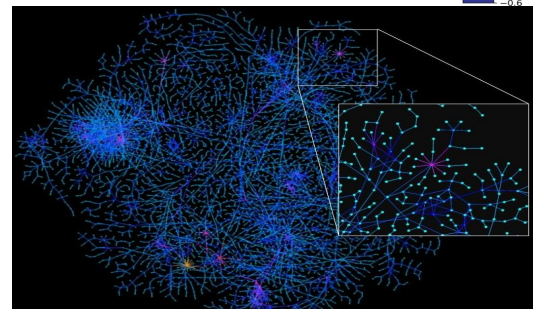
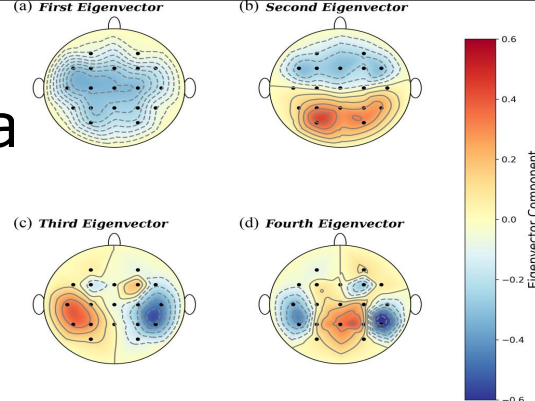


Phase Transitions, Scale Invariance and Criticality in Self Organized Systems



Alejandro Frank

ECN, C3, ICN



Symmetry and conservation Laws

Translation (all times and/or positions are equivalent)----- Energy and momentum conservation

All directions equivalent : Rotation ---- angular momentum conservation

“Internal symmetries” quantum numbers: spin, isospin, up, down, strange,..

Gauge Symmetries in Field Theory.....

Symmetries in Nuclear Physics

Elliot's SU(3)

Isospin Symmetry

IBM1 and IBM2

IBFM

Nuclear Susy

Electron-Vibron Model

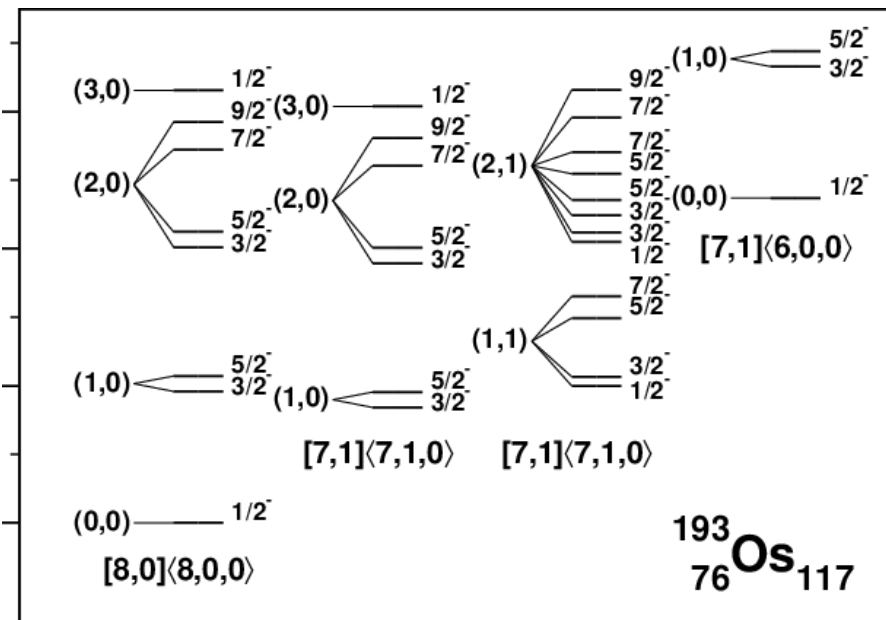
Algebraic Scattering Theory

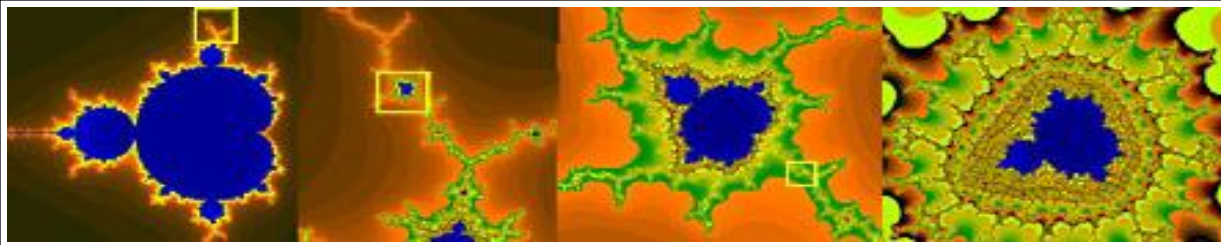
F-Spin Symmetry

Symmetry Adapted Polyatomic Molecules

E(5), X(5)

-





Similarity and Scale
Invariance

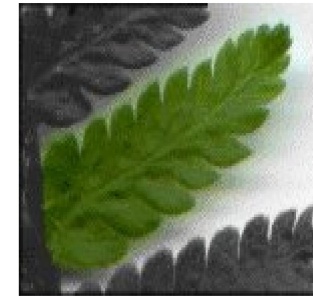
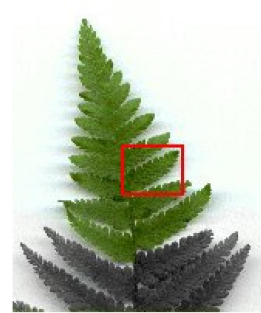
Scale
Invariance
Symmetry

What about Biology?

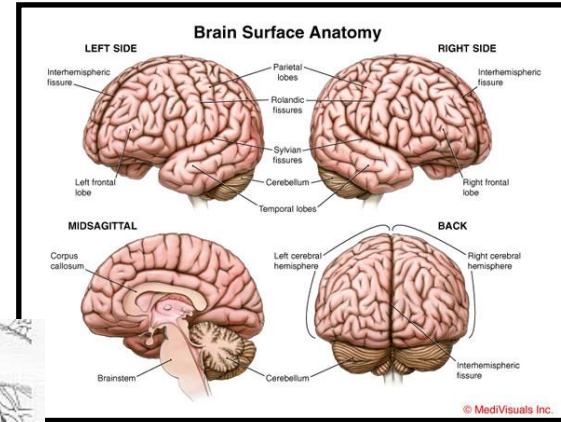
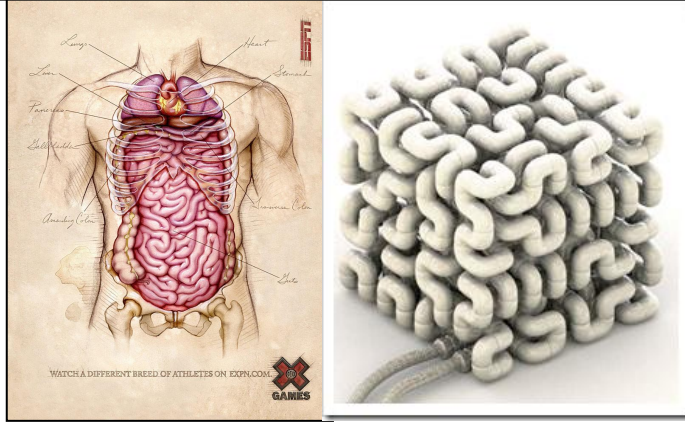
brócoli



living systems : self similarity,fractality



Fractal Structure of organs: An OPTimization strategy of nature

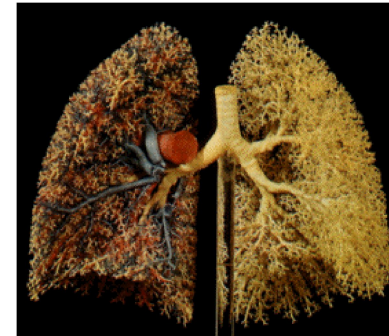
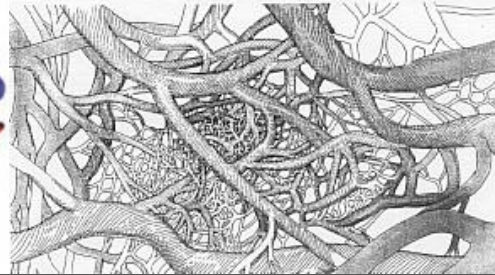
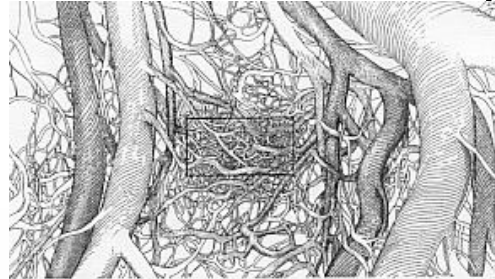
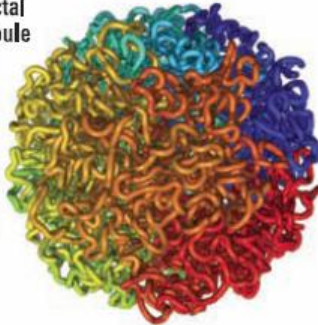


Fractality Self similarity in units and subunits.

Cardiovascular and nervous
systems

DNA

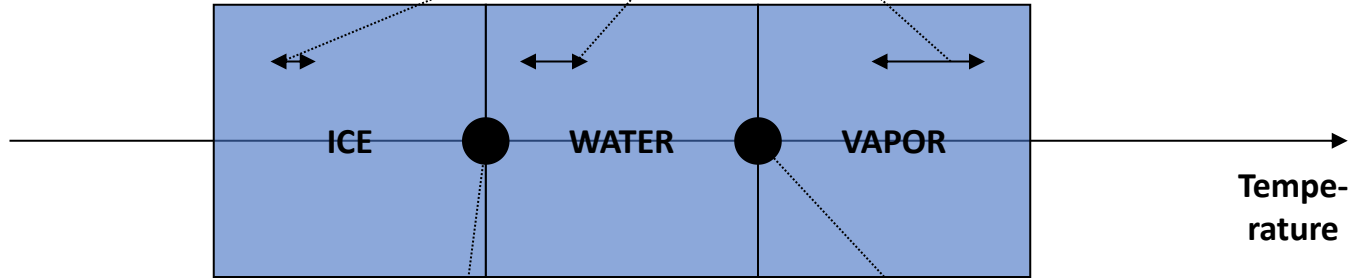
Fractal
globule



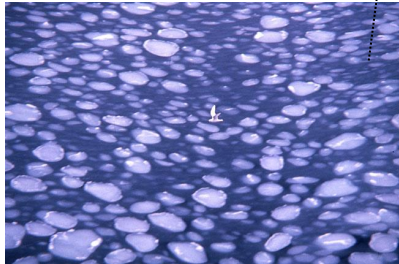
Adult lungs
gas exchange
Area:

From Physics: Phase transitions and critical points

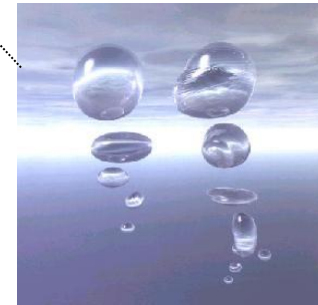
A single scale defines the molecular interactions in each phase.



Critical point: ice-water :all scales

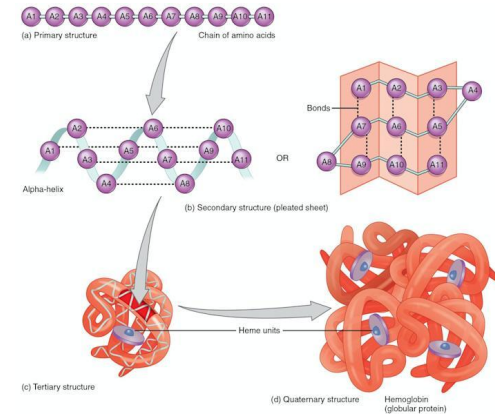


Critical mixing of water and vapor : all scales appear.



What about function? Phase transitions in biological systems.

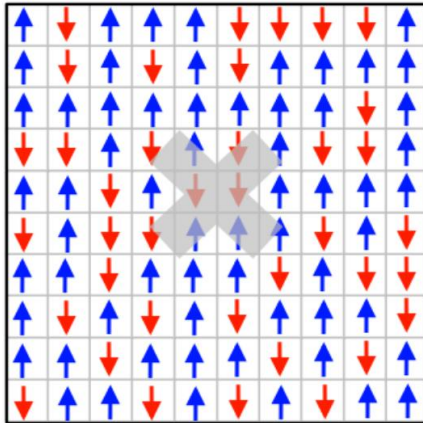
- It has been proposed that some biological systems might lie near critical points.
- Examples include neural networks in the salamander retina, bird flocks, gene expression networks in *Drosophila* and protein folding.
- Biological organisms share two key properties of phase transitions: the change of macroscopic behavior and the



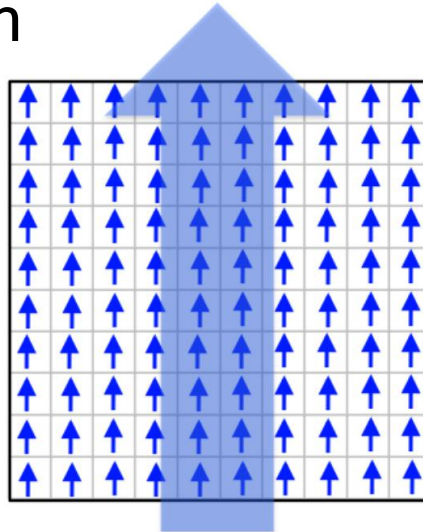
How to define criticality in time series

Ising model and criticality

Simple model for magnetization

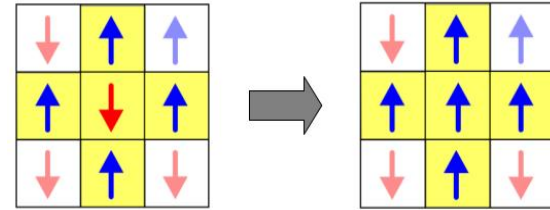


Not
magnetized

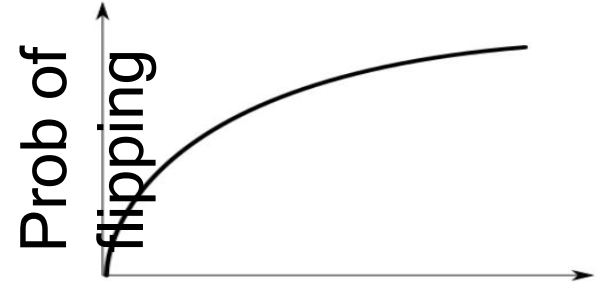


Magnetized

Two competing
1. Local interactions define spin alignments



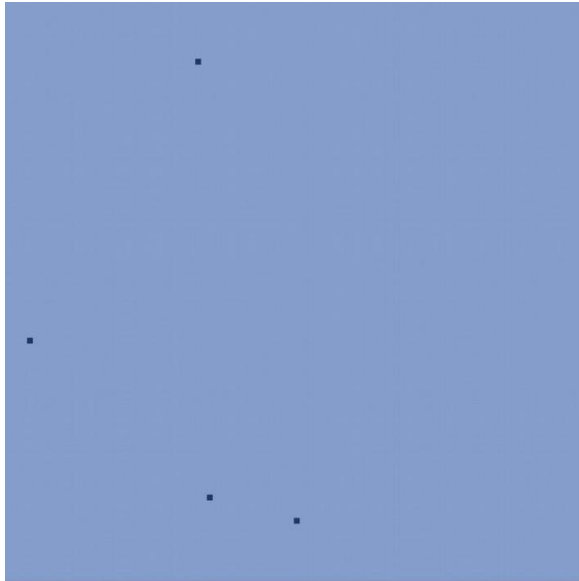
2. Random spin flipping defined by temperature with Boltzmann distribution



Criticality: Ising Model

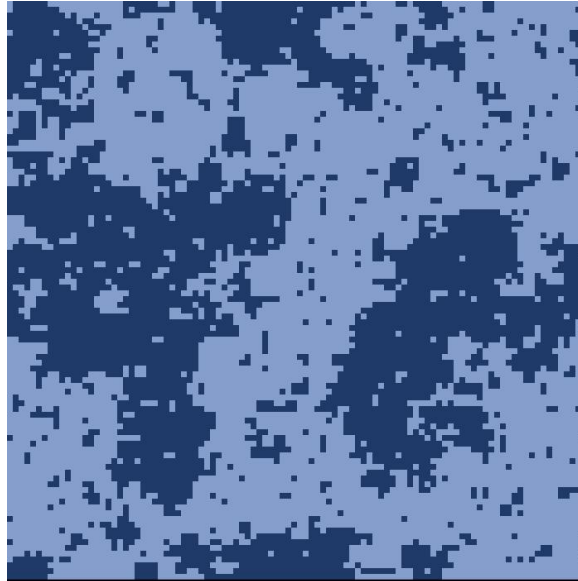
Space reference frame

$T < T_c$



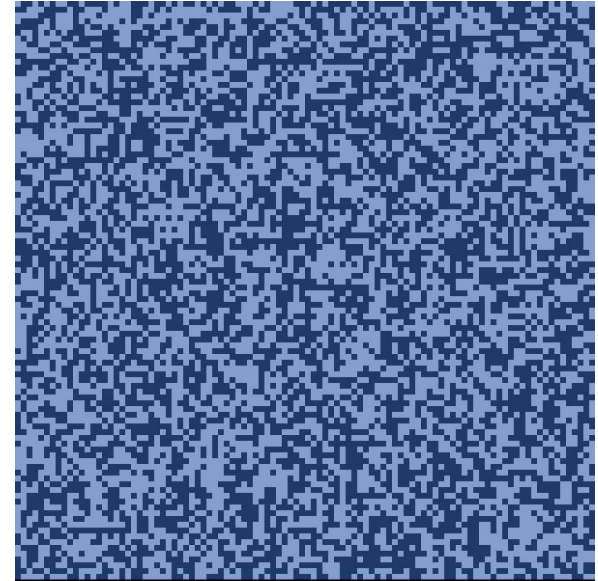
Order
Uniformity

$T = T_c$



Criticality
Fractality, scale invariance

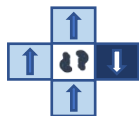
$T > T_c$



Disorder
Random fluctuations

THE ISING MODEL

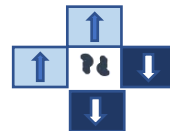
LESS FAMILIAR : THE DYNAMICS CAN BE STUDIED VIA PERTURBATION
METROPOLIS-MONTECARLO METHODS



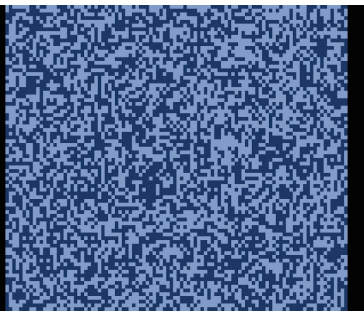
subcritical



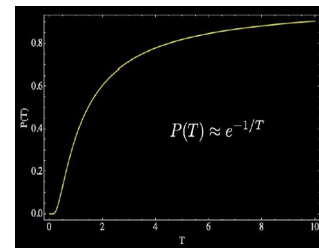
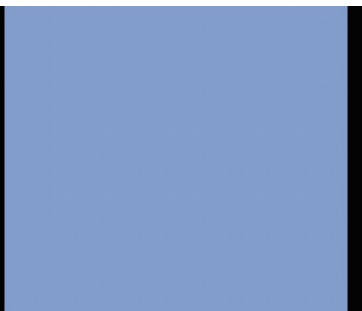
Balance



criticality.



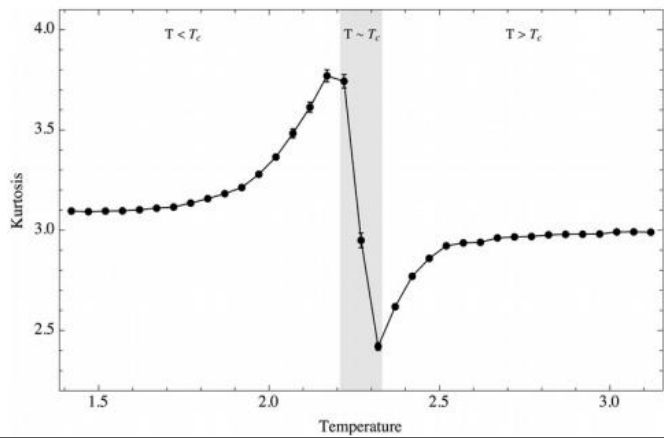
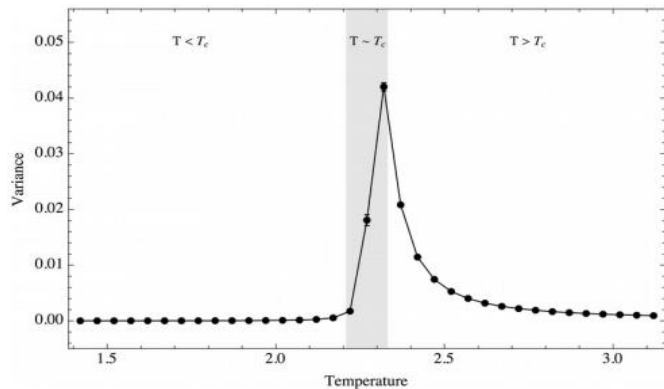
supercritical



Morales et al. (2015) *PLoS ONE* 10:e0130751.

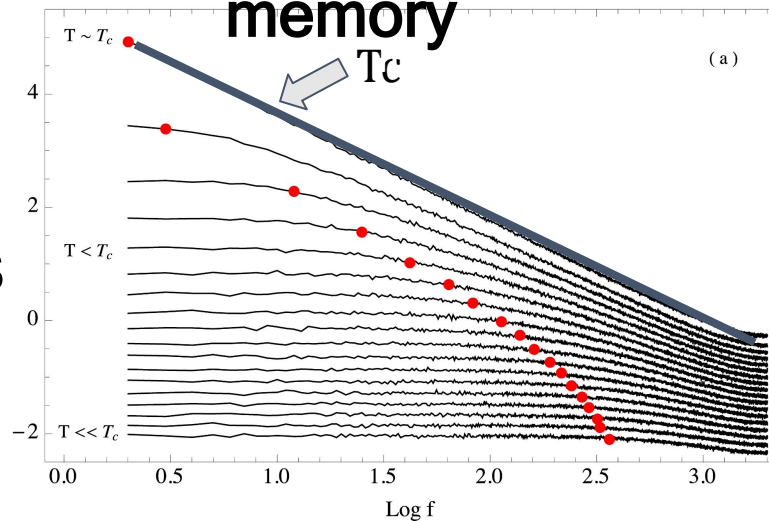
Total magnetization as a function of temperature: Ising Model

Behavior at criticality



variance autocorrelation & memory

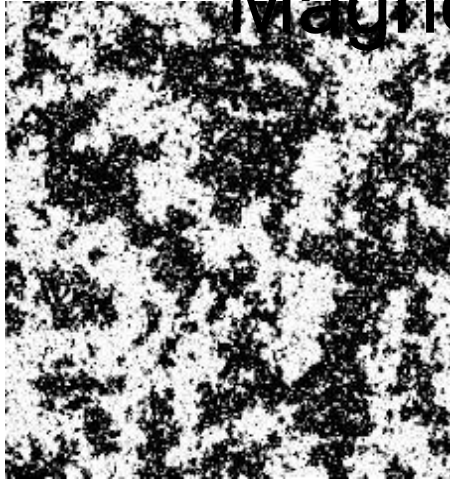
kurtosis



Complexity Approach: criticality Compute total Magnetization

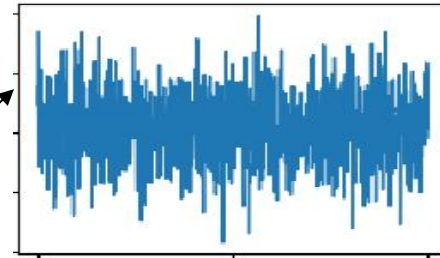
Time series and

Power Spectra

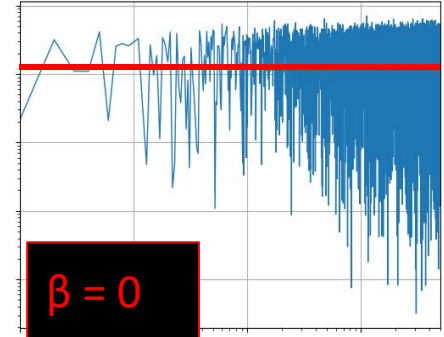
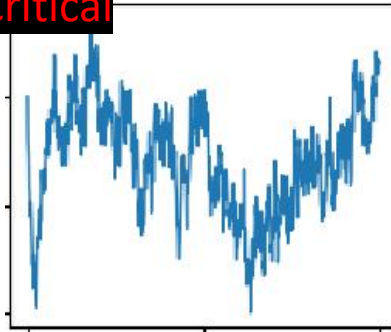


Ising Model

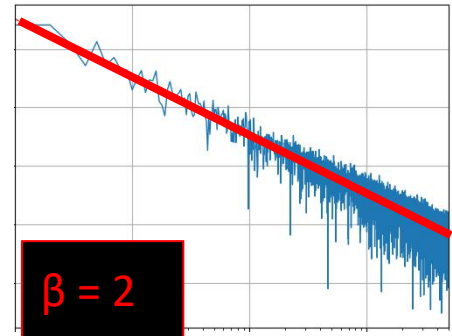
subcritical



Critical



$\beta = 0$



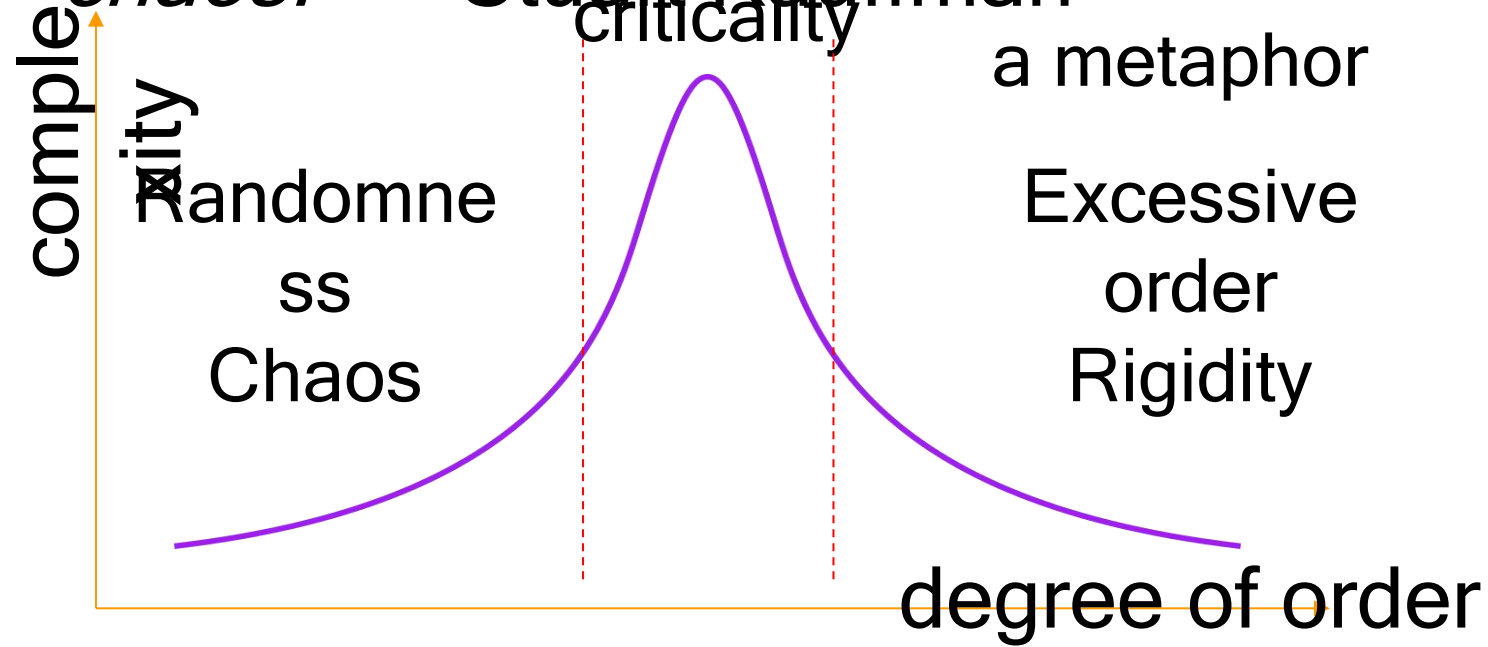
$\beta = 2$

Complexity: at the border between order and chaos

Living systems exist at the border of

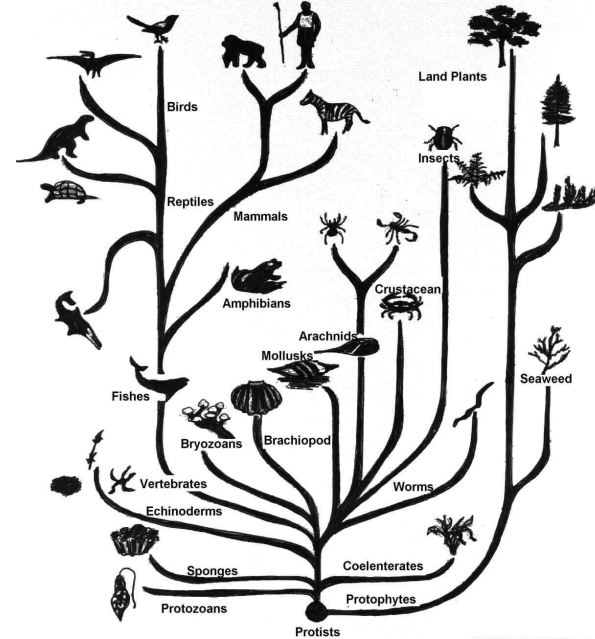
chaos." –Stuart Kauffman

Turns out not to be
a metaphor

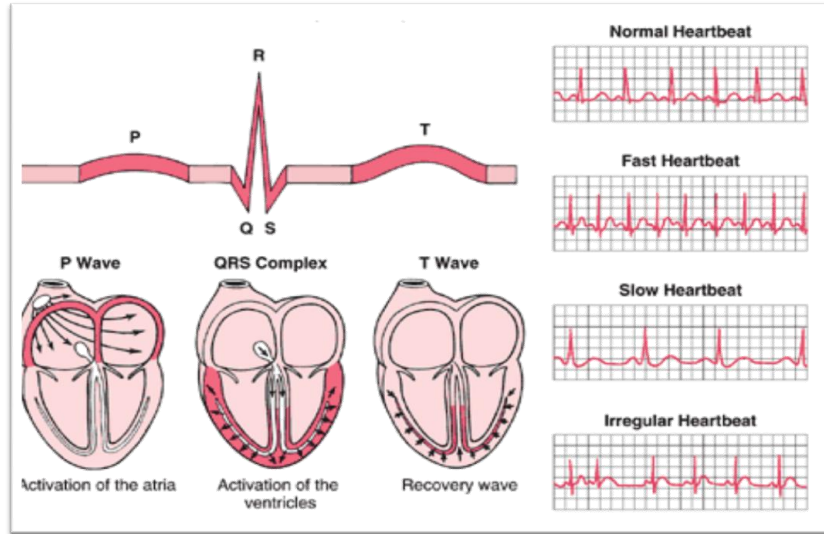


Main Hypothesis, Living Organisms: evolution and criticality

- Organisms and their organs develop under constantly changing environments.
- Two **antagonistic** requirements:
 - **Robustness**: phenotypes should be robust as they evolve.
 - **Adaptability**: Individuals must adapt to changes
 - **Criticality** : optimal equilibrium (alostasis).
- Darwinian **selection** leads to criticality



A one dimensional example: Heartbeats ECG



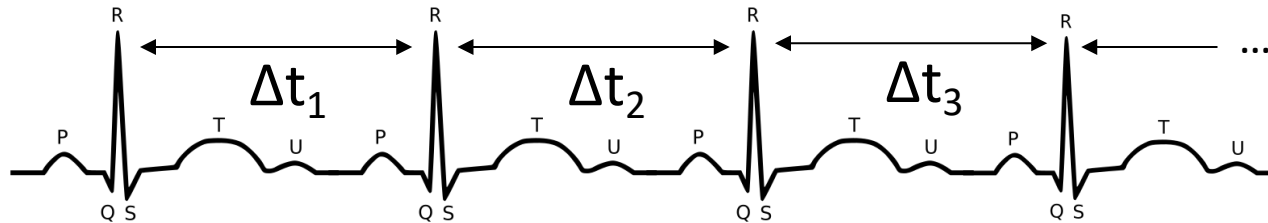
Serie de tiempo

Intervalos RR

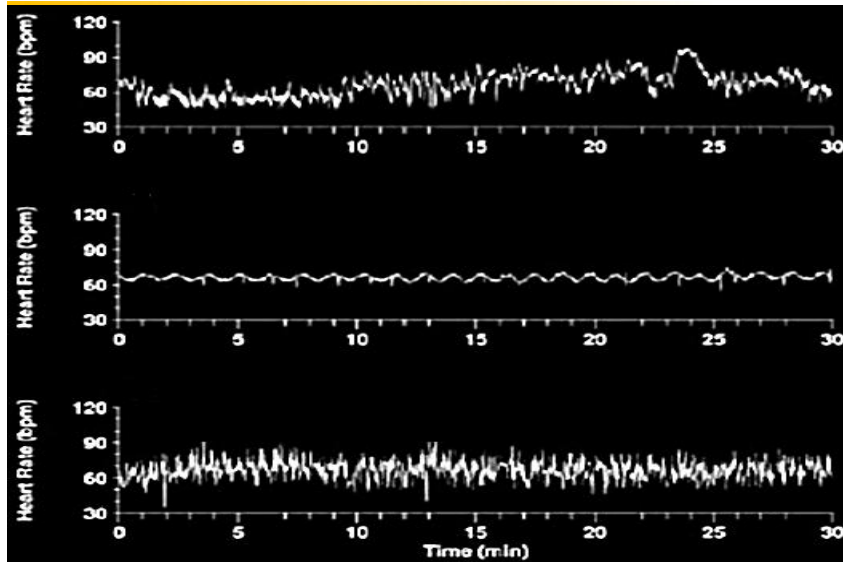
$$\Delta t_1, \Delta t_2, \Delta t_3, \dots, \Delta t_n$$

Fluctuaciones

$$\frac{\Delta t_i - \overline{\Delta t}}{\Delta t}$$



CARDIAC VARIABILITY



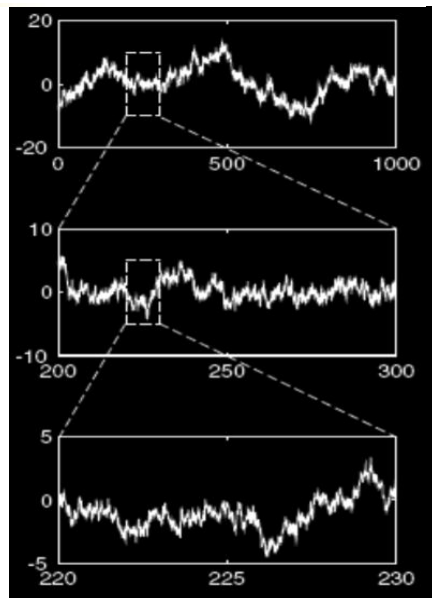
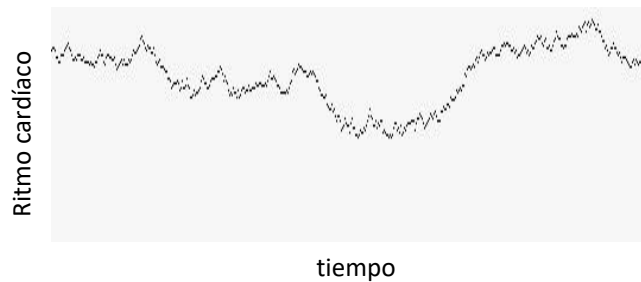
healthy heart

cardiac insufficiency

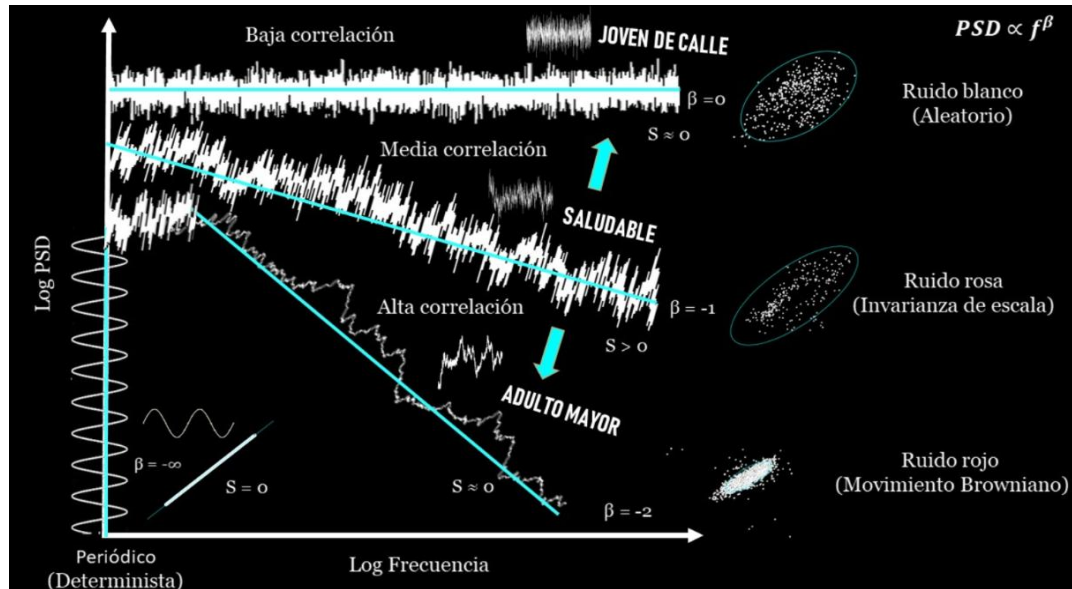
ventricular fibrillation

Goldberger et al. (2002) *PNAS* 99:2466

SCALE INVARIANCE AT CRITICALITY.



POWER SPECTRA



Power Spectra of R-R intervals

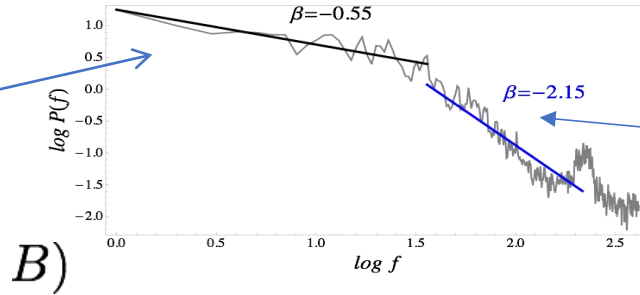
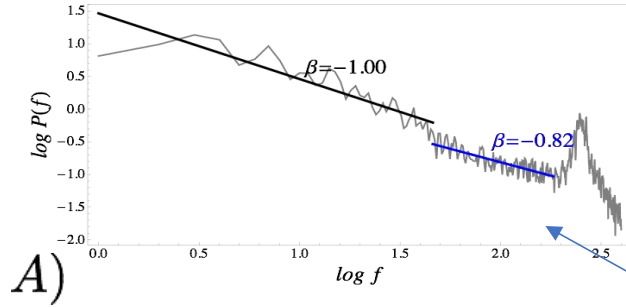
Collaboration Hospital of Nutrition and Cardiology: Prof. Dr. Bruno Estañol:



Detection of Pre-diabetes and Diabetes, Metabolic Syndrome, for early warnings: IBI and Blood Pressure. **Coronary Obstruction**, Intensive Therapy, Epilepsy.

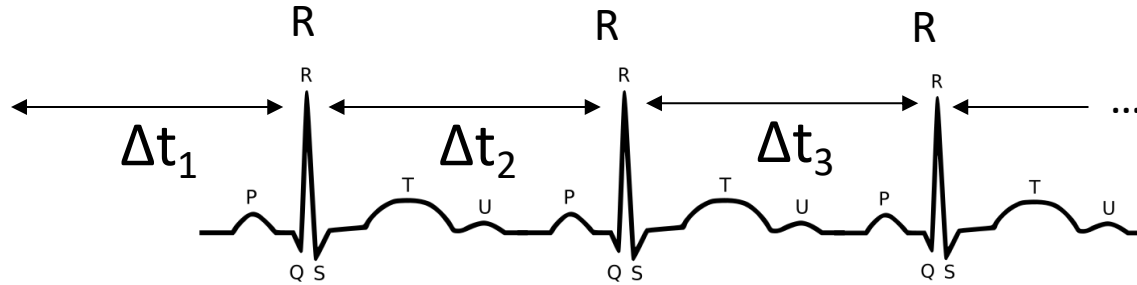
A) young (21-34 years old),

B) older (68-84 years old). Physionet.



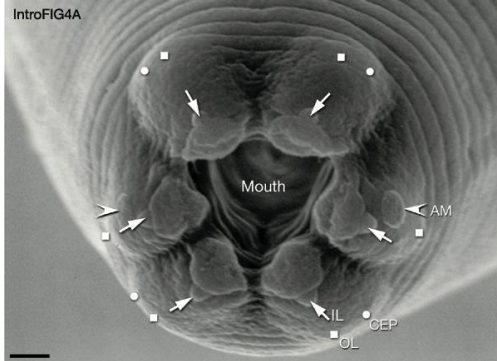
Change of slopes

Correlation loss at low frequencies (longer times)



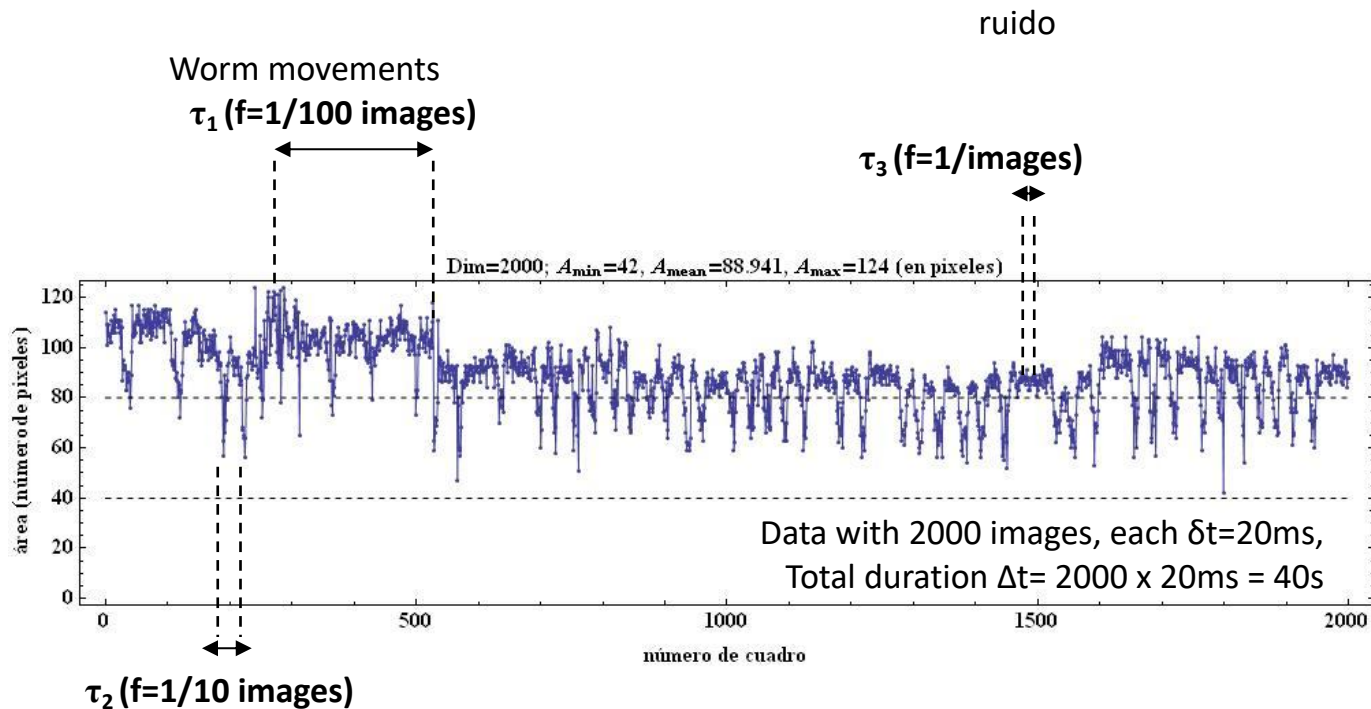
LAENPA2024

A simple model: C Elegans



Pharynx oscillations display perfect scale invariance for young healthy worms

High speed camera . Area measurements

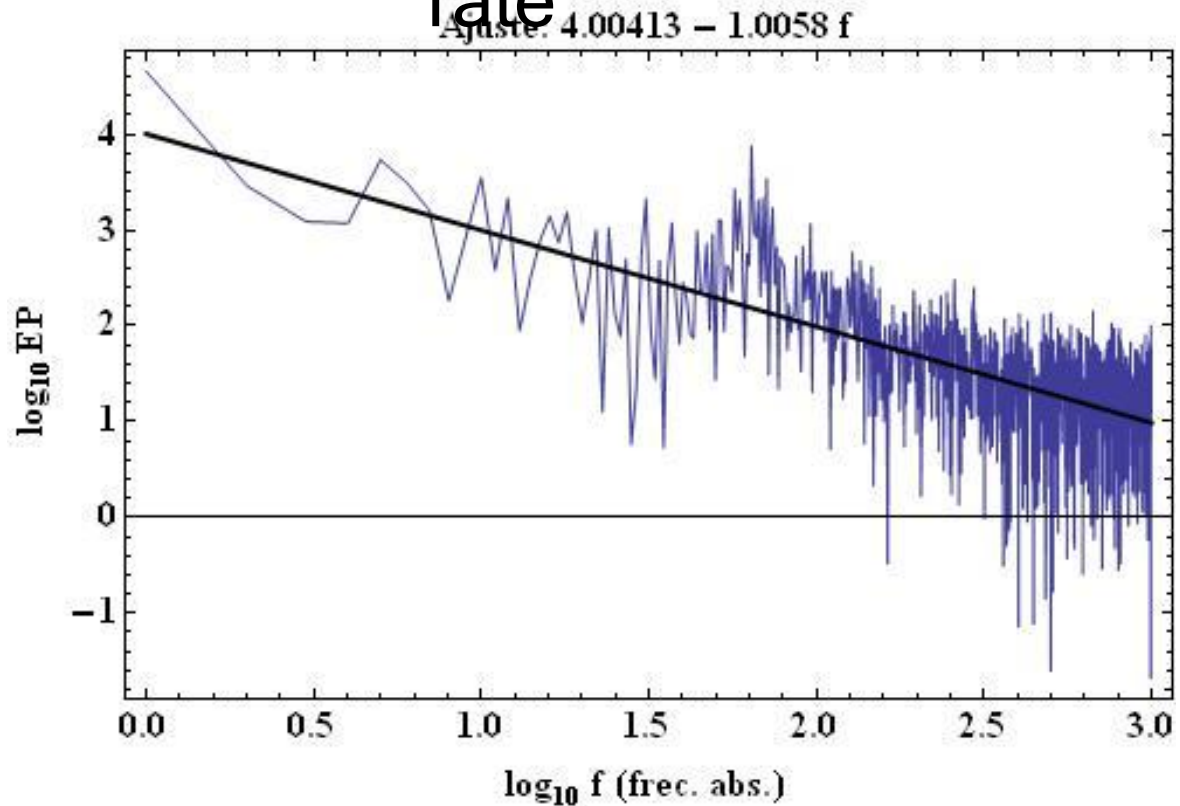


C-elegans

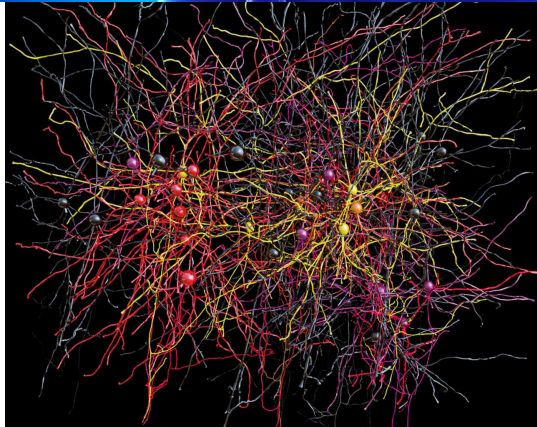
Pharynx oscillation rate

Young C-elegans

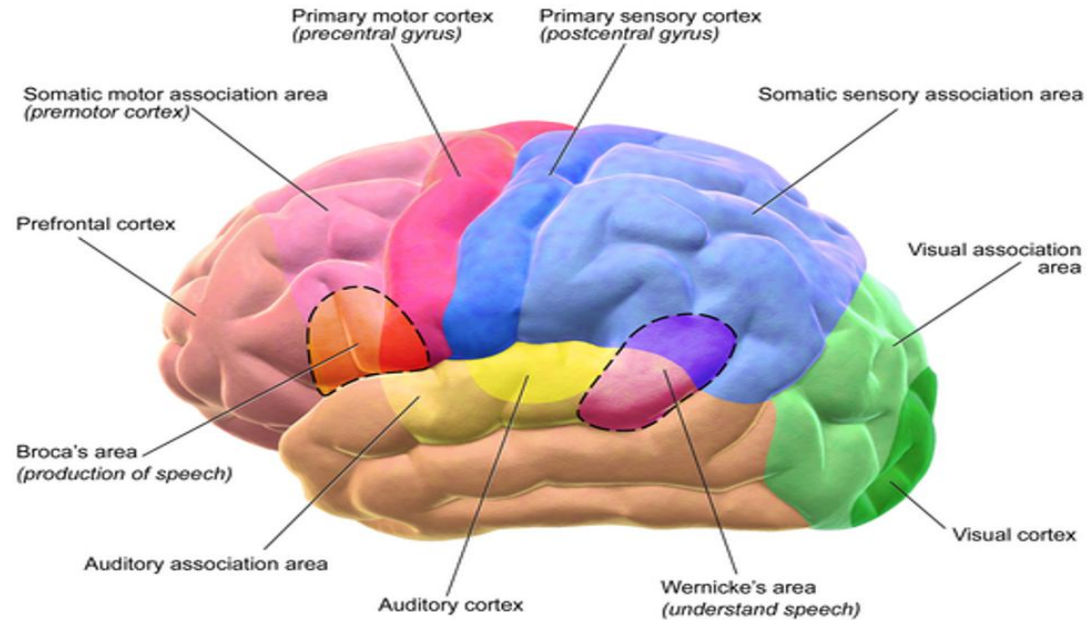
Older become
irregular



Organic growth of brain networks produces areas of functional specialization



Motor and Sensory Regions of the Cerebral Cortex



Brain Criticality ?

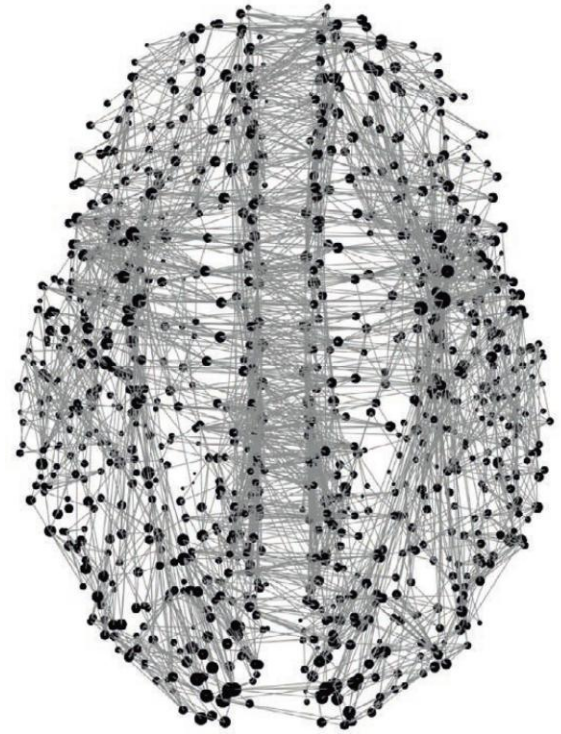
There is evidence that brain behavior is critical.

Scale invariance helps explain the emergence of complex structures

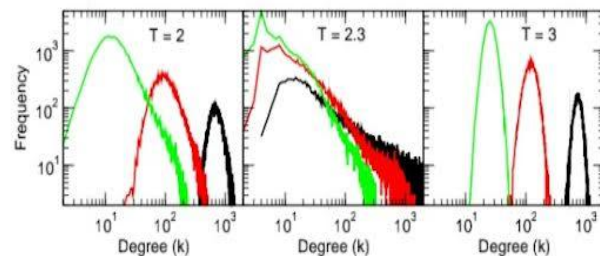
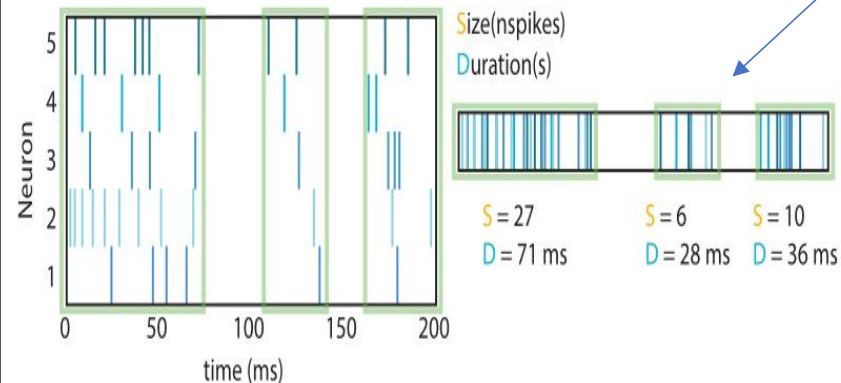
Optimizes information processing
(transmission, storage)

Produces diversity and functional flexibility

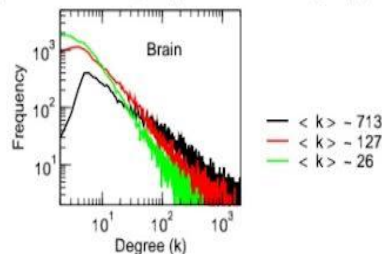
Provides functional robustness in the face of serious alterations
(injuries)



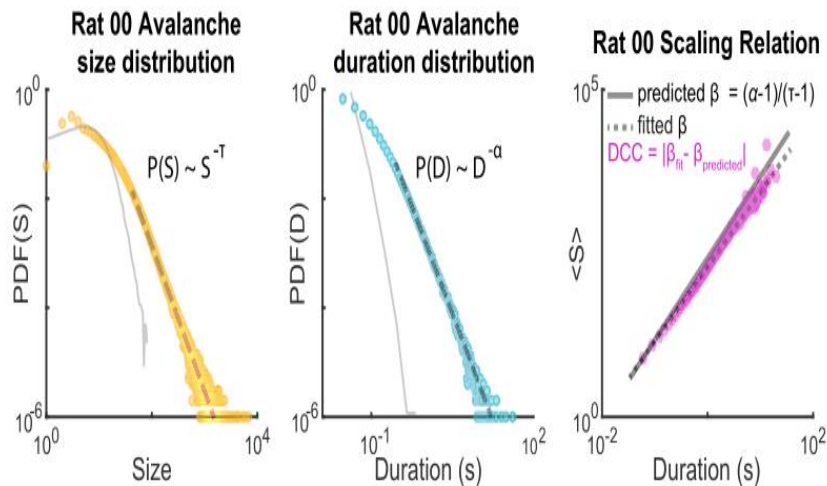
Evidence through Experiments with rats (invasive) AND Models



Ising



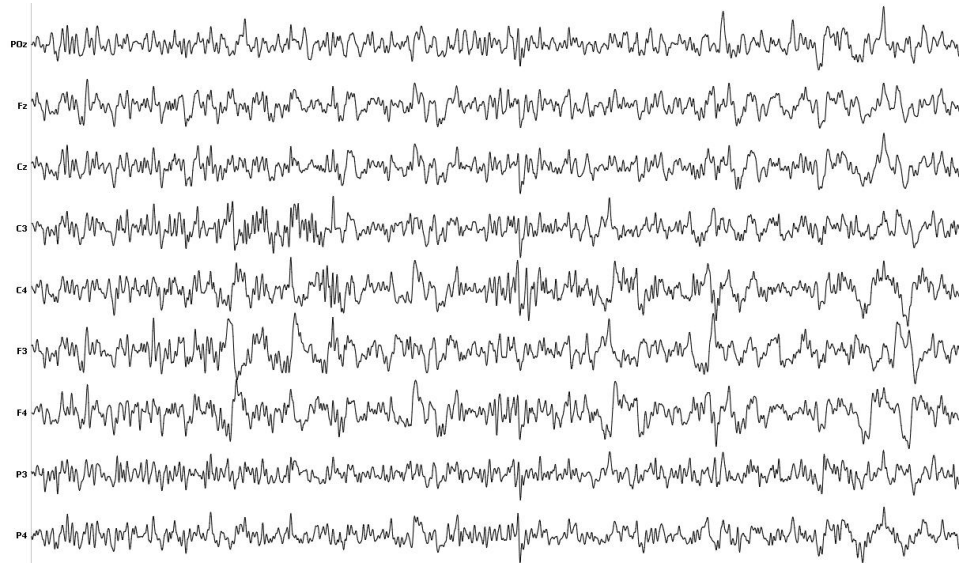
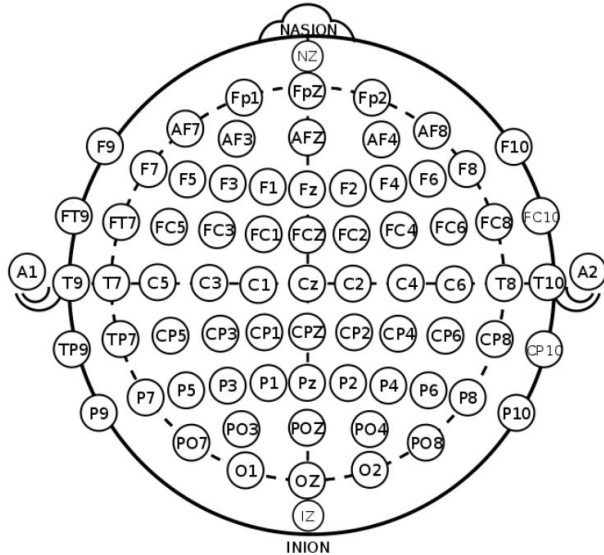
Brain



At criticality, brain and Ising networks are indistinguishable from each other. The graphs show a comparison of the link density distributions computed from correlation networks extracted from brain data (bottom panel) and from numerical simulations of the Ising model (top three panels) at three temperatures: critical ($T = 2.3$), sub ($T = 2$) and supercritical ($T = 3$). Top three panels depict the degree distribution for the Ising networks at $T = 2$, $T = 2.3$ and $T = 3$ for three representative values of $\langle k \rangle \approx 26, 127$, and 713 . Bottom panel: Degree distribution for correlated brain network for the same three values of $\langle k \rangle$. Figure redrawn from Fraiman et al [42].

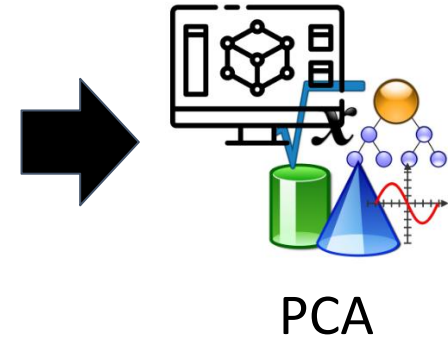
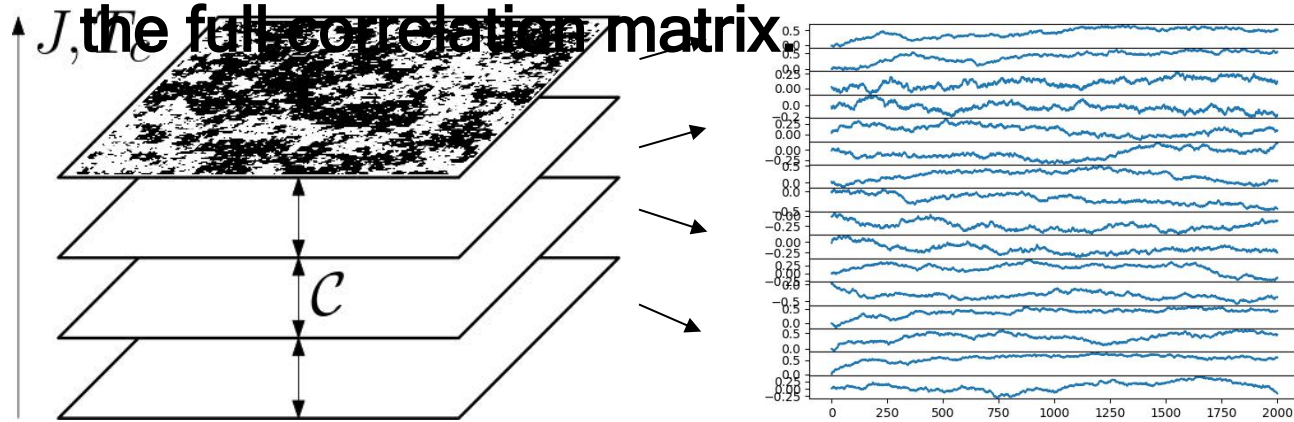
Multicriticality (multisignal criticality)

How to define and study criticality in a multiple signal system?



Ising multilayer model

Multilayer Ising model, each layer has a different coupling strength, vertically coupled to its nearest neighbors. We evaluate each layer's magnetization time series and compute



Criticality in a multisignal system using principal component analysis

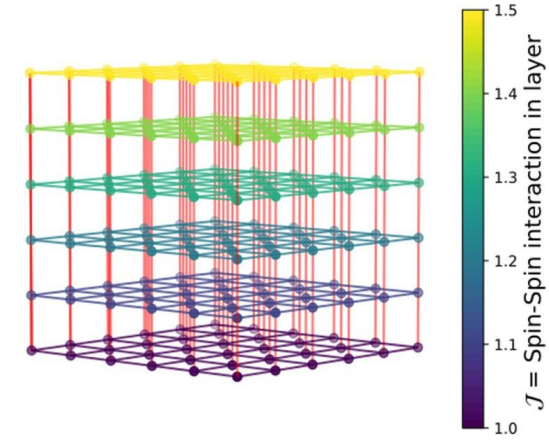
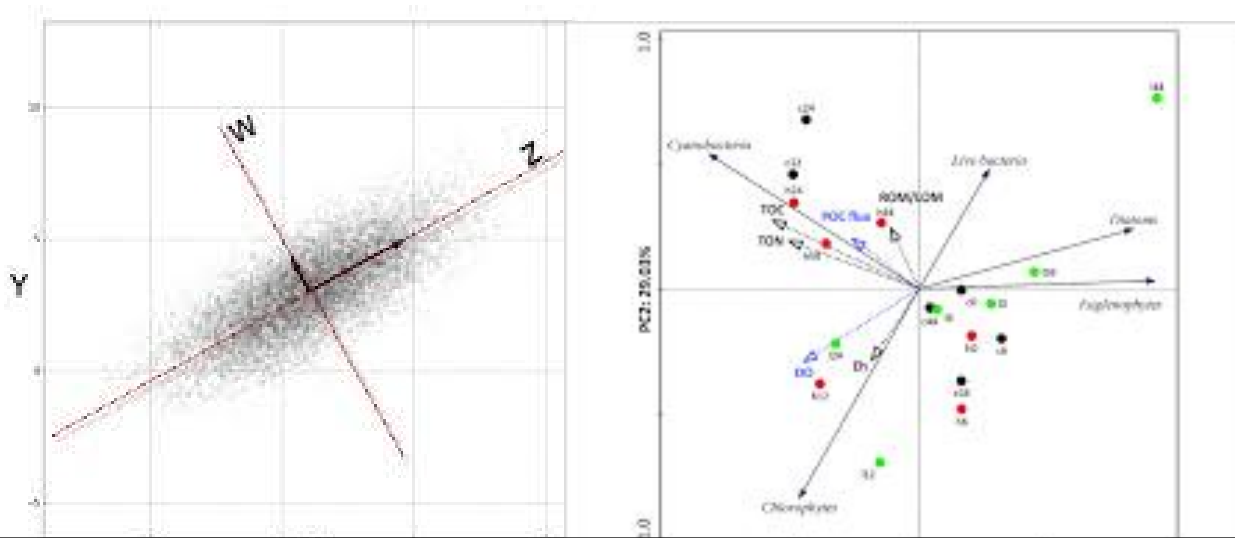
Miguel Sánchez-Islas^{1,*}, Juan Claudio Toledo-Roy^{2,1} and Alejandro Frank^{2,1,3}

¹Centro de Ciencias de la Complejidad, Universidad Nacional Autónoma de México, Mexico

²Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Mexico

³El Colegio Nacional, Mexico City, Mexico

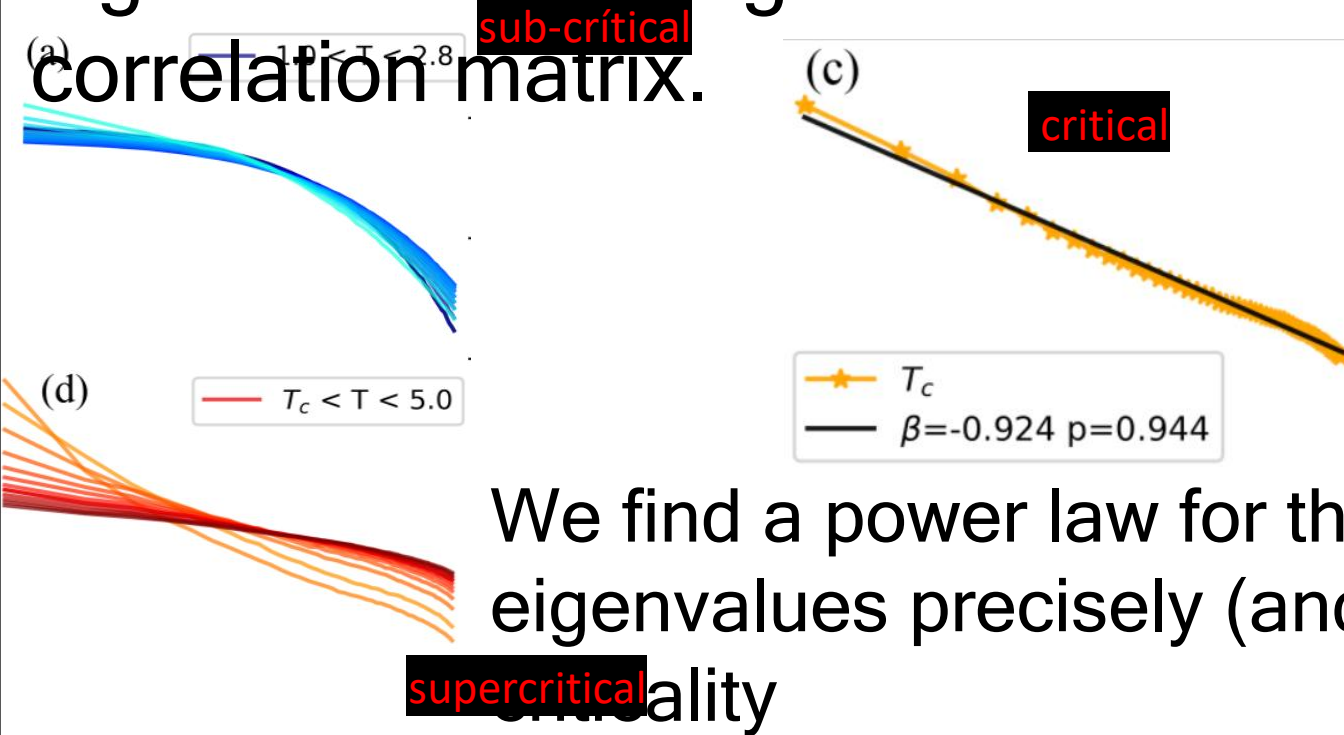
(accepted 11 March 2021; published 6 April 2021)



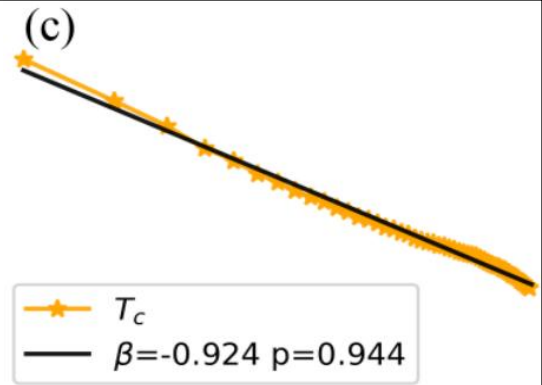
C = Constant of interaction between layers

Multilayer Ising Model

Using PCA we calculate the ordered eigenvalues and eigenfunctions of the correlation matrix.



What is the meaning of this result?



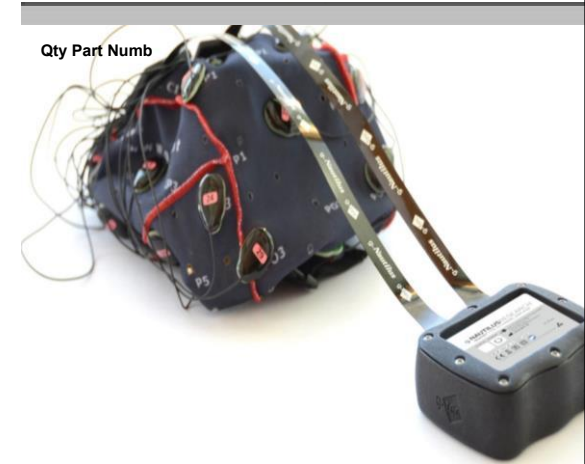
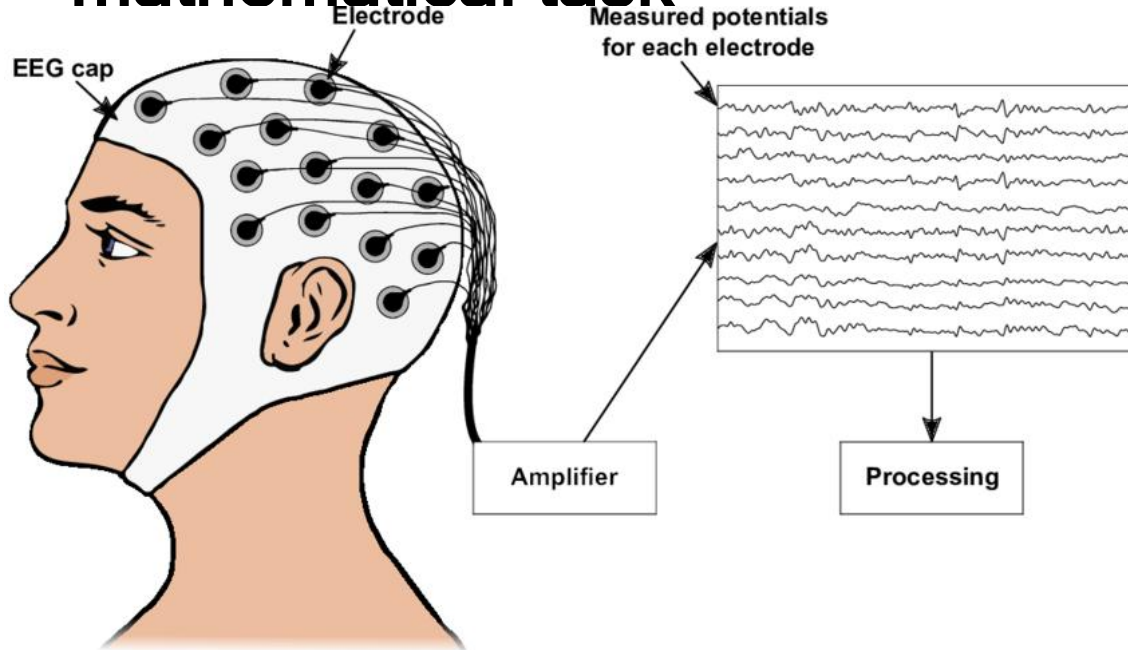
- Our result implies scale invariance of the correlation matrix.

We postulate the following conjectures:

- 1): Given a complex system where n different signals are simultaneously measured, the system is in a critical state if the associated correlation matrix is scale invariant.
- 2): A (symmetric) matrix is scale invariant if its

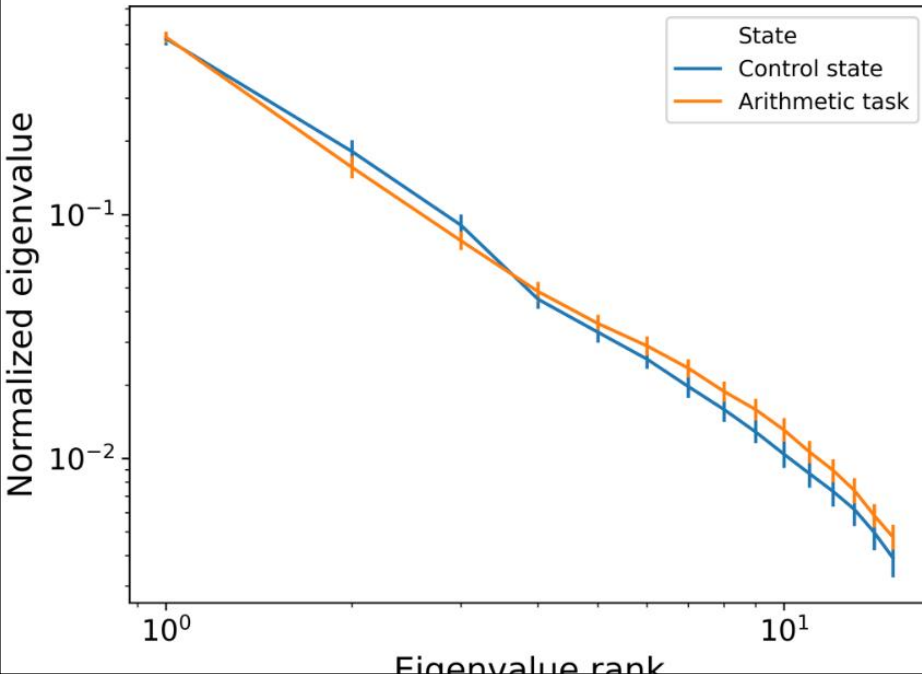
Brain criticality

We tested our methodology by taking eeg's of young people **at rest** or **carrying out a mathematical task**



Criticality of the brain

We find almost exact power laws in both cases,

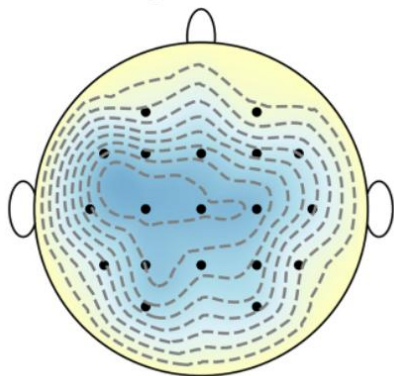


ishable by this
n.

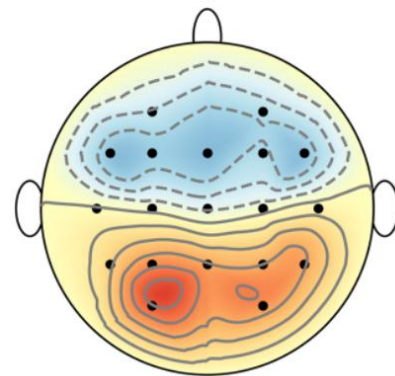
Power laws in
eigenvalues of
brain eeg's

The brain's principal (normal) modes display ~~symmetric~~ modes

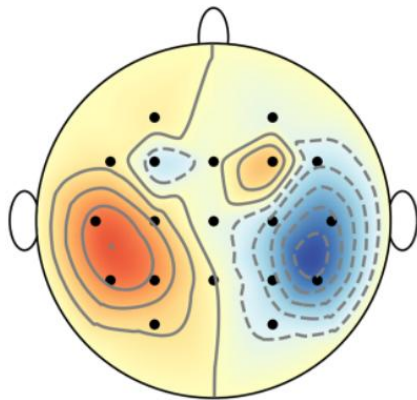
Mode 1:
Average



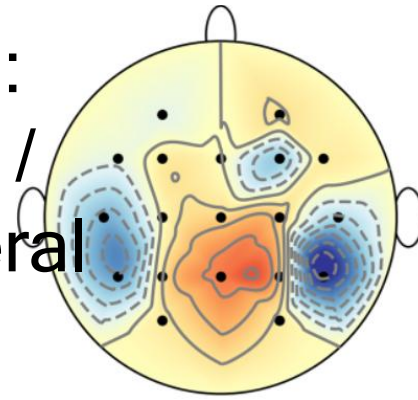
Mode 2:
back/forth



Mode 3:
left/right



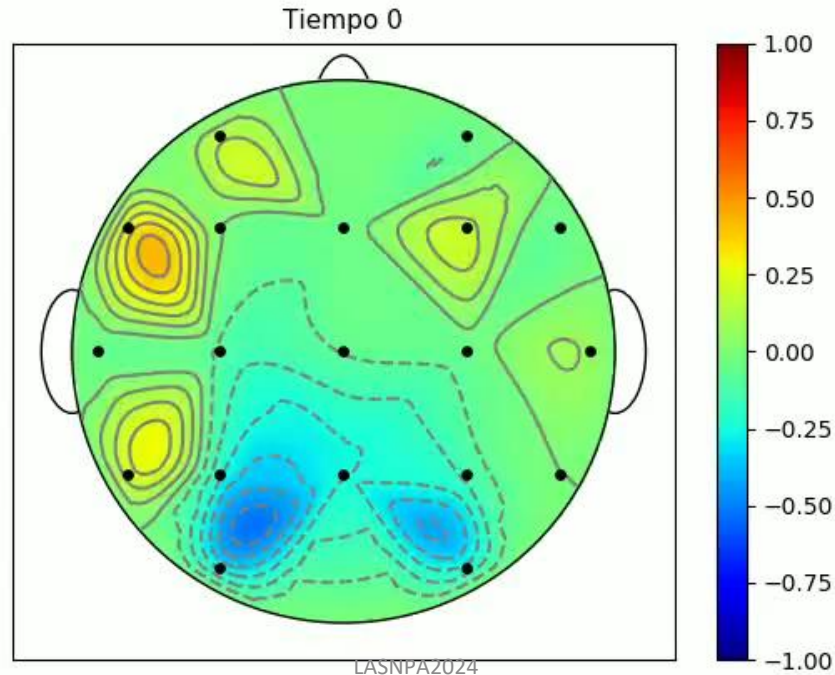
Mode 4:
Central /
peripheral



The critical brain: Time dependent modes

Full EEG of resting brain:

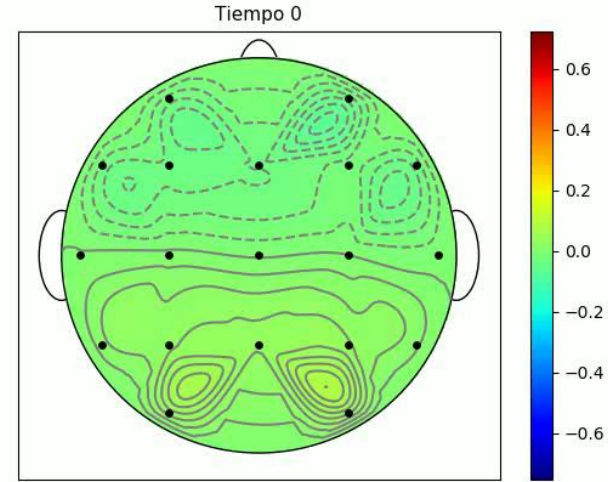
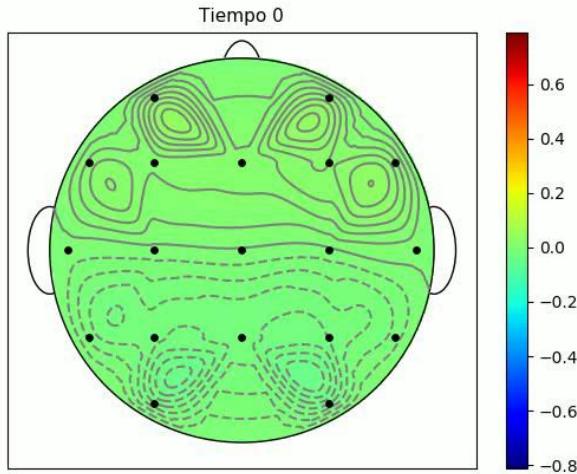
▪



task

Front-Back Mode

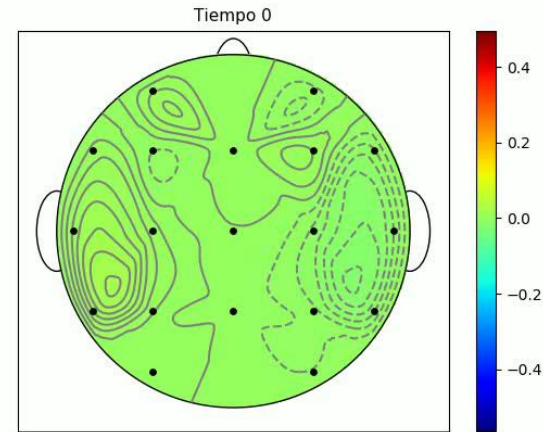
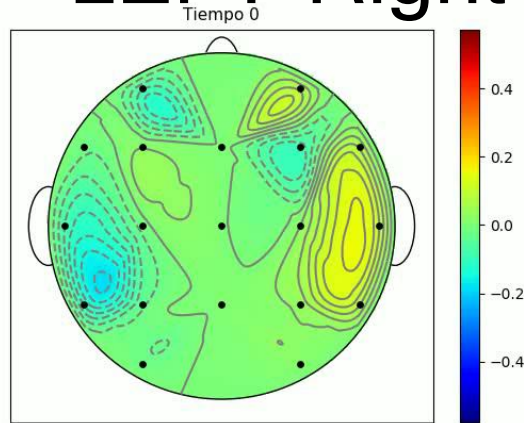
pc2



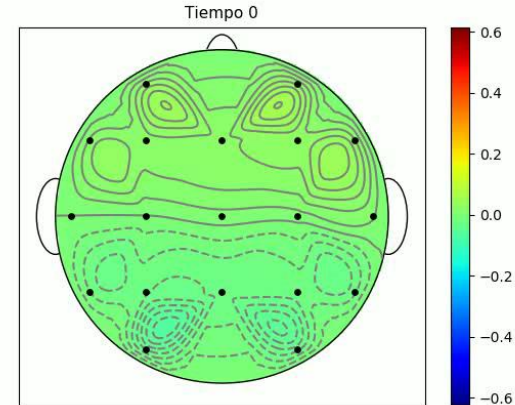
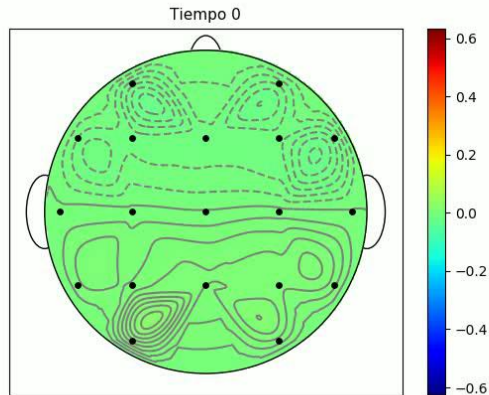
LASNPA2024

Subject no.10 at rest and with task LEFT-Right MODE

pc3



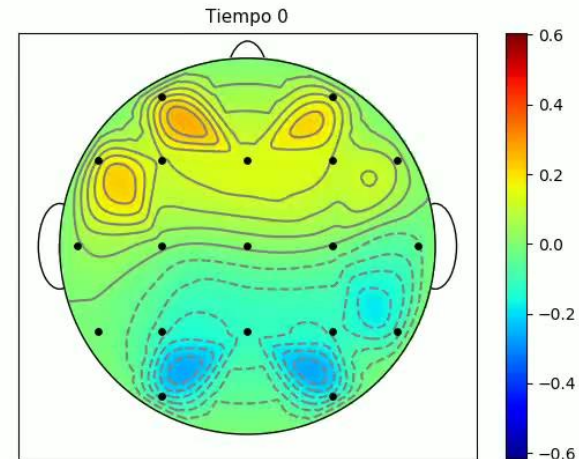
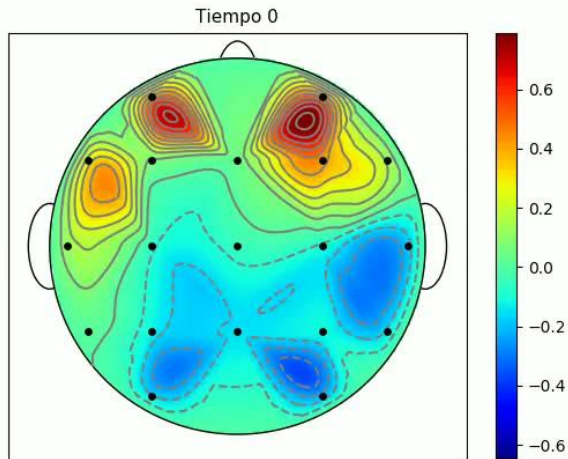
THESE MODES ARE SEEN IN ALL OUR SUBJECT Subject no.12 at rest and with task



Behavior is almost identical between different people.

Critical Brain. SUBJECT 15 , REST AND WITH TASK

► SECOND PRINCIPAL COMPONENT **PC2**



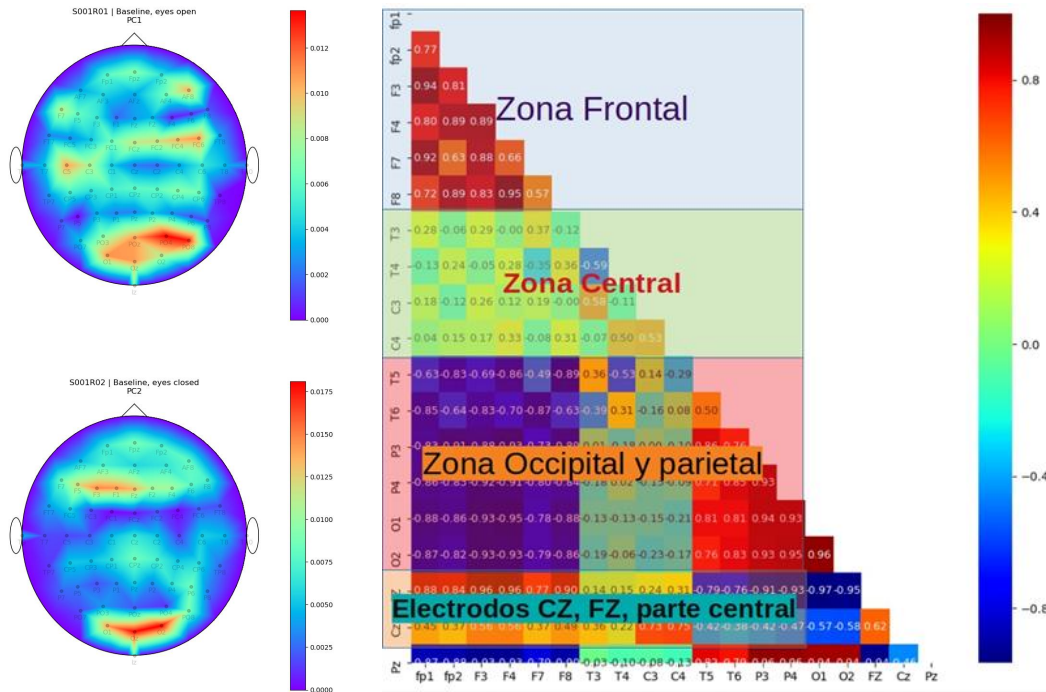
Multicriticality in the brain

PHYSICAL REVIEW E **103**, 042111 (2021)

Criticality in a multisignal system using principal component analysis

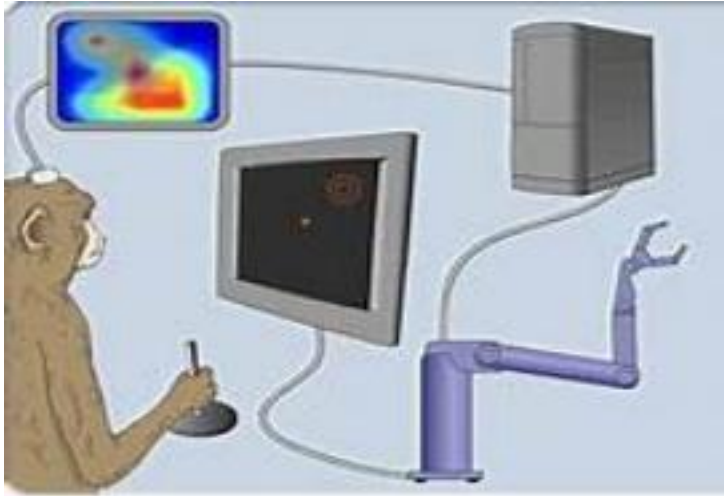
Miguel Sánchez-Islas, Juan Claudio Toledo-Roy, and Alejandro Frank

Lower modes PC4, PC5 can distinguish brain activity



Machine learning algorithm recognizes ~100% of cognitive task in these modes

Brain Modes, Correlation Matrix

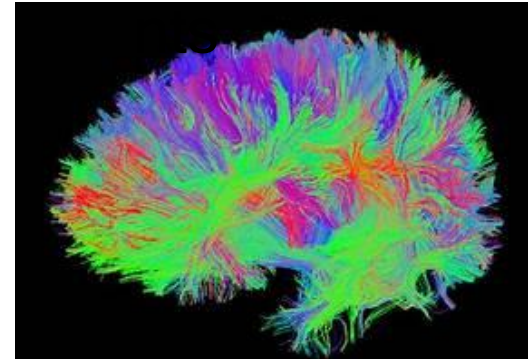


Prof. Ranulfo
Romo

Neurophysiology Laboratory,
Unam, Research on monkey
connectome learning
modifications



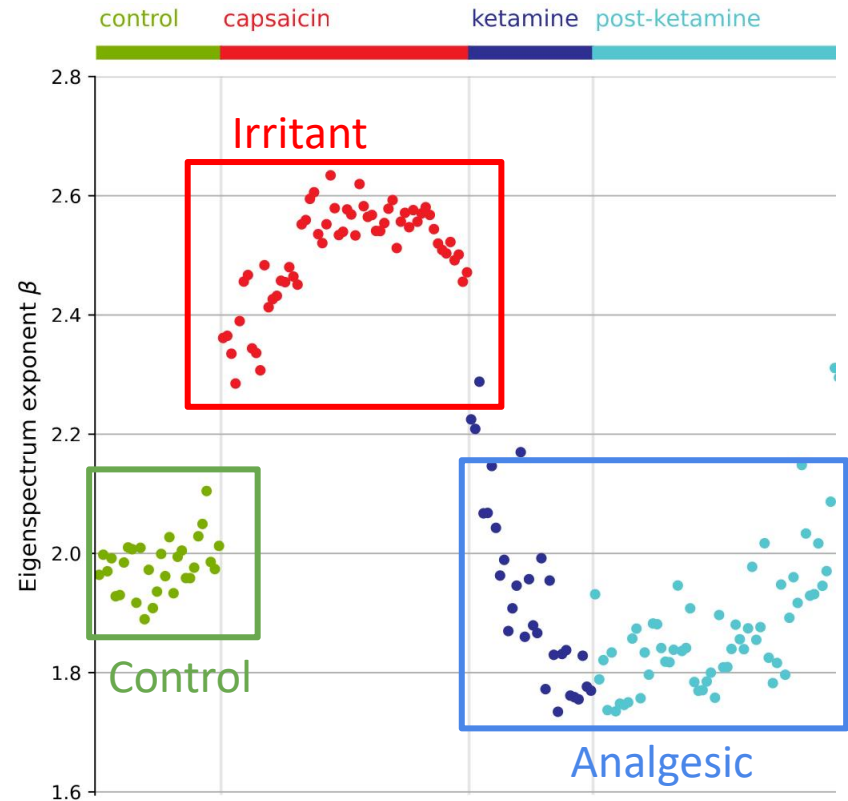
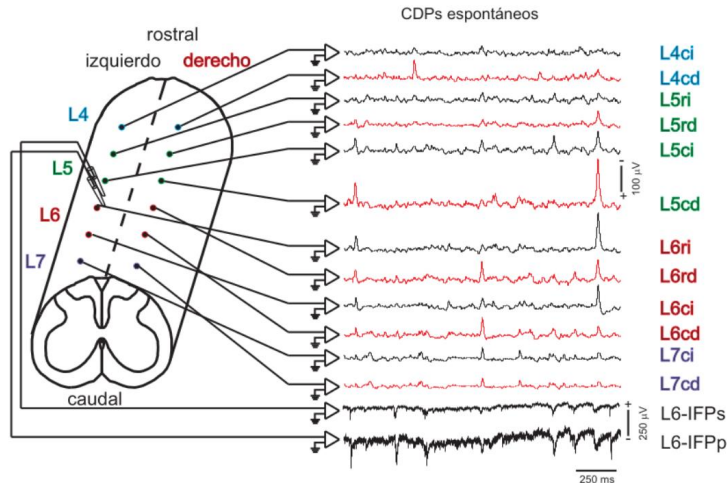
EEG's, NMR,
Laser
Opto-
measureme



Other nervous system experiments

Neurophysiology Lab,
CINVESTAV

Prof, P. Domín,

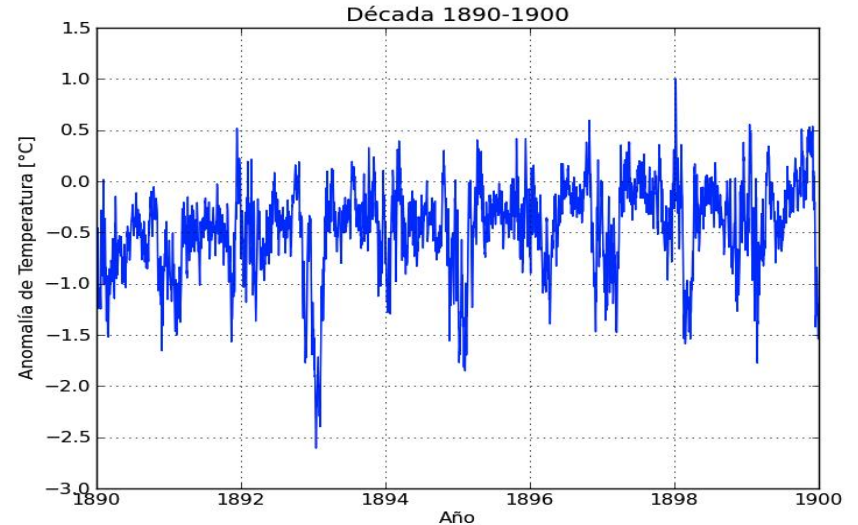
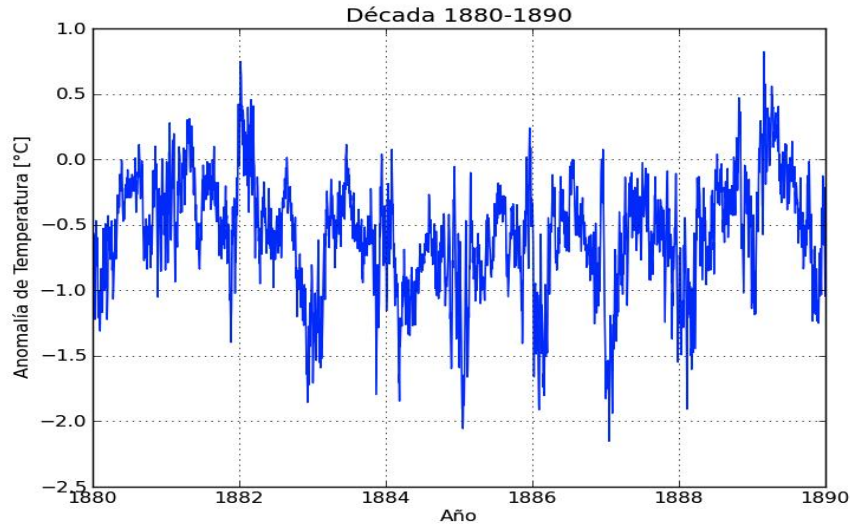
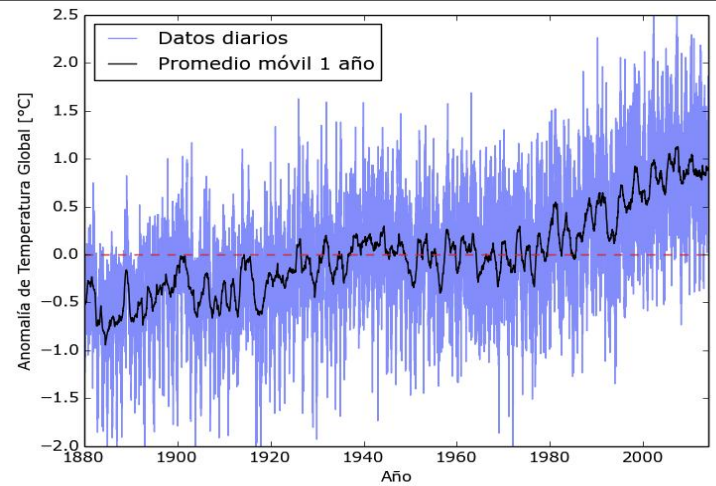


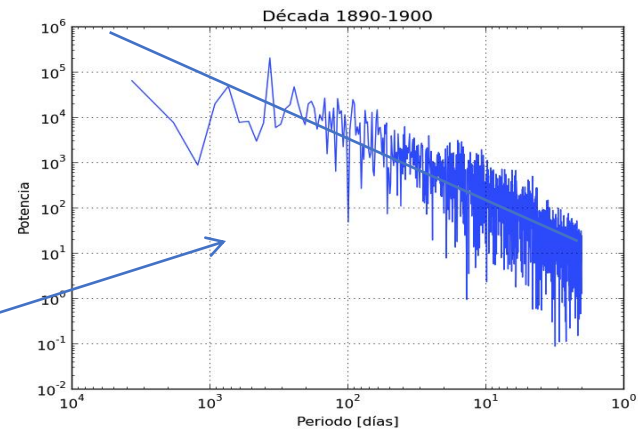
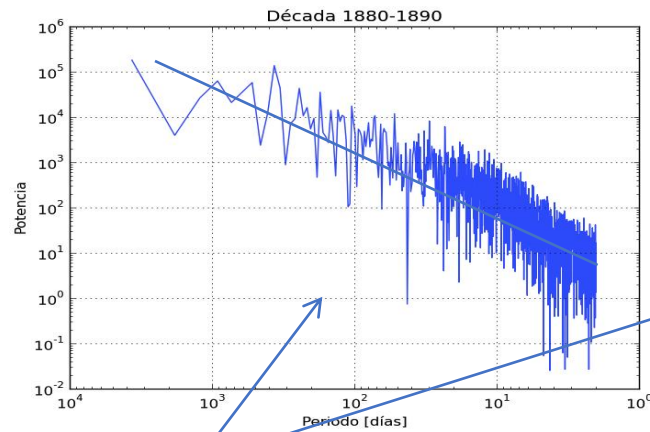
Other signals

Earth's average temperature: 1880-2010

Fluctuation Analysis by decade

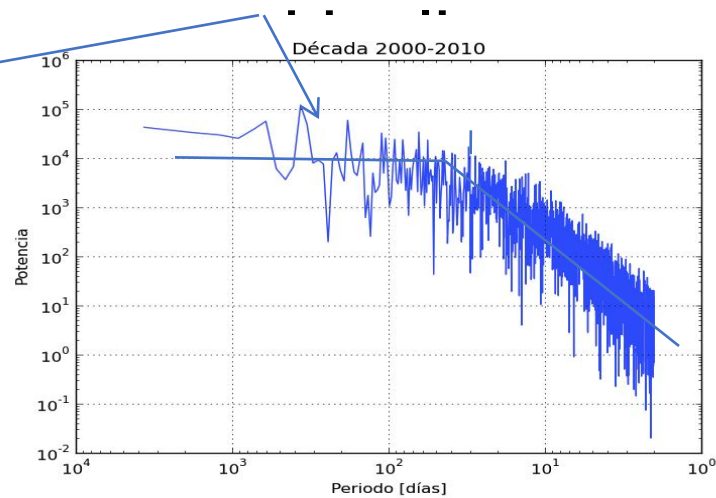
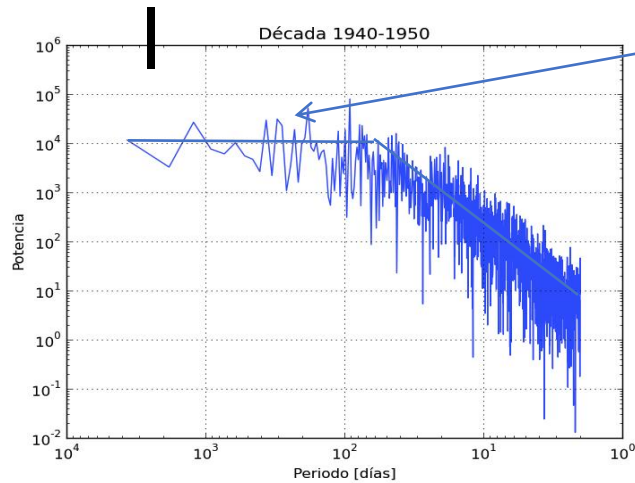
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Critica

Loss of



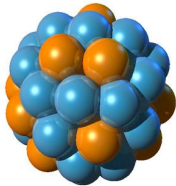
Microscopic systems versus self organized systems (a pond))

Elementary



Dominant Scale

Fundamental Symmetries: The system remains isolated from other scales



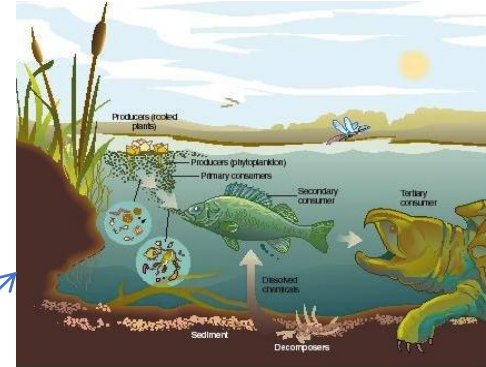
Symmetries= conserved quantities

Multiple scales

Self similarity, scale invariance. Cascading communication among different scales

Criticality=.
Evolutionary equilibrium

Complex

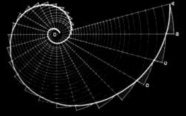


Conclusions:

1. Symmetry ideas have had few applications in the biological domain, particularly beyond the static fractal nature of some living organisms.

2. Self organized biological systems are characterized by and evolve towards critical points and dynamical self similarity.

3. Evolution leads to self similar (or scale invariant) behavior, which signals homeostatic dynamical equilibrium in living organisms. Optimization of



$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618033988749894848204586834365$$

SUMMARY

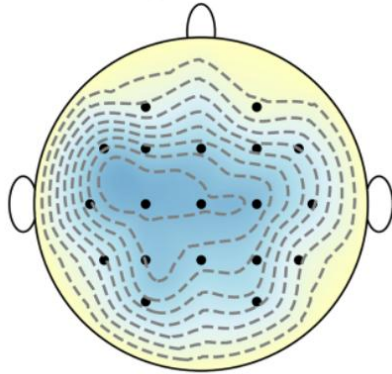
Criticality in biological systems refers to the delicate balance and interplay of various components within living organisms that allows for optimal functioning and adaptability.

The concept of criticality suggests that biological systems operate at a point where they are neither too ordered (rigid) nor too disordered (chaotic), but rather at a state of dynamic equilibrium that enables efficient information processing and response to external stimuli.

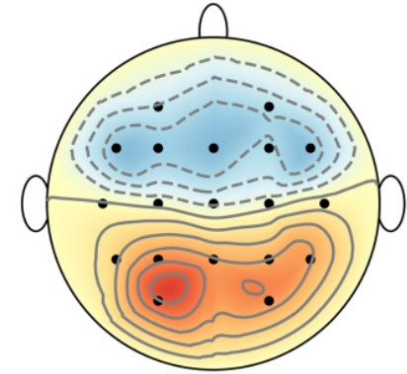
Studying criticality in physical or biological systems with many variables, such as EEGs of the brain, can provide valuable insights into the underlying principles of brain function, cognitive processes, and the mechanisms that govern complex behaviors. By investigating the dynamics of neural activity at criticality, researchers aim to uncover the fundamental principles that govern brain dynamics and contribute to our understanding of brain health, cognition, and neurological disorders.

THANK YOU!

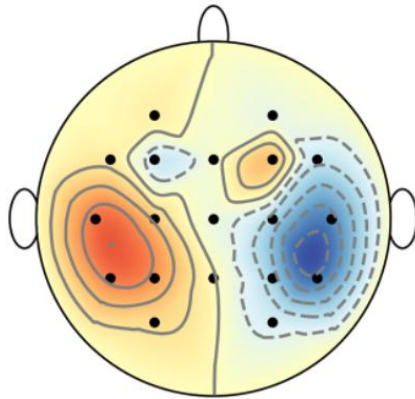
Mode 1:
Average



Mode 2:
back/forth



Mode 3:
left/right



Mode 4:
Central /
peripheral

