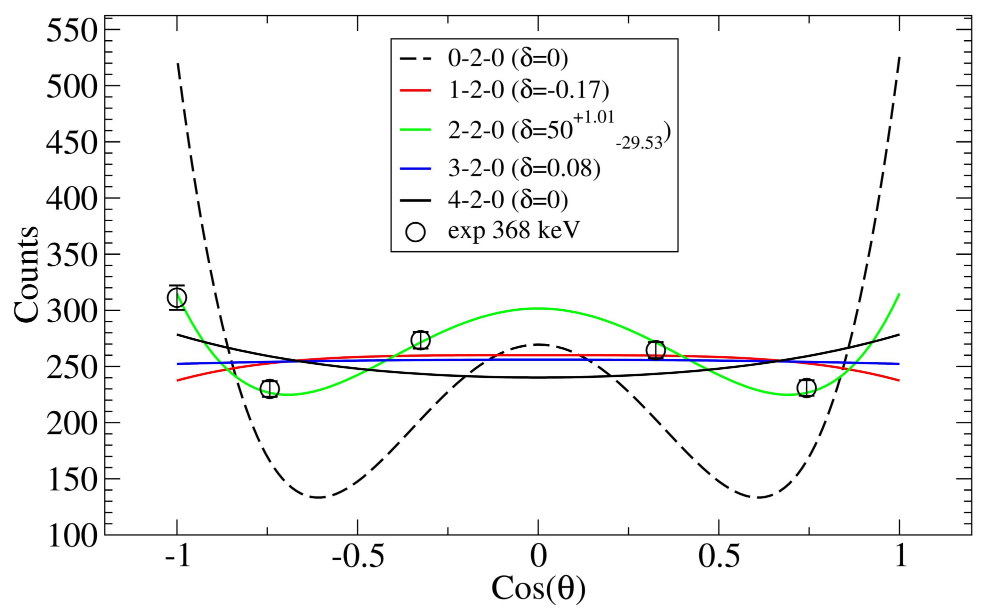


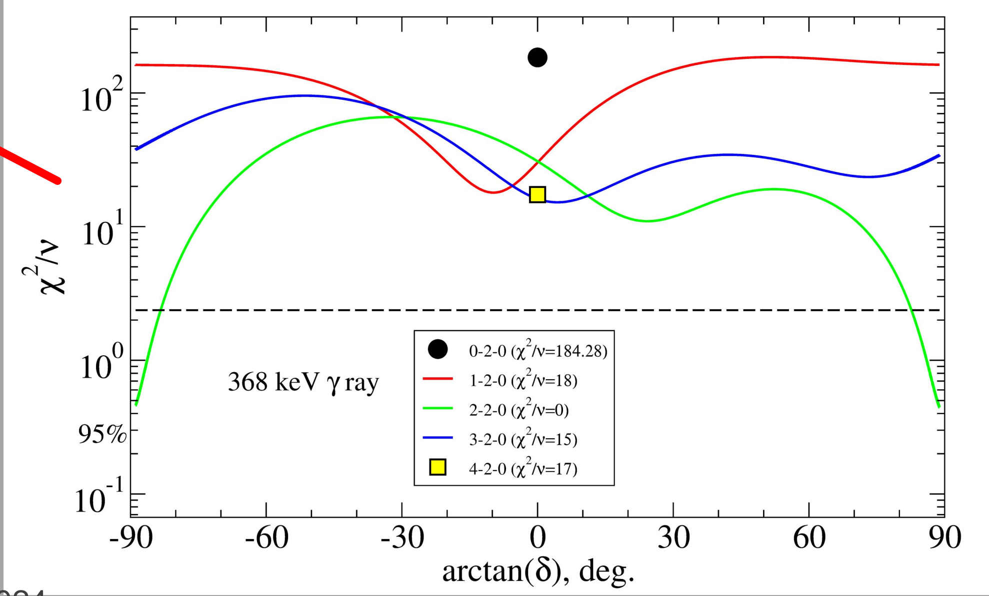


New results from β decay of ^{98}Y at TRIUMF-ISAC with the 8π spectrometer: γ-γ angular correlations

- $J^{\pi} \rightarrow 2_1^+ \rightarrow 0_1^+$ cascade, examine intensity $\gamma\gamma(\theta)$ to determine both $J^{(\pi)}$ and $E2/M1$ mixing ratio δ





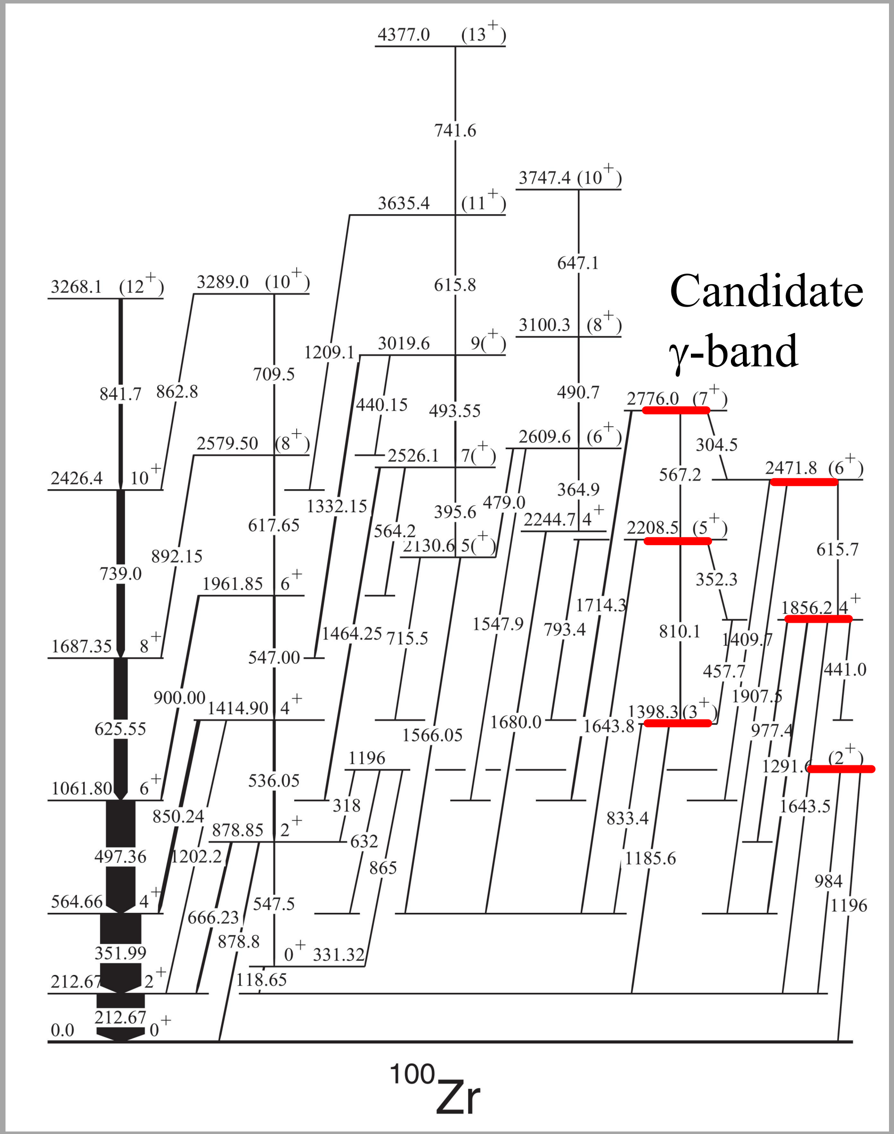


- 368 keV $2_2^+ \rightarrow 2_1^+$ transition pure E2

Analysis by K. Mastakov, Guelph

Recent work on ^{100}Zr

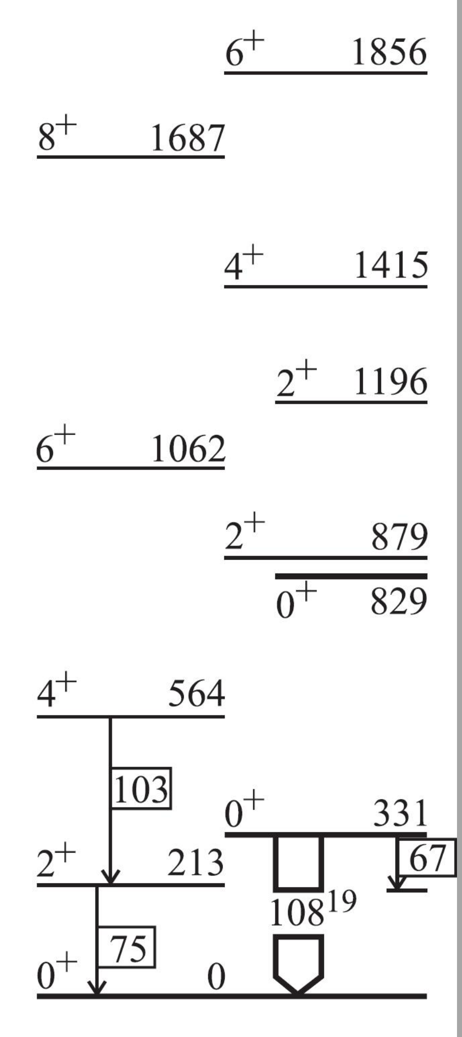
- ^{100}Zr from ^{248}Cm and ^{252}Cf fission, W. Urban *et al.*, PRC 100, 014319 (2019)



- **Identified candidate γ-band in ^{100}Zr, but band-head not established**
  - **Only the level at 1856 keV has firm spin-parity assignment, 4$^+$**
  - **Hinted that 1196-keV level may be band head**

- **K. Heyde and J. Wood, RMP 83, 1467 (2011) assign 1196 keV (2_3^+) state as member of 0_3^+ band**

Spectroscopy vs. systematics



Results from β-decay of ^{100}Y with the GRIFFIN spectrometer

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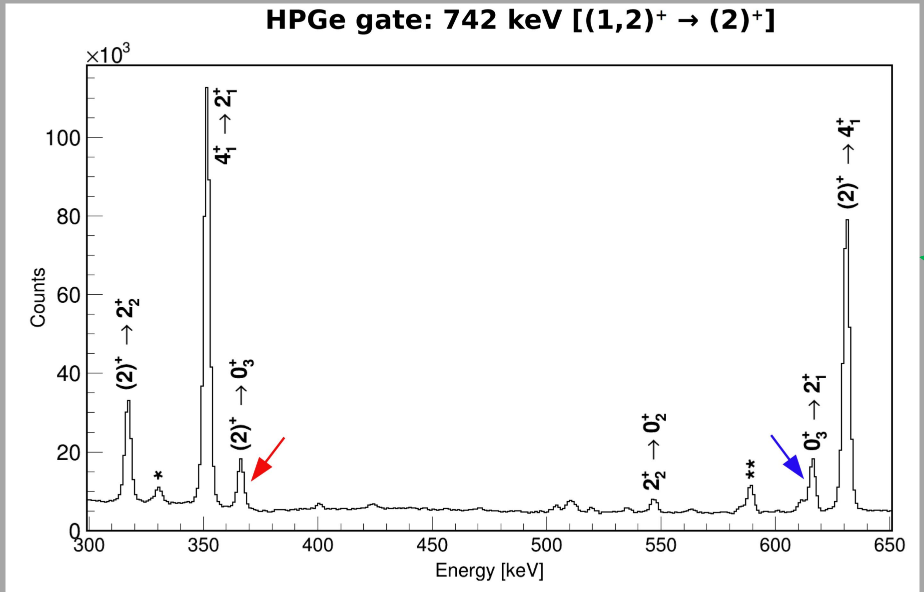
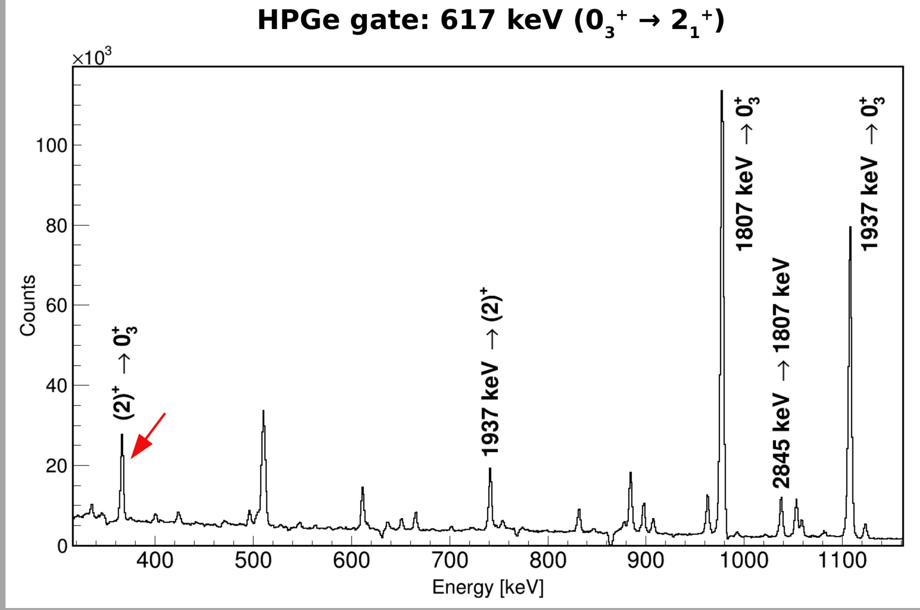
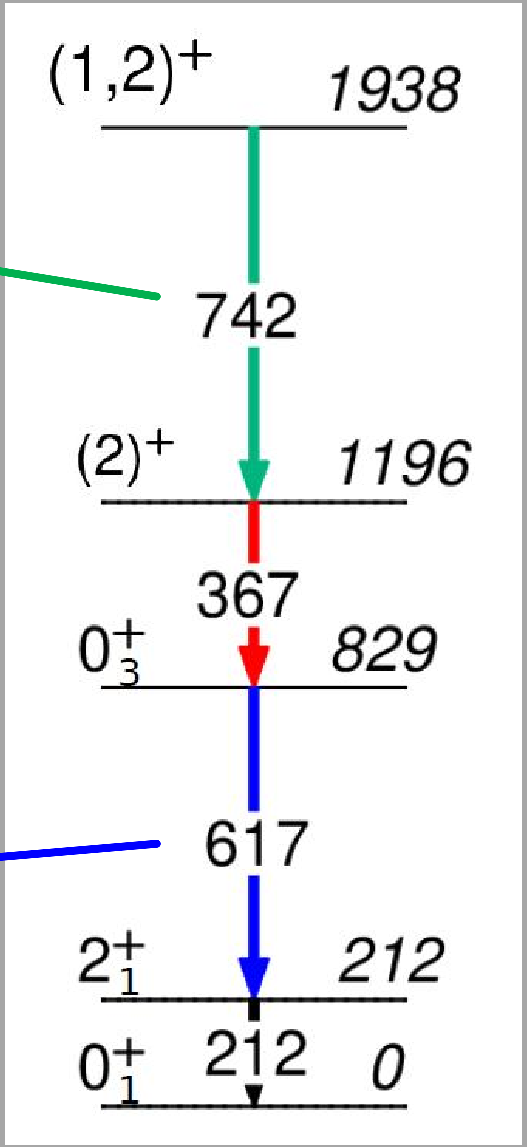


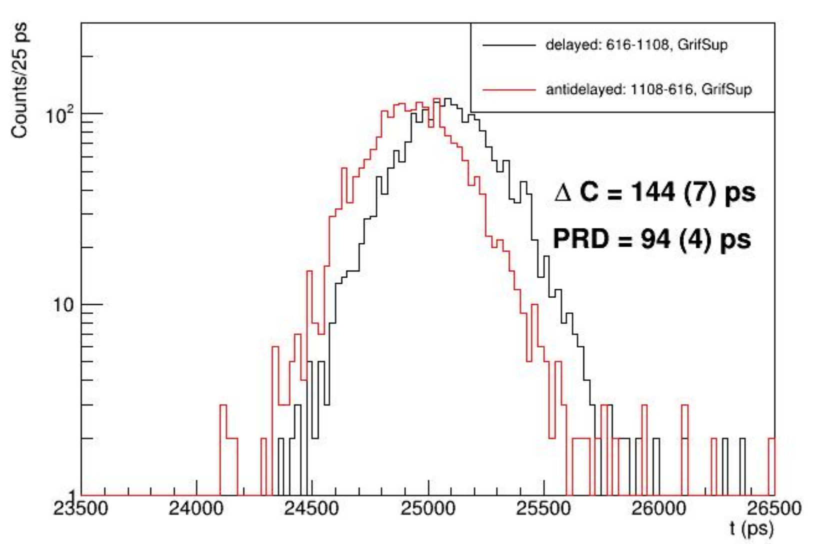


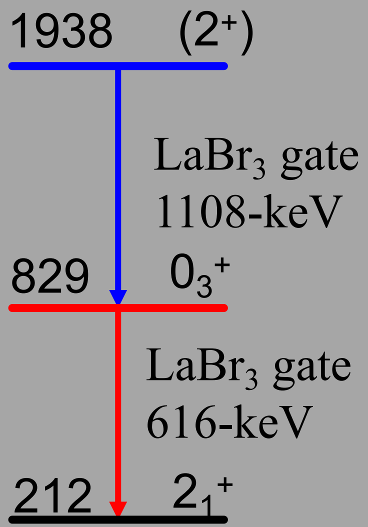


- Key transition: 367-keV $2_3^+ \rightarrow 0_3^+$ observed in gating from above and below
- 367-keV transition is ~1.2% branch
- Result suggests 1196 keV level is 2^+ band member
  - Spin now firmly established as 2^+

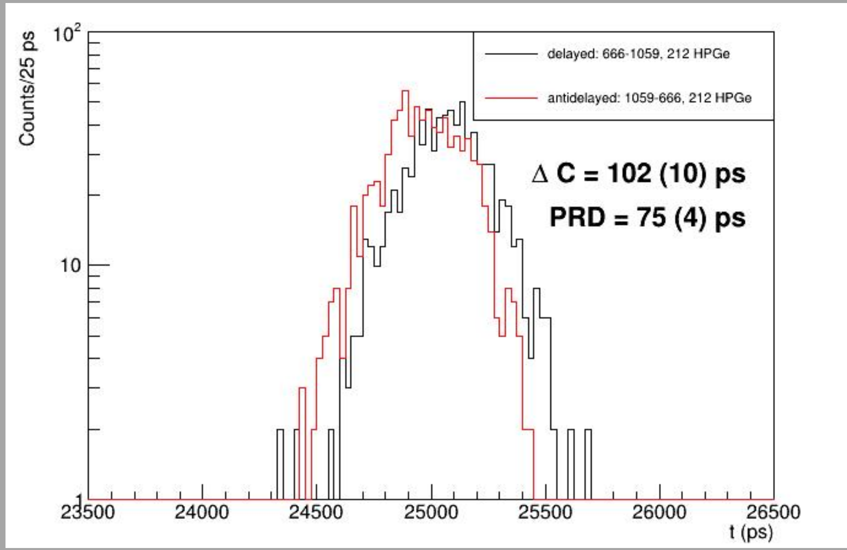
Level scheme analysis: D. Kalaydjieva, M. Zielinska (CEA Saclay)

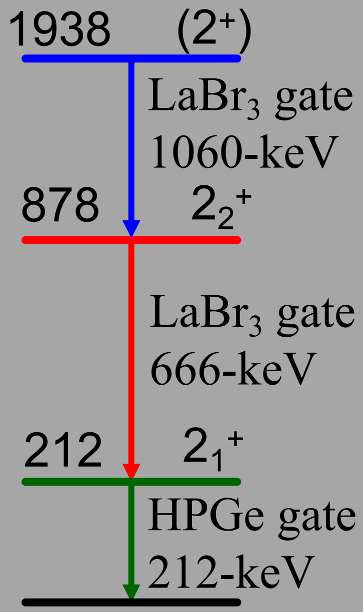
New results: lifetime of 0_3^+ and 2_2^+ levels





$\tau = 23(6)$ ps





$\tau = 16(3)$ ps

- **$B(E2;0_2^+ \rightarrow 2_1^+) = 70(8)$ W.u., $B(E2;2_2^+ \rightarrow 0_2^+) = 8(3)$ W.u.**
  - **0_2^+ band appears to be weakly collective**
- **$B(E2;0_3^+ \rightarrow 2_1^+) = 14(4)$ W.u**

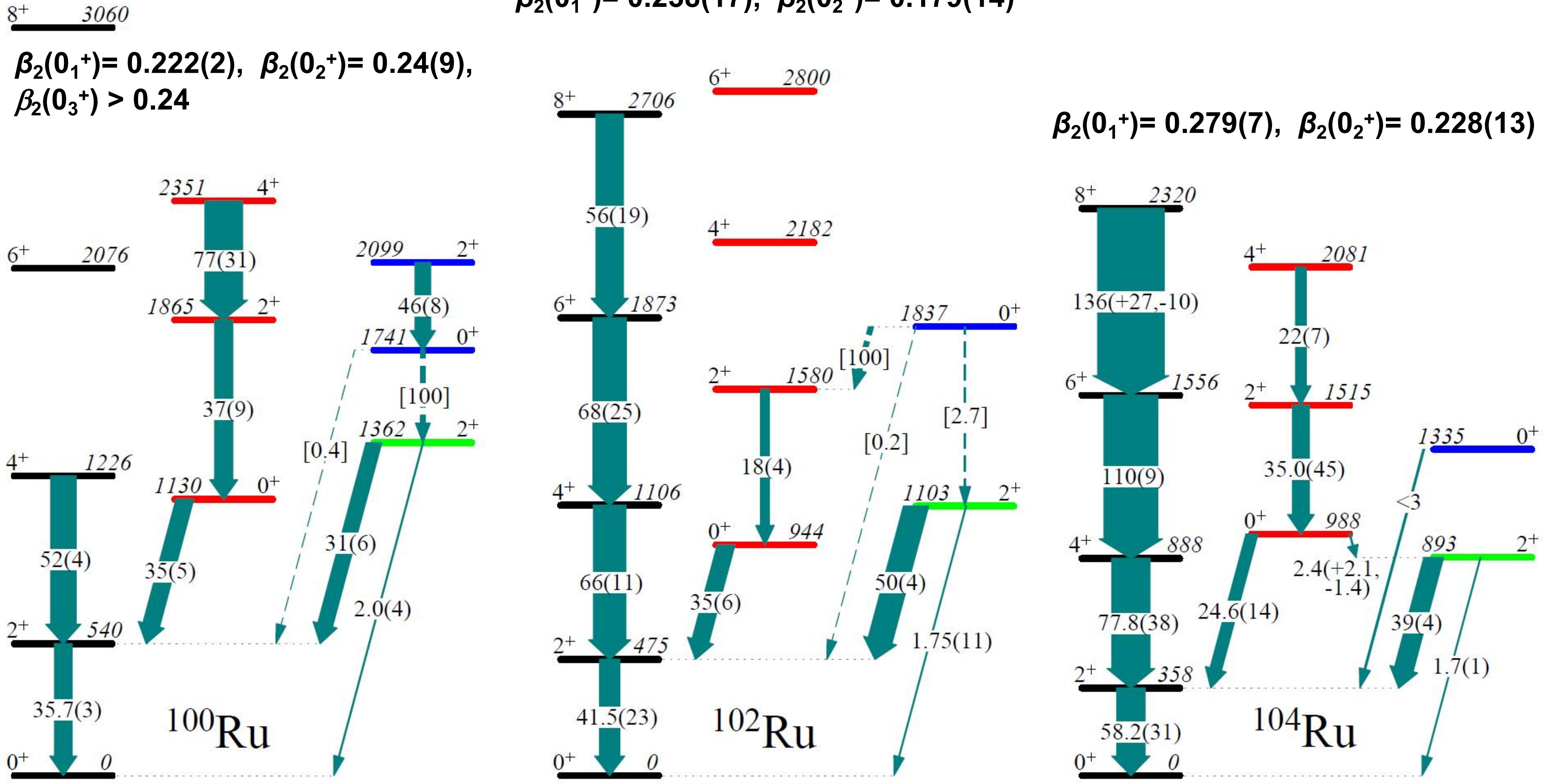
Lifetime analysis:
H. Bidaman (Guelph)

Results on excited 0^+ bands in ^{100,102}Ru

$\beta_2(0_1^+)$= 0.238(17), $\beta_2(0_2^+)$= 0.179(14)

$\beta_2(0_1^+)$= 0.222(2), $\beta_2(0_2^+)$= 0.24(9), $\beta_2(0_3^+)$ > 0.24

$\beta_2(0_1^+)$= 0.279(7), $\beta_2(0_2^+)$= 0.228(13)

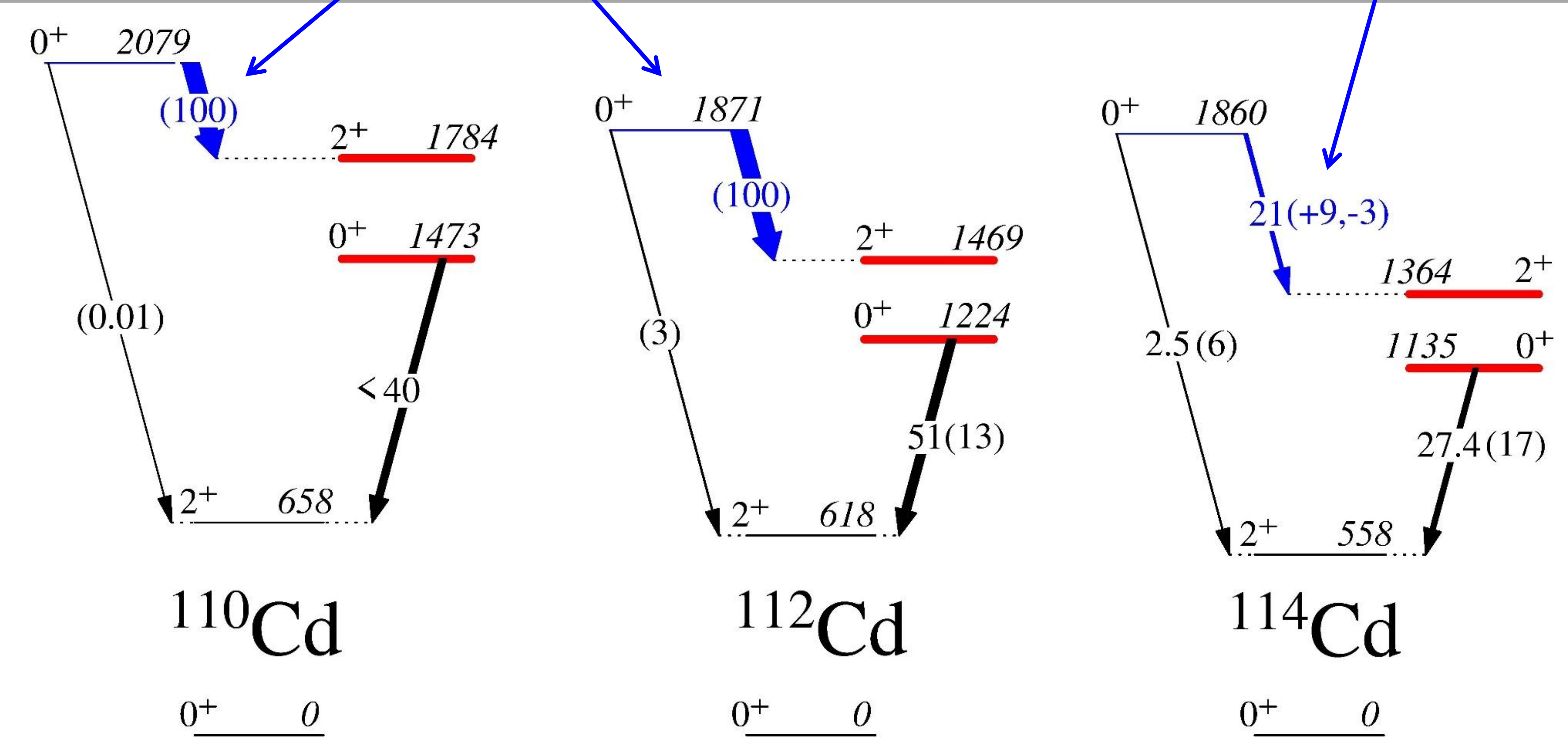


0_4^+ level preferentially decays to 2^+ intruder band member in the Cd isotopes

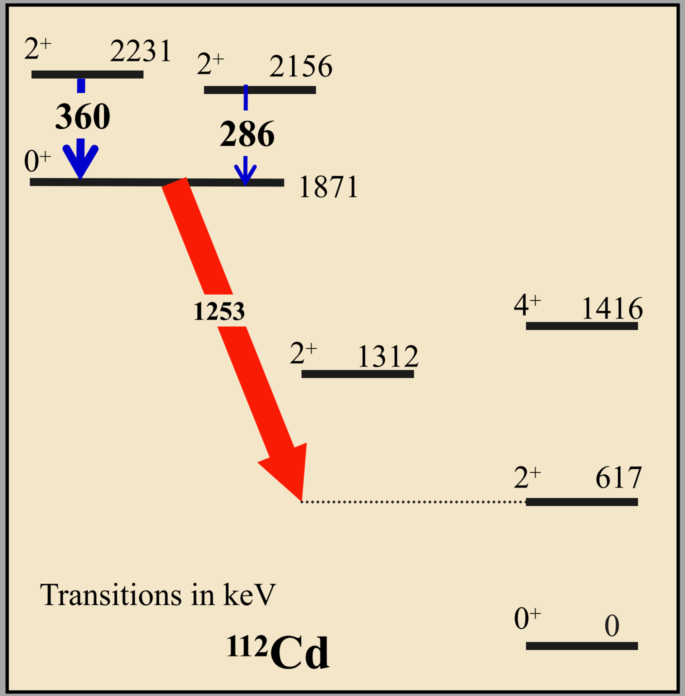
- Enhanced $0_4^+ \rightarrow 2_3^+$ decay observed in ^{114}Cd

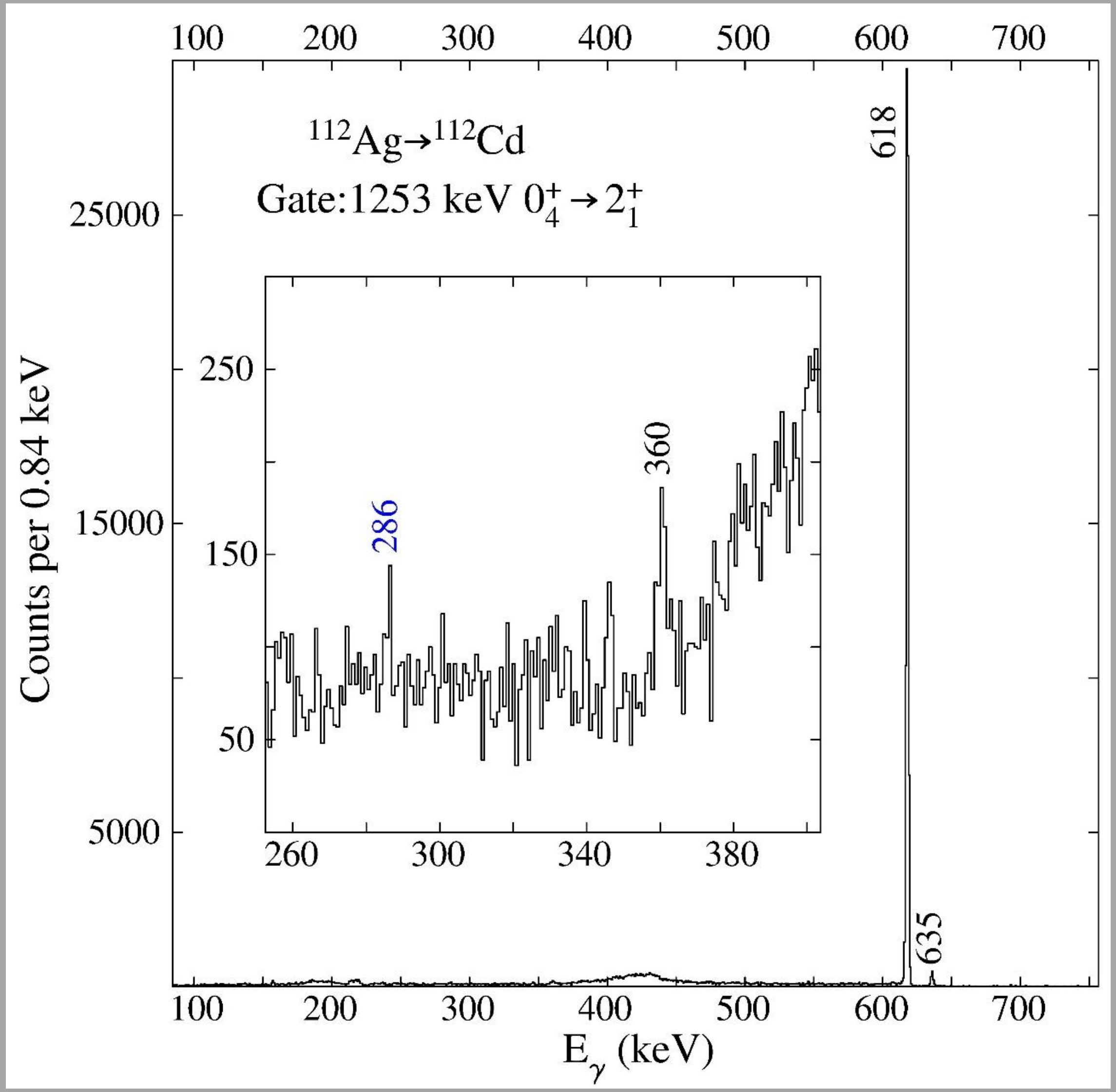
Relative B(E2)values

Absolute B(E2) in W.u.



Search for very weak, low-energy γ branches in ^{112}Cd via β-decay



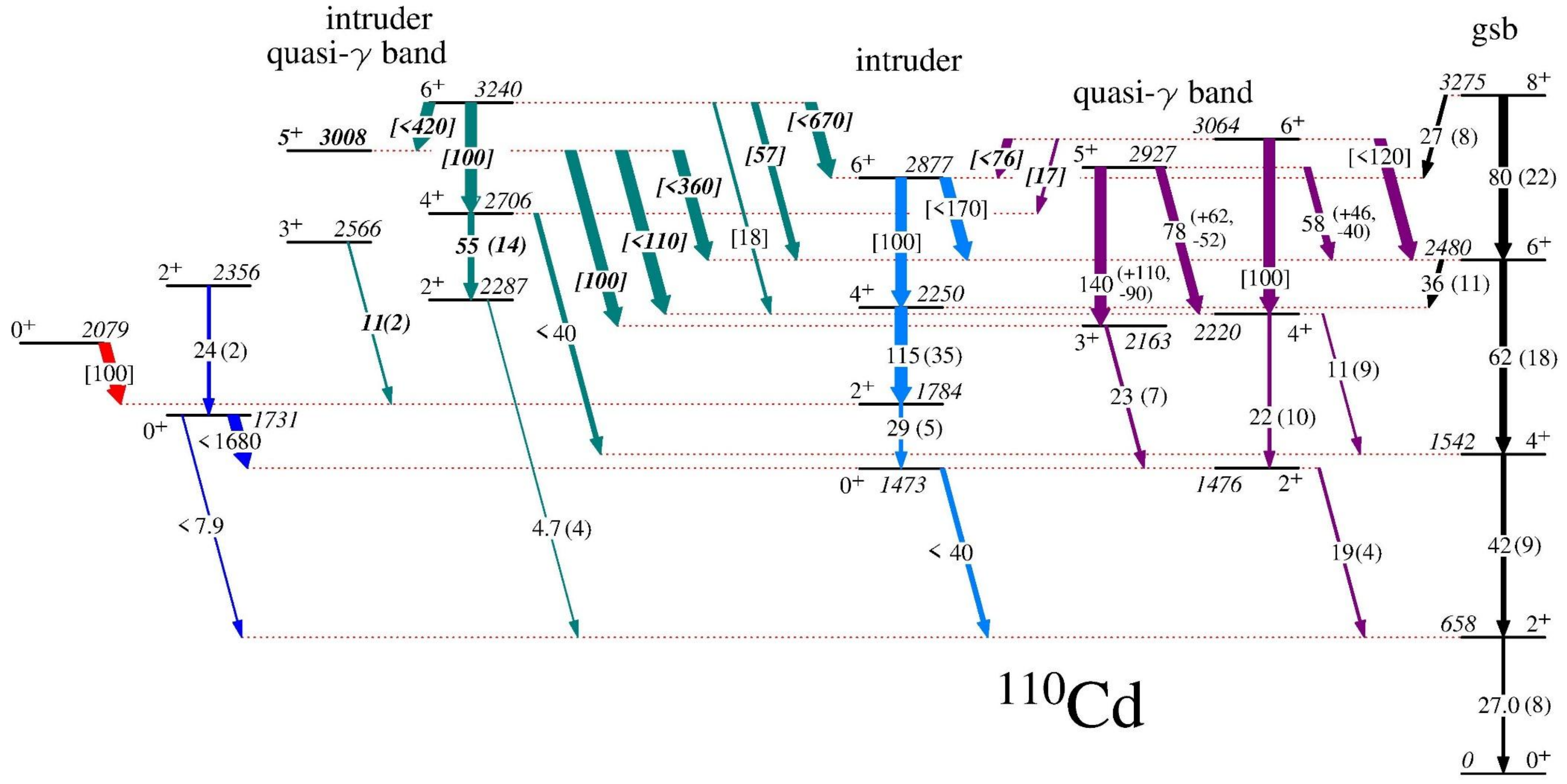


- **286-keV transition is a 7.9(33)×10^{-4} γ branch from 2156-keV level, → 34(15) W.u. transition**

Garrett et al., PRC **101**, 044302 (2020)

^{110}Cd band structure

- Transitions labelled with *B(E2)* in W.u.
- Square brackets indicate relative *B(E2)* values
- Very weak transitions removed
  - Most of the upper limits are due to unknown *E*2/*M*1 mixing ratios



^{112}Cd band structure

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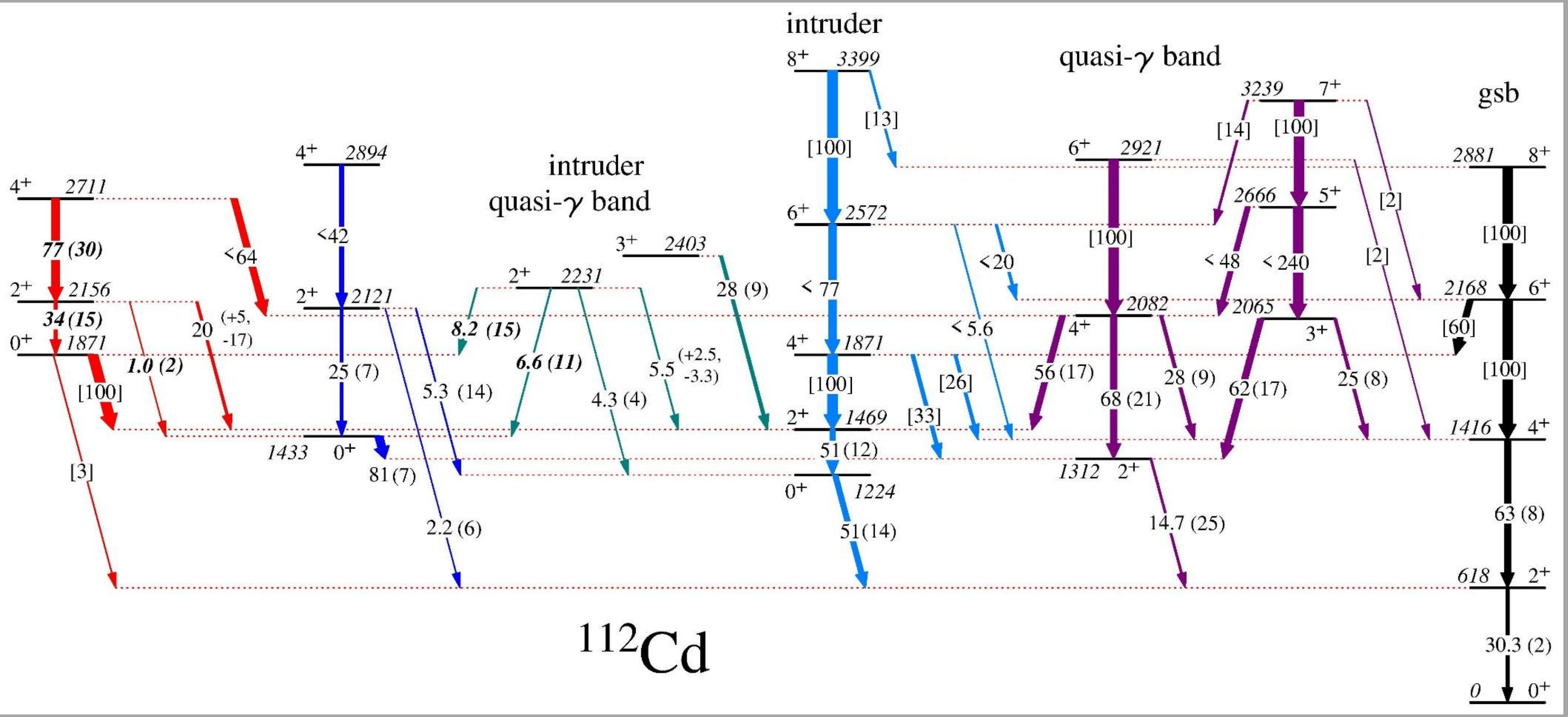

Results interpreted with aid of BMF calculations

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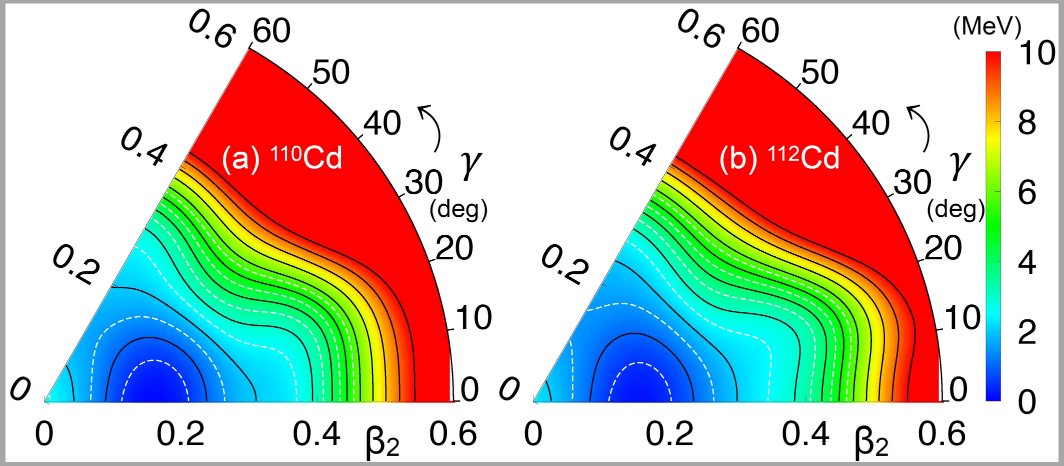
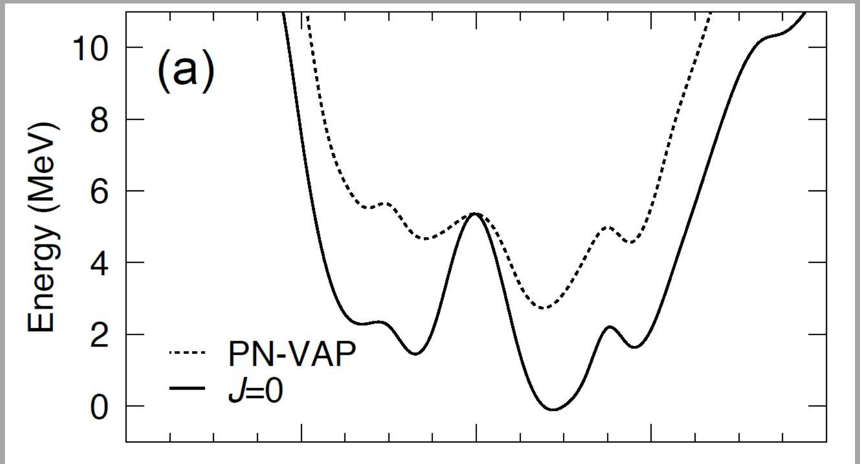
- BMF calculations using symmetry-conserving configuration mixing method (SCCM) with Gogny D1S energy-density functional



- "Shoulders" on PES, but the rich variety of shapes not readily apparent



Calculations by T. Rodriguez

Garrett et al., PRC **101**, 044302 (2020)

- Exact angular momentum and particle number restoration