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Breakup effects on fusion. Reduction methods and quantum-mechanical methods for the CF and ICF calculation

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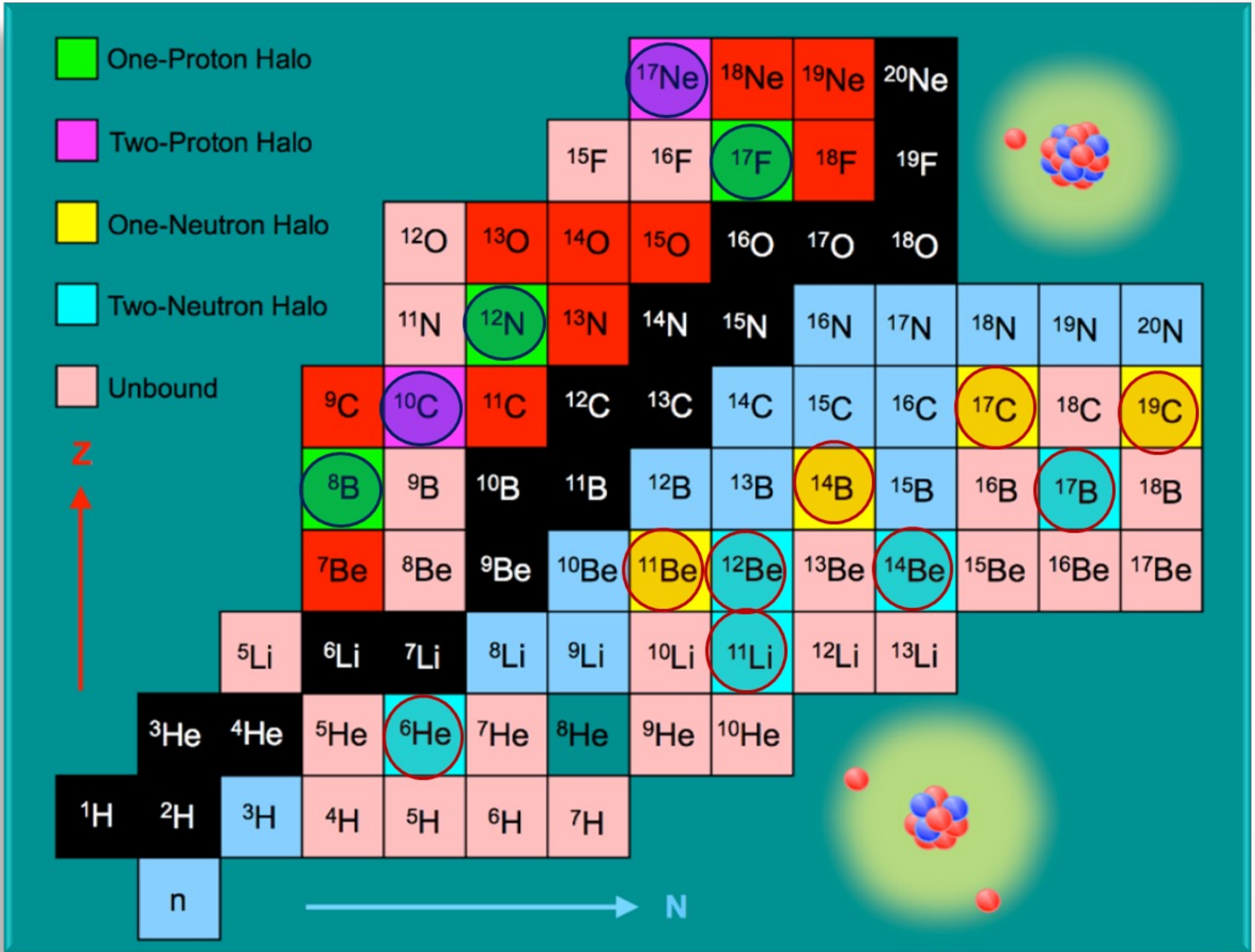
Outline

- Brief introduction
- Reduction of fusion cross section. The fusion function method
- Improved Wong formula and improved fusion function method
- Quantum-mechanical method to derive CF and ICF.
- Conclusions and perspectives

Light exotic nuclei



Investigation of fusion induced by exotic nuclei is important to:

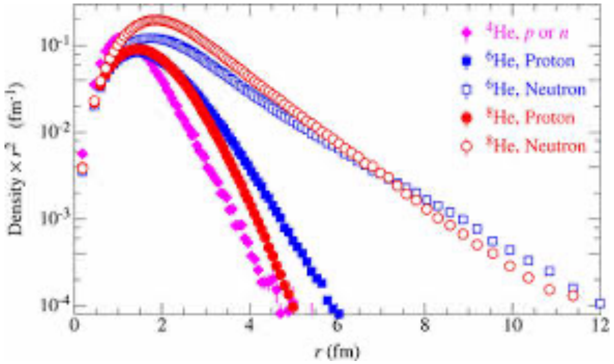
- ❑ Understand the role of nuclear structure in fusion mechanism.
- ❑ Understand the sub-barrier mechanism for astrophysics and superheavy element production.

❑ Exotic nuclei (static effects)	⇨	anomalous feature weakly-bound strong cluster configuration, extended spatial distribution
❑ Exotic nuclei (dynamic effect)	⇨	influence on reaction process couplings continuum states (breakup): Fusion Elastic scattering
❑ Exotic nuclei fusion measurement	⇨	weakly-bound, proton and neutron halos

Light exotic nuclei

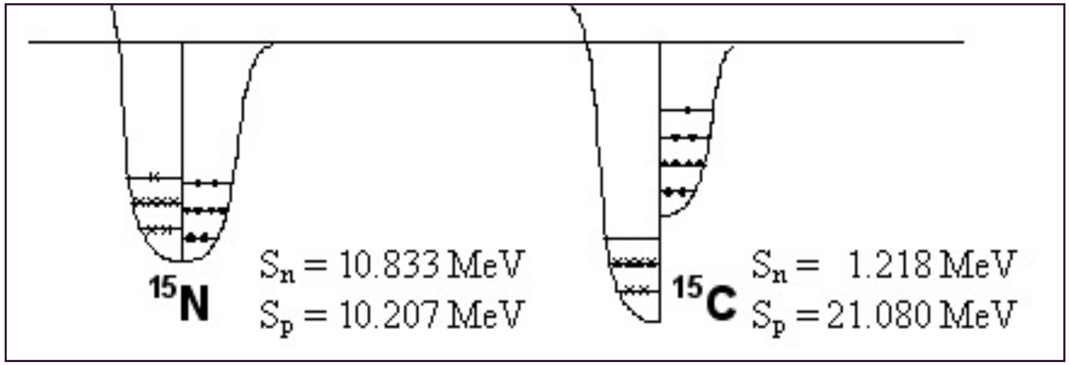
Low binding energy

Extended matter distribution



Nucleon distribution

^{15}N stable **^{15}C exotic, neutron-rich (drip-line)**



Exotic nuclei

^{8}B = ^{7}Be+p	(0.137 MeV)
^{6}He = α + 2n	(0.973 MeV)
^{11}Li = ^{9}Li + 2n	(0.369 MeV)
^{11}Be = ^{10}Be+n	(0.502 MeV)
^{15}C = ^{14}C+n	(1.218 MeV)

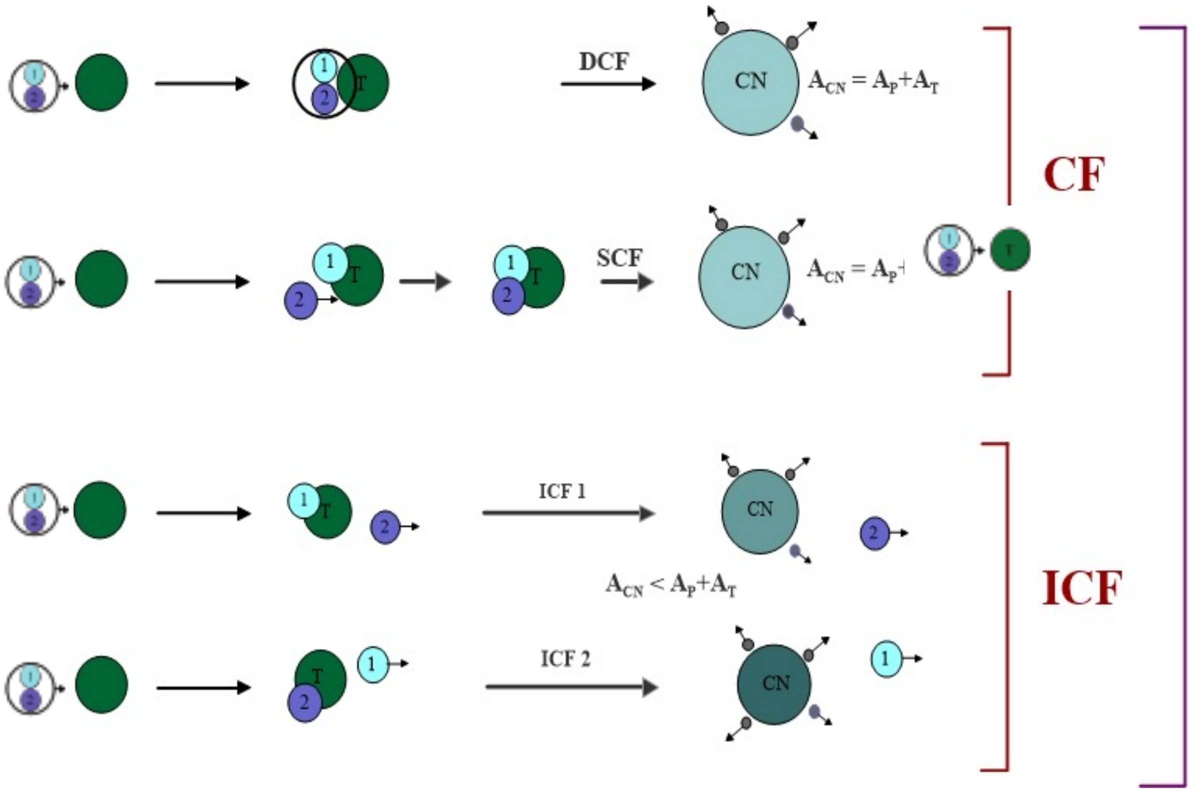
Weakly bound nuclei

^{7}Be = α + ^{3}He	(1.587 MeV)
^{8}He = ^{6}He + 2n	(2.140 MeV)
^{9}Be = ^{8}Be + n	(1.665 MeV)
^{6}Li = α + d	(1.474 MeV)
^{7}Li = α + t	(2.467 MeV)
^{8}Li = ^{7}Li + n	(2.032 MeV)

Tightly bound nuclei

^{16}O = ^{12}C + α	(7.192 MeV)
^{10}B = ^{6}Li + α	(4.461 MeV)
^{11}B = ^{7}Li + α	(8.664 MeV)
^{12}B = ^{11}B+n	(3.370 MeV)
^{10}C = ^{8}Be+2p	(3.821 MeV)

Collisions of weakly nuclei (different fusion processes)



$$\sigma_{\mathrm{CF}} = \sigma_{\mathrm{DCF}} + \sigma_{\mathrm{SCF}}$$

$$\sigma_{\mathrm{TF}} = \sigma_{\mathrm{CF}} + \sigma_{\mathrm{ICF}}$$

$$\sigma_{\mathrm{ICF}} = \sigma_{\mathrm{ICF}_1} + \sigma_{\mathrm{ICF}_2}$$

Other processes: elastic scattering, quasi-elastic scattering, transfer reactions, quasi-fission, deep inelastic, fission, break-up triggered by transfer .

Frequently used procedures to answer “Enhancement or suppression in relation to what?

a) Comparison of data with theoretical predictions.

b) Comparison of data for weakly and tightly bound systems. => a benchmark is mandatory

Effects to be considered

- **Static effects:** longer tail of the optical potential arising from the weakly bound nucleons.

- **Dynamical effects:** strong coupling between the elastic channel and the continuum states representing the break-up channel.

1. Experiment vs. theory

$\Delta\sigma_F \equiv \sigma_F^{exp} - \sigma_F^{theo} \Rightarrow$ 'ingredients' missing in the theory

Theoretical possibilities:

a) Single channel - standard densities

$\Delta\sigma_F$ arises from all static and dynamic effects

b) Single channel - realistic densities

$\Delta\sigma_F$ arises from couplings to all channels

c) CC calculation with all relevant bound channels

$\Delta\sigma_F$ arises from continuum couplings

d) CDCC

no deviation expected. Details of the reaction mechanism can be studied

2. Compare with σ_F of a similar tightly bound system

1. Gross dependence on size and charge:

Z_P, Z_T, A_P, A_T – affects V_B and R_B

$$V_B \sim Z_P Z_T e^2 / R_B; \quad \sigma_{geo} \sim \pi R_B^2, \quad R_B \propto (A_P^{1/3} + A_T^{1/3})$$

Fusion data reduction required !

Fusion functions $F(x)$ (our reduction method)

$$E \rightarrow x = \frac{E - V_B}{\hbar\omega} \quad \text{and} \quad \sigma_F^{\exp} \rightarrow F_{\exp}(x) = \frac{2E}{\hbar\omega R_B^2} \; \sigma_F^{\exp}$$

The São Paulo potential is used to determine barrier parameters
Inspired in Wong's approximation

$$\sigma_F^W = R_B^2 \, \frac{\hbar\omega}{2E} \, \ln\left[1 + \exp\left(\frac{2\pi\left(E - V_B\right)}{\hbar\omega}\right)\right]$$

$$\text{If} \quad \sigma_{\mathrm{F}}^{\exp} = \sigma_{\mathrm{F}}^{\mathrm{W}} \quad \Rightarrow \quad F(x) = F_0(x) = \ln\left[1 + \exp\left(2\pi x\right)\right]$$

***$F_0(x)$* = Universal Fusion Function (UFF)**

system independent !

Direct use of the reduction method

Compare $F_{\rm exp}(x)$ with UFF for x values where $\sigma_{\rm F}^{\rm opt} = \sigma_{\rm F}^{\rm W}$

Deviations are due to couplings with bound channels and breakup

Refining the method

Eliminate the failure of the Wong model for light systems at sub-barrier energies

Eliminate influence of couplings with bound channels

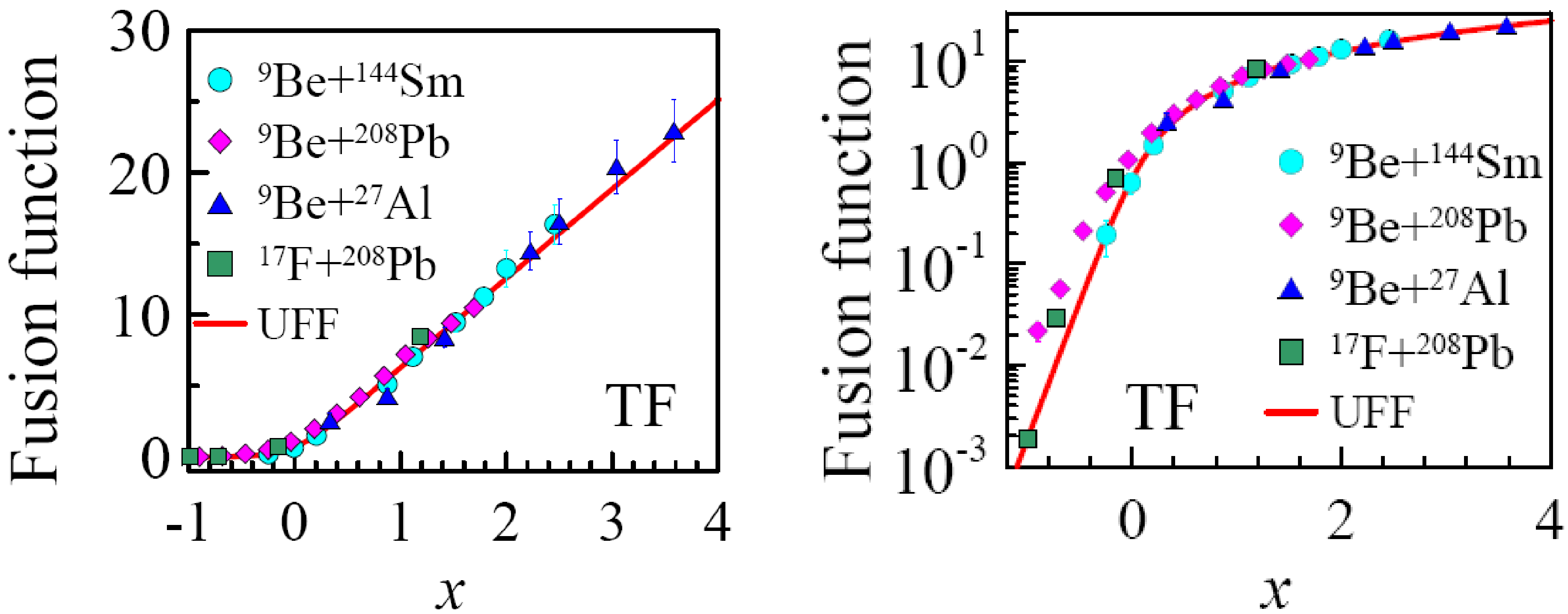
Renormalized fusion function

$$F_{\rm exp}(x) \rightarrow \bar{F}_{\rm exp}(x) = \frac{F_{\rm exp}(x)}{R(x)}, \quad \text{with } R(x) = \frac{\sigma_{\rm F}^{\rm CC}}{\sigma_{\rm F}^{W}} = \frac{\sigma_{\rm F}^{\rm CC}}{\sigma_{\rm F}^{\rm opt}}$$

If CC calculation describes data $\rightarrow \bar{F}_{\rm exp} = {\rm UFF}$

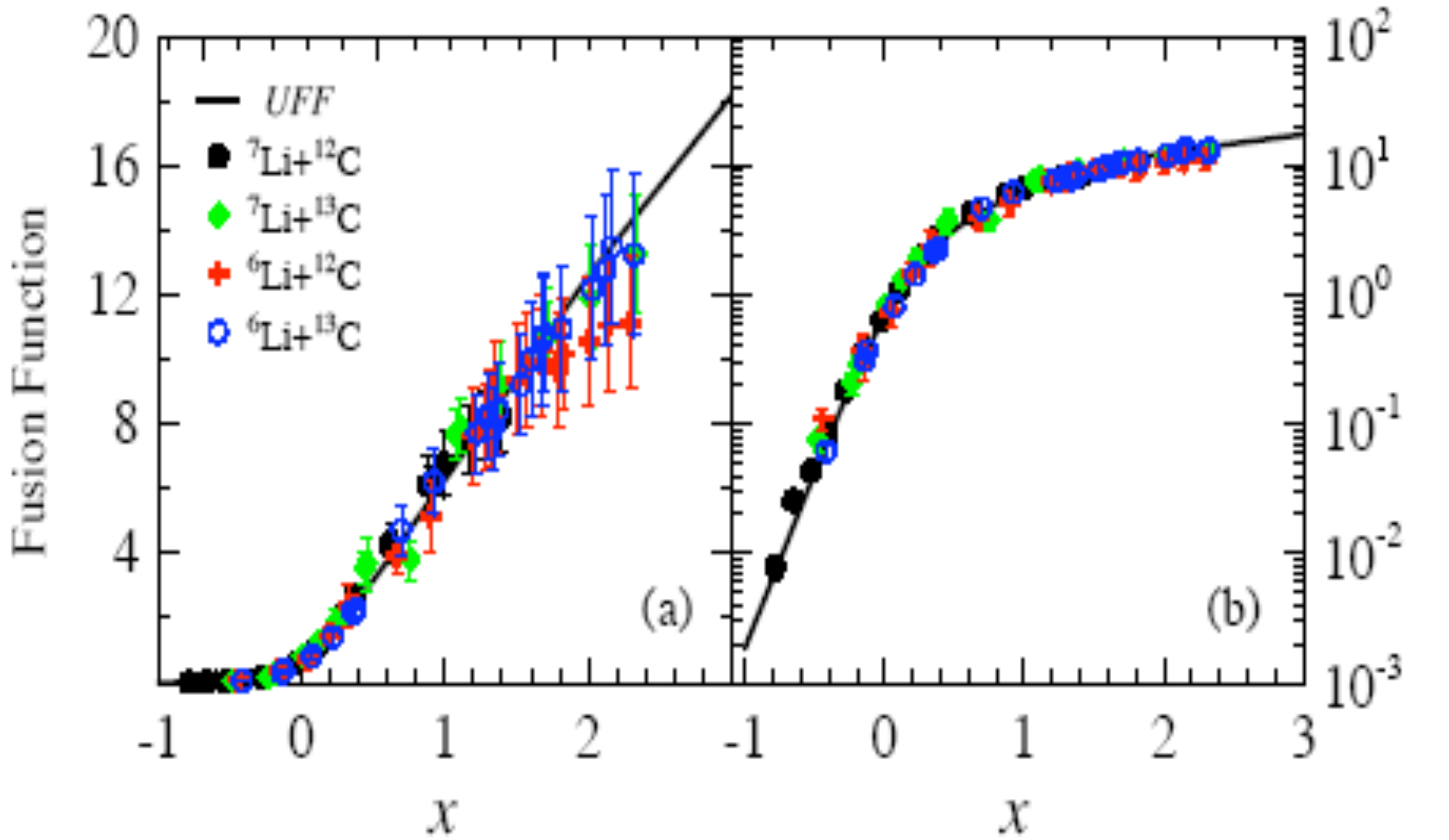
L.F. Canto et al. JPG 36,015109 (2009) & NPA 821 (2009) 51

Use of UFF for investigating the role of BU + transfer dynamical effects on the **total fusion of heavy** weakly bound systems



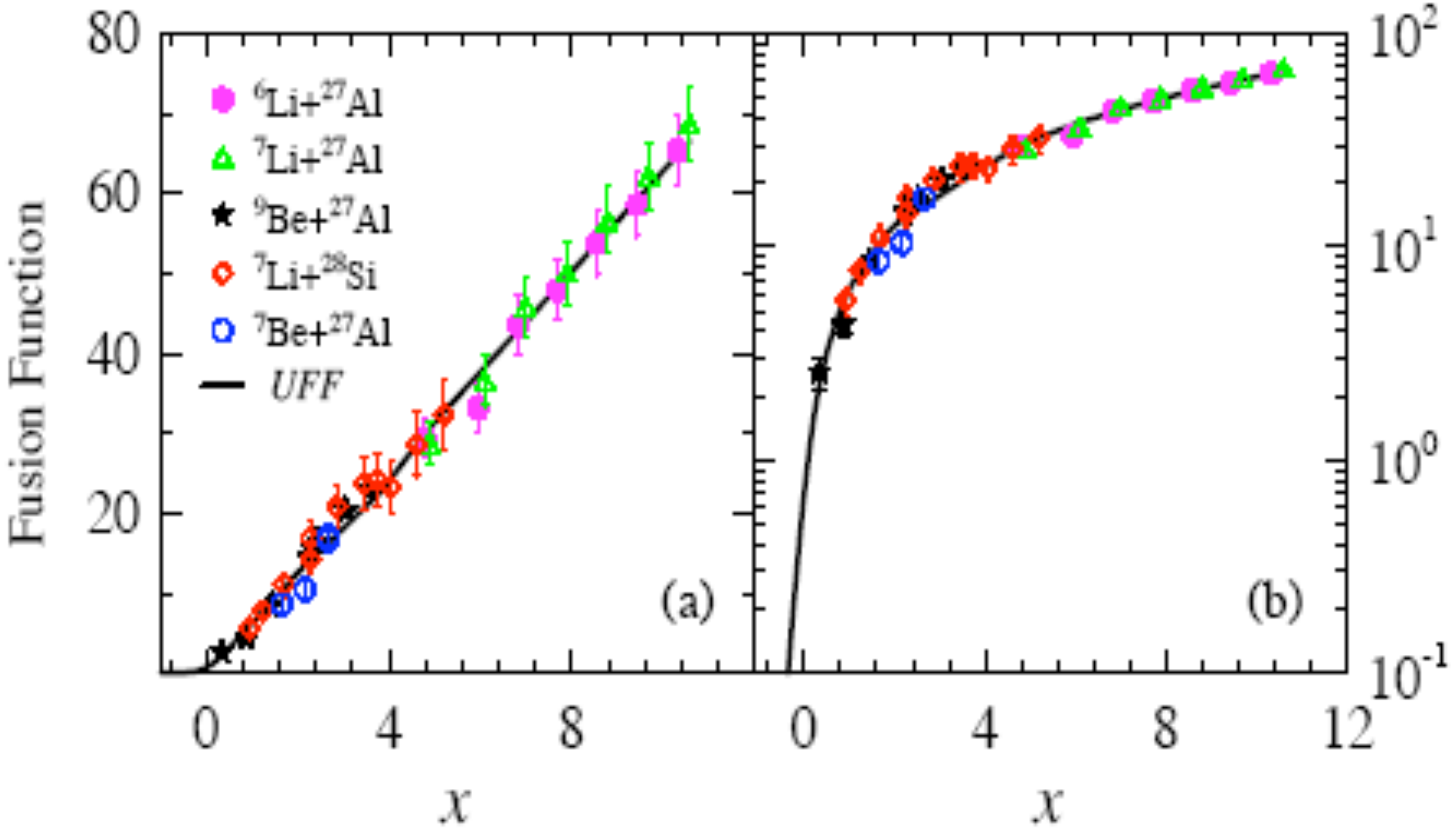
No effect above the barrier- enhancement below the barrier

Use of UFF for investigating the role of BU + transfer dynamical effects on the **total fusion of very light** weaklv bound svstems



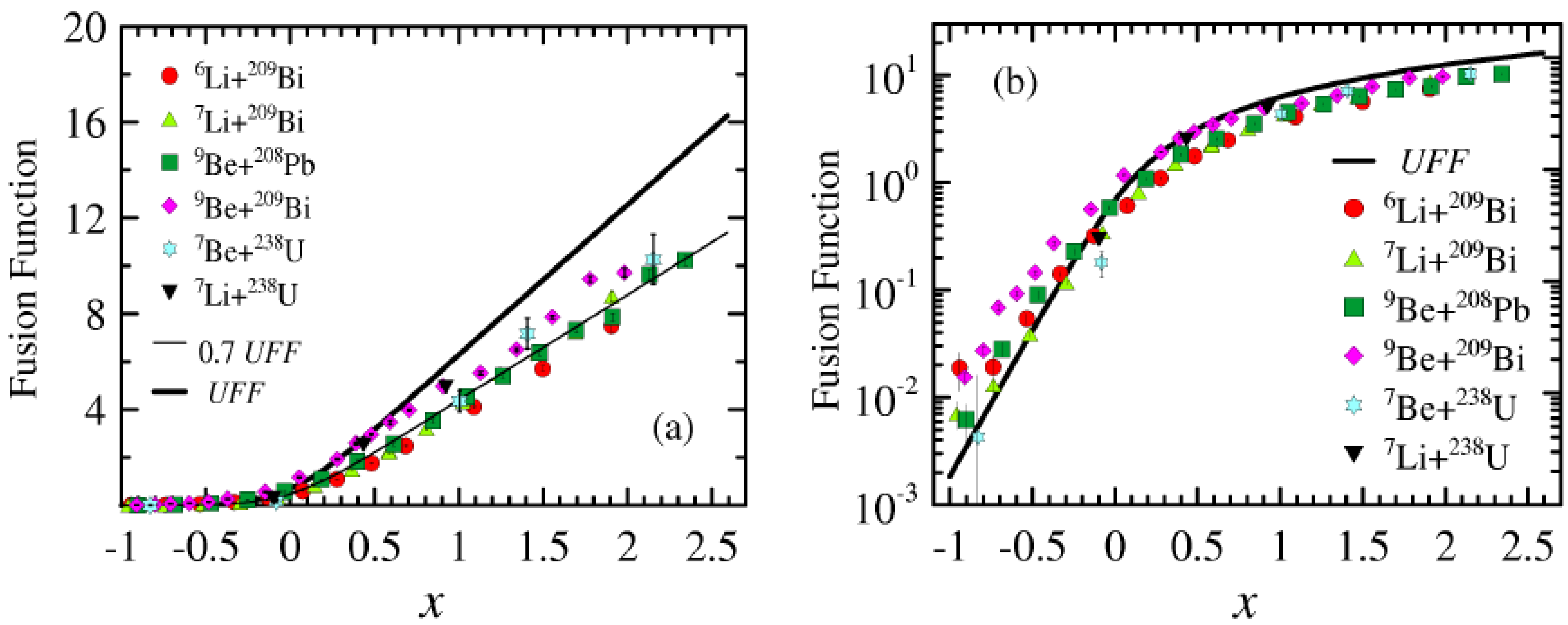
No effect above the barrier- almost no data below the barrier

Use of UFF for investigating the role of BU + transfer dynamical effects on the **total fusion of light** weakly bound svstems



No effect above the barrier- no data below the barrier

Use of UFF for investigating the role of BU + transfer dynamical effects on the complete fusion of stable weakly bound heavy systems



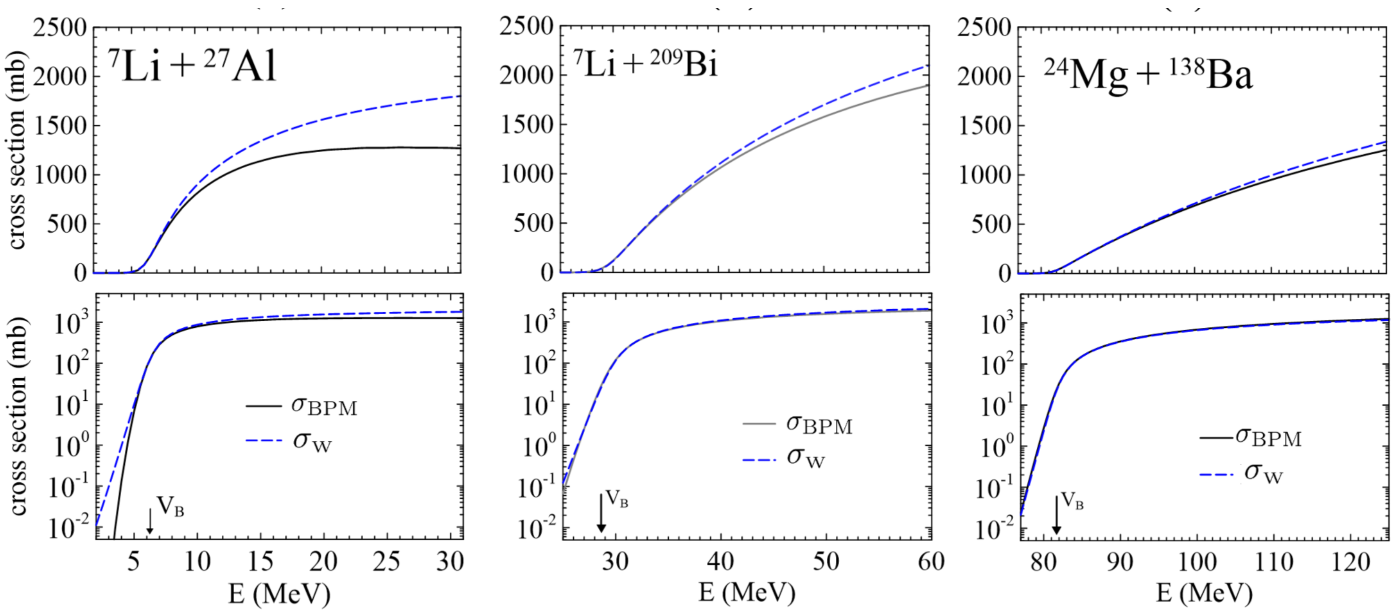
We did not include any resonance of the projectiles in CC calc.

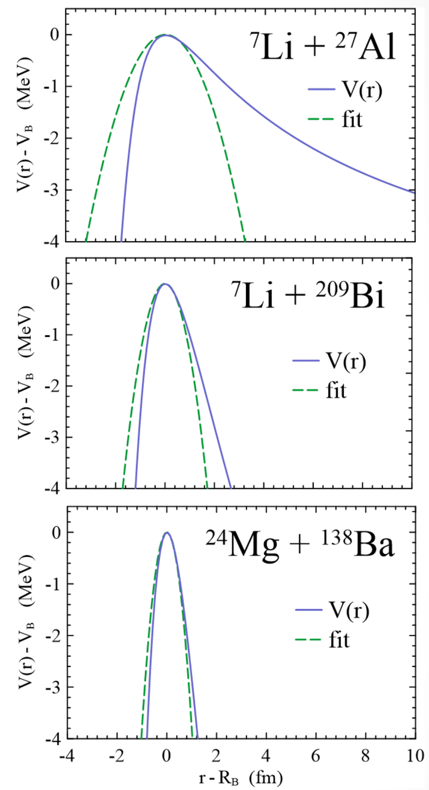
Suppression above the barrier- enhancement below the barrier

Improvement of Wong formula for fusion cross section

$$\sigma_W = \frac{\hbar\omega R_B^2}{2E} \ln\left\{1 + \exp\left[\frac{2\pi}{\hbar\omega}(E - V_B)\right]\right\}$$

Wong formula





Wong formula does not work properly above and below V_B especially for light systems

Reason: parabolic approximation is not good for ligth systems

Improvement of Wong formula for fusion cross section

$$\sigma_{\mathrm{W}}^{\mathrm{cl}}(E) = \pi R_{\mathrm{B}}^2 \left(1 - \frac{V_{\mathrm{B}}}{E}\right) \text{ for } E \geqslant V_{\mathrm{B}}$$

Wong formula (classical limit above V_B

System	$E_{\max}$ (MeV)	λ_{g}	R_{g}	R_{B}	$R_{\mathrm{g}}^2/R_{\mathrm{B}}^2$
$^{7}\mathrm{Li} + {}^{27}\mathrm{Al}$	36	20	6.2	8.5	0.53
$^{7}\mathrm{Li} + {}^{209}\mathrm{Bi}$	60	34	10.4	11.4	0.83
$^{24}\mathrm{Mg} + {}^{138}\mathrm{Ba}$	112	59	10.6	11.1	0.91

V_B slightly depend on ℓ

R_B stronly depend on ℓ

$$\sigma_{\mathrm{W}}^{\mathrm{g}} = \frac{\hbar\omega_{\mathrm{g}} R_{\mathrm{g}}^2}{2E} \ln\left\{1 + \exp\left[\frac{2\pi}{\hbar\omega_{\mathrm{g}}}(E - V_{\mathrm{B}})\right]\right\}$$

Rowley and Hagino PRC 91, 044615 (2015)

Overestimates the role of λ_g ($\lambda = \ell + ½$)

As the Hamiltonian is proporcional to λ^2 we assume $R_\lambda \simeq R_{\mathrm{B}} - \gamma\lambda^2$

We define the effective barrier radius as $\overline{R} = \langle R_\lambda \rangle_\lambda = \frac{1}{N}\int_0^{\lambda_{\mathrm{g}}} 2\lambda d\lambda [R_{\mathrm{B}} - \gamma\lambda^2]$

$$N = \int_0^{\lambda_{\mathrm{g}}} 2\lambda d\lambda$$

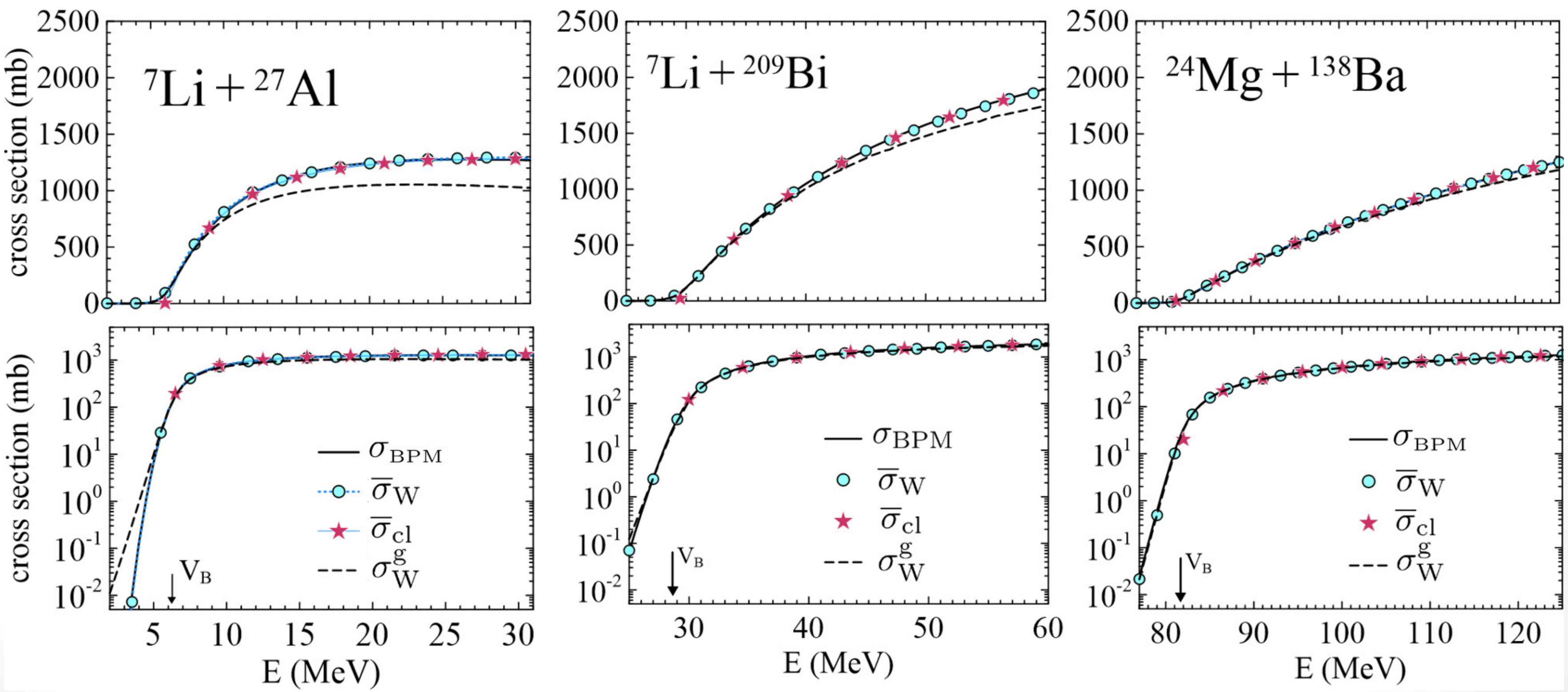
We get: $\overline{R} = R_{\rm B} - \gamma \dfrac{\lambda_{\rm g}^2}{2}$ and $\lambda_{\rm rms} = \sqrt{\langle \lambda^2 \rangle_\lambda} = \dfrac{\lambda_{\rm g}}{\sqrt{2}}$

$\overline{R} = R_{\lambda_{\rm rms}}$ and $\hbar\overline{\omega} = \langle \hbar\omega_\lambda \rangle_\lambda = \hbar\omega_{\lambda_{\rm rms}}$

So we do not need to determine γ

$$\overline{\sigma}_{\rm W} = \frac{\hbar\bar{\omega}\overline{R}^2}{2E} \ln\left\{1 + \exp\left[\frac{2\pi}{\hbar\bar{\omega}}(E - V_{\rm B})\right]\right\}$$

Modified Wong formula
L.F. Canto et al. PRC **109**, 054609 (2024)

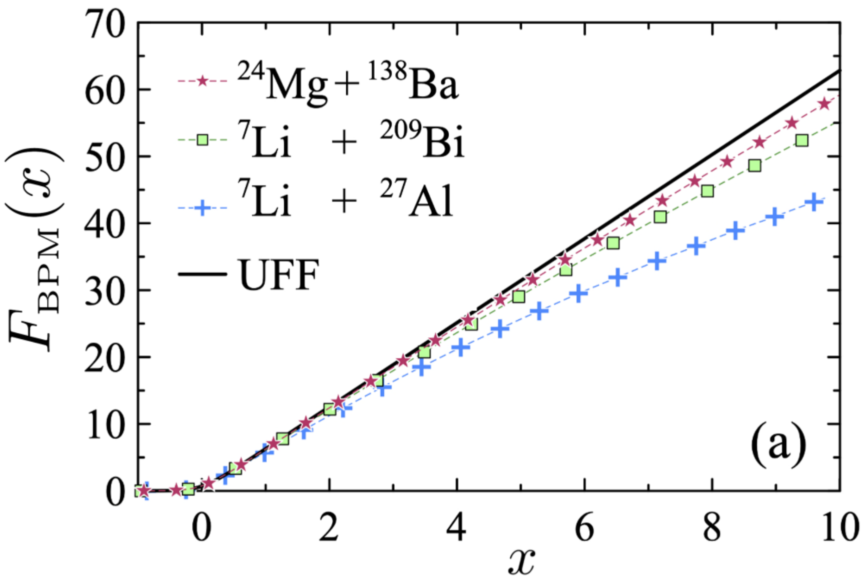


What about redution methods?

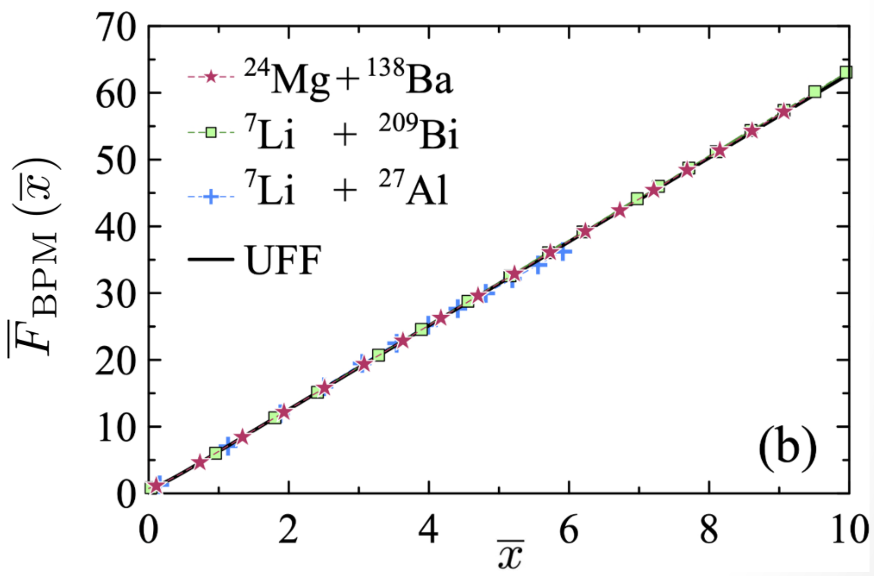
UFF method: $E \rightarrow x = \dfrac{E - V_B}{\hbar\omega}, \quad \sigma_F \rightarrow F(x) = \dfrac{\sigma_F}{\sigma_0} \quad ; \qquad \sigma_0 = \dfrac{\hbar\omega R_B^2}{2E}$

Improved fusion funciton (IFF) method: $E \rightarrow \bar{x} = \dfrac{E - V_B}{\hbar\bar{\omega}}, \quad \sigma_F \rightarrow \bar{F}(\bar{x}) = \dfrac{\sigma_F}{\bar{\sigma}_0}$

$$\bar{\sigma}_0 = \frac{\hbar\bar{\omega}\bar{R}^2}{2E}$$



UFF



IFF

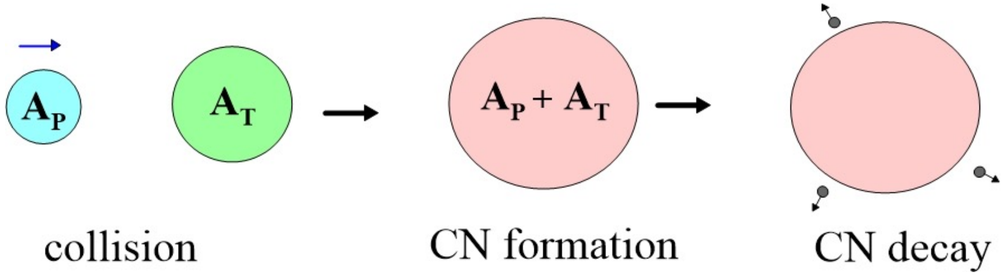
The IFF method can be used to study of the effect of any channel on fusion

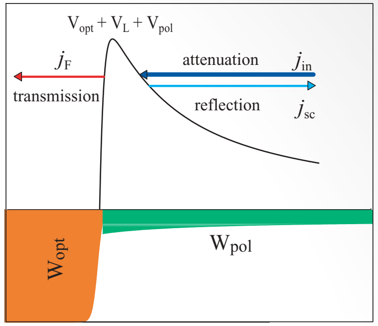
J. Lubian, XIV LASNPA2024, Mexico

Conclusions

- We proposed the UFF reduction method. It has a universal function as a benchmark.
- The CF is enhanced below V_B and hindered above V_B for the reactions of weakly bound projectiles with heavy targets
- The Wong formula was improved to consider the angular momentum dependence of barrier parameters.
- The improved fusion method was introduced that allows to study of the effect of any reaction channel on fusion cross section.

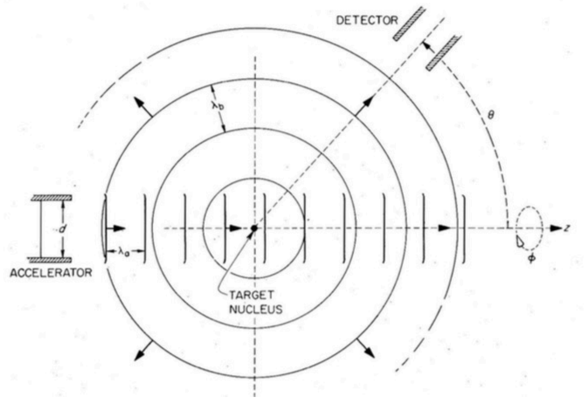
Fusion of tightly bound nuclei





Theoretical estimation

- Potential Scattering approach
- Fusion is not a channel



$$\sigma_{\mathrm{F}}(E) = \frac{\pi}{K^2} \sum_{l=0} (2l+1)\, P_l^{\mathrm{F}}(E)$$

$$P_l^{\mathrm{F}}(E) = 1 - |S_l(E)|^2$$

- Alternative version (from continuity equation)

$$V(R) = U(R) - iW^F(R)$$

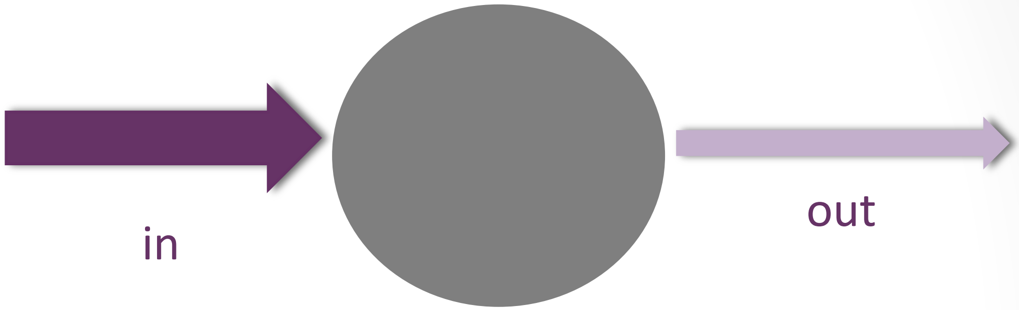
$$\sigma_{\mathrm{F}} = \frac{k}{E} \langle \psi | W^F | \psi \rangle$$

Theoretical estimation

- Potential Scattering approach (almost never works)

Alternative?

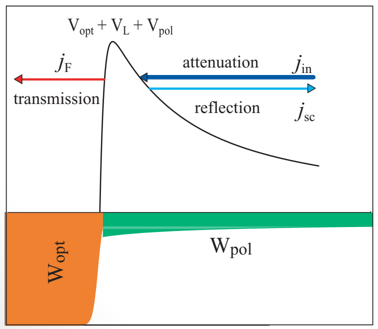
- Coupled Channels (inelastic, rearrangement)



$$\Big[E_\alpha - H_\alpha\Big]\psi_\alpha^{(+)} = \sum_{\alpha' \neq \alpha} V_{\alpha\alpha'}\ \psi_{\alpha'}^{(+)}, \quad \alpha, \alpha' = 1, N$$

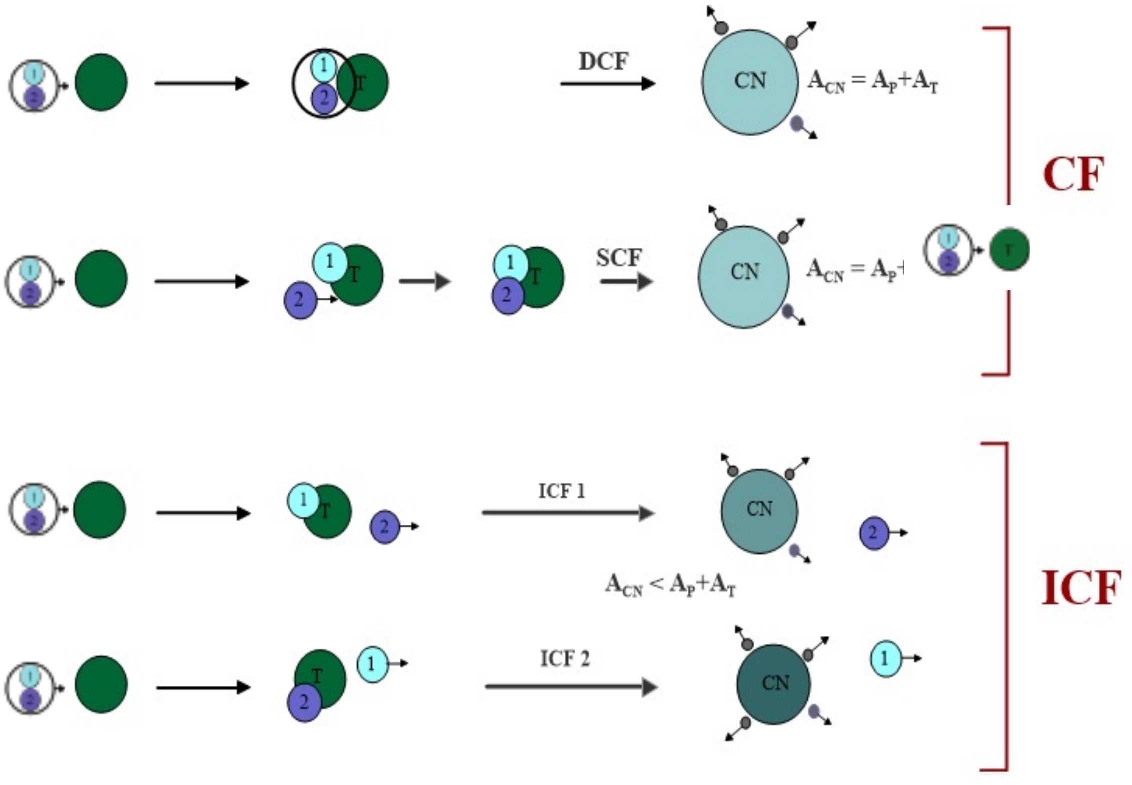
$$\Rightarrow \quad \sigma_{\mathrm{F}} = \frac{K}{E} \sum_{\alpha\alpha'=1} \langle \psi_\alpha | W_{\alpha\alpha'} | \psi_{\alpha'} \rangle$$

$$\sigma_{\mathrm{R}} = \frac{\pi}{k^2} \sum_l (2l+1)\ \Big[1 - |S_{0l}(E)|^2\Big]$$



$$\Rightarrow \quad \sigma_{\mathrm{F}} = \sigma_{\mathrm{R}} - \sum_{\alpha \neq 0} \sigma_\alpha$$

Collisions of weakly nuclei (different fusion processes)



$$\sigma_{\rm CF} = \sigma_{\rm DCF} + \sigma_{\rm SCF}$$

$$\sigma_{\rm TF} = \sigma_{\rm CF} + \sigma_{\rm ICF}$$

$$\sigma_{\rm ICF} = \sigma_{\rm ICF_1} + \sigma_{\rm ICF_2}$$

Other processes: elastic scattering, quasi-elastic scattering, transfer reactions, quasi-fission, deep inelastic, fission, break-up triggered by transfer .

Finding CF and ICF cross section is a great challenge (both for experimentalists and theorists)

Experiment:

- σ_{CF} absorption of all projectile charge (^{11}Be = ^{10}Be +n)
- Most experiments determine only σ_{TF}
- Individual σ_{CF} and/or σ_{ICF} have been measured for some particular stable and radioactive P-T combinations:

Some examples:

Stable

^{6}Li: B = 1.47 MeV $(\alpha + d)$

^{7}Li: B = 2.45 MeV $(\alpha + t)$

^{9}Be: B = 1.65 MeV $(^{8}\mathrm{Be} + n)$

$^{6,7}\mathrm{Li} + {}^{209}\mathrm{Bi},\ {}^{159}\mathrm{Tb},\ {}^{197}\mathrm{Au},\ {}^{124}\mathrm{Sn}.$

$^{9}\mathrm{Be} + {}^{208}\mathrm{Pb},\ {}^{197}\mathrm{Au}$

radioactive

^{6}He: B = 0.973 MeV α + 2n

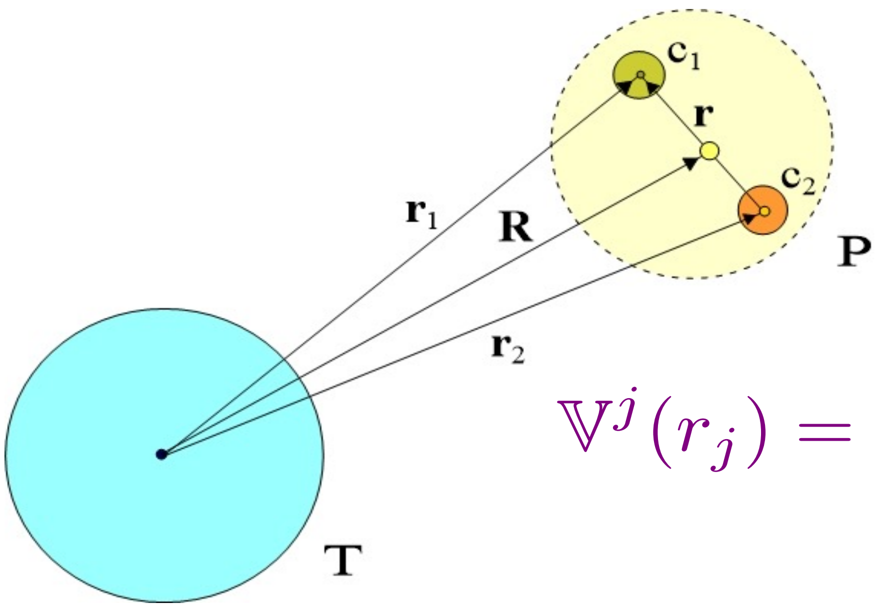
^{11}Be: B = 0.502 MeV ^{10}Be+n

$^{6}\mathrm{He} + {}^{209}\mathrm{Bi},\ {}^{238}\mathrm{U}$

$^{11}\mathrm{Be} + {}^{209}\mathrm{Bi}$

Theory (quantum mechanic):

Projectiles of two-fragment



$$\mathbb{V}(\mathbf{r},\mathbf{R}) = \mathbb{V}^1(r_1) + \mathbb{V}^2(r_2)$$

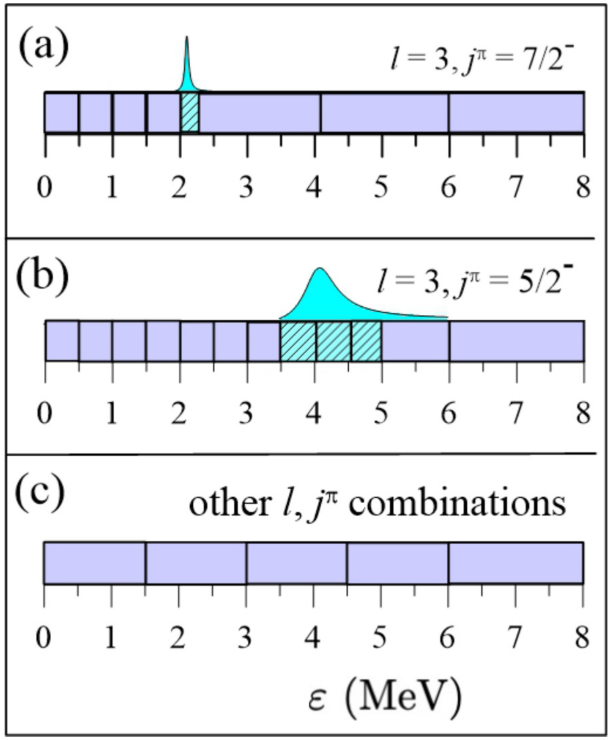
$$\mathbb{V}^j(r_j) = \mathbb{U}^j(r_j) - i\mathbb{W}^j(r_j), \quad j = 1, 2$$

Difficulty:

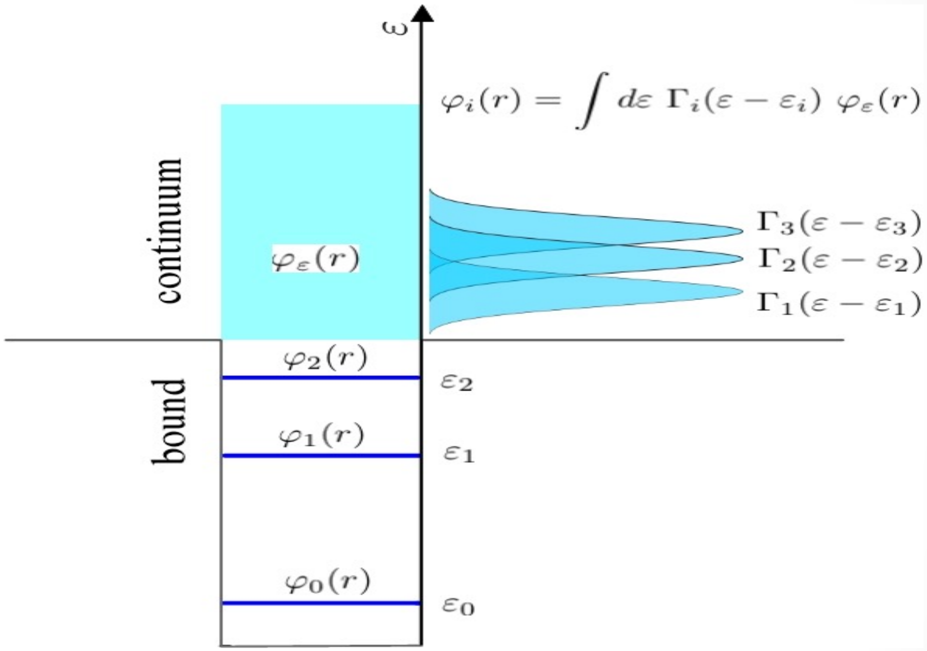
Continuous energy label $\Longrightarrow$ **Infinite set of equations (even with truncation)**

Solution: discretize the continuum

CDCC method with bins $\{\varphi_\varepsilon\} \Longrightarrow \{\varphi_i\}$



Bins adopted for ^{7}Li



Reduces to a standard CC problem,
(finite number of coupled equations)

- Project angular momentum
- Solve CC equations, get S-matrices and radial w.f.

Calculation of fusion cross sections

- Indirect calculation:

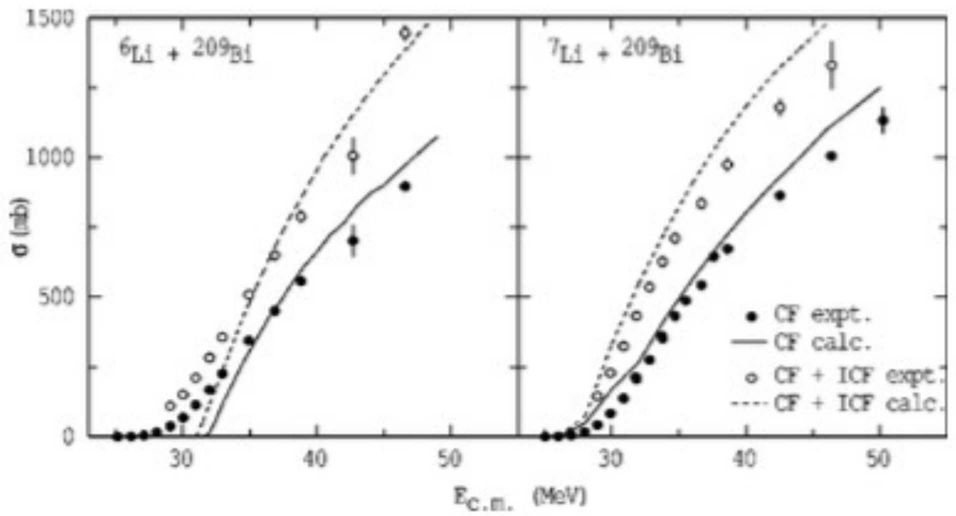
$$\sigma_{\mathrm{R}} = \frac{\pi}{k^2} \sum_l (2l+1) \left[1 - \left|S_{0l}(E)\right|^2\right] \Longrightarrow \sigma_{\mathrm{F}} = \sigma_{\mathrm{R}} - \sum_{\alpha \neq 0} \sigma_\alpha$$

- Direct calculation using radial wave functions

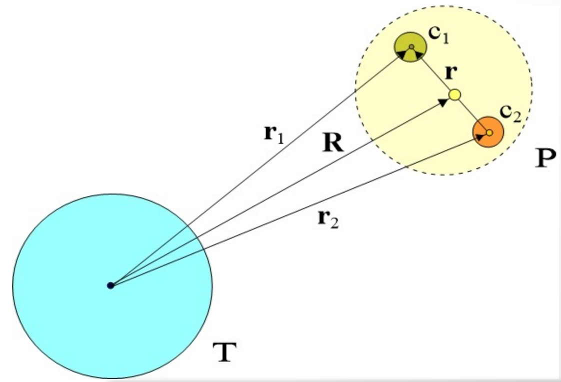
$$\sigma_{\mathrm{F}} = \frac{k}{E} \sum_{\alpha\alpha'=1} \langle \psi_\alpha | W^1_{\alpha,\alpha'} + W^2_{\alpha,\alpha'} | \psi_{\alpha'} \rangle \qquad ??????$$

Fusion Estimations: classical picture

Hagino et al., NPA **238**, 475 (2004), Dasgupta et al., PRC **66**, 041602 (2002),



- Classical picture with stochastic parameters.



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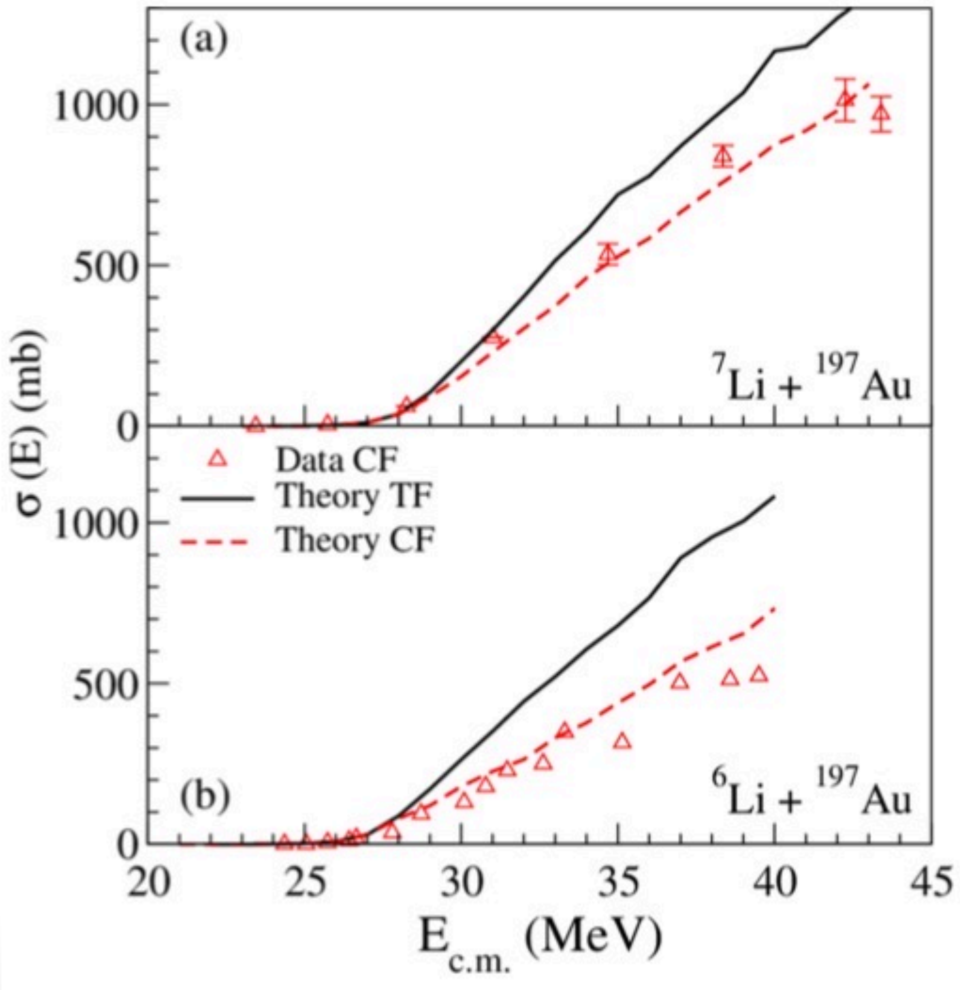
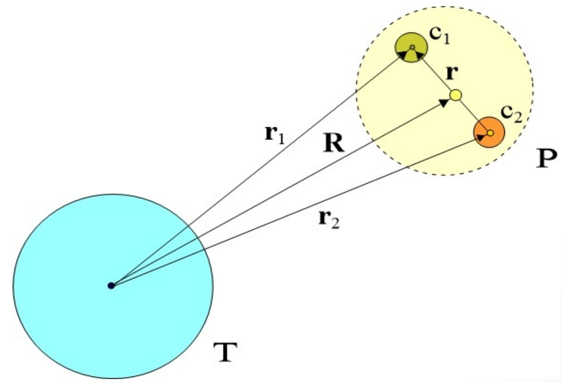
Fusion Estimations: semi-classical models

* Marta et al., PRC **89**, 034625 (2014), Kolinger et al., PRC **98**, 044604 (2018)



- Classical trajectory
- Intrinsic dynamic: time dependent Schrodinger equation
- Fusion: tunnelling trough the barrier

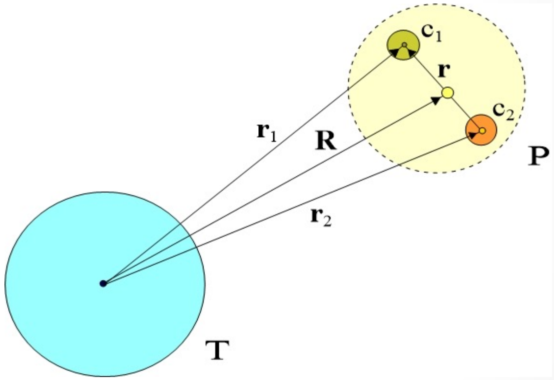


The method of Hagino, Vitturi, Dasso and Lenzi (HVDL)

Hagino et al., PRC **61**, 037602 (2000)
A. Diaz-Torres and I. J. Thompson, PRC **65**, 024606 (2002).

- **P-T imaginary potential (instead of $W^{(1)}$ + $W^{(2)}$)**

$$W(\mathbf{R},\mathbf{r}) = W^1(r_1) + W^2(r_2) \rightarrow W(R) = W_\alpha \delta_{\alpha,\alpha'}$$



Then,

$$\sigma_{\rm TF} = \frac{k}{E} \sum_{\alpha=1}^{N} \langle \psi_\alpha \,|\, W_\alpha \,|\, \psi_\alpha \rangle = \sum_{\alpha=1}^{N} \sigma_\alpha$$

Or,

$$\sigma_{\rm TF} = \sigma_{\rm B} + \sigma_{\rm C}$$

$$With\ \sigma_{\rm B} = \frac{k}{E} \sum_{\alpha\ \epsilon\ bound} \langle \psi_\alpha \,|\, W_\alpha \,|\, \psi_\alpha \rangle$$

Contributions from from bound channels

$$And\ \sigma_{\rm C} = \frac{k}{E} \sum_{\alpha\ \epsilon\ cont.} \langle \psi_\alpha \,|\, W_\alpha \,|\, \psi_\alpha \rangle$$

From continuum channels (*bins*)

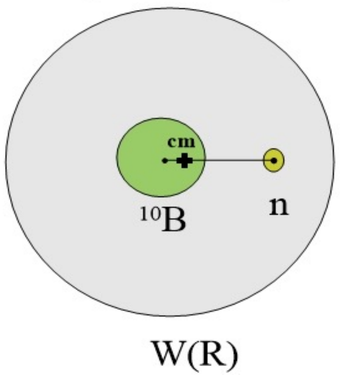
Basic Assumption: $\sigma_{\mathrm{CF}} = \sigma_{\mathrm{B}}\ , \sigma_{\mathrm{ICF}} = \sigma_{\mathrm{C}}$

Limitation: works for a fragment much heavier than the other

$^{11}\mathrm{Be}\ (^{10}\mathrm{Be}-\mathrm{n}) + {}^{208}\mathrm{Pb}$

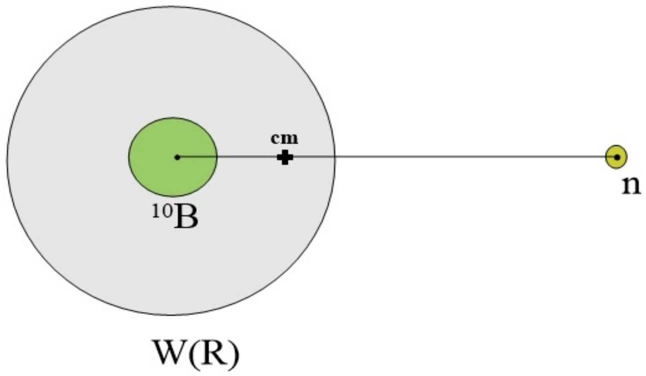
CF

Absorption in B space



ICF

Absorption in C space



Works fine !

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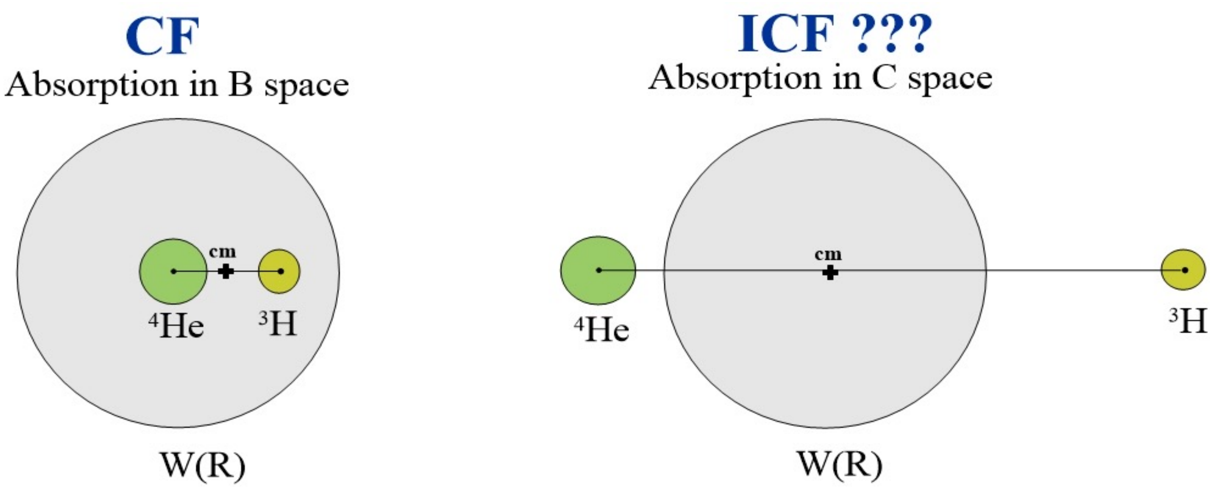
Basic Assumption: $\sigma_{\mathrm{CF}} = \sigma_{\mathrm{B}}$, $\sigma_{\mathrm{ICF}} = \sigma_{\mathrm{C}}$

Limitation: works for a fragment much heavier than the other

^{7}Li (^{4}He - ^{3}H) + ^{209}Bi



Does not work !

Indirect determination of CF using the spectator model*

* Lei and Moro, PRL 122, 042503 (2019)

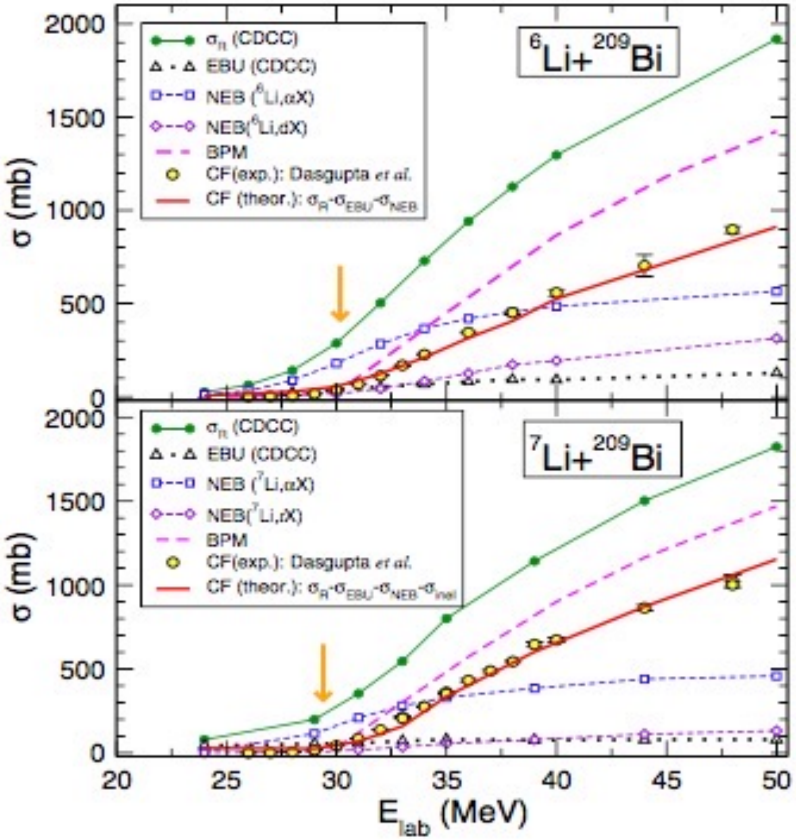
Extract σ_{CF} from the relation:

$$\sigma_{\mathrm{R}} = \sigma_{\mathrm{CF}} + \sigma_{\mathrm{inel}} + \sigma_{\mathrm{EBU}} + \sigma^{(1)}_{\mathrm{NBU}} + \sigma^{(2)}_{\mathrm{NBU}}$$

- $\sigma_{R\,:}$ from CDCC calculation or opt. model analysis
- σ_{inel} :from standard CC calculation (only bound channels)
- σ_{EBU} : from CDCC calculation:
- σ_{NEB1} ,σ_{NEB2} : from inclusive spectator- participant model (IAV)

Quantum mechanical methods

1. **Indirect determination of CF using the spectator model***



Nice model, … but cannot evaluate ICF

* Lei and Moro, PRL 122, 042503 (2019)

Other methods found in the literature

- S. Hashimoto et al., Prog. Theor. Phys. **122**, 1291 (2009): Radial integrals of the imaginary potentials with CDCC w.f.s over the coordinates of the fragments, r1 and r2. They picked contribution from proper regions to determine individual cross sections for each fusion process. ICF the neutron and the proton in the d + ^{7}Li collision.

- M. Boseli and Diaz-Torres, JPG 41 (2014) 094001, PRC 92 (2015) 044610: Used position projection operators to describe the time-evolution of wave packets. Used to estimate CF and ICF cross sections for the ^{6}Li +^{209}Bi system. The method is promising but so far it has not been used in realistic calculations involving weakly bound projectiles.

- V.V. Parkar et al., PRC 94, 024606 (2016): Performed separate CDCC calculations with short-range W to determine CF, ICF, TF (no self-consistent) ^{6,7}Li +^{209}Bi,^{198}Pt

A new QM method to evaluate CF and ICF*

(Based on the HVDL method, but with abs. of each fragment)

Instead of absorption of the cm of the projectile:

$$W(\mathbf{R},\mathbf{r}) = W^1(r_1) + W^2(r_2) \rightarrow W(R) = W_\alpha \delta_{\alpha,\alpha'}$$

Individual absorption of each fragment:

$$W(\mathbf{R},\mathbf{r}) = W^1(r_1) + W^2(r_2), \quad W_{\alpha,\alpha'} \neq W_\alpha$$

Assumption:

W^i does not connect spaces B and C

$$\mathbb{W}^{(i)}(r_i) = \frac{W_0}{1+\exp[(r_i - R_\mathrm{w})/a_\mathrm{w}]}, \qquad i = 1, 2, \qquad (44)$$

with the following parameters:

$$W_0 = 50\ \mathrm{MeV}, \quad R_\mathrm{w} = 1.0\left[A_i^{1/3} + A_\mathrm{T}^{1/3}\right]\ \mathrm{fm}; \quad a_\mathrm{w} = 0.2\ \mathrm{fm}.$$

* J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)

$$\mathbb{H}(\mathbf{R},\mathbf{r}) = h(\mathbf{r}) + \hat{K} + \mathbb{U}^{(1)}(r_1) + \mathbb{U}^{(2)}(r_2),$$

where

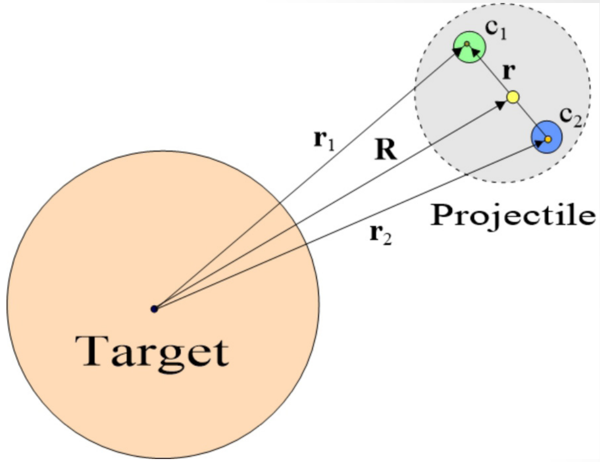
$$\mathbb{U}^{(i)}(r_i) \equiv \mathbb{V}^{(i)}(r_i) - i\mathbb{W}^{(i)}(r_i)$$

we split the wave function as

$$\Psi^{(+)}(\mathbf{R},\mathbf{r}) = \Psi^{\mathrm{B}}(\mathbf{R},\mathbf{r}) + \Psi^{\mathrm{C}}(\mathbf{R},\mathbf{r}),$$

$$\Psi^{\mathrm{B}}(\mathbf{R},\mathbf{r}) = \sum_{\beta\in\mathrm{B}} [\psi_\beta(\mathbf{R}) \otimes \phi_\beta(\mathbf{r})]$$

$$\Psi^{\mathrm{C}}(\mathbf{R},\mathbf{r}) = \sum_{\gamma\in\mathrm{C}} [\psi_\gamma(\mathbf{R}) \otimes \phi_\gamma(\mathbf{r})],$$



CDCC leads the x-sect.s

Experiment

$$\sigma_{\mathrm{DCF}} = \frac{\pi}{K^2} \sum_J (2J+1)\,\mathcal{P}^{\mathrm{DCF}}(J),$$

CF=DCF+SCF

$$\sigma_F^{(1)} = \frac{\pi}{K^2} \sum_J (2J+1)\,\mathcal{P}^{(1)}(J),$$

ICF1

$$\sigma_F^{(2)} = \frac{\pi}{K^2} \sum_J (2J+1)\,\mathcal{P}^{(2)}(J).$$

ICF2

For the real parts the São Paulo potential is used

$P^{(i)}(J)$ = abs. probability of fragment i in the C-space

Contribution from the B-space: $\sigma_{\rm B} = \sigma_{\rm DCF}$

(as in the HVDL method)

$$\sigma_{\rm B} = \frac{k}{E} \sum_{\alpha\ \epsilon\ bound} \left\langle \psi_\alpha \middle| W^1(r_1) + W^2(r_2) \middle| \psi_\alpha \right\rangle$$

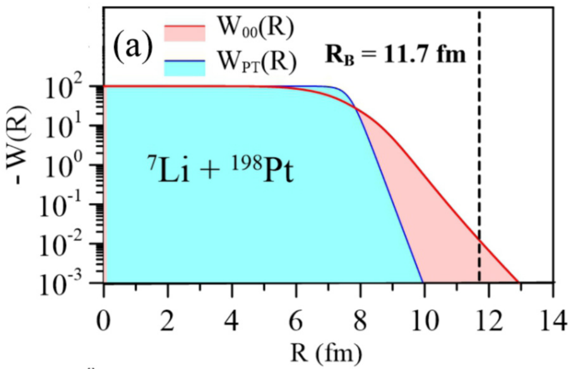
BUT ...... !!!!!!

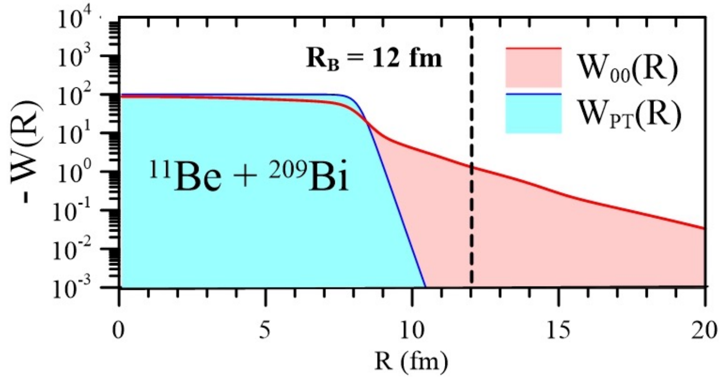
* J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)

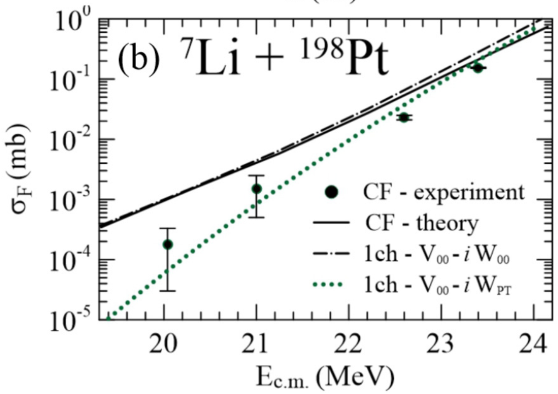
Our first approach was this $$\sigma_{\mathrm{B}} = \frac{k}{E} \sum_{\alpha\ \epsilon\ bound} \langle \psi_\alpha \,|\, W^1(r_1) + W^2(r_2) \,|\, \psi_\alpha \rangle$$

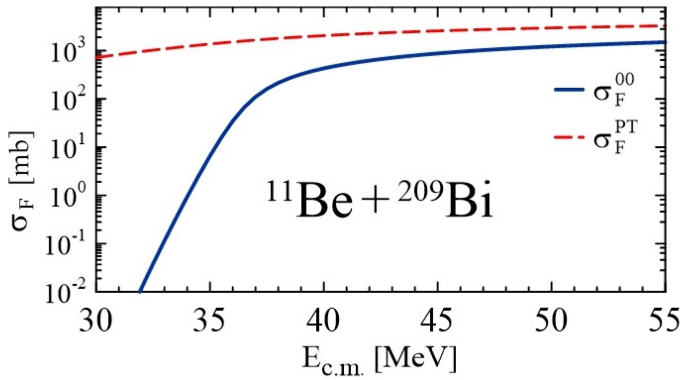
But we realized that that istead of using $$W_{00}(R) = \int d^3\mathbf{r} |\phi_0(\mathbf{r})|^2 [\mathbb{W}^{(1)}(r_1) + \mathbb{W}^{(2)}(r_2)]$$

we should use W_{PT} for bound states:









J. Lubian, XIV LASNPA2024, Mexico

Contribution from channels in the continuum to TF

(here is the difference to HVDL)

$$\sigma_{\mathrm{C}} = \frac{k}{E} \sum_{\alpha\alpha' \,\epsilon\, C} \left[\left\langle \psi_\alpha \,\middle|\, W^1_{\alpha,\alpha'}(r_1) \,\middle|\, \psi'_\alpha \right\rangle + \left\langle \psi_\alpha \,\middle|\, W^2_{\alpha,\alpha'}(r_2) \,\middle|\, \psi_{\alpha'} \right\rangle \right]$$

Performing ang. mom. projection and the summing over α and α' (in C),

$$\sigma_{\mathrm{C}} = \frac{\pi}{k^2} \sum_J (2J+1) \left[P^1(J) + P^2(J) \right]$$

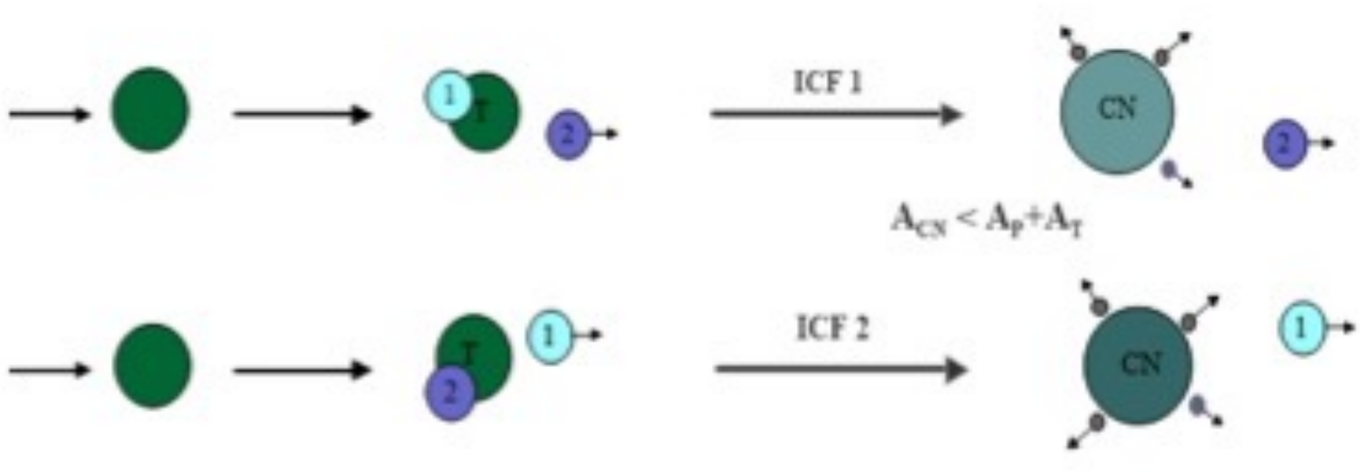
$P^{(i)}(J)$ = abs. probability of fragment *i* in the C-space

* J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)

ICF (ICF1, ICF2), SCF cross sections

$$\sigma_{\mathrm{ICF1}} = \frac{\pi}{k^2}\sum_J (2J+1)P^1(J)\left[1 - P^2(J)\right]$$

$$\sigma_{\mathrm{ICF2}} = \frac{\pi}{k^2}\sum_J (2J+1)P^2(J)\left[1 - P^1(J)\right]$$

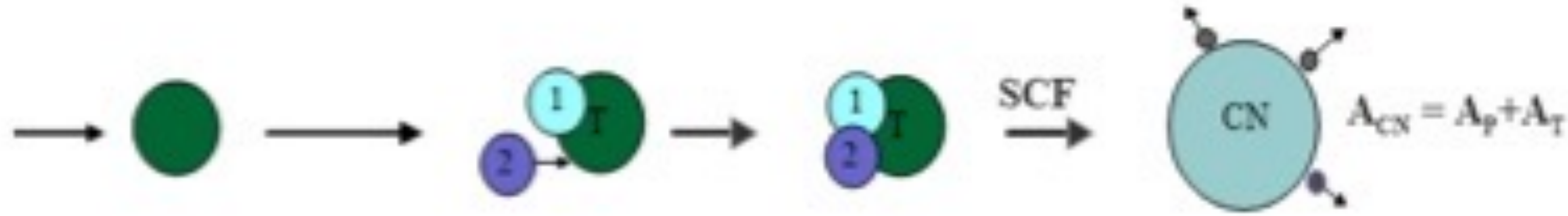


$$\sigma_{\mathrm{ICF}} = \sigma_{\mathrm{ICF1}} + \sigma_{\mathrm{ICF2}}$$

* J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)

ICF (ICF1, ICF2), SCF cross sections

$$\sigma_{\rm SCF} = \sigma_{\rm C} - \sigma_{\rm ICF} = \frac{\pi}{k^2}\sum_J \left(P^1(J) \times P^2(J)\right)$$



$$\sigma_{\rm CF} = \sigma_{\rm DCF} + \sigma_{\rm SCF}$$

$$\sigma_{\rm TF} = \sigma_{\rm CF} + \sigma_{\rm ICF}$$

$$\mathcal{P}^{\rm TF}(J) = \mathcal{P}^{\rm DCF}(J) + \mathcal{P}^{\rm SCF}(J) + \mathcal{P}^{\rm ICF1}(J) + \mathcal{P}^{\rm ICF2}(J)$$
$$= \mathcal{P}^{\rm DCF}(J) + \mathcal{P}^{(1)}(J) + \mathcal{P}^{(2)}(J) - \mathcal{P}^{(1)}(J) \times \mathcal{P}^{(2)}(J)$$

- J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)
- M.R Cortes, J. Rangel. J.L. Ferreira, J. Lubian., L.F Canto (PRC 102, 06428 (2020)
- J. Lubian, J.L. Ferreira, J. Rangel, M. Cortes, L.F. Canto (PRC 105, 054601 (2022)

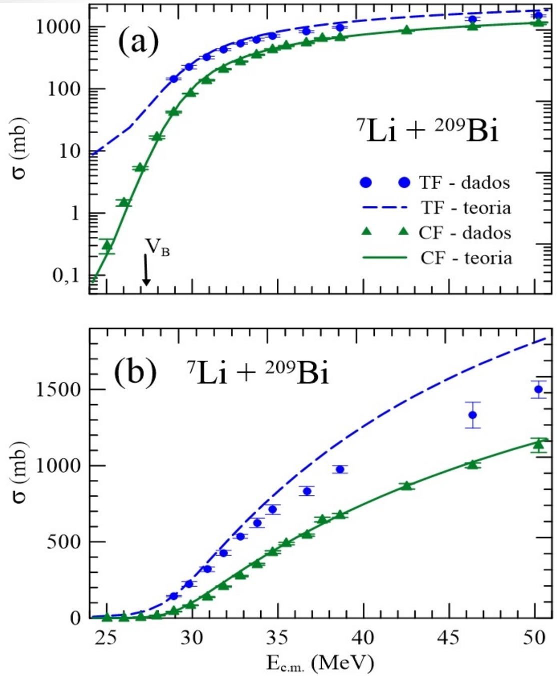
Application: **Fusion cross sections in $\underbrace{{}^{7}\mathrm{Li}+}_{{}^{4}\mathrm{He}+t}$ A**

^{4}He + t BE= 2.47 MeV

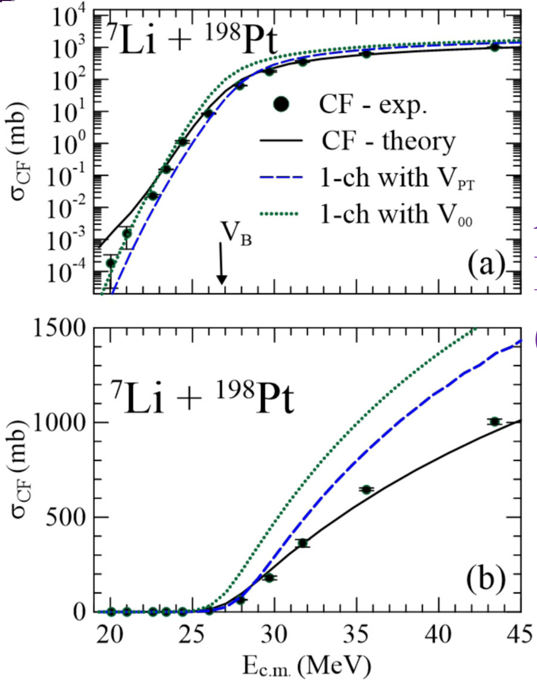
Procedure:

- Perform CDCC calculations running FRESCO, with options to export intrinsic and radial w.f.
- !!! We need radial w.f. converged inside V_B too. Hard task!!!
- Use them in the the angular momentum projected expressions for the cross sections (Code CF-ICF, unpublished)
  - J. Rangel, M.R. Cortes, J. Lubian, L.F.Canto (Phys. Let. B, 803- 2020)
  - M.R Cortes, J. Rangel. J.L. Ferreira, J. Lubian., L.F Canto (PRC 102, 06428 (2020)

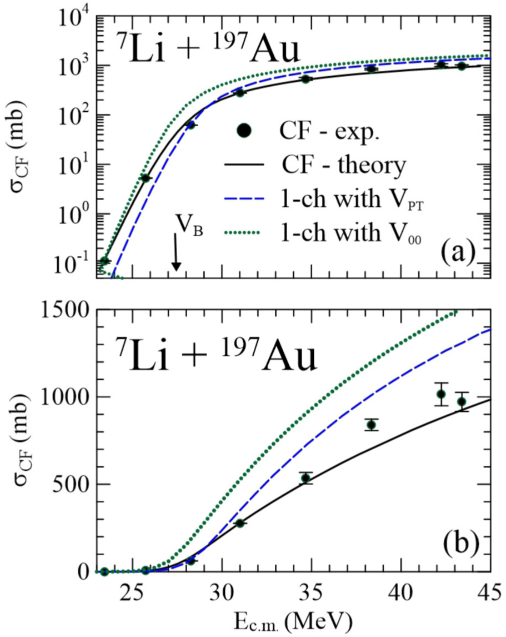
^{7}Li + A CF – theory vs. experiment

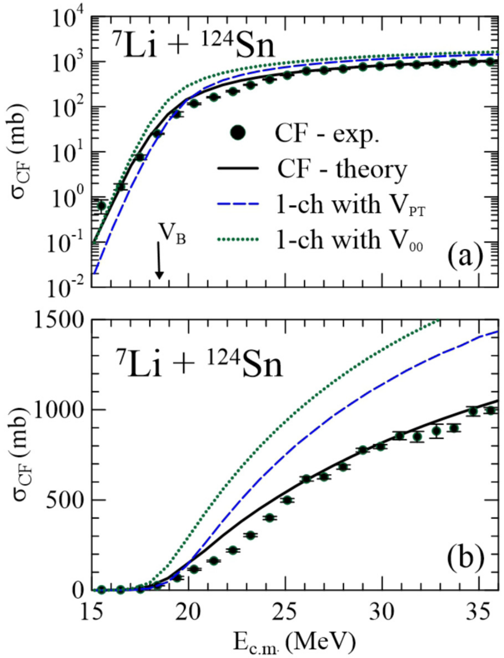


Dasgupta *et al.*, PRC **66**, 041602(R) (2002); PRC **70**, 024606 (2004)



A.Shrivastava et al, PRL, 103, 232701, (2009)

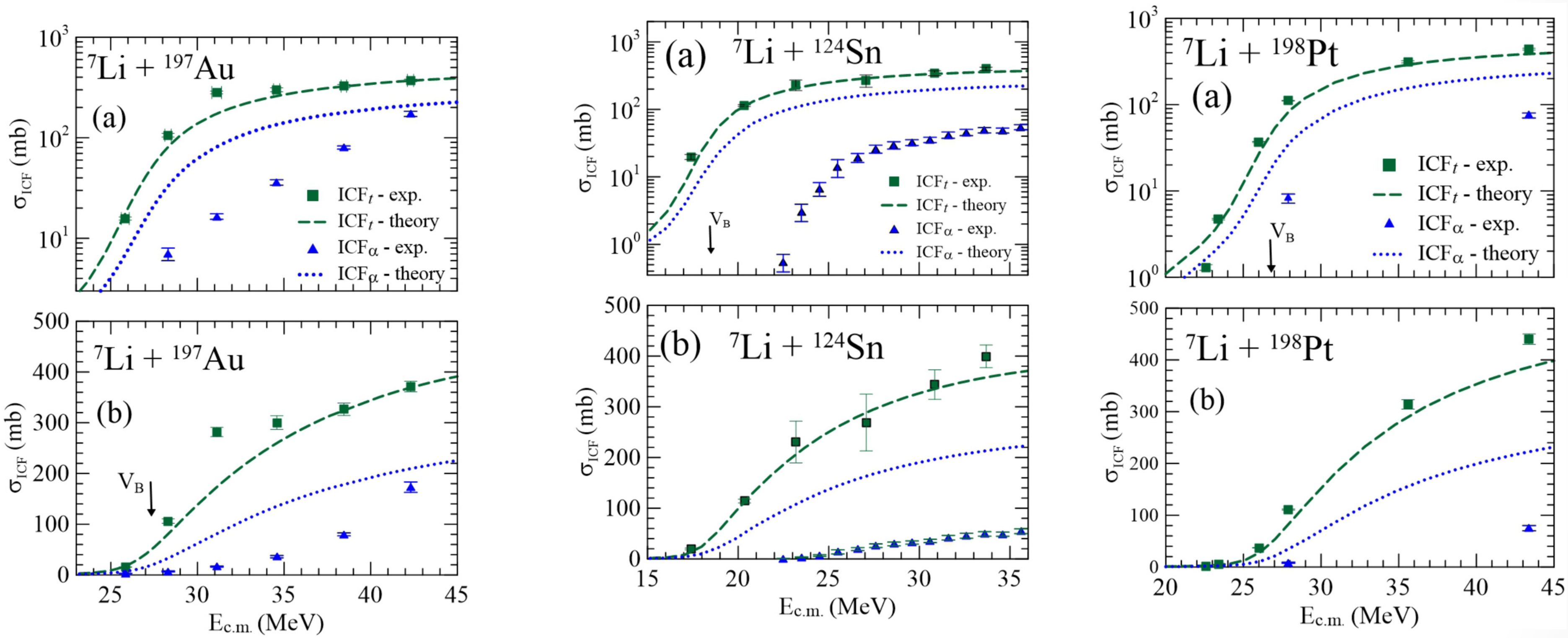




V. V. Parkar, et al. PRC 97, 014607 (2018)

C. S. Palshetkar et al., PRC 89, 024607 (2014).

ICF: theory versus experiment

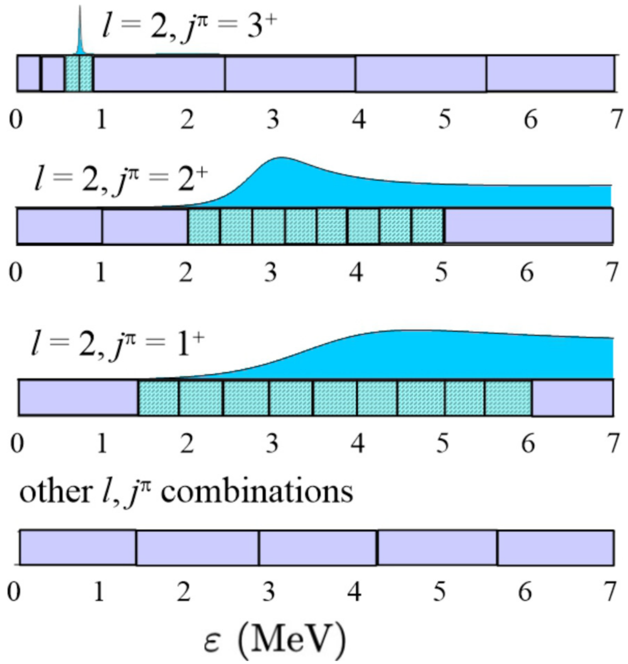


ICF$_t$ very well described

ICF$_\alpha$ overpredicted. Why? Problem of the theory, of the data?????

Results for ^{6}Li on heavy targets: J. Lubian, et al. (PRC 105, 054601 (2022)

$S = 0.7$

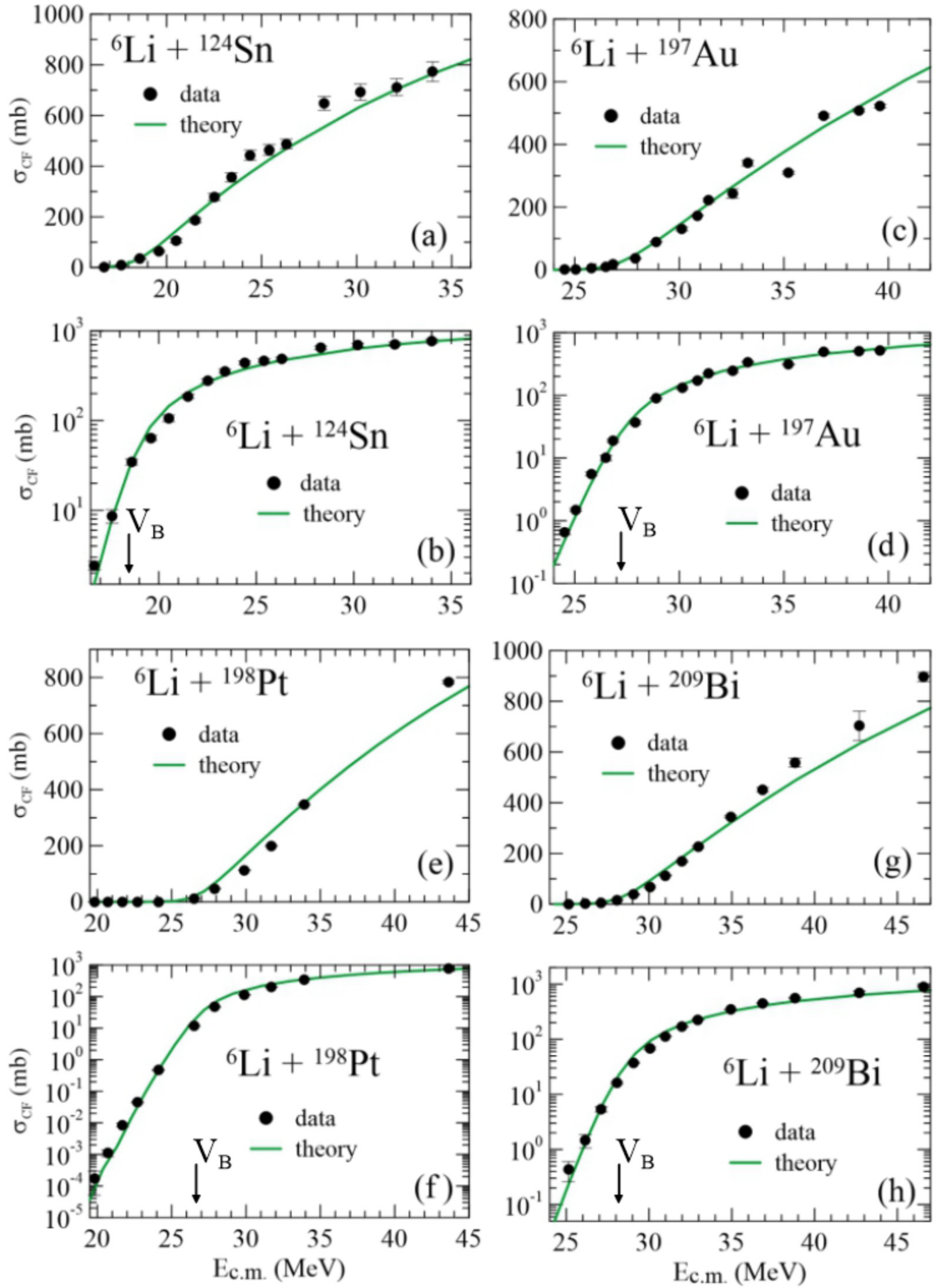


l	j^π	E^{th}_{res}	Γ^{th}_{res}	E^{exp}_{res}	Γ^{exp}_{res}
2	3^+	0.717	0.020	0.716	0.024
2	2^+	3.14	1.88	2.84	1.7
2	1^+	4.06	3.5	4.18	1.5

Reaonably well described the width and the position of the resonaces

CF theory vs experiment

Excelent agreement!!!!



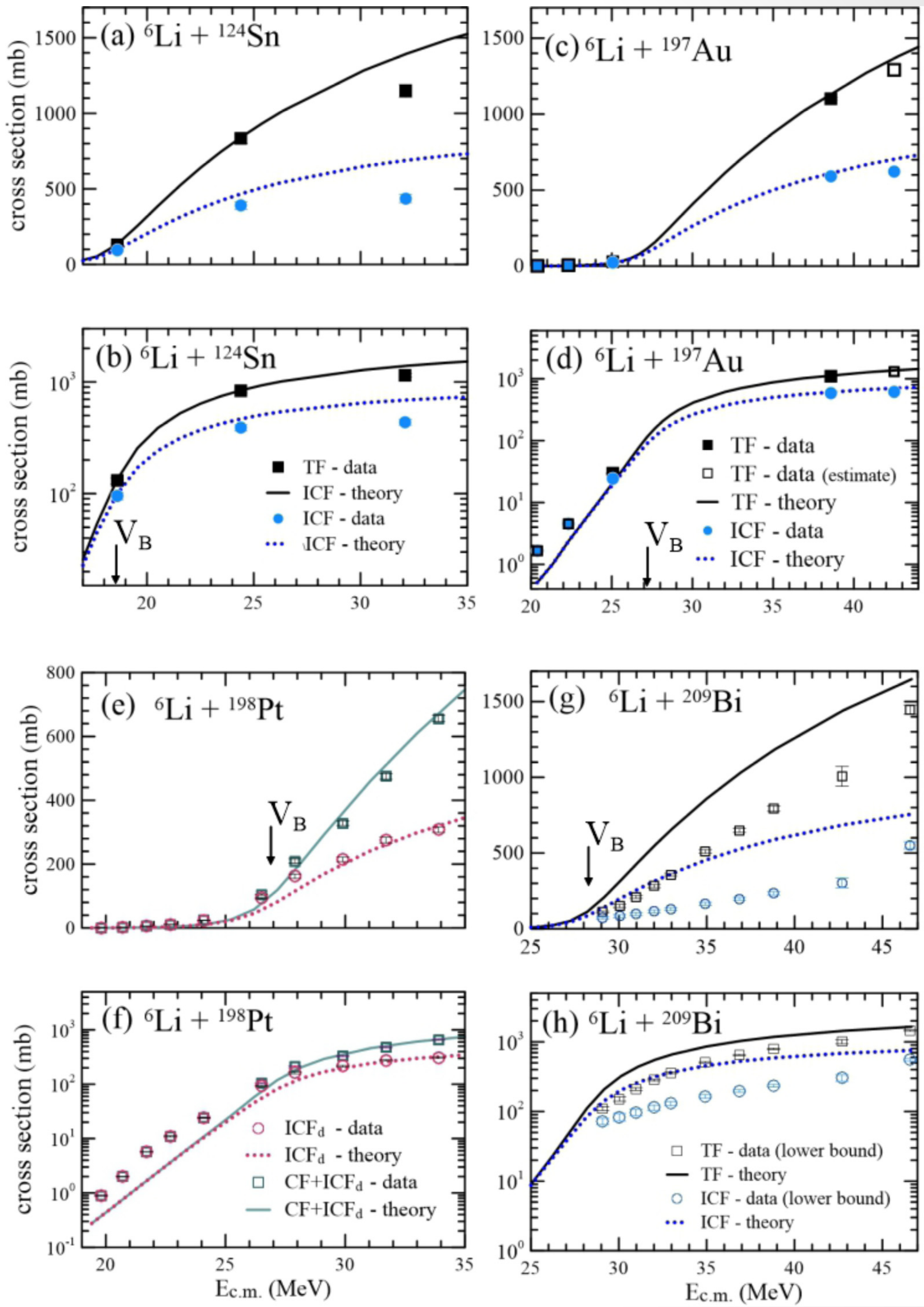
J. Lubian, XIV LASNPA2024, Mexico

TF, ICF theory vs experiment

Good agreement!!

For ^{6}Li +^{209}Bi experimental data are lower bund only.

^{6}Li + ^{198}Pt are underpredicted below V_B … why?

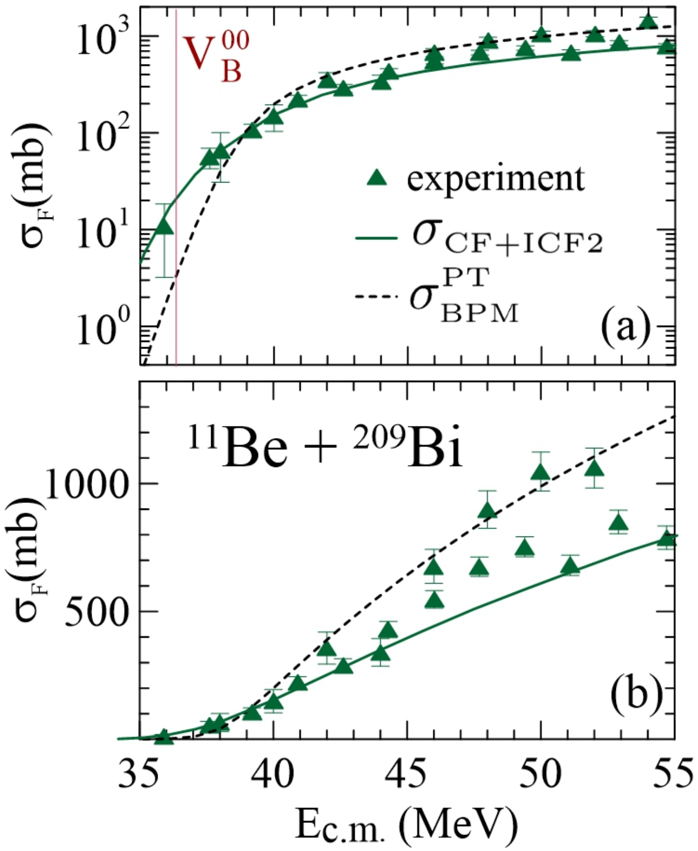


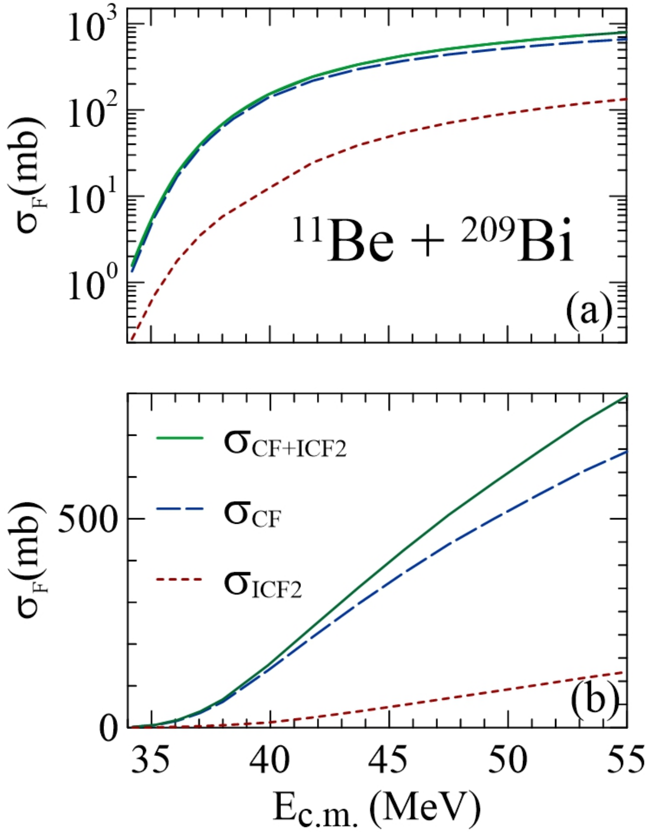
Results for ^{6}He and ^{11}Be neutron halo projectiles on heavy targets:

J.L. Ferreira, et al. (PRC 107, 034603 (2023))

^{11}Be + ^{209}Bi.

$$\mathcal{S} \simeq \sqrt{B(E1)_{\rm exp}/B(E1)_{\rm calc}} = 0.66.$$



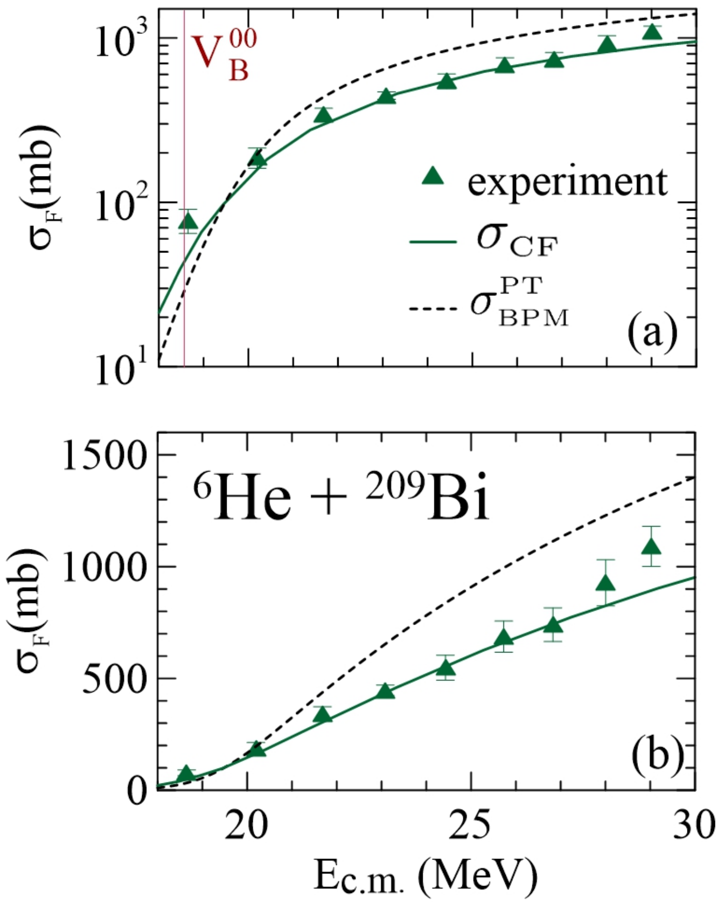


Good agreement!!

Hindered above and enhanced above V_B

C. Signorini, et al., NPA735, 329 (2004).

^{6}He + ^{209}Bi. $\mathcal{S} = 0.7$



ICF2 was very unlikely because the CN Q-value for α + ^{209}Bi => ^{213}As is -15 MeV

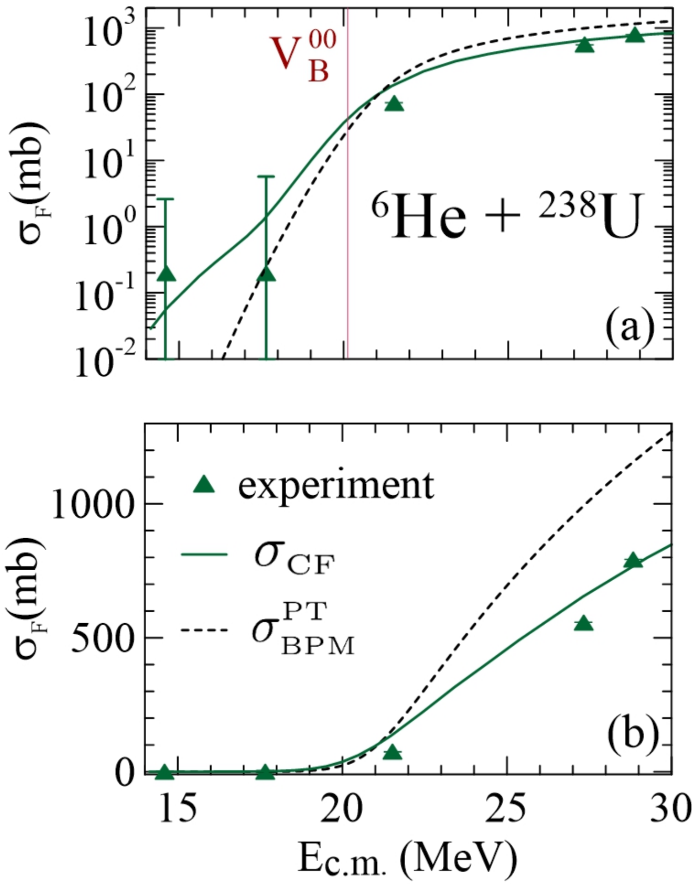
Very endotermic !!!!

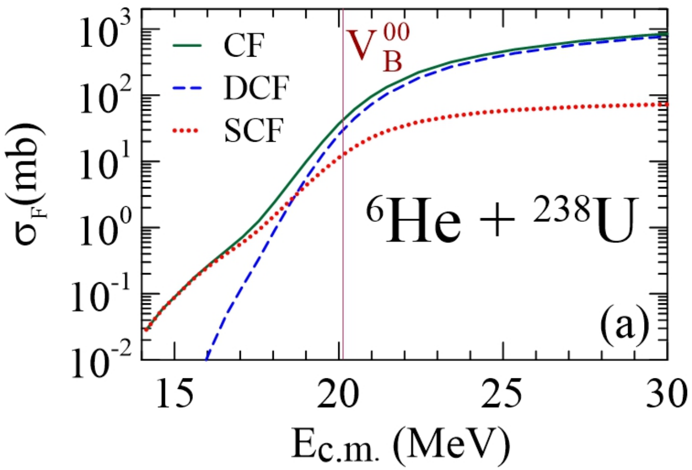
Good agreement!!
Hindered above and enhanced above V_B

J. J. Kolata, et al., PRC57, R6 (1998).

^{6}He + ^{238}U

$$\mathcal{S} = 0.7$$





ICF2 was very unlikely because the CN excitation energy is well below the fission barrier for α + ^{238}U => ^{242}Pu

Good agreement!!
Hindered above and enhanced above V_B

R. Raabe et al., Nature 431, 823 (2004).

Conclusions

- We have proposed a new quantum mechanical method to evaluate CF and ICF in collisions of weakly bound nuclei
- The method was applied to the ^{6,7}Li + heavy target system and the results were compared with the data.
- Considering that our calculations use standard interaction and have no free parameters, the agreement between theory and experiment is excellent
- Calculations for neutron halo induced reactions were in very good agreement with the experimental data too. For other systems are in progress

Future plans

- Study other systems (e.g. ^{8}B on light targets, ^{6,7}Li on medium mass targets)
- Include spectroscopic factors* (cluster structure of g.s. is just an approximation)
- Include target excitation (important in fusion of deformed targets)*
- Include core-excitations*
- Extension to 4-body CDCC (ex: ^{9}Be, ^{6}He collisions)*
- Include transfer channels ??????
- Include BU triggered by transfer ?????

* Requires another version of the CDCC code

Team members

- Luiz Felipe Canto (UFRJ)
- Jeannie rangel Borges (UERJ)
- Jonas Leonardo Ferreira (UFF)
- Mariane Rodrigues Cortes (UFF)
- Vinicius Zagatto (UFF)
- Paulo Roberto Silveiura Gomes (UFF) diceased
- Luiz Carlos Chamon (USP)
- Edilson Crema(USP)

Thank you ☺

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de ciência e tecnologia

CAPES

CNPq

1. Experiment vs. theory

$\Delta\sigma_F \equiv \sigma_F^{exp} - \sigma_F^{theo} \Rightarrow$ 'ingredients' missing in the theory

Theoretical possibilities:

a) Single channel - standard densities

$\Delta\sigma_F$ arises from all static and dynamic effects

b) Single channel - realistic densities

$\Delta\sigma_F$ arises from couplings to all channels

c) CC calculation with all relevant bound channels

$\Delta\sigma_F$ arises from continuum couplings

d) CDCC

no deviation expected

2. Compare with σ_F of a similar tightly bound system

Differences due to static effects:

1. Gross dependence on size and charge:

   $Z_P,\ Z_T, A_P, A_T -$ affects V_B and R_B

   $V_B \sim Z_P Z_T e^2 / R_B; \quad \sigma_{geo} \sim \pi R_B^2, \quad R_B \propto (A_P^{1/3} + A_T^{1/3})$

2. Different barrier parameters due to diffuse densities

   (lower and thicker barriers)

Fusion data reduction required !

Fusion functions $F(x)$ (our reduction method)

$$E \rightarrow x = \frac{E - V_B}{\hbar\omega} \quad \text{and} \quad \sigma_F^{\exp} \rightarrow F_{\exp}(x) = \frac{2E}{\hbar\omega R_B^2}\,\sigma_F^{\exp}$$

Inspired in Wong's approximation

$$\sigma_F^W = R_B^2 \frac{\hbar\omega}{2E} \ln\left[1 + \exp\left(\frac{2\pi\left(E - V_B\right)}{\hbar\omega}\right)\right]$$

$$F(x) = F_0(x) = \ln\left[1 + \exp\left(2\pi x\right)\right]$$

$F_0(x)$ = Universal Fusion Function (UFF)

system independent !

Direct use of the reduction method

Compare $F_{\text{exp}}(x)$ with UFF for x values where $\sigma_{\text{F}}^{\text{opt}} = \sigma_{\text{F}}^{\text{W}}$

Deviations are due to couplings with bound channels and breakup

Refining the method

Eliminate the failure of the Wong model for light systems at sub-barrier energies

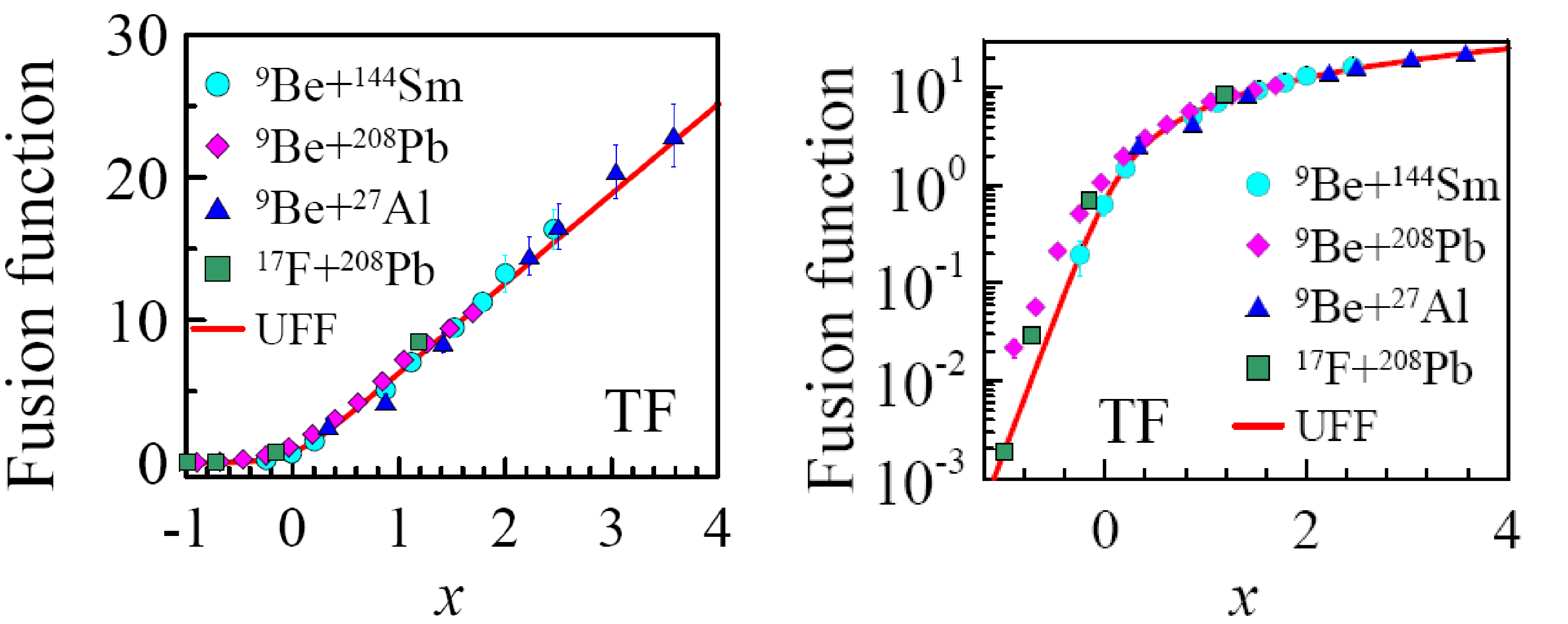
Eliminate influence of couplings with bound channels

Renormalized fusion function

$$F_{\text{exp}}(x) \rightarrow \bar{F}_{\text{exp}}(x) = \frac{F_{\text{exp}}(x)}{R(x)}, \quad \text{with } R(x) = \frac{\sigma_{\text{F}}^{\text{CC}}}{\sigma_{\text{F}}^{W}} = \frac{\sigma_{\text{F}}^{\text{CC}}}{\sigma_{\text{F}}^{\text{opt}}}$$

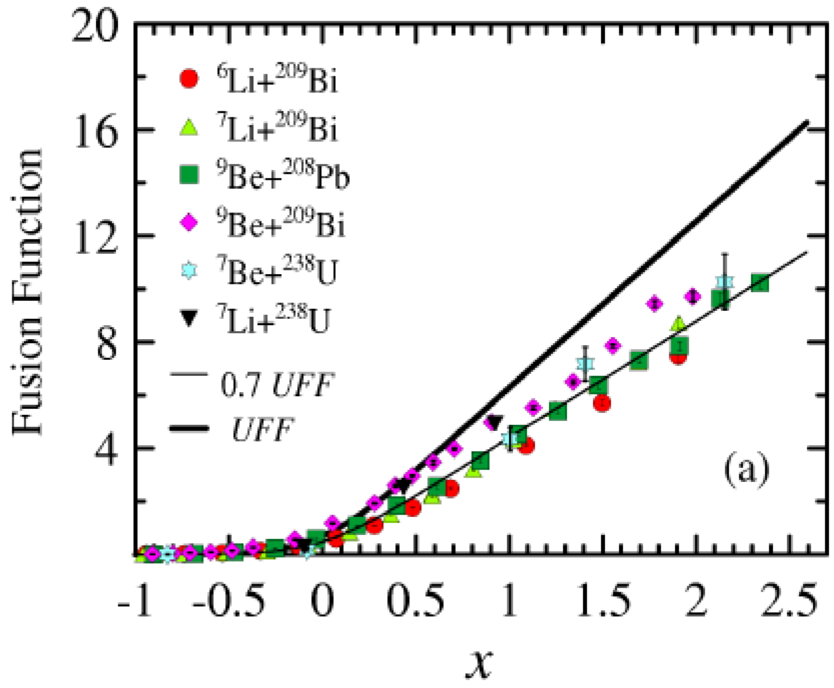
If CC calculation describes data $\rightarrow \bar{F}_{\text{exp}} = \text{UFF}$

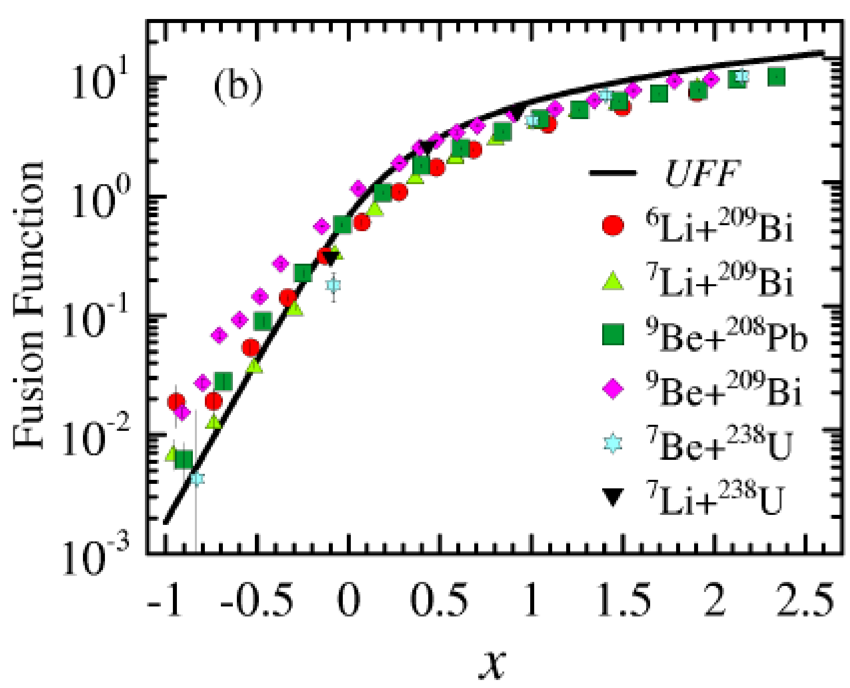
Use of UFF for investigating the role of BU dynamical effects on the total fusion of heavy weakly bound systems



No effect above the barrier- enhancement below the barrier

Use of UFF for investigating the role of BU dynamical effects on the complete fusion of stable weakly bound heavy systems

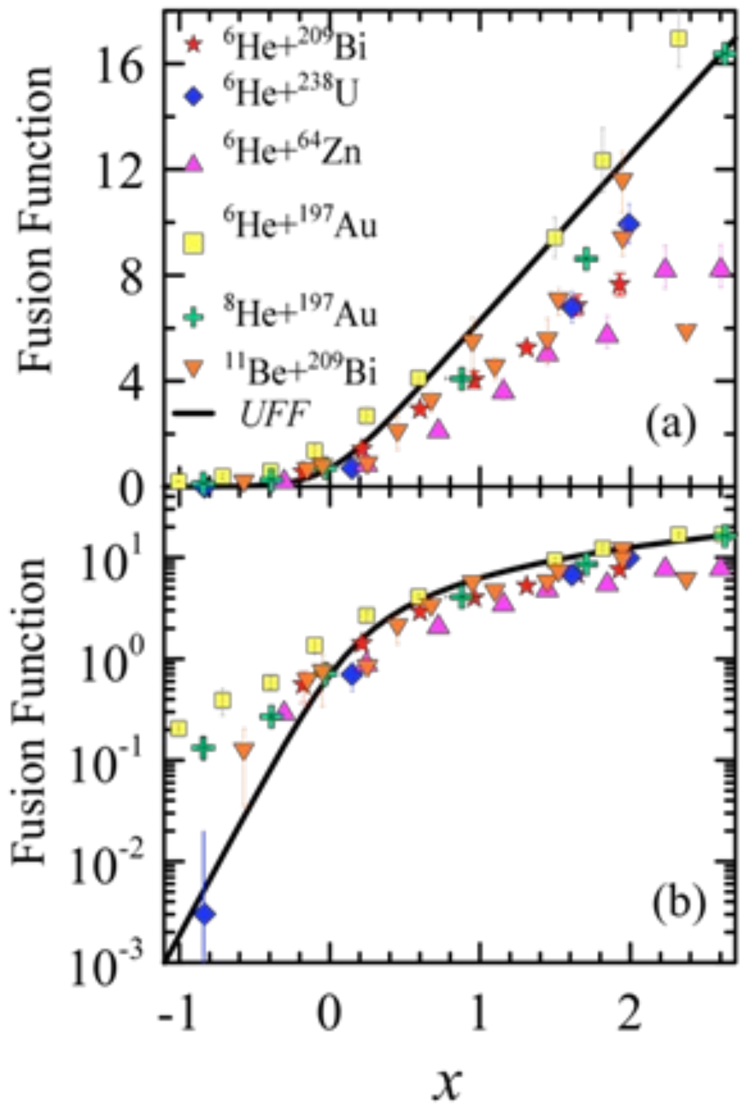




We did not include any resonance of the projectiles in CC calc.

Suppression above the barrier- enhancement below the barrier

Fusion of neutron halo ^{6,8}He, ^{11}Be



Conclusion from the systematic (several systems): CF enhancement at sub-barrier energies and suppression above the barrier, when compared with what it should be without any dynamical effect due to breakup and transfer channels.

How to measure and calculate CF, and ICF?

Procedures used to answer: "Enhancement or suppression in relation to what?

a) **Comparison of data with theoretical predictions.**
b) **Comparison of data for weakly and tightly bound systems.**
(reduction of x-sect is mandatory => UFF method!!

$$E \to x = \frac{E - V_B}{\hbar\omega} \qquad \text{and} \qquad \sigma_F^{\exp} \to F_{\exp}(x) = \frac{2E}{\hbar\omega R_B^2}\ \sigma_F^{\exp}$$

Inspired on Wong's formula $\quad \sigma_F^W = R_B^2\ \frac{\hbar\omega}{2E}\ \ln\left[1 + \exp\left(\frac{2\pi\left(E - V_B\right)}{\hbar\omega}\right)\right]$

Reducing gives: $\quad F(x) = F_0(x) = \ln\left[1 + \exp\left(2\pi x\right)\right]$

Renormalized fusion function

$$F_{\exp}(x) \to \bar{F}_{\exp}(x) = \frac{F_{\exp}(x)}{R(x)}, \quad \text{with } R(x) = \frac{\sigma_{\mathrm{F}}^{\mathrm{CC}}}{\sigma_{\mathrm{F}}^{W}} = \frac{\sigma_{\mathrm{F}}^{\mathrm{CC}}}{\sigma_{\mathrm{F}}^{\mathrm{opt}}}$$

L.F. Canto et al. JPG 36,015109 (2009) & NPA 821 (2009) 51