

Shape coexistence studied with Coulomb excitation and AGATA

Magda Zielińska

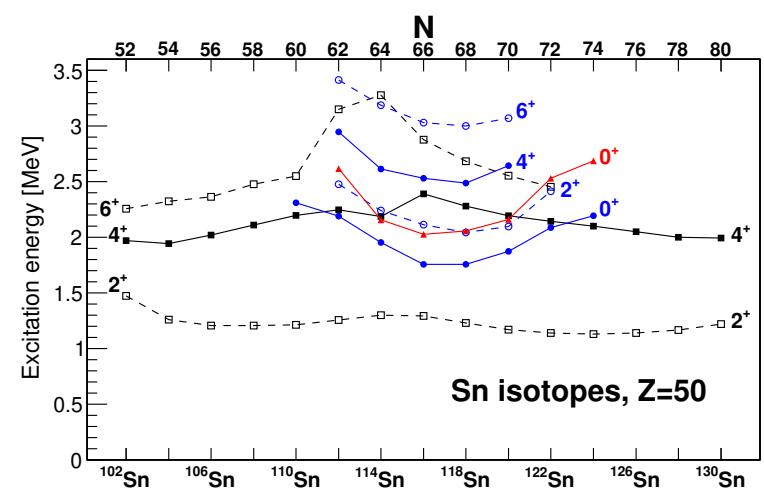
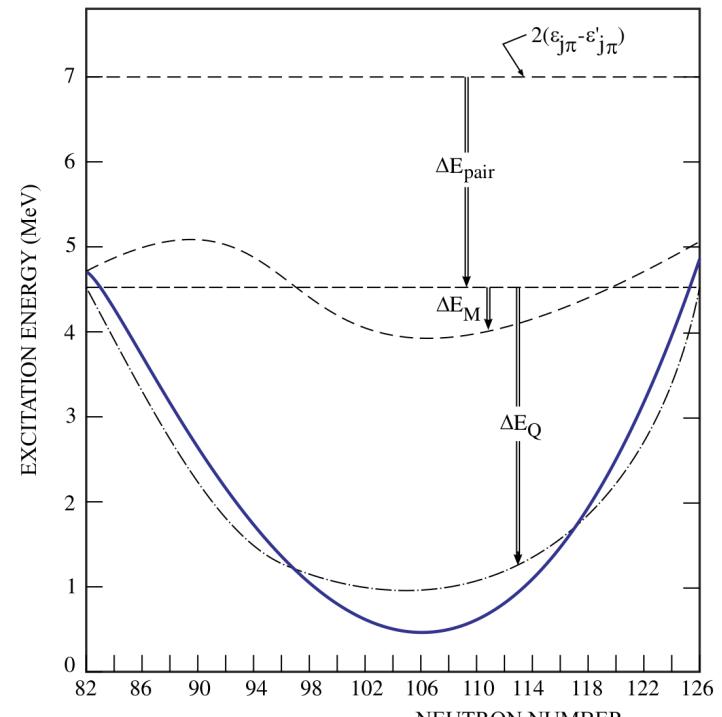


- superdeformation in ^{42}Ca :
K. Hadyńska-Klęk *et al.*, Phys. Rev. Lett. 117 (2016) 062501
K. Hadyńska-Klęk *et al.*, Phys. Rev. C 97 (2018) 024326
- shape coexistence in ^{106}Cd :
D. Kalaydjieva, PhD thesis, Université Paris-Saclay, 2023
- quadrupole shapes in ^{96}Zr :
data under analysis (Uni Guelph, INFN Firenze)

Shape coexistence

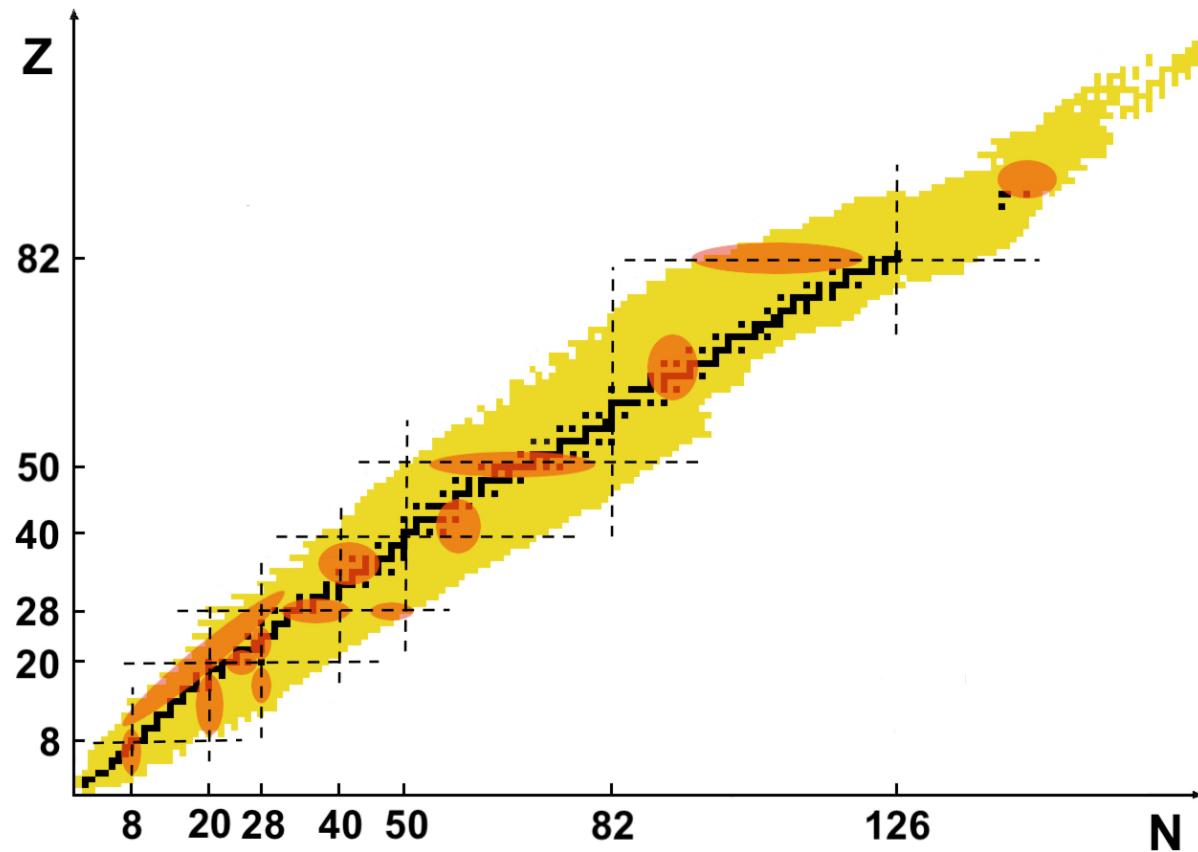
K. Heyde and J. Wood, Rev. Mod. Phys. 83, 1467 (2011)

- appearance of states characterised by different shapes closely lying in energy
- proposed mechanism: gain from correlations offsets the shell gap and multiparticle-multiparticle excitations go down in excitation energy
- effect increases towards mid shell – characteristic parabolic behaviour of intruder states energies
- depends on a delicate balance of macroscopic, liquid-drop-like properties of the nuclear matter and microscopic shell effects – provides stringent tests of modern nuclear structure models



Shape coexistence

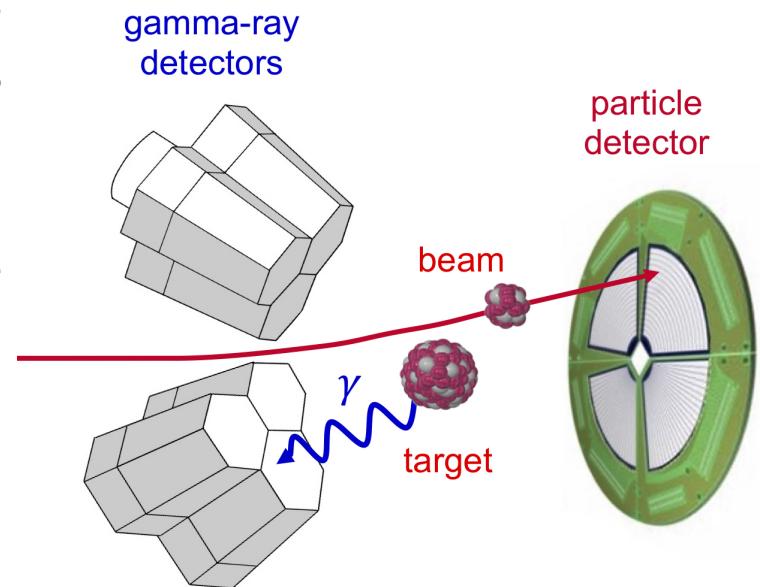
- a widespread phenomenon in areas close to proton and neutron shell closures



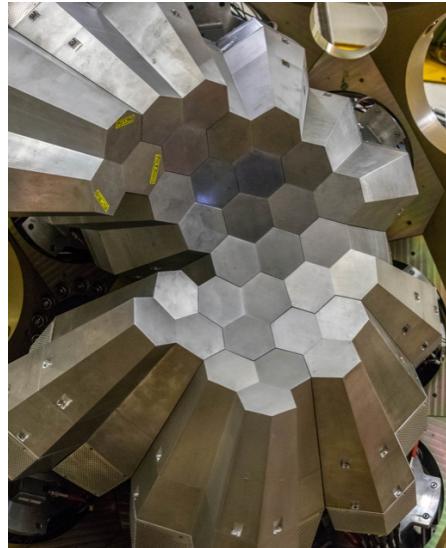
- difficult to establish experimentally as nuclear shape is not an observable
- Coulomb excitation: perfect tool to study shape coexistence as it is the only technique sensitive to charge distribution in excited nuclear states

Coulomb excitation

- population of excited states via **purely electro-magnetic interaction** between the collision partners in the process of quasi-elastic scattering
- we observe **gamma-ray decay** of Coulomb-excited states in coincidence with **scattered beam ions or target recoils**
- the decay intensities, measured as a function of particle scattering angle, are related to **reduced transition probabilities** and **spectroscopic quadrupole moments** determined via a multi-dimensional fit performed using dedicated analysis codes (e.g. GOSIA)
- they are related to the nuclear shape and collectivity – from extensive sets of E2 matrix elements **quadrupole invariants** can be formed in order to deduce deformation parameters for individual states defined in the intrinsic frame of the nucleus



AGATA

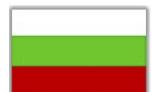


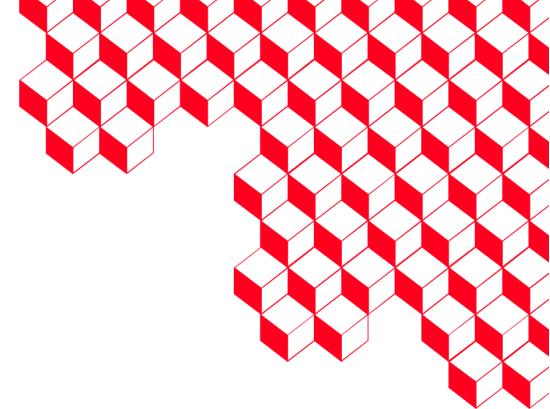
11 ATC, Feb 2022

- new-generation gamma-ray tracking array developed by a European collaboration involving 13 countries and over 40 institutions
- currently at LNL after campaigns at GANIL and GSI
- angular resolution: $\sim 1^\circ$
- large inner radius to accommodate ancillary devices
- final configuration: 180 segmented crystals (60 ATC), 35% efficiency

<http://www.agata.org>

S. Akkoyun et al., Nucl. Instrum. Methods Phys. Res. A 668, 26 (2012).





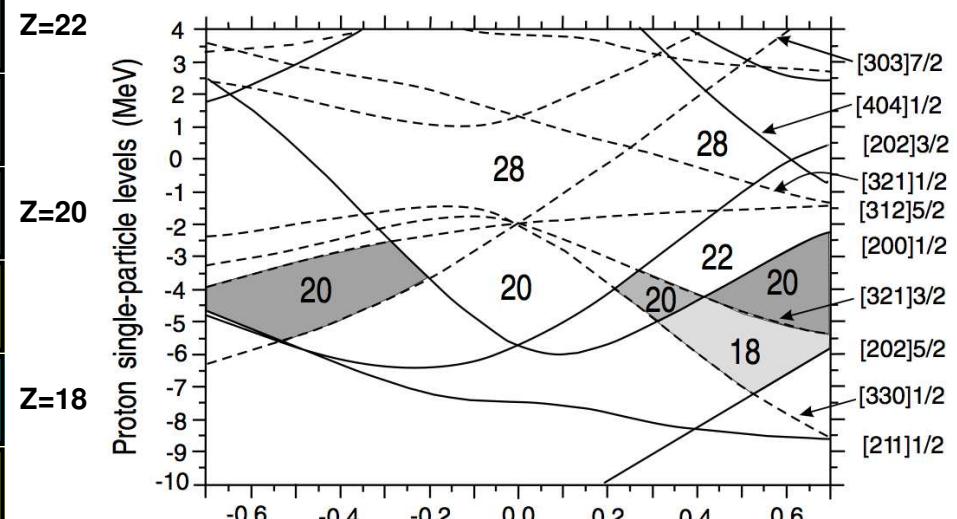
Part I: AGATA Demonstrator at LNL

superdeformation in ^{42}Ca

Highly-deformed structures in the $A \sim 40$ region

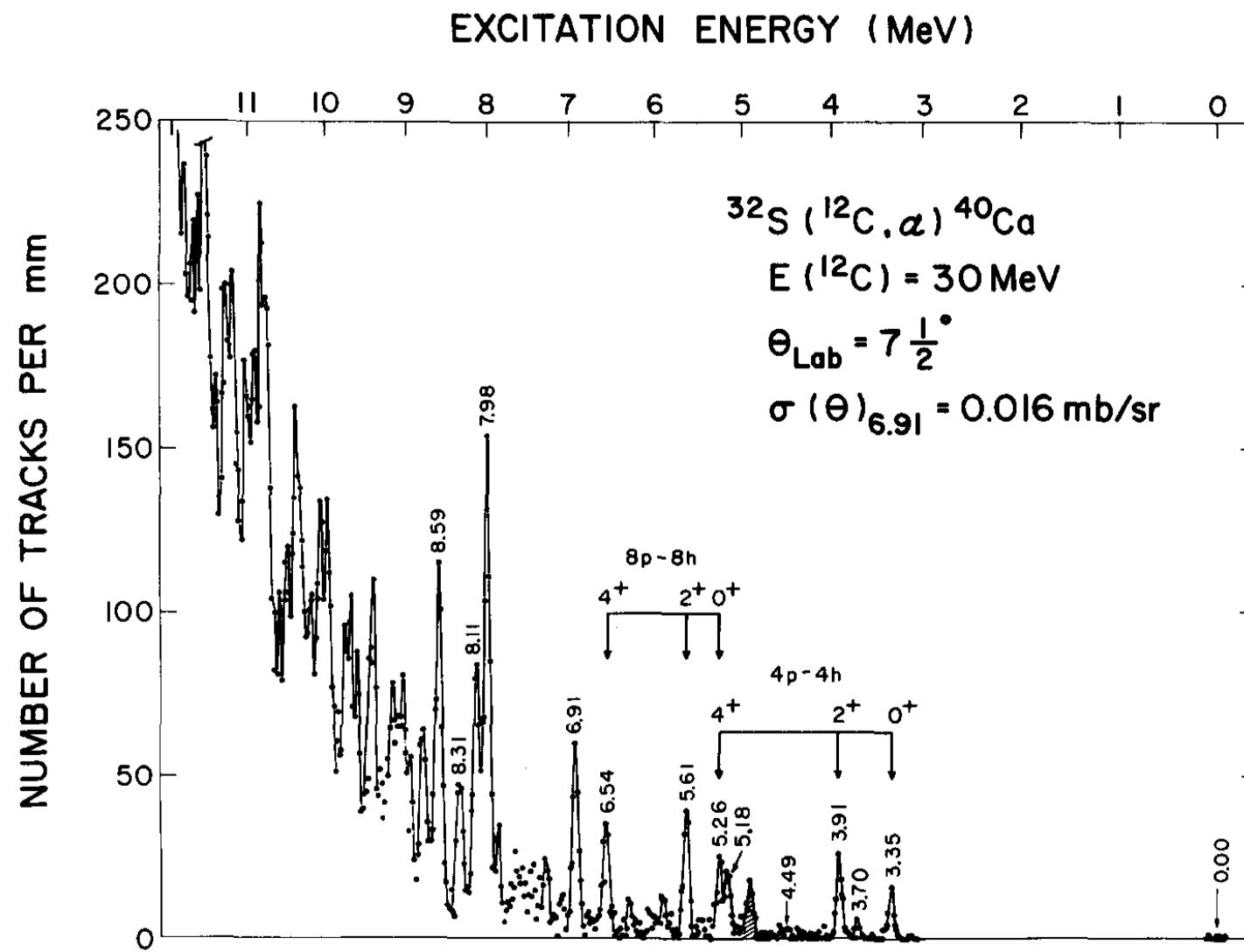
		N=18			N=20			N=22		
		^{42}V	^{43}V	^{44}V	^{45}V	^{46}V	^{47}V			
^{39}Ti	^{40}Ti	^{41}Ti	^{42}Ti	^{43}Ti	^{44}Ti	^{45}Ti	^{46}Ti			
^{38}Sc	^{39}Sc	^{40}Sc	^{41}Sc	^{42}Sc	^{43}Sc	^{44}Sc	^{45}Sc			
^{37}Ca	^{38}Ca	^{39}Ca	^{40}Ca	^{41}Ca	^{42}Ca	^{43}Ca	^{44}Ca			
^{36}K	^{37}K	^{38}K	^{39}K	^{40}K	^{41}K	^{42}K	^{43}K			
^{35}Ar	^{36}Ar	^{37}Ar	^{38}Ar	^{39}Ar	^{40}Ar	^{41}Ar	^{42}Ar			
^{34}Cl	^{35}Cl	^{36}Cl	^{37}Cl	^{38}Cl	^{39}Cl	^{40}Cl	^{41}Cl			

E. Ideguchi et al., PRL 81 (2001) 222501



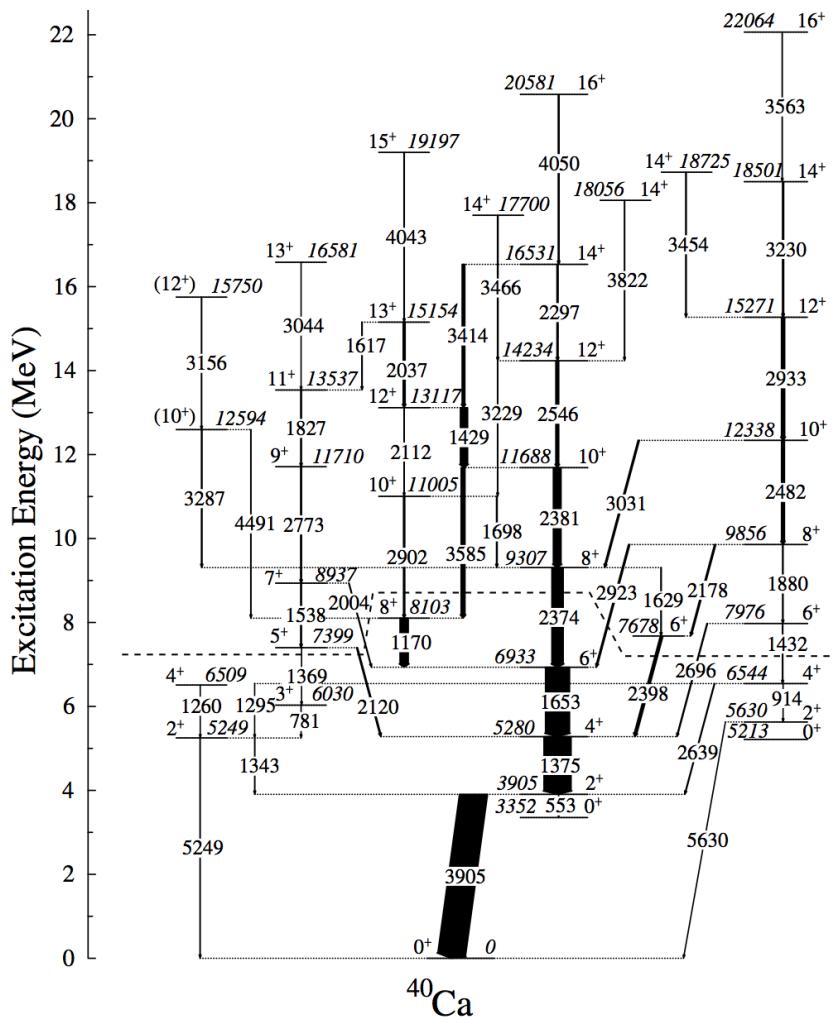
- spherical and highly-deformed magic numbers appear at similar particle numbers – dramatic shape coexistence

Identification of 4p-4h and 8p-8h structures in ^{40}Ca



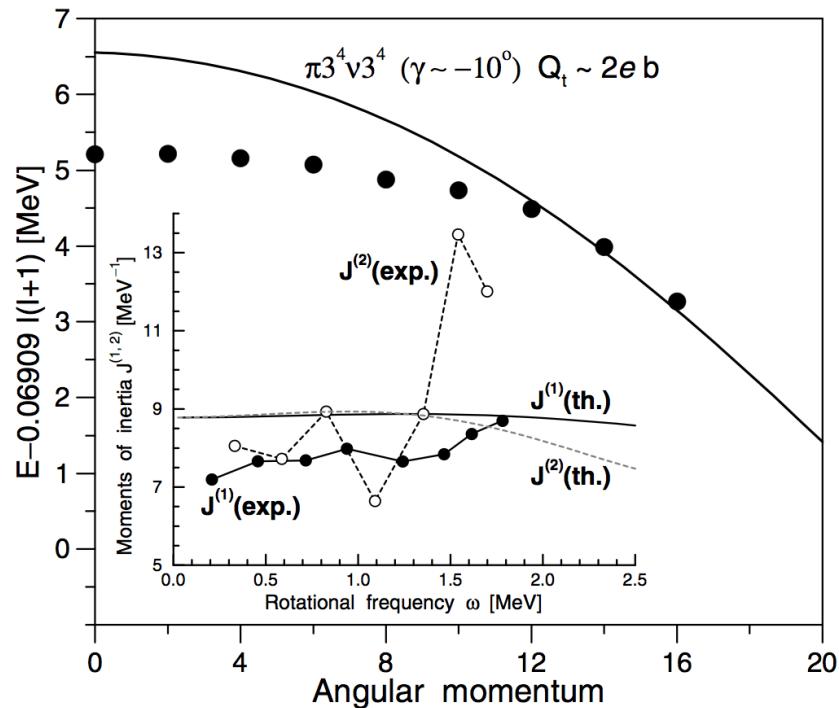
R. Middleton et al, Phys. Lett. 39B (1972) 339

High-spin spectroscopy around ^{40}Ca



- regular rotational bands built on 0^+ states observed up to spin $14^+ - 16^+$ in ^{40}Ca , $^{36,38,40}\text{Ar}$, ...

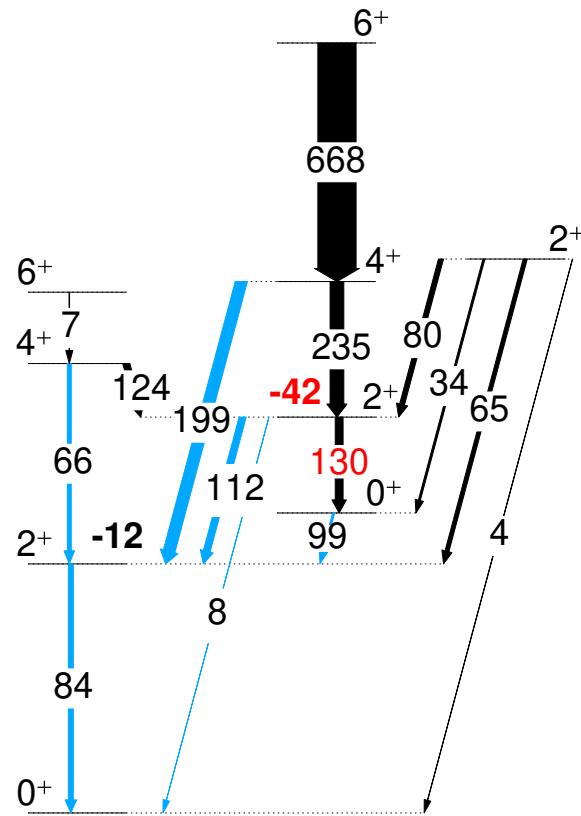
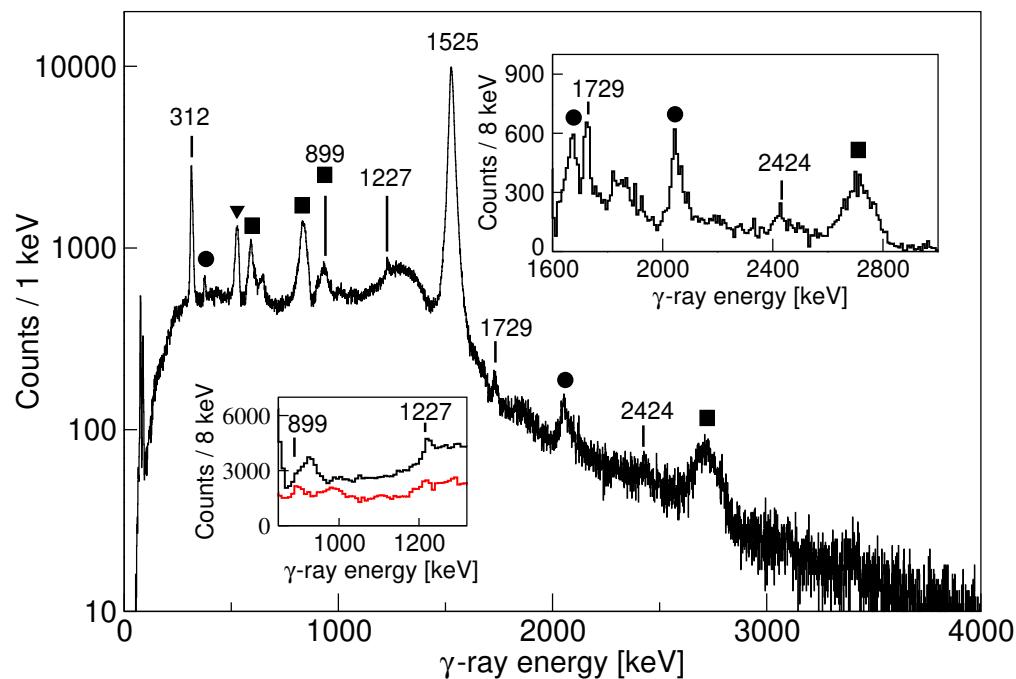
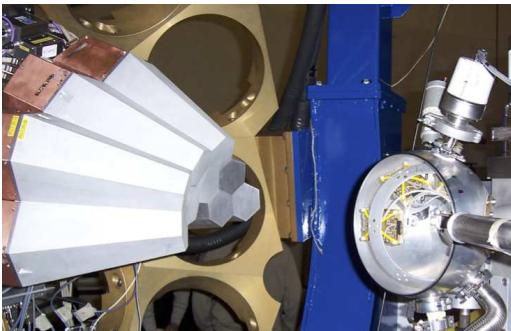
E. Ideguchi et al., PRL 81 (2001) 222501



- intense transitions linking very deformed structures to ground-state bands – mixing of configurations

Coulomb excitation of ^{42}Ca at LNL

- Targets: ^{208}Pb , ^{197}Au , 1mg/cm²
- AGATA: 3 triple clusters
- DANTE: 3 MCP detectors, θ range: 100-144°



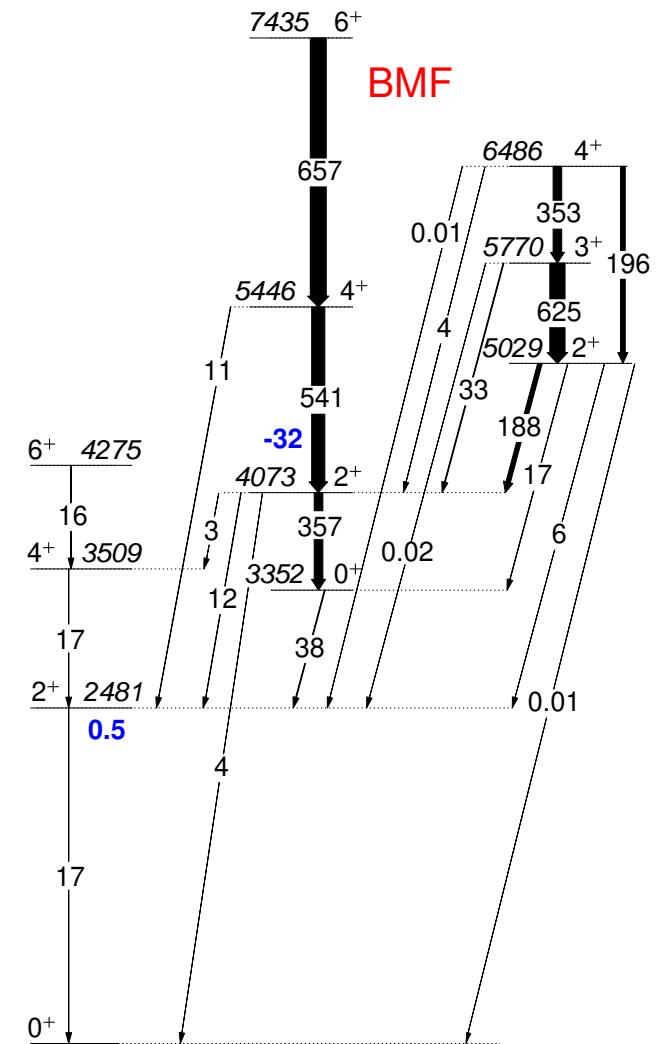
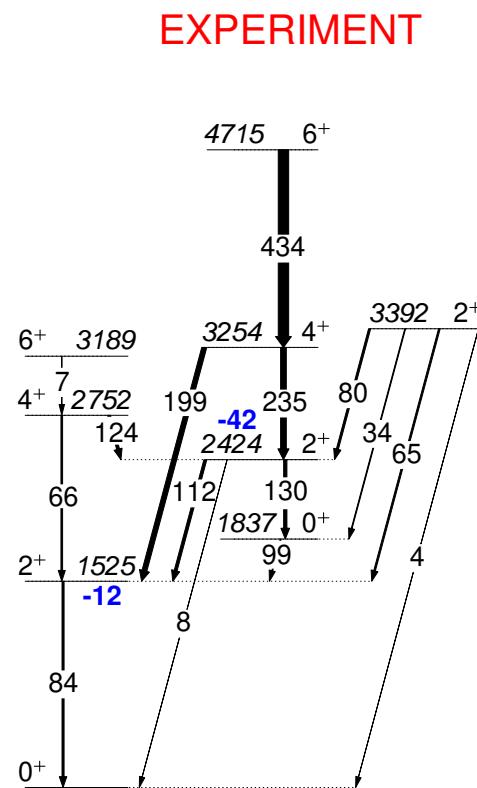
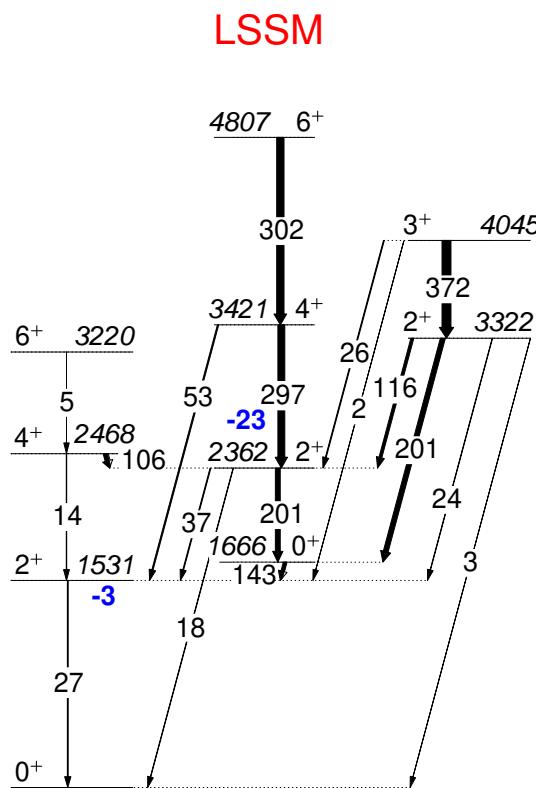
- first population of a superdeformed band in Coulomb excitation
- measured quadrupole moment of 2_2^+ corresponds to $\beta = 0.48(14)$

K. Hadyńska-Klek et al, PRL 117 (2016) 062501

Comparison with theoretical calculations

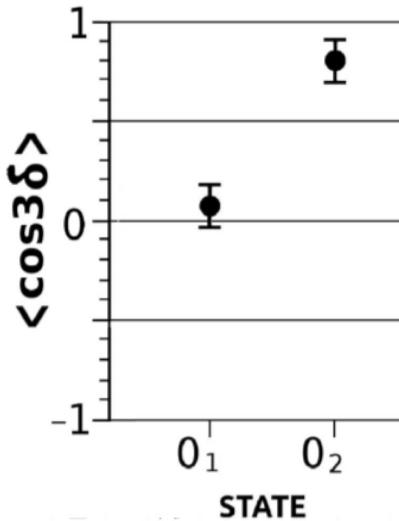
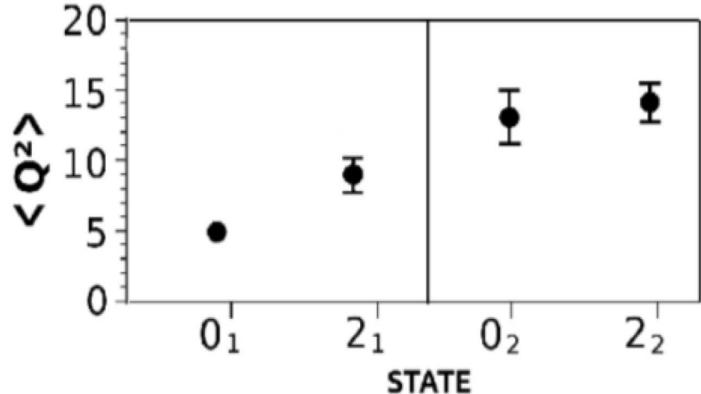
K. Hadyńska-Klek,
PRL 117 (2016) 062501

- Large-Scale Shell Model: F.Nowacki, H.Naïdja, B.Bouthong (Strasbourg)
- Beyond Mean Field calculations with Gogny D1S: T. R. Rodriguez (Madrid)



Shape parameters of 0^+ and 2^+ states in ^{42}Ca

K. Hadyńska-Klęk, PRC 97 (2018) 024326



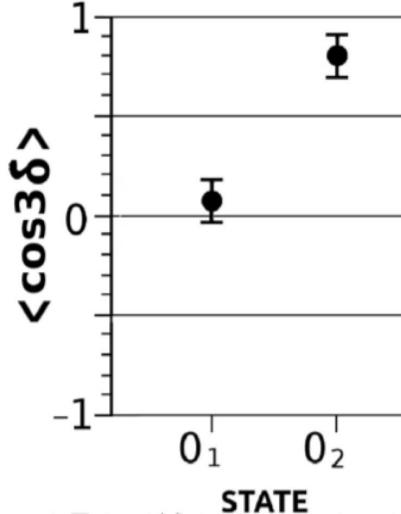
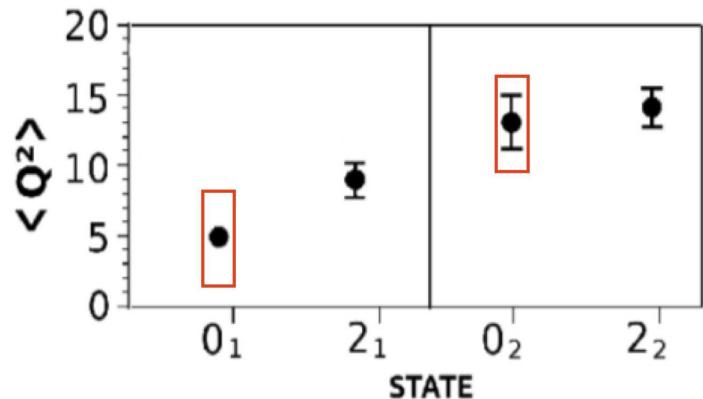
$$\bar{\beta} = \sqrt{\langle \beta^2 \rangle} = \sqrt{\frac{\langle Q^2 \rangle}{q_0^2}}$$
$$\bar{\gamma} = \arccos \langle \cos(3\delta) \rangle$$

- deformation parameters:
 - side band: $\bar{\beta}=0.43(4)$, $\bar{\gamma}=13(6)^\circ$
 - ground-state band: $\bar{\beta}=0.26(2)$, $\bar{\gamma}=29(2)^\circ$ (?)
- are these static deformations, or fluctuations?
 - what about softness in β : $\sigma(Q^2) = \sqrt{\langle Q^4 \rangle - \langle Q^2 \rangle^2}$?

Shape parameters of 0^+ and 2^+ states in ^{42}Ca

K. Hadyńska-Klęk, PRC 97 (2018) 024326

red rectangles: $\sigma(Q^2)$



$$\bar{\beta} = \sqrt{\langle \beta^2 \rangle} = \sqrt{\frac{\langle Q^2 \rangle}{q_0^2}}$$
$$\bar{\gamma} = \arccos \langle \cos(3\delta) \rangle$$

$\sigma(Q^2)$ comparable with $\langle Q^2 \rangle$ for the ground-state band

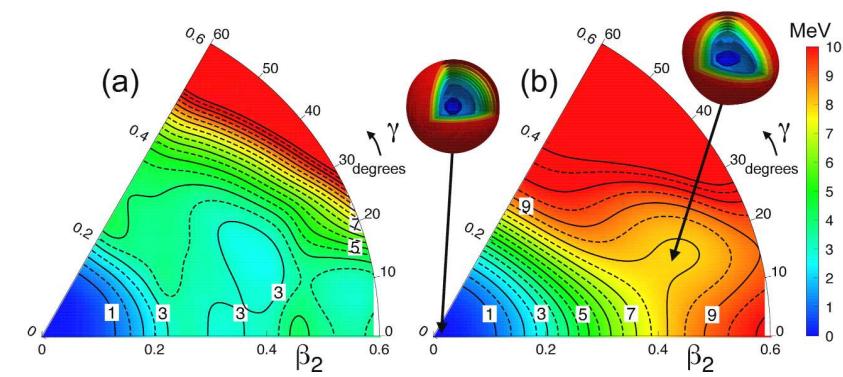
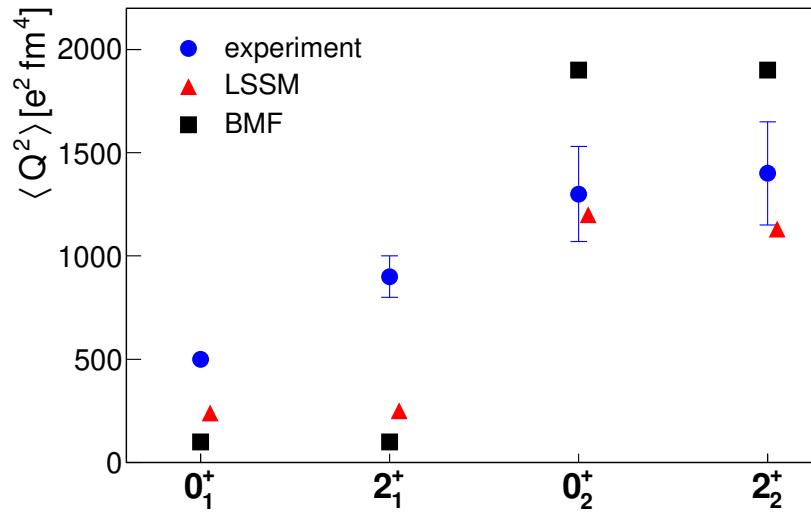
→ fluctuations about a spherical shape; $\langle \cos(3\delta) \rangle = 0$ resulting from averaging over all possible quadrupole shapes ranging from prolate to oblate

excited band: $\sigma(Q^2)$ few times lower than $\langle Q^2 \rangle$

→ static deformation

Comparison with theoretical calculations

K. Hadyńska-Klęk, PRL 117 (2016) 062501



- coexistence of two very different structures reproduced by both theories, slightly triaxial SD minimum present in both potential maps
- deformation in the ground-state band increases with spin contrary to theoretical predictions → mixing seems to be underestimated by calculations

Two-states mixing model applied to ^{42}Ca

- mixing of the underlying configurations can be deduced from measured E2 matrix elements, or alternatively for 0^+ states from E0 strengths:

$$\begin{aligned}\rho^2(E0) &= \frac{Z^2}{R^4} \cos^2\theta_0 \sin^2\theta_0 (\langle r^2 \rangle_A - \langle r^2 \rangle_B)^2 \\ &= \left(\frac{3Z}{4\pi}\right)^2 \cos^2(\theta_0) \sin^2(\theta_0) \cdot \left[(\beta_1^2 - \beta_2^2) + \frac{5\sqrt{5}}{21\sqrt{\pi}} (\beta_1^3 \cos\gamma_1 - \beta_2^3 \cos\gamma_2) \right]^2\end{aligned}$$

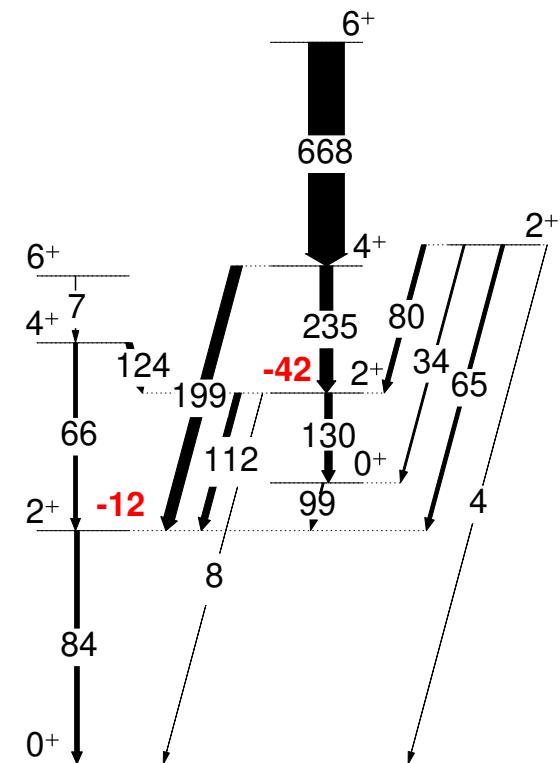
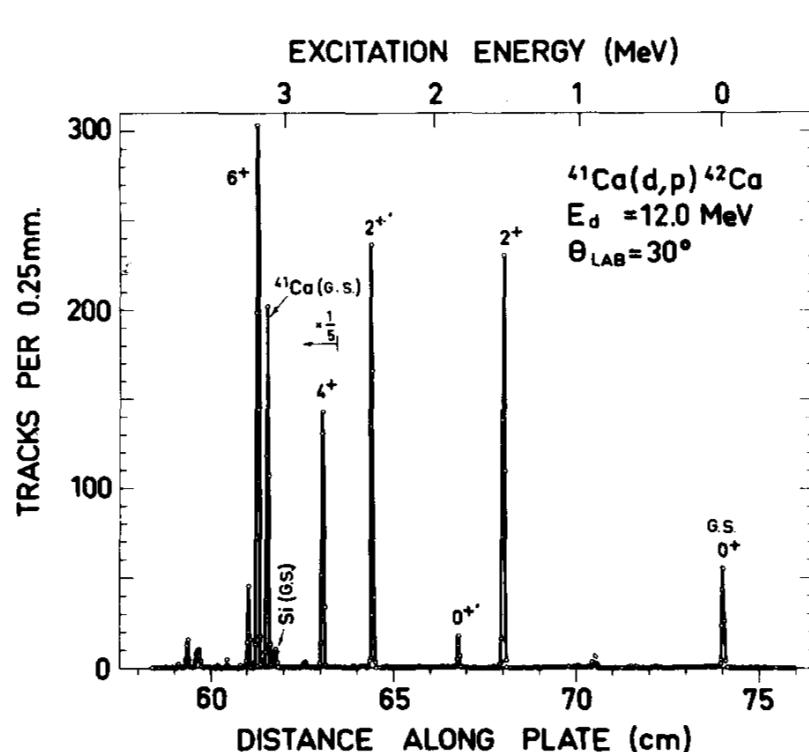
J.L. Wood *et al.*, NPA 651, 323 (1999)

K. Hadyńska-Klek *et al.*, PRC 97 (2018) 024326 (Coulomb excitation),
J.L. Wood *et al.*, NPA 651, 323 (1999) (E0)

	from E2 matrix elements [KHK]	from $\rho^2(E0)$ [JLW] + sum rules results [KHK]	SM	BMF
$\cos^2(\theta_0)$	0.88(4)	0.84(4)	0.85	0.96
$\cos^2(\theta_2)$	0.39(8)	-	0.83	0.97

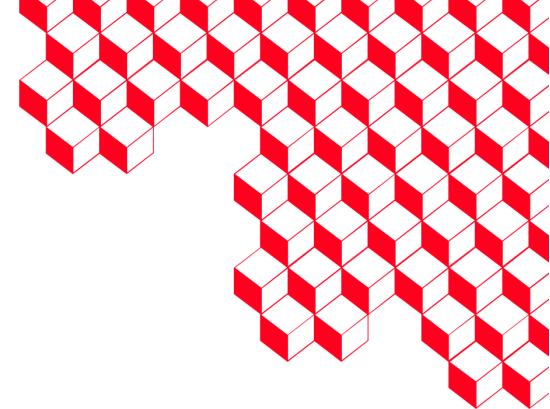
- good agreement of the $\cos^2(\theta_0)$ values obtained with the two methods
- $\cos^2(\theta_2) < 0.5$: two-state mixing model cannot be applied to 2^+ states in ^{42}Ca

Population of the deformed structure in one-neutron transfer



C. Ellegaard *et al.*, Phys. Lett. 40B (1972) 641

- equal population of 2_1^+ and 2_2^+ in $^{41}\text{Ca}(\text{d},\text{p})^{42}\text{Ca}$ – **the same** admixture of $(f_{7/2})^2$, while the **quadrupole moments are very different!**
- the remaining admixtures to the 2_1^+ and 2_2^+ wave functions must be different → another configuration must enter the mixing

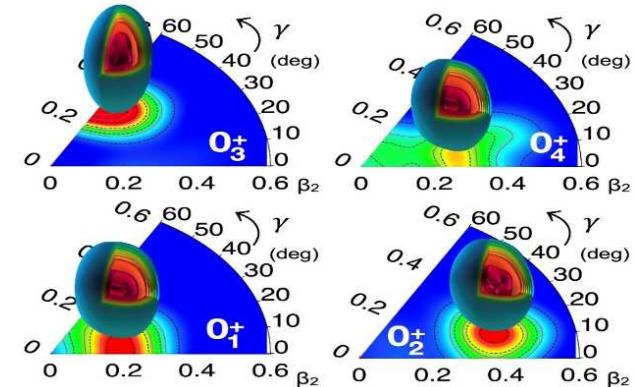


Part II: AGATA at GANIL

shape coexistence in ^{106}Cd

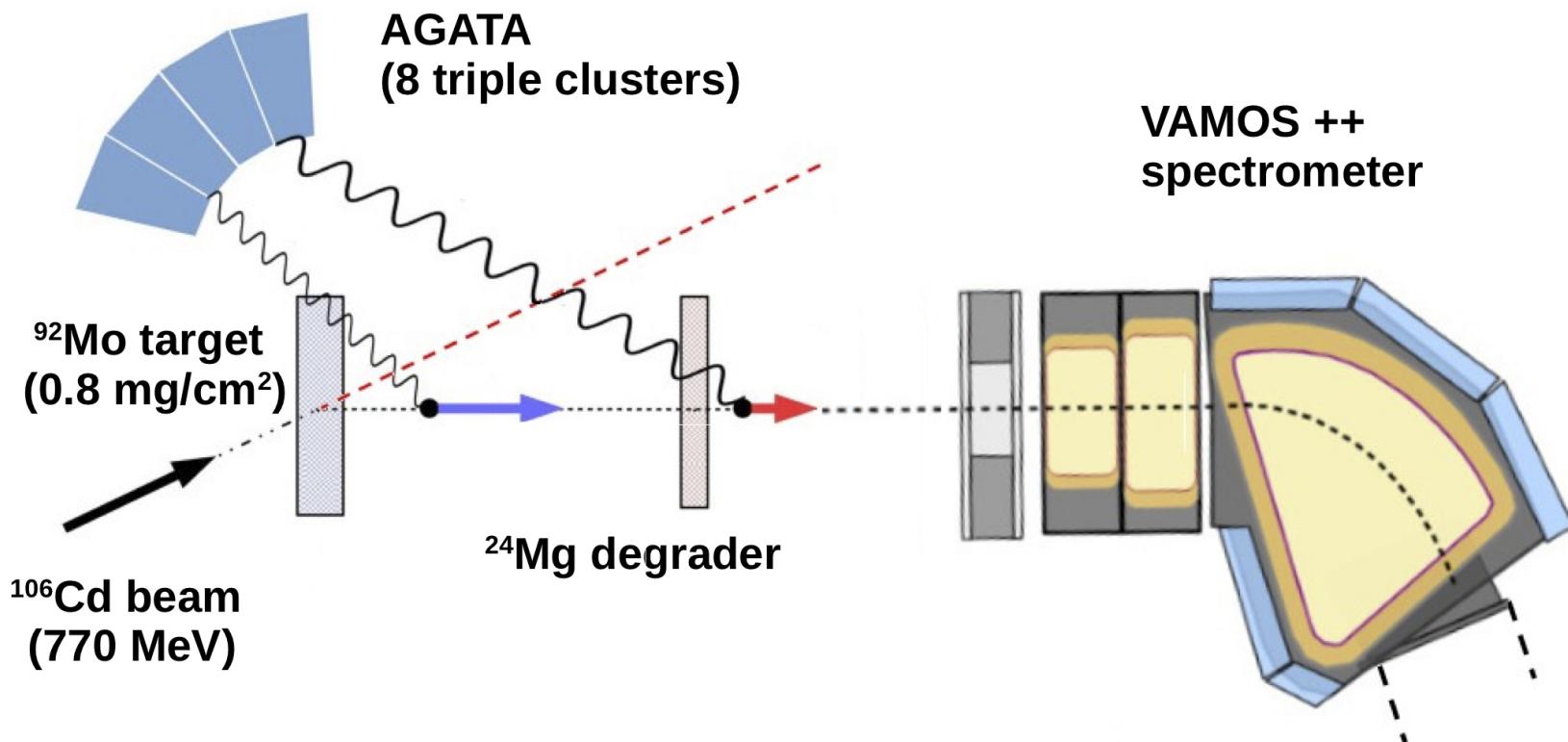
Shapes of Cd nuclei – context

- departure from the surface-vibration paradigm towards a multiple shape-coexistence scenario
 - β decay ([TRIUMF](#)) + DSAM lifetime measurements ([Kentucky](#)) in $^{110,112}\text{Cd}$ with guidance from BMF calculations ([P.E. Garrett et al, Phys. Rev. Lett. 123, 142502 \(2019\)](#))
- data can be reconciled with the vibrational picture using partial dynamical symmetry in the IBM ([N. Gavrielov et al, Phys. Rev. C 108, L031305 \(2023\)](#))
- triggered a multitude of new measurements:
 - high-precision beta decay into ^{110}Cd ([GRiffin, TRIUMF – 2022](#))
 - Coulomb excitation of ^{110}Cd ([AGATA, LNL; GRETINA, ANL – 2022](#))
- also for neighbouring nuclei, in particular ^{106}Cd :
 - Coulomb excitation of ^{106}Cd : ([ReA3, MSU – D. Rhodes et al, Phys. Rev. C 103, L051301 \(2021\); GRETINA, ANL – T. Gray et al, Phys. Lett. B 834, 137446 \(2022\)](#))
 - RDDS lifetime measurement in $^{102-108}\text{Cd}$: ([AGATA, GANIL – M. Siciliano et al, Phys. Rev. C 104, 034320 \(2021\)](#))



Experiment

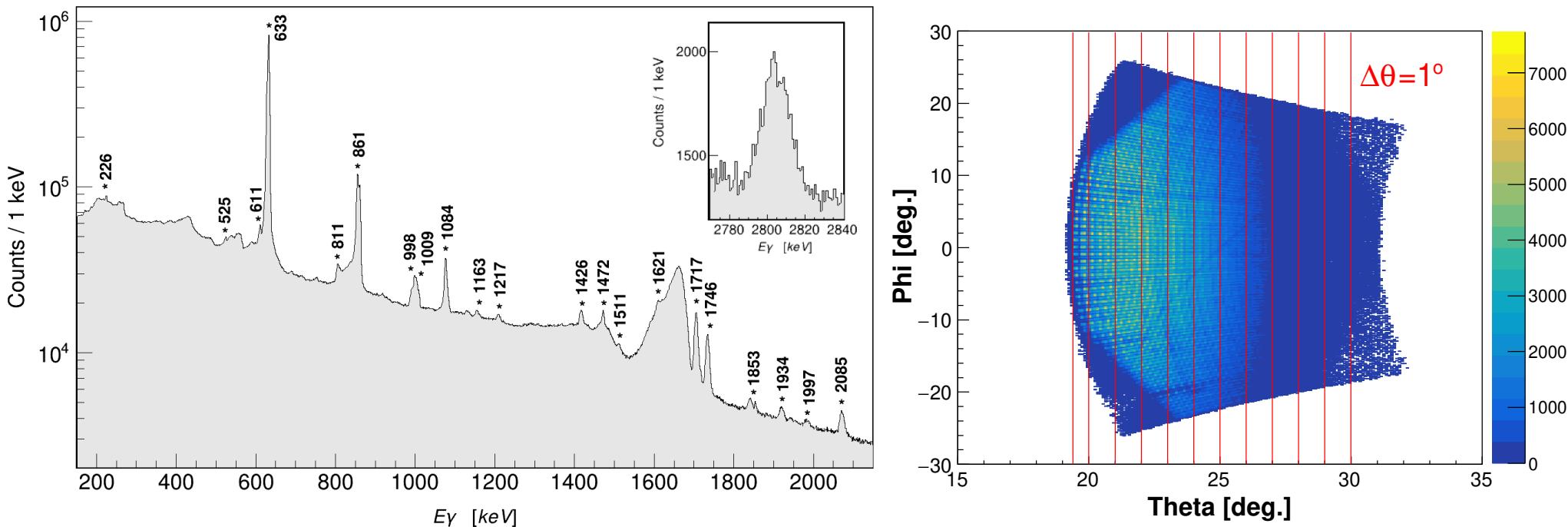
- inelastic scattering data on ^{106}Cd : byproduct of a RDDS lifetime measurement following multinucleon transfer in the $^{106}\text{Cd} + ^{92}\text{Mo}$ reaction at 7 MeV/A
 - M. Siciliano et al., Phys. Lett. B 806, 135474 (2020)
 - M. Siciliano et al., Phys. Rev. C 104, 034320 (2021)



- VAMOS at grazing angle (25°); lowest observed scattering angle (19.4°) corresponding to 107% of Cline's safe energy

Experiment

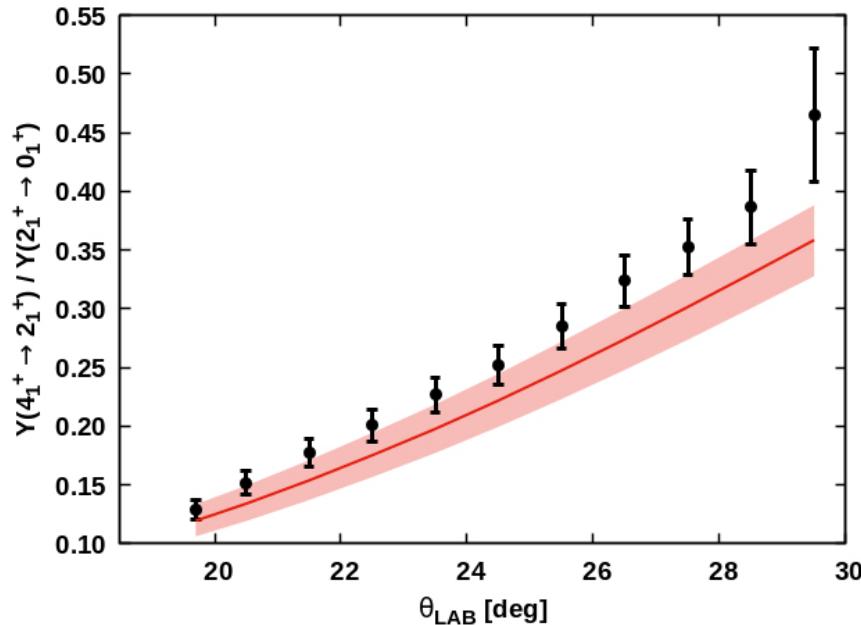
- population of 21 excited states observed (up to spin 6^+)



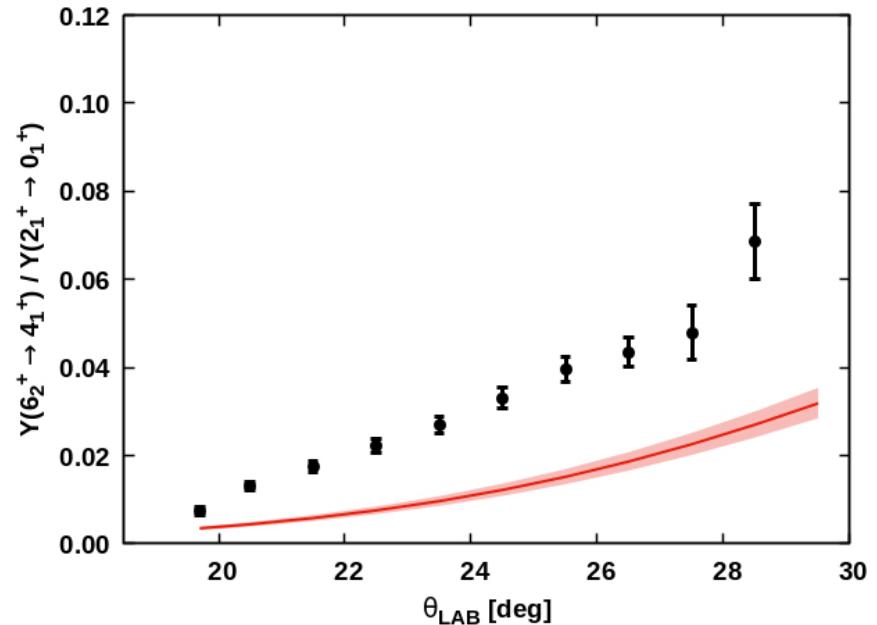
- ^{106}Cd ions identified in VAMOS with $19.4^\circ \leq \theta_{\text{LAB}} \leq 30^\circ$ (Cline's criterion fulfilled for $\theta_{\text{LAB}} \leq 18^\circ$)
- we apply gates on θ_{LAB} with 1° width to study the dependence of the excitation cross sections on scattering angle
- due to complicated acceptance of the spectrometer as a function of θ , we normalise the measured γ -ray intensities to that of the $2_1^+ \rightarrow 0_1^+$ transition

Sample results (strongly populated states)

$4_1^+ \rightarrow 2_1^+$



$6_2^+ \rightarrow 4_1^+$

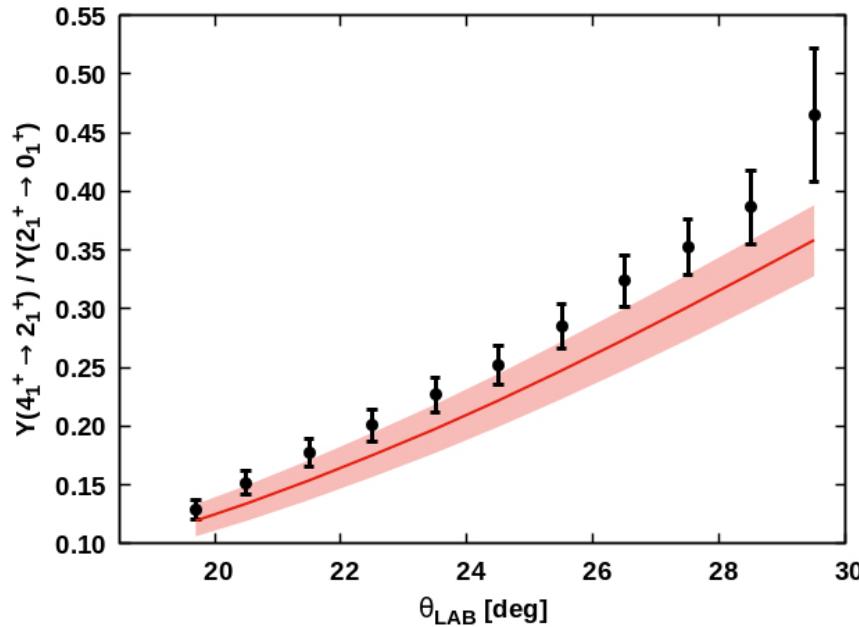


- reasonable agreement with literature data for 4_1^+ (weighted average of measured lifetimes)
- lifetime of the 6_2^+ state deduced from the same data as our transition intensities ([M. Siciliano et al., Phys. Rev. C 104, 034320 \(2021\)](#) is not consistent with the measured intensity ratios

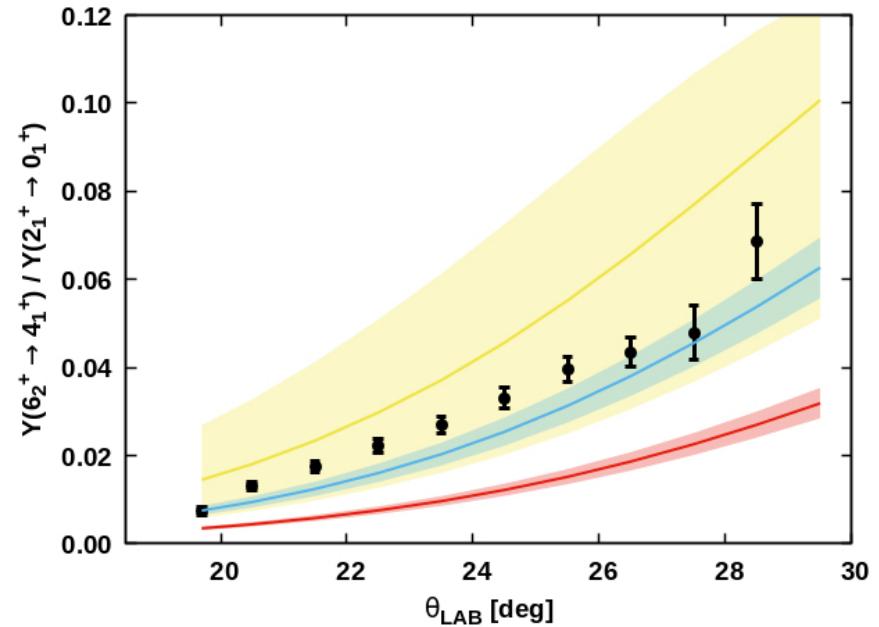
D. Kalaydjieva, PhD thesis, 2023

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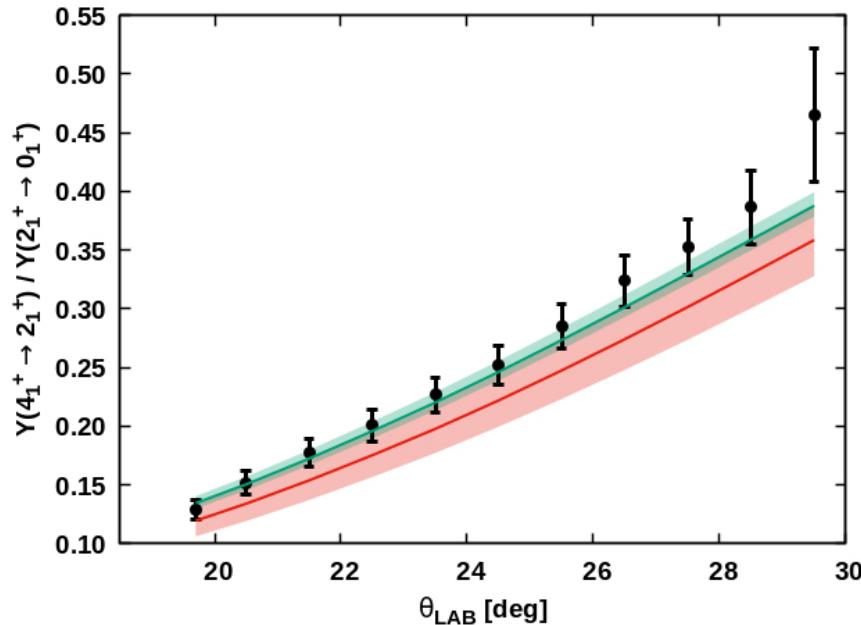


- much better agreement for the 6_2^+ state if we assume:
 - $\langle 6_2^+ || E2 || 4_1^+ \rangle$ matrix element from Coulomb excitation (D. Rhodes et al., Phys. Rev. C 103, L051301 (2021))
 - or 6_2^+ lifetime from $(n, n' \gamma)$ (A. Linnemann, PhD thesis, University of Cologne, 2005 – but here the uncertainty is very large ($\tau = 0.26^{+0.44}_{-0.14}$ ps))

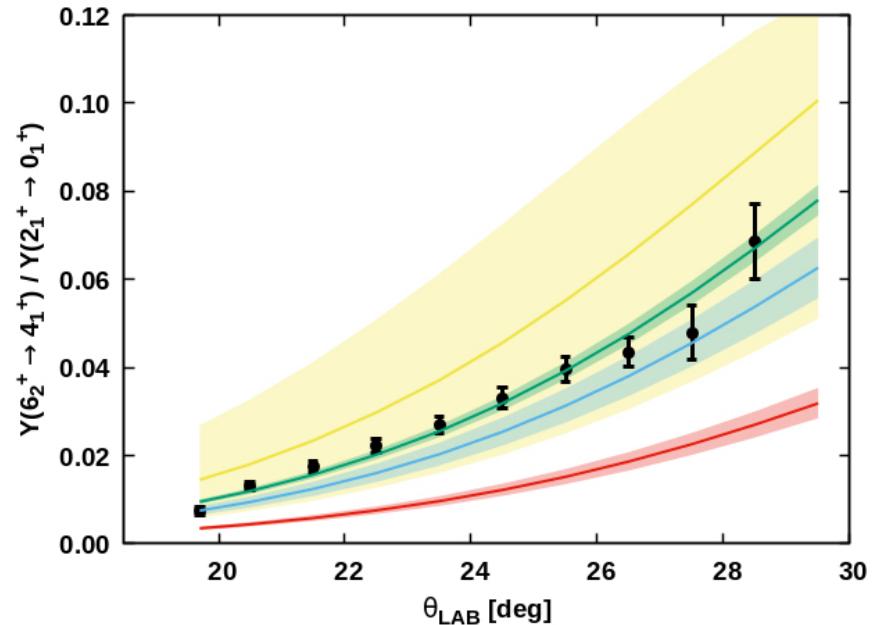
D. Kalaydjieva, PhD thesis, 2023

Sample results (strongly populated states)

$4_1^+ \rightarrow 2_1^+$



$6_2^+ \rightarrow 4_1^+$



- finally, we can try to fit a set of matrix elements to the first few points of the cross-section distribution, and compare the resulting lifetimes:

4_1^+ – GOSIA fit: 1.23(7) ps

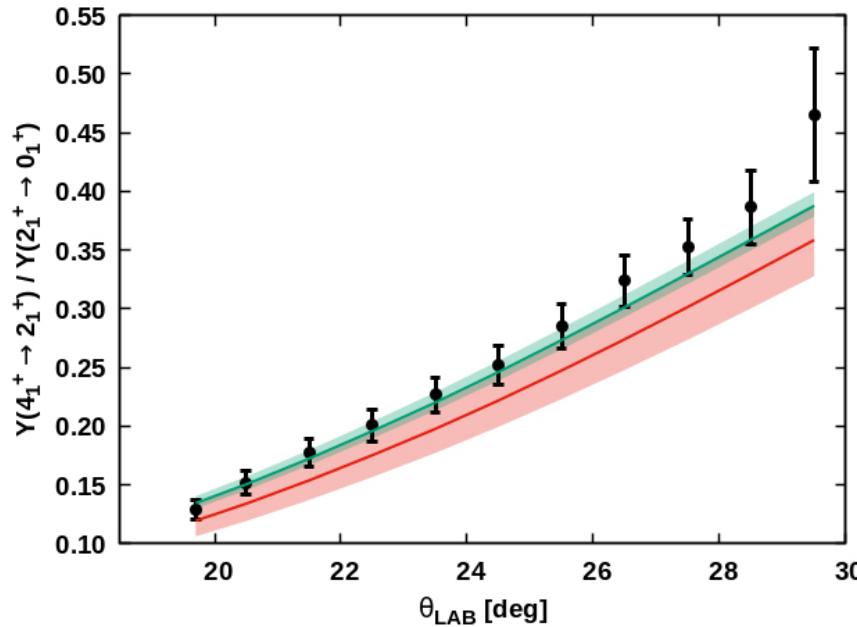
weighted average of lifetimes:
1.32(12) ps

6_2^+ – GOSIA fit: 0.48(3) ps

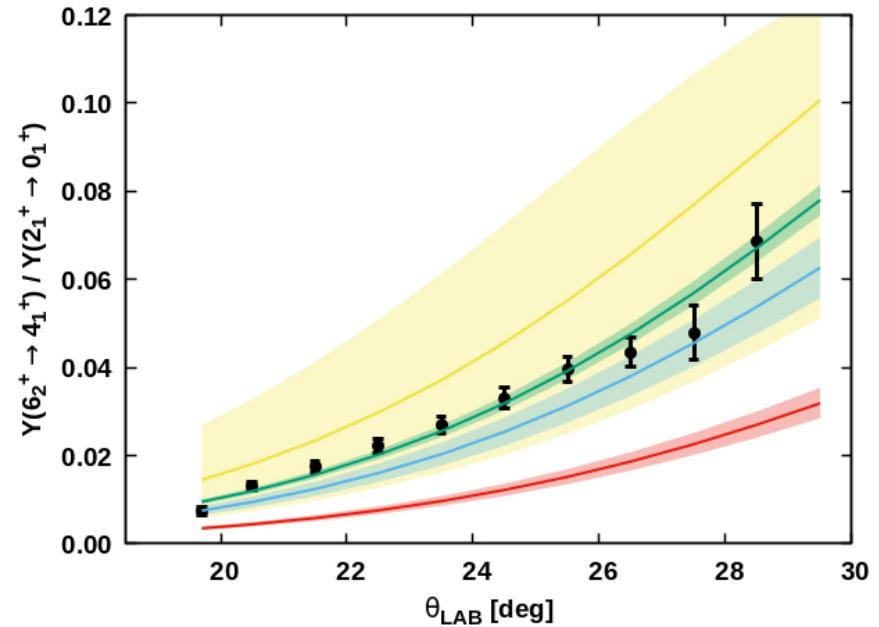
M. Siciliano et al., Phys. Rev. C 104, 034320 (2021): 1.22(15) ps
D. Rhodes et al., Phys. Rev. C 103, L051301 (2021): 0.54(8) ps

Sample results (strongly populated states)

$4_1^+ \rightarrow 2_1^+$



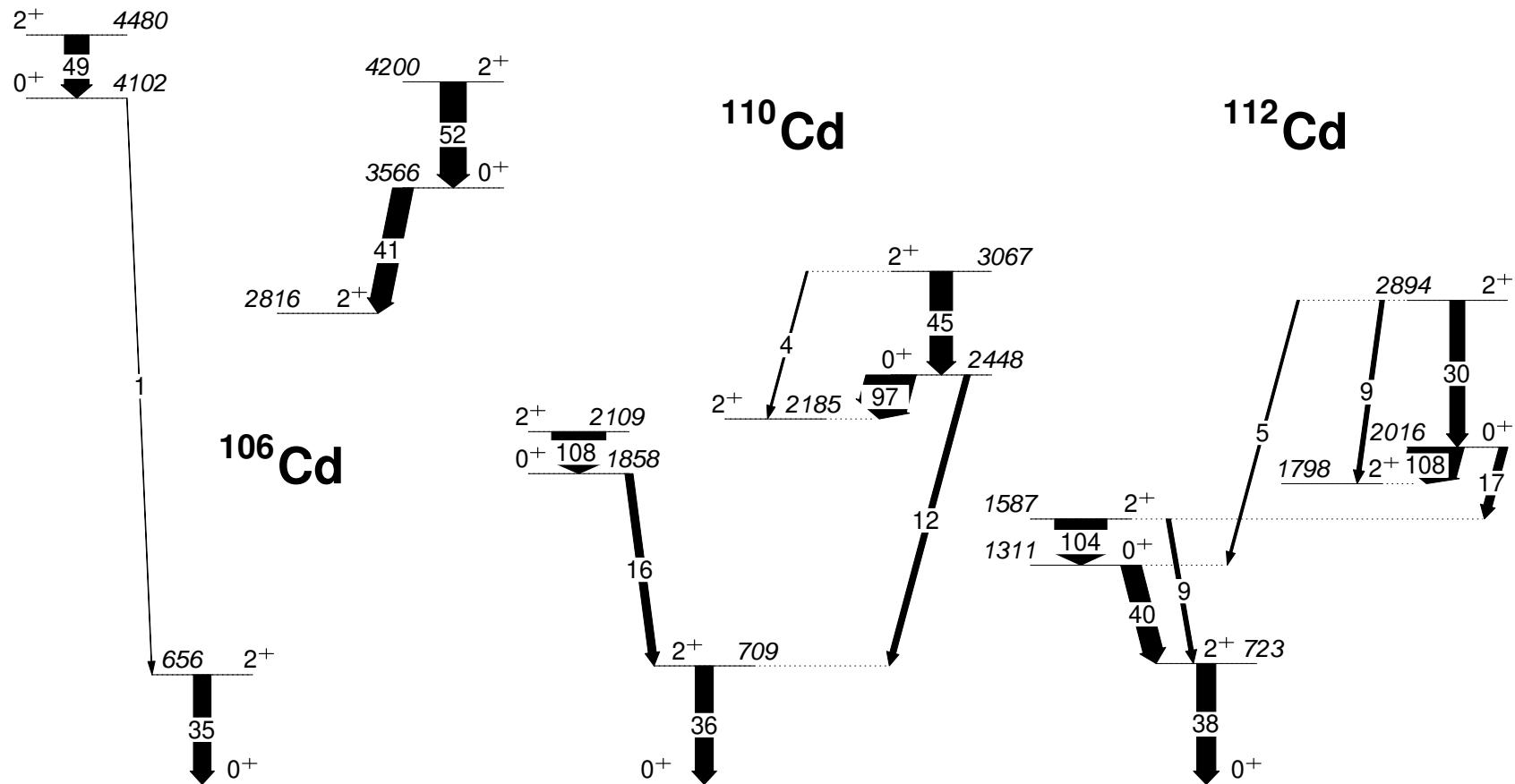
$6_2^+ \rightarrow 4_1^+$



- similar analysis has been applied to all observed states, yielding B(E2) values complementary to those obtained from the RDDS analysis of the same date
- contrary to RDDS, it was possible to obtain B(E2) values for the decay of states that have lifetimes shorter than 1 ps

Shape coexistence in Cd isotopes: BMF predictions

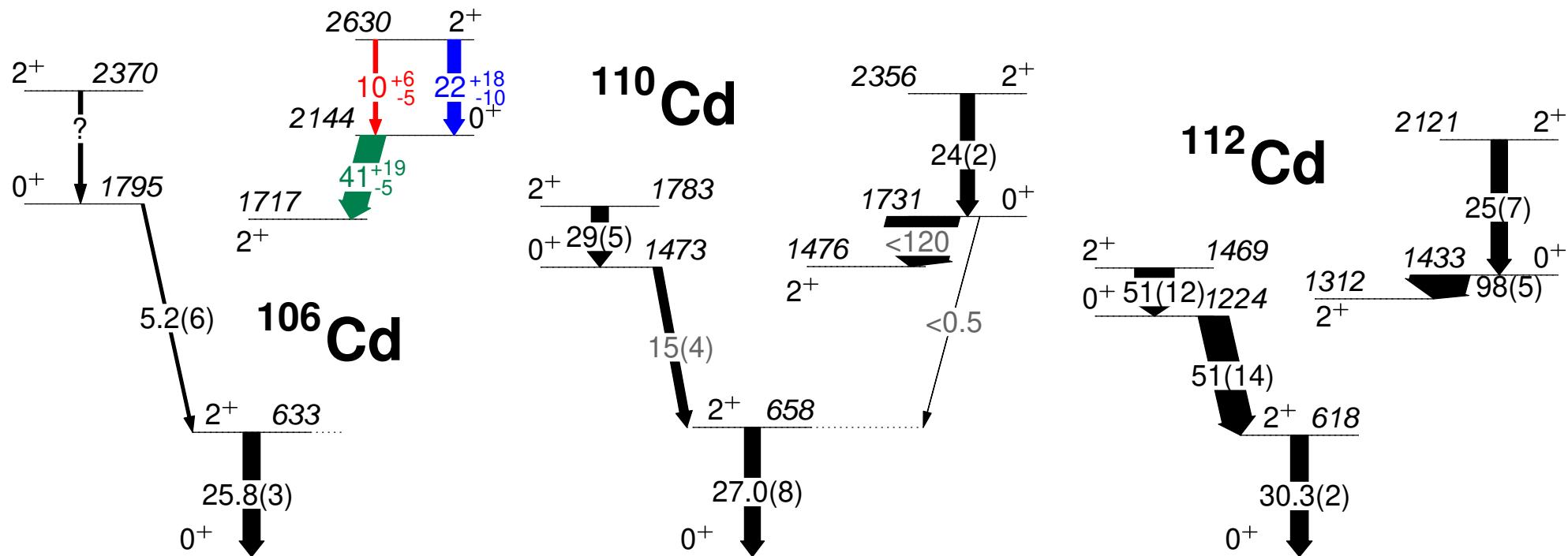
- similar shape-coexisting structures as in $^{110,112}\text{Cd}$ are predicted in ^{106}Cd
- in-band transition strength in the oblate structure predicted to increase with decreasing N, while the $B(E2; 0_3^+ \rightarrow 2_2^+)$ value decreases



SCCM calculations: T.R. Rodriguez

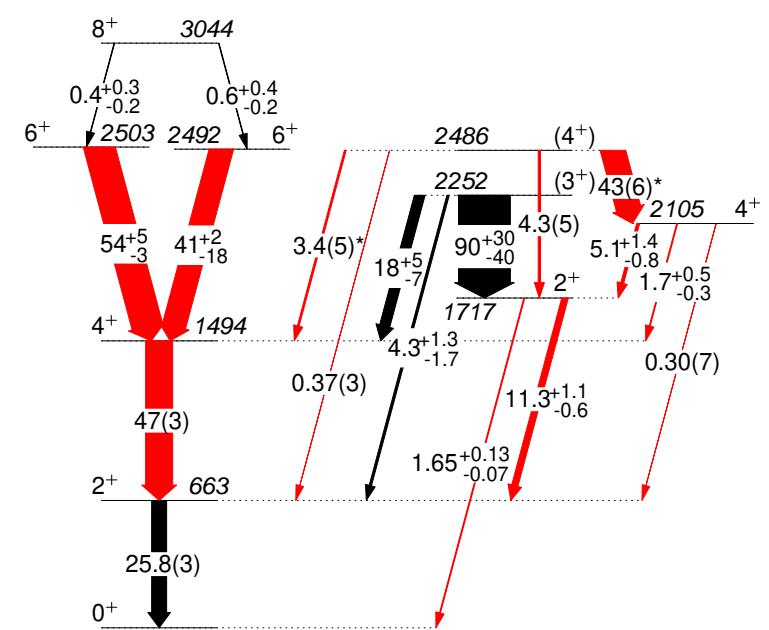
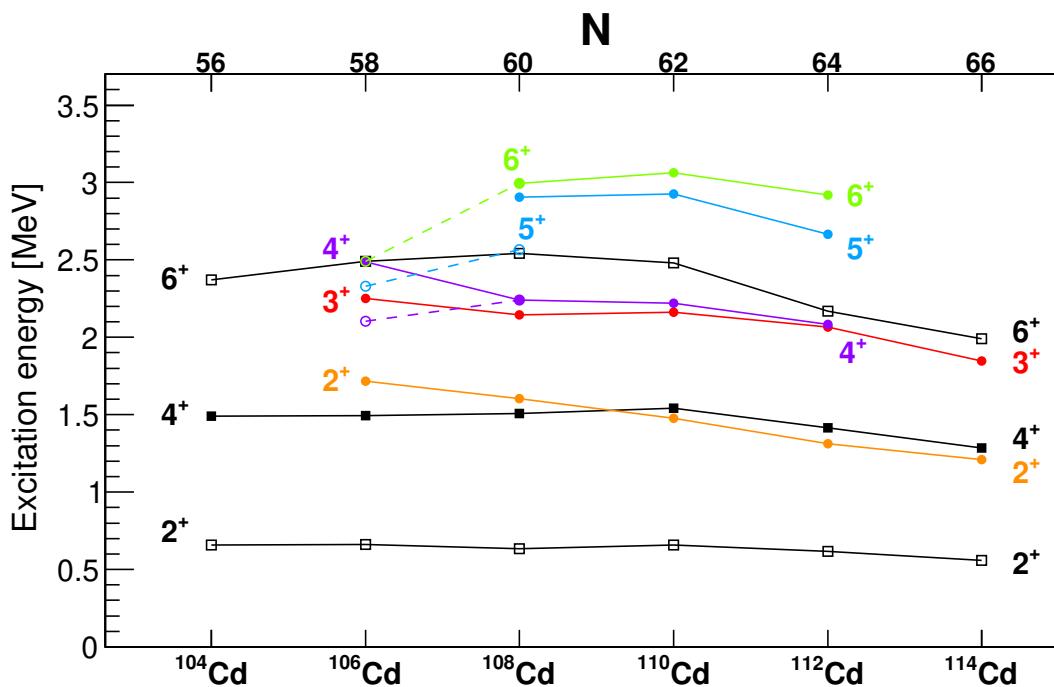
Unsafe Coulomb-excitation results

- decay of the presumably oblate 0_3^+ state agrees well with the SCCM prediction, but the in-band transition strength has a very different trend
- larger $B(E2; 2_5^+ \rightarrow 0_3^+)$ (similar to that in the ground-state band) if the branching ratio from A. Linnemann PhD (Cologne, 2005) is assumed instead of the more precise value from T. Schmidt PhD (Cologne, 2019)

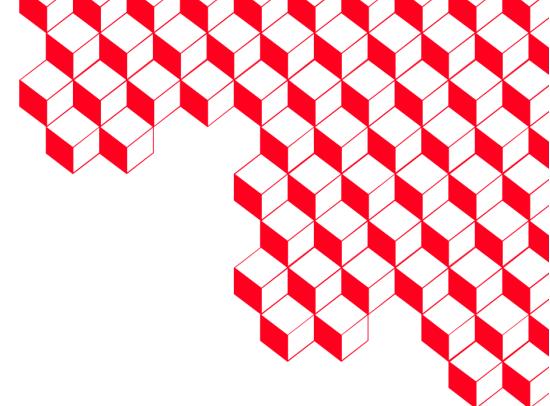


^{106}Cd : D. Kalaydjieva, PhD thesis, 2023; ^{110}Cd : preliminary values (K. Wrzosek-Lipska) in gray

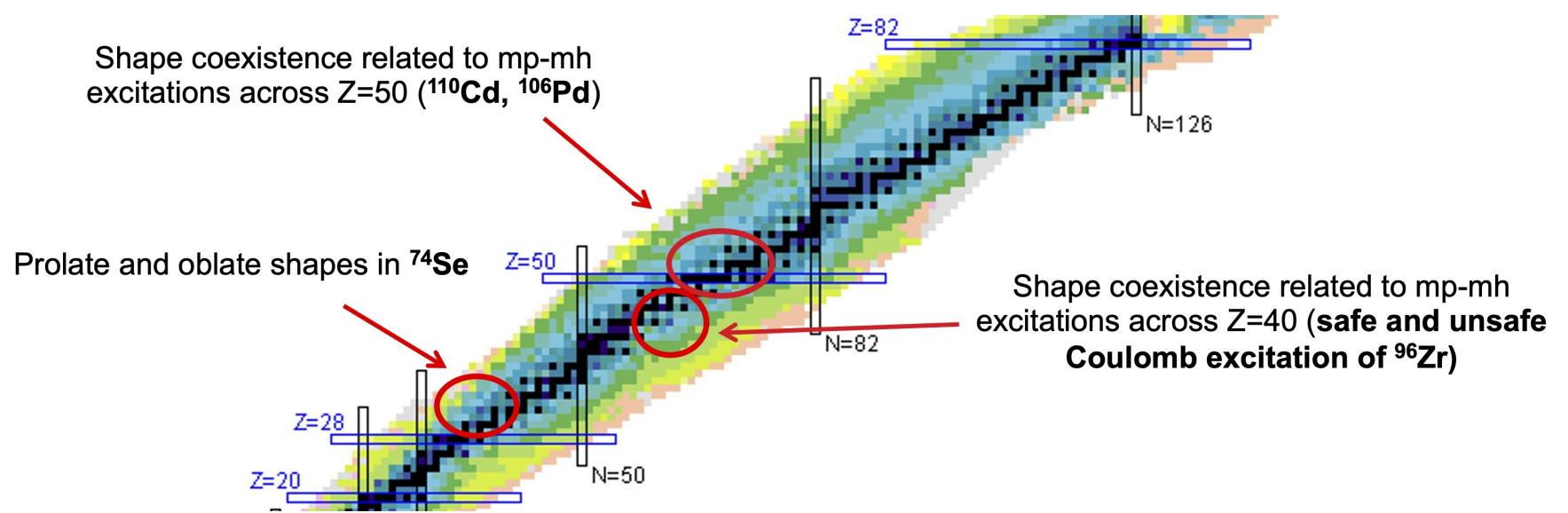
Proposed reorganisation of the level scheme



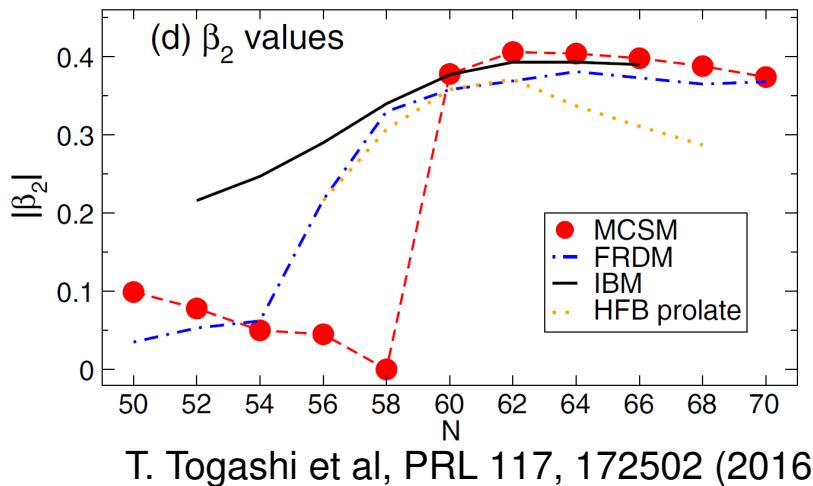
- new K=2 3⁺ and 4⁺ and K=4 4⁺ band members proposed that have expected decay patterns and excitation energies consistent with the systematics
- closely spaced 6⁺ states suggested to result from a strong mixing of the rotational band member with a seniority state
- non-observation of the 2252-keV state in the present data supports its 3⁺ spin-parity (Coulomb excitation of odd-spin positive parity states is strongly hindered)
- firm spin assignments will be obtained from a high-statistics β decay study into ^{106}Cd recently approved at TRIUMF



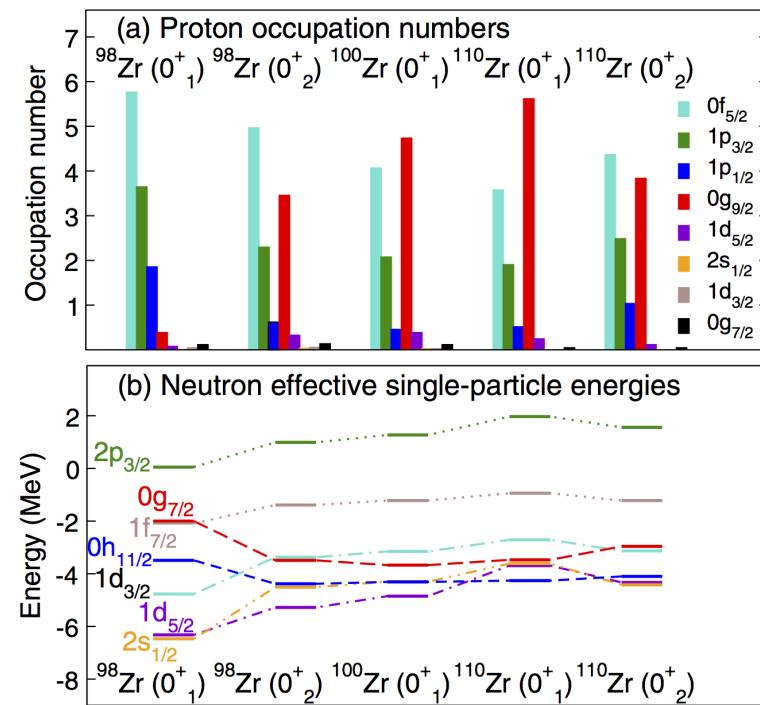
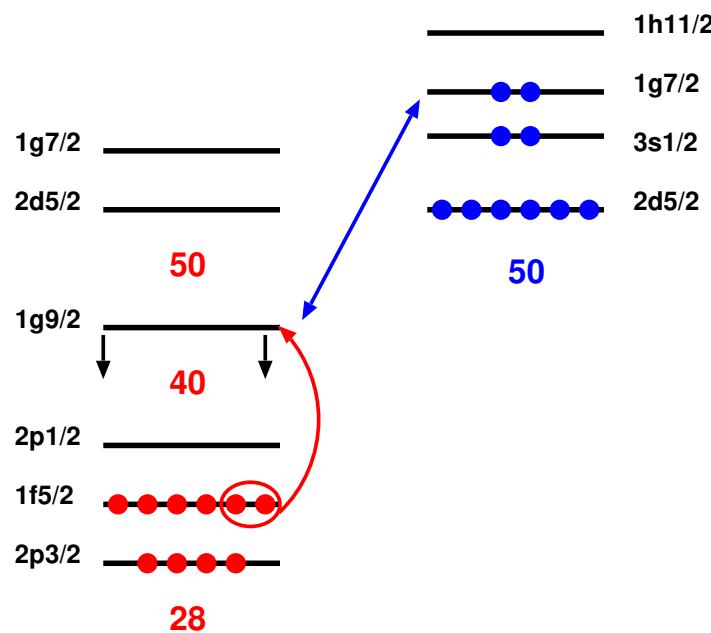
Present and future: AGATA at LNL



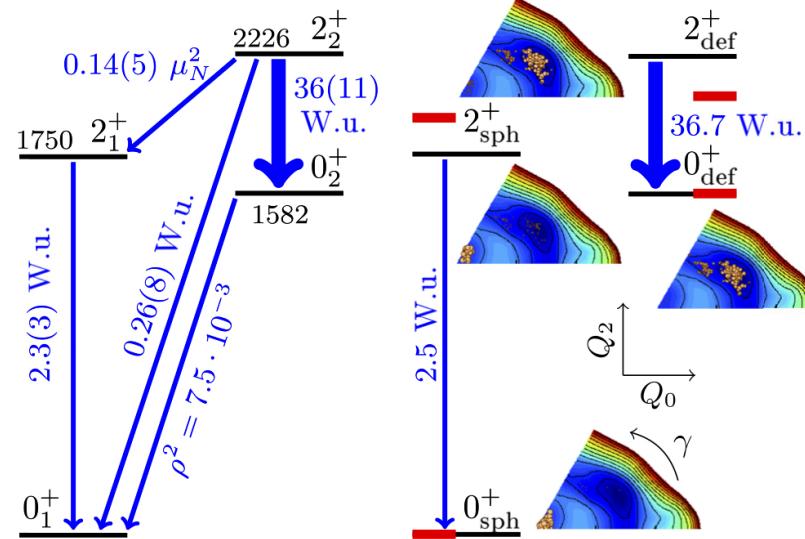
Shape coexistence and type-II shell evolution in Zr isotopes



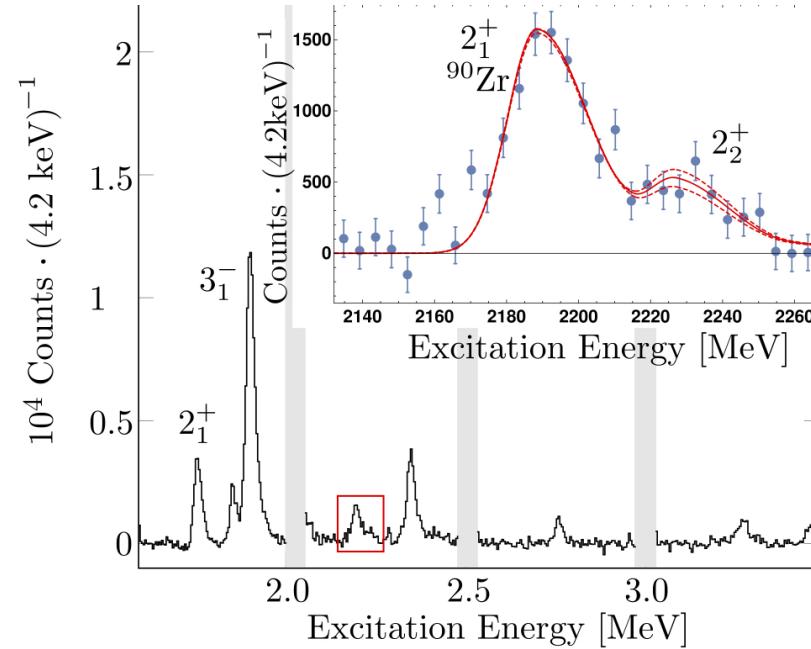
- p-n tensor interaction reduces the $Z=40$ gap when $\nu g_{7/2}$ is being filled
- 0_2^+ states created by $2p-2h$ (+ $4p-4h\dots$) excitation across $Z=40$
- very different configurations and small mixing of 0_1^+ and 0_2^+



Shape coexistence in ^{96}Zr – experimental information



S. Kremer et al, Phys. Rev. Lett. 117 (2017) 172503

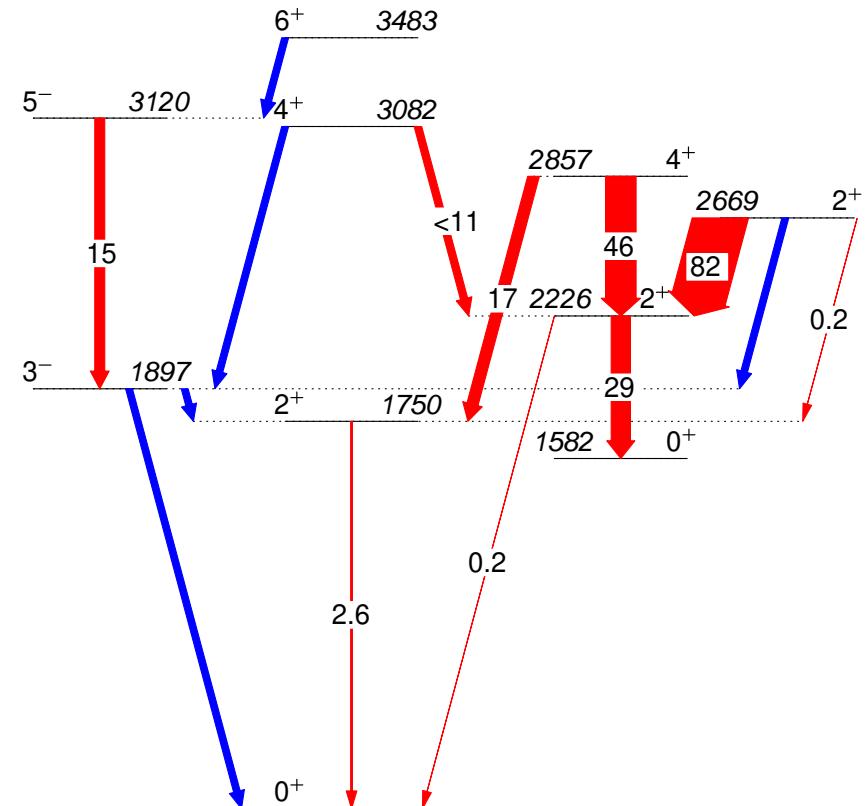


- $B(E2; 2_2^+ \rightarrow 0_1^+)$ measured using electron scattering, combined with known branching and mixing ratios:
→ transition strengths from the 2_2^+ state
- $B(E2; 2_1^+ \rightarrow 0_1^+) = 2.3(3)$ Wu vs $B(E2; 2_2^+ \rightarrow 0_2^+) = 36(11)$ Wu: nearly spherical and a well-deformed structure ($\beta \approx 0.24$)

^{96}Zr decay scheme with revised branching and mixing ratios

J. Wiśniewski et al, Phys. Rev. C 108, 024302 (2023)

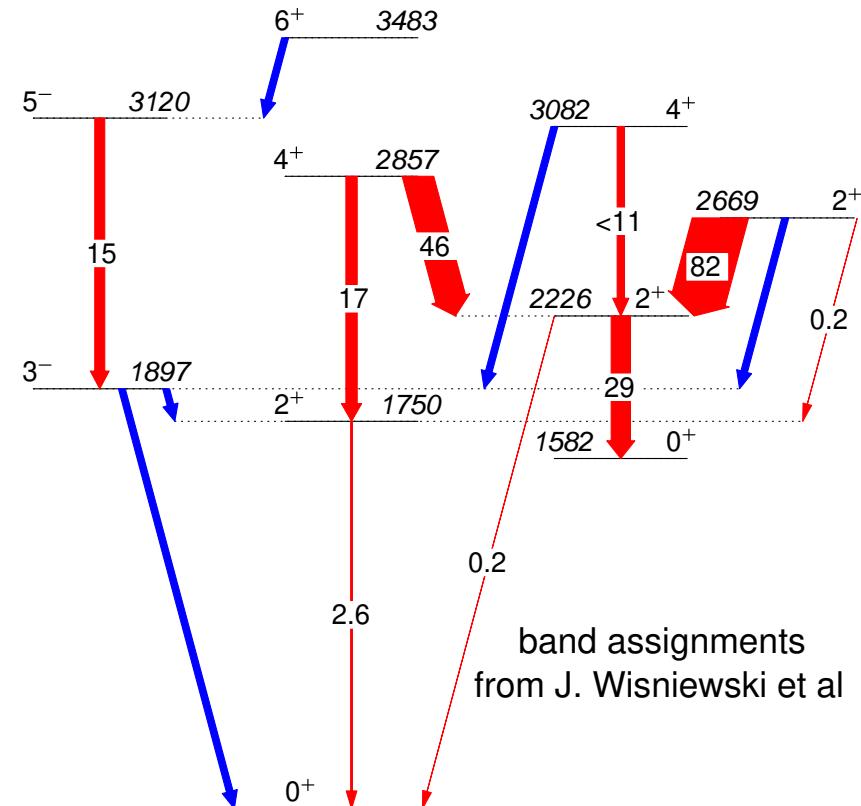
- which 4^+ belongs to which band? if 4_1^+ is part of the deformed structure, why is its decay to the 2_1^+ so strong (mixing between bands should be weak)?
- the $2_3^+ \rightarrow 2_2^+$ decay seems surprisingly enhanced
- E1 transitions from presumably collective states compete with E2 ones



^{96}Zr decay scheme with revised branching and mixing ratios

J. Wiśniewski et al, Phys. Rev. C 108, 024302 (2023)

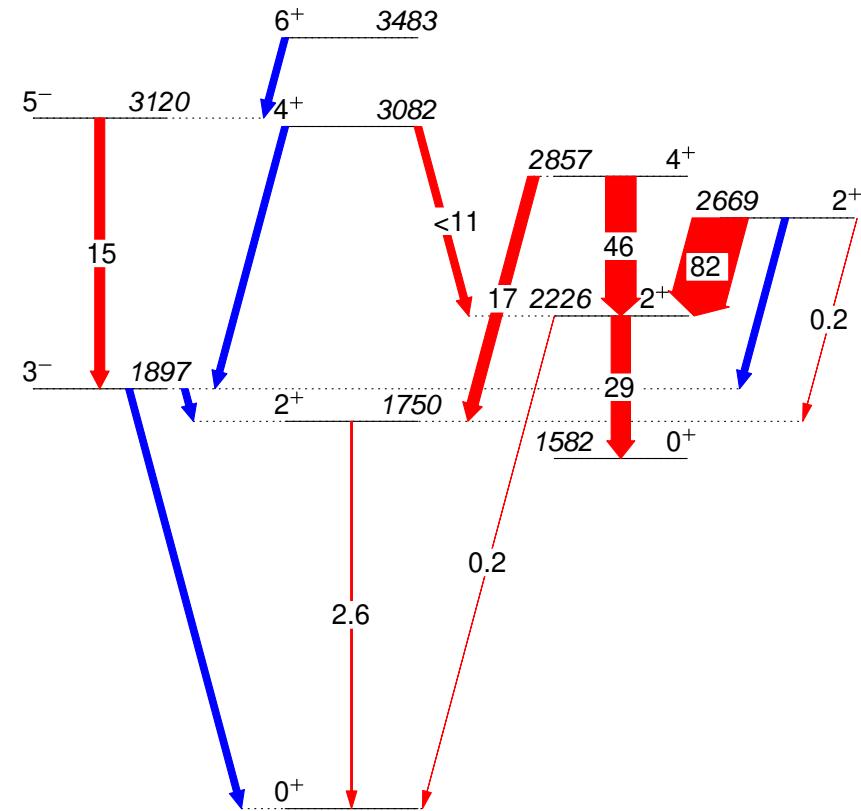
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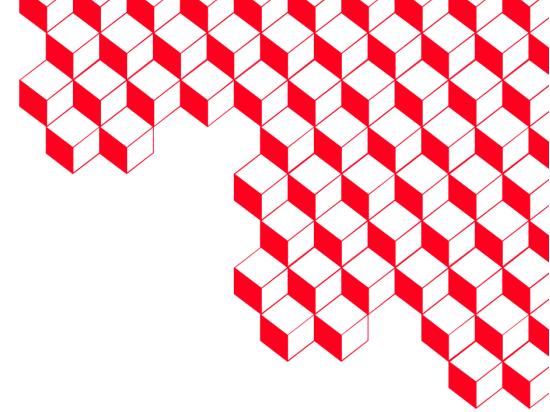
- “unsafe” Coulomb-excitation experiment with AGATA coupled to PRISMA magnetic spectrometer (May 2024)
- notable result: unexpectedly strong population of the 4_1^+ state – consistent with mixing of $4_{1,2}^+$ states

data analysis: Z.T. Ahmed, University of Guelph

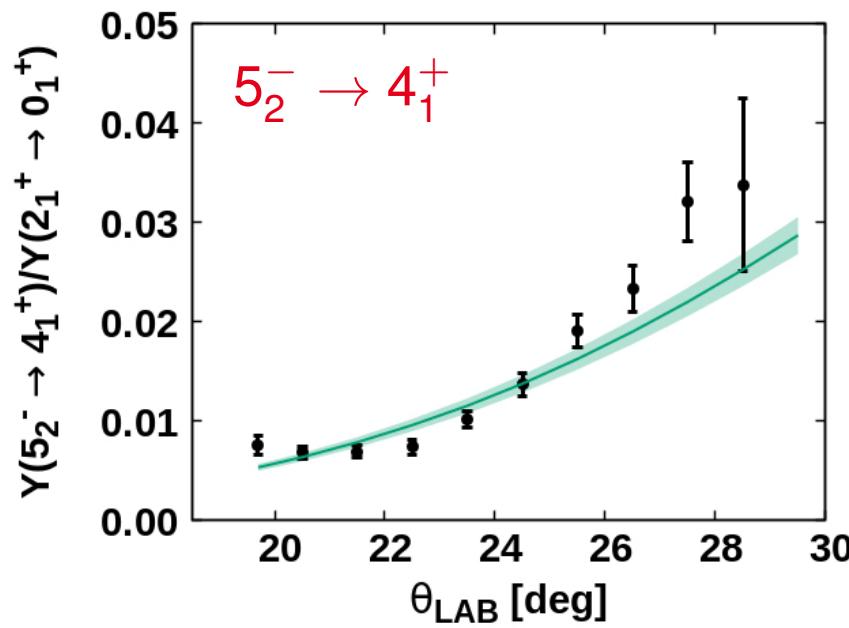
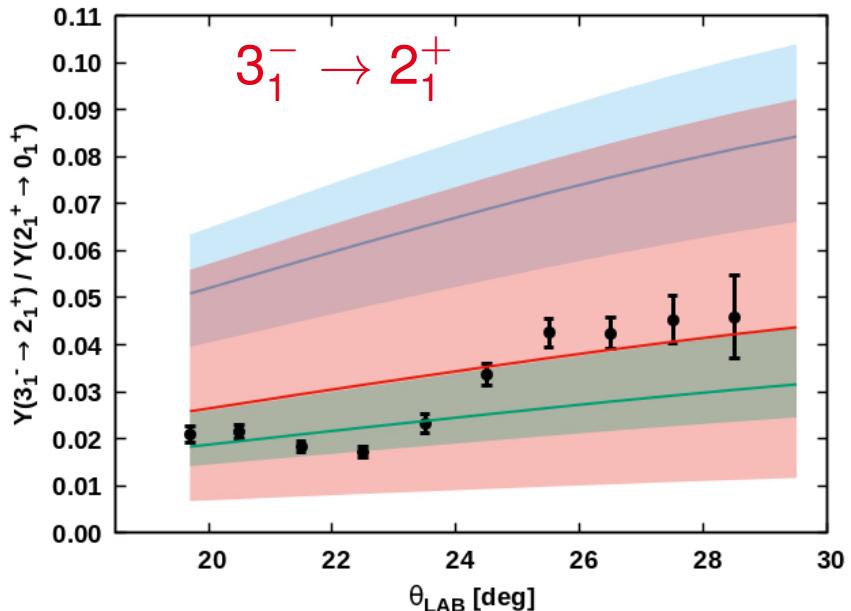
Summary

- **^{42}Ca :** large static deformation of $\beta(0_2^+)=0.43(4)$ and $\beta(2_2^+)=0.45(4)$ is consistent with the superdeformed character of the side band
- $\langle \cos(3\delta) \rangle$ obtained for the 0_2^+ state in ^{42}Ca brings the first experimental evidence for the non-axial character of SD structures in the $A \sim 40$ mass region
- **^{106}Cd :** we obtained new experimental information on the presumably oblate 0_3^+ and 2_5^+ states and proposed a rearrangement of the level scheme involving $K=2$ and $K=4$ structures
- **^{96}Zr (preliminary!):** we observed a surprisingly strong population of the 4_1^+ state, important in the context of shape coexistence with weak mixing postulated for this nucleus

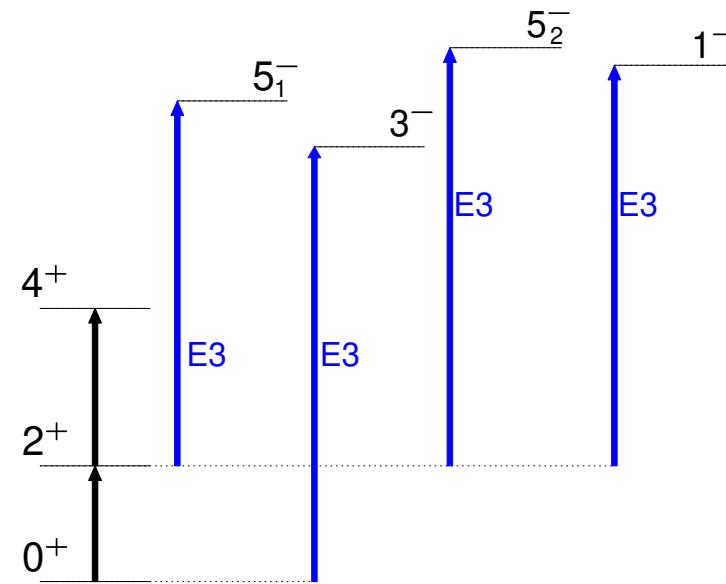




Negative-parity states



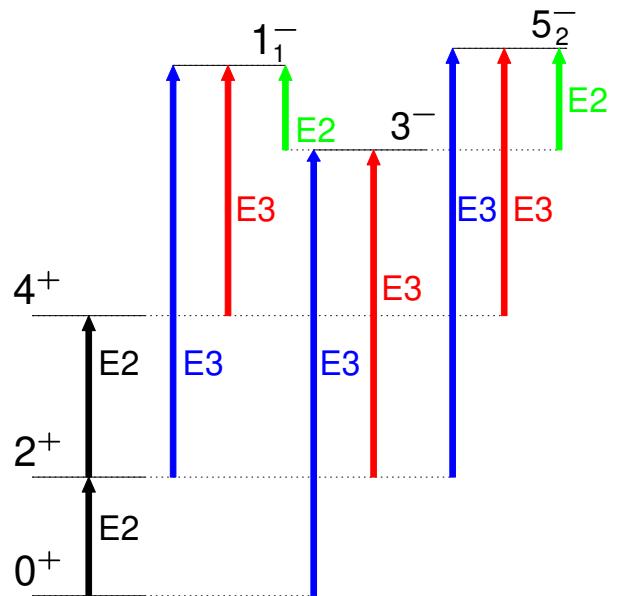
- oscillatory behaviour observed for the 3_1^- and 5_2^- excitation cross sections
- initially only a single E3 matrix element is assumed to be responsible for the population of each of the 3_1^- , 5_1^- , 5_2^- and 1_1^- states



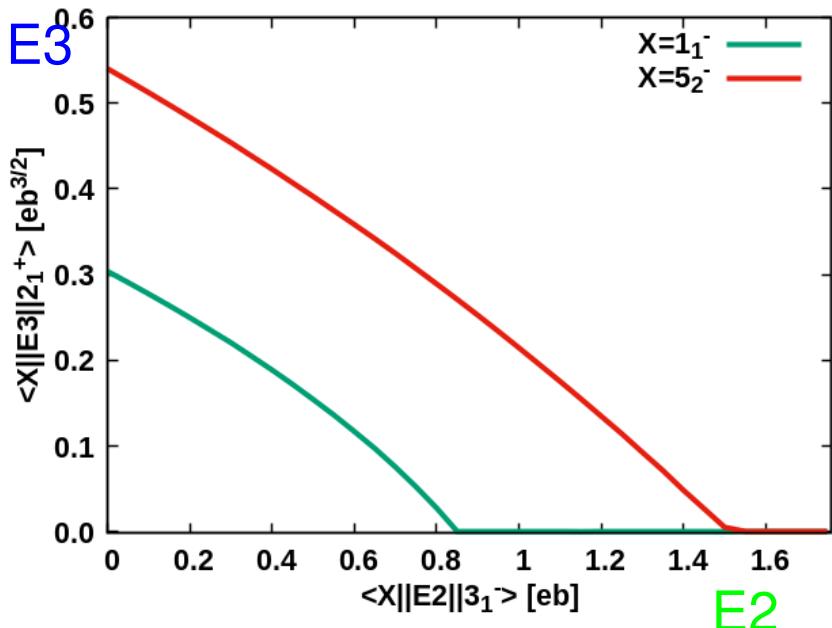
Results:

- $B(E3; 3_1^- \rightarrow 0_1^+) = 11^{+4}_{-3}$ W.u.
- $B(E3; 5_2^- \rightarrow 2_1^+) = 40(3)$ W.u. (a lot!)
- $B(E3; 1_1^- \rightarrow 2_1^+) = 45(6)$ W.u. (also a lot!)

Negative-parity states: quadrupole-octupole coupling



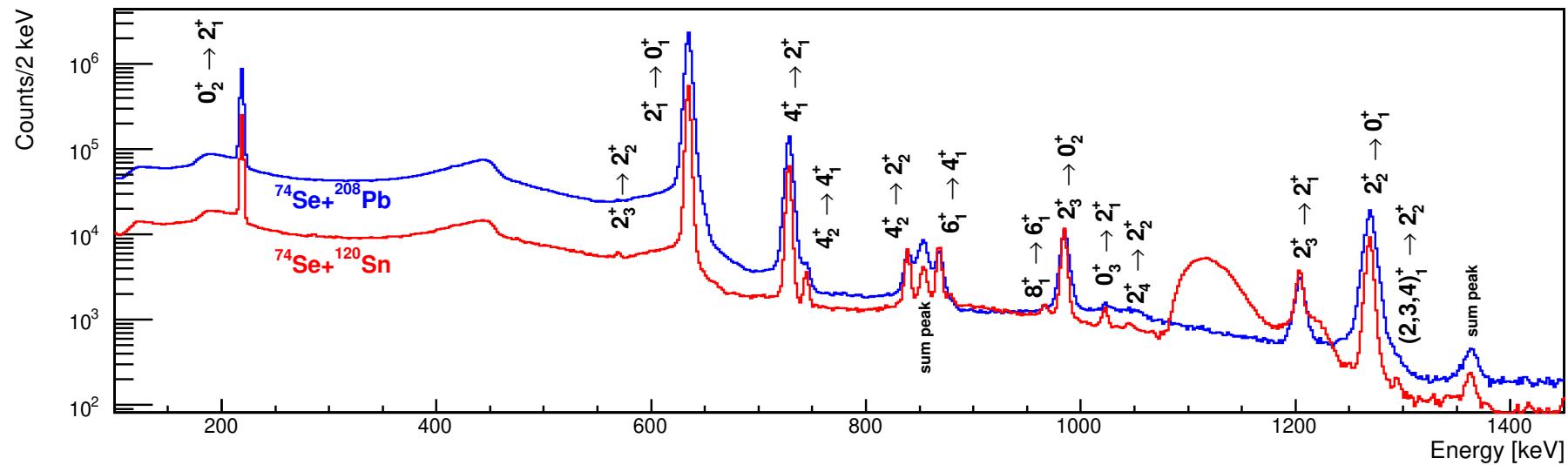
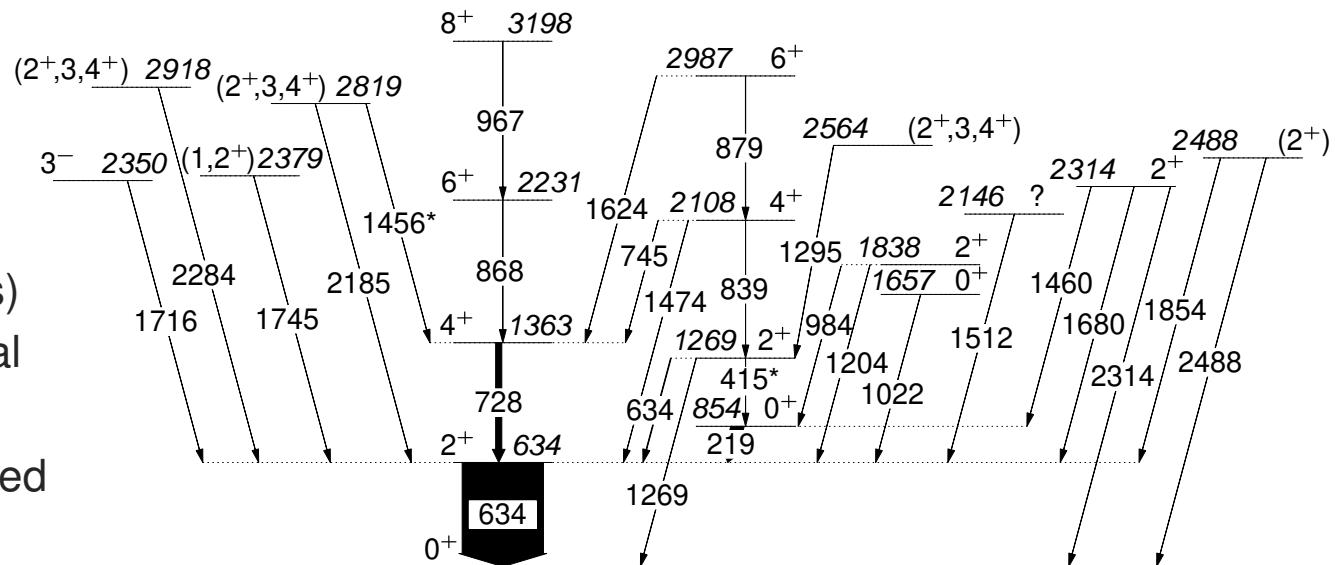
- it is necessary to introduce a more complicated coupling scheme to describe populations of the 5_2^- and 1_1^- states
- additional E3 transitions change the populations by a few percent, even if values of many tens of W.u. are assumed
- E2 transitions to the 3^- state prove to be very important



- we can extract the correlation between the E3 and E2 strength involved in the population of the negative parity states
- assuming $B(E2; 5_2^- \rightarrow 3_1^-) = B(E2; 1^- \rightarrow 3_1^-) = B(E2; 2_1^+ \rightarrow 0_1^+) = 26$ W.u. we obtain:
 - $B(E3; 5_2^- \rightarrow 2_1^+) = 7.8(1.3)$ W.u
 - $B(E3; 1_1^- \rightarrow 2_1^+) = 13(3)$ W.u.
- while $B(E3; 3_1^- \rightarrow 0_1^+) = 11_{-3}^{+4}$ W.u.

Shapes in ^{74}Se

- two interpretations:
 - prolate-oblate shape
coexistence (0_1^+ , 0_2^+ states)
 - weakly deformed vibrational
(ground-state band, 0_2^+)
coexisting with well deformed
states (0_3^+ , 2_4^+)



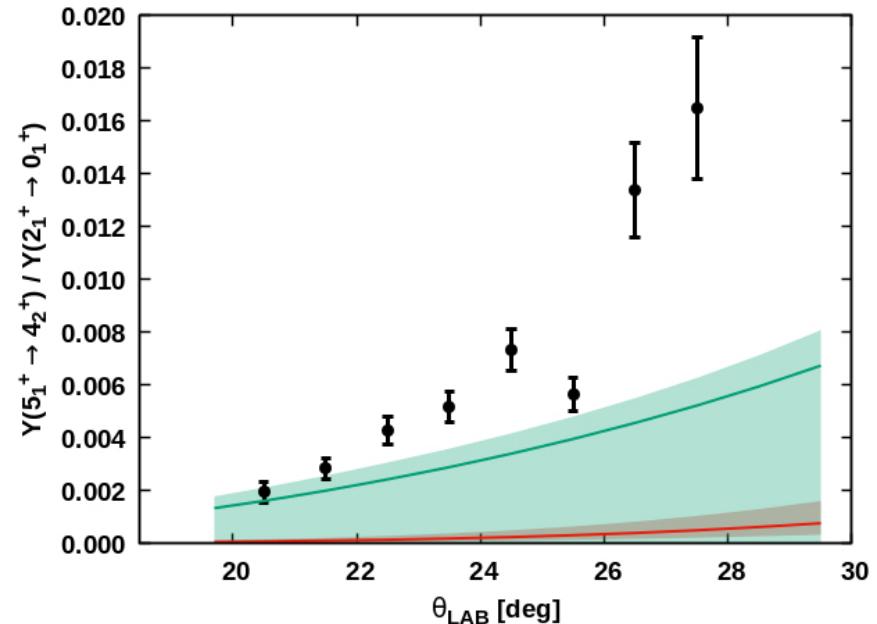
^{120}Sn and ^{208}Pb targets used to enhance sensitivity to the $0_3^+ \rightarrow 2_4^+$ excitation path

data analysis: R. Kjus, MZ, CEA Saclay

Ambiguities regarding the K=2 structure in ^{106}Cd

$$5_1^+ (2331 \text{ keV}) \rightarrow 4_2^+$$

- the observed population of the 5_1^+ state would require $B(E2; 5_1^+ \rightarrow 4_2^+)$ over 300 W.u.
- lifetime: $9(1)$ ps (GOSIA fit)
 $870(290)$ ps (ENSDF)
- we suspect the 226-keV $5_1^+ \rightarrow 4_2^+$ transition is part of a doublet

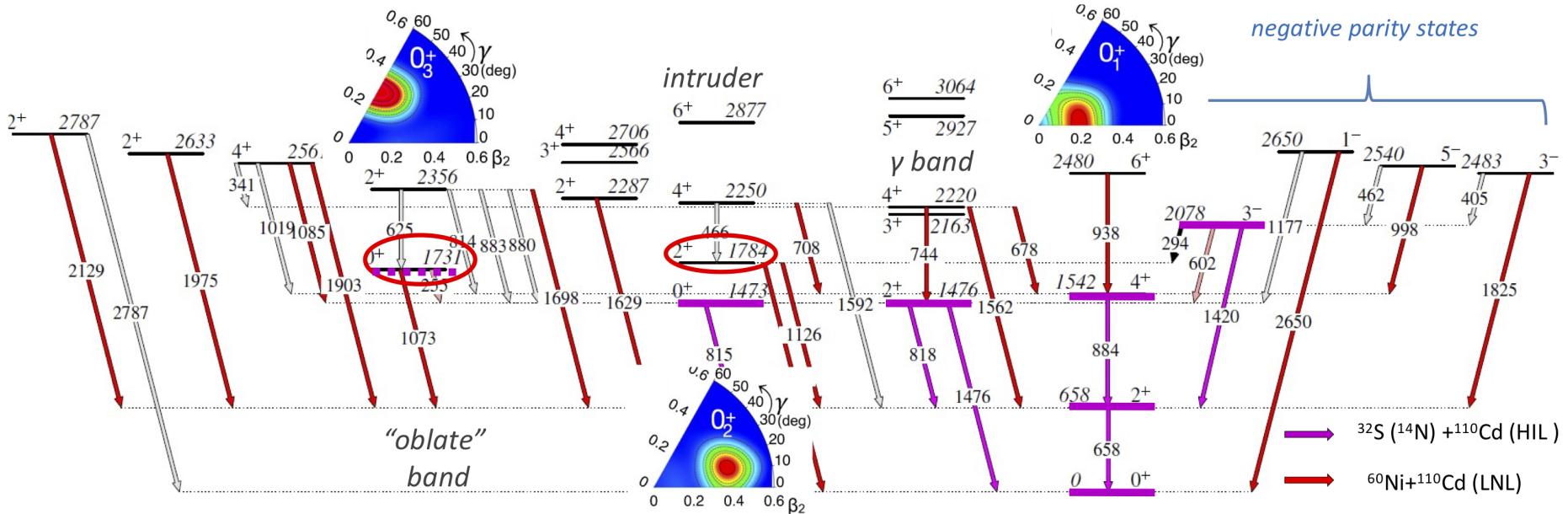
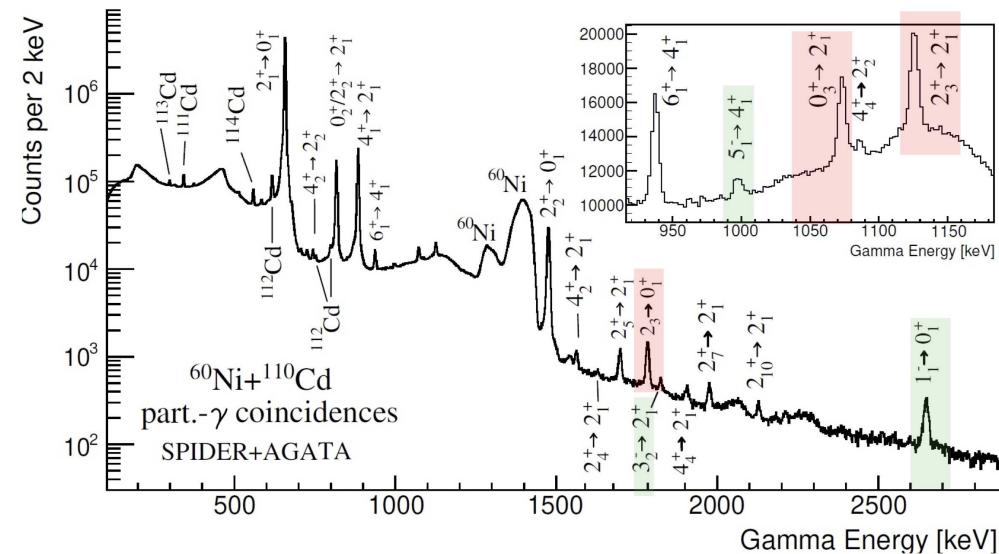


- K=2 band proposed in M. Siciliano et al., Phys. Rev. C 104, 034320 (2021) has a much more narrow energy spacing than those in heavier Cd nuclei
- multiple 3^+ candidates (2252, 2254, 2710, 2718-keV), none of them with a firm spin assignment
- strong discrepancies in the literature regarding branching ratios in the 4_2^+ and 2_2^+ decay

D. Kalaydjieva, PhD thesis, 2023

Multiple shape coexistence in ^{110}Cd

- aim: determination of β and γ parameters of 0_1^+ , 0_2^+ and 0_3^+ states in ^{110}Cd using quadrupole sum rules
- verification of the multiple shape-coexistence scenario (P.E. Garrett et al, Phys. Rev. Lett. 123, 142502 (2019))



5 days of ^{58}Ni beam with AGATA in 2022, complementary measurements at HIL Warsaw

data analysis: I. Piętka, K. Wrzosek-Lipska, HIL Warsaw