The beryllium isotopic chain: a microcosm of nuclear structure

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UNAM, Mexico City, Mexico
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Outline

- Ab initio calculations of energies and radii in reasonable agreement with experimental values
- Goal: extract a more intuitive (approximate) pictures for understanding the structure of $^{8-14}$Be
- With deformation also see rotational bands
- Look for signatures of rotational dynamics (characteristic energies, enhanced transition strengths, etc.)
- Occupations of single particle orbitals (natural orbitals)

No-core shell model

Solve many-body Schrodinger equation

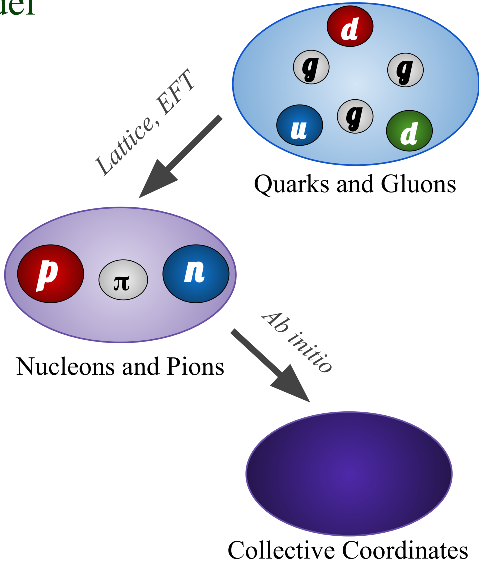
$$\sum_{i}^{A} -\frac{\hbar^2}{2m_i}\nabla_i^2 \Psi + \frac{1}{2}\sum_{i,j=1}^{A} V(|r_i - r_j|)\Psi = E\Psi$$

Expanding wavefunctions in a basis

$$\Psi = \sum_{k=1}^{\infty} a_k \phi_k$$

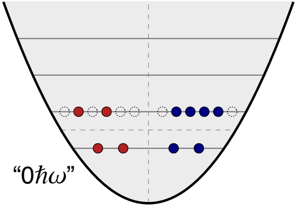
Reduces to matrix eigenproblem

$$\begin{pmatrix} H_{11} & H_{12} & \dots \\ H_{21} & H_{22} & \dots \\ \vdots & \vdots & \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix} = E\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix}$$



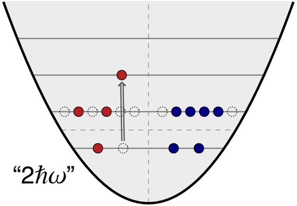
Harmonic oscillator basis

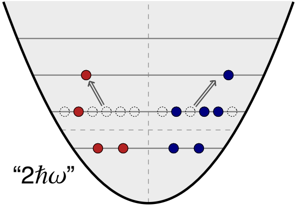
$N = 2n + l$



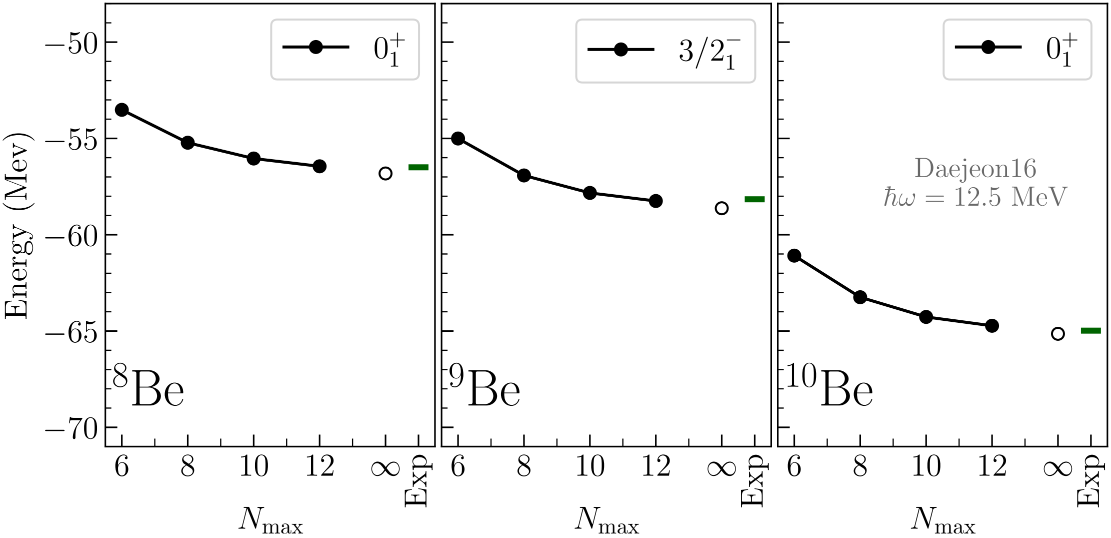
- Basis states are configurations, i.e., distributions of particles over harmonic oscillator shells (*nlj substates*)
- States are organized by total number of oscillator quanta above the lowest Pauli allowed number N_{ex}
- States with higher N_{ex} contribute less to the wavefunction
- Basis must be truncated:
  Restrict $N_{\mathrm{ex}} \leq N_{\max}$

Want results that are approximately independent of $N_{\max}$





Binding energies



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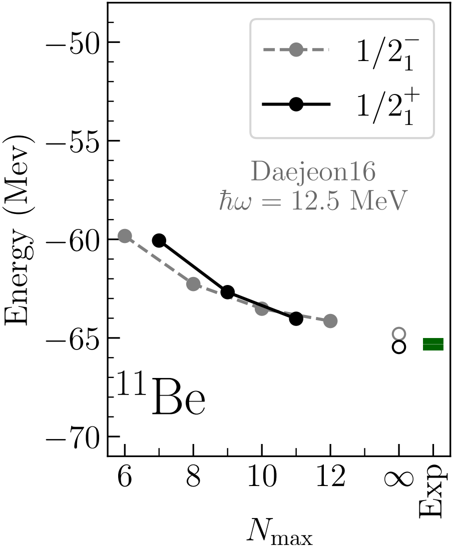

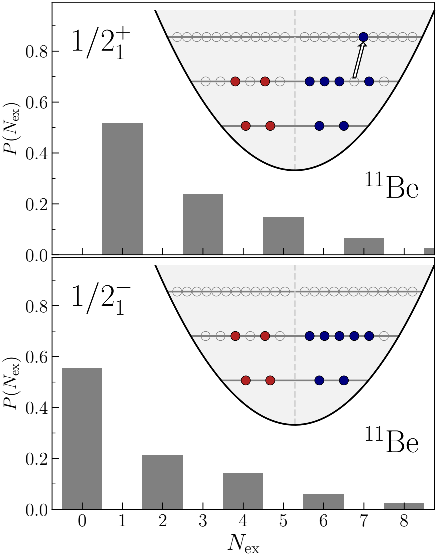

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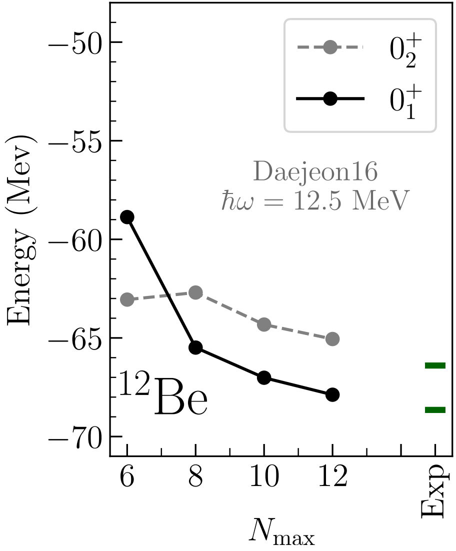

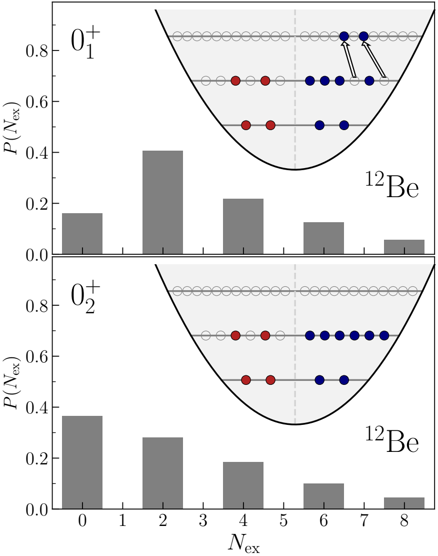

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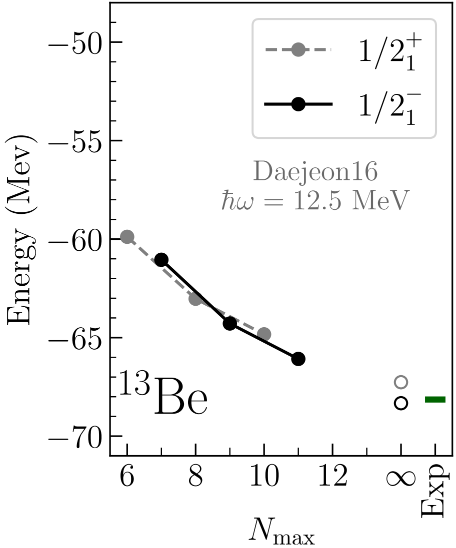

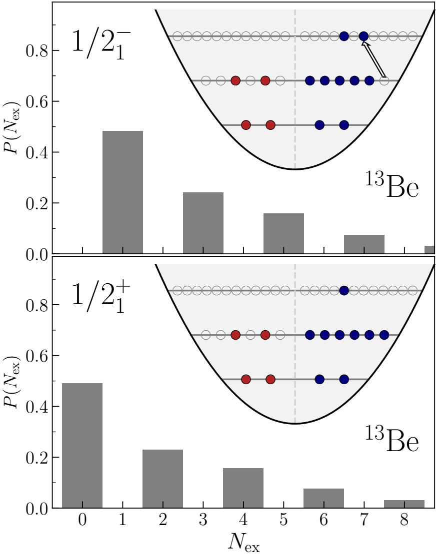

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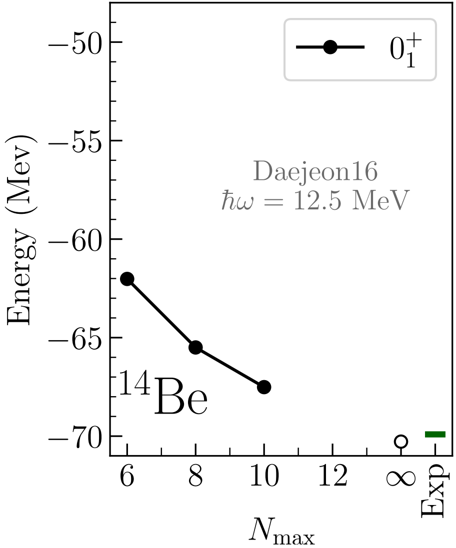
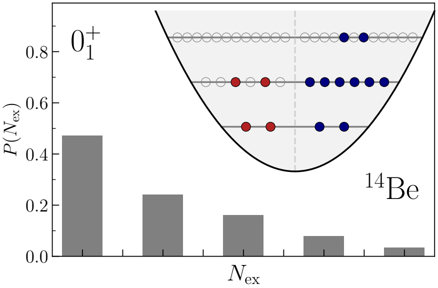
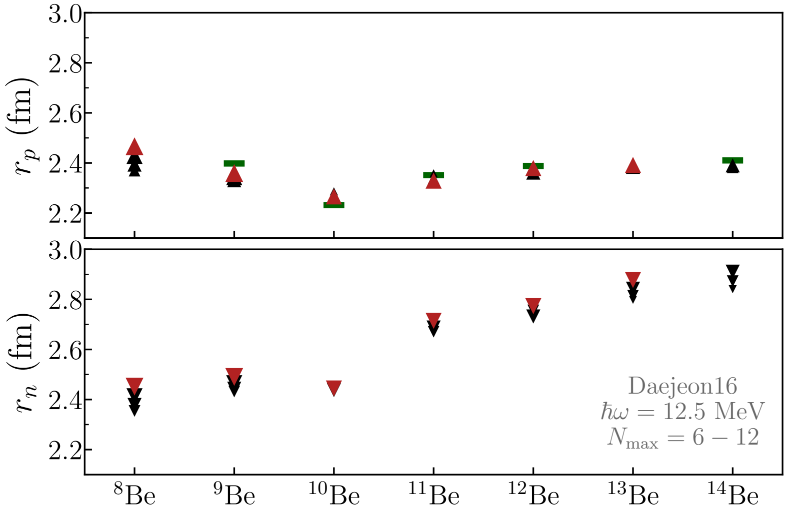




Radii


3.0
2.8
2.6
2.4
2.2
r_p (fm)
r_n (fm)
^{8}Be
^{9}Be
^{10}Be
^{11}Be
^{12}Be
^{13}Be
^{14}Be
Daejeon16
$\hbar\omega = 12.5$ MeV
$N_{\text{max}} = 6 - 12$

Beryllium isotopes

- Island of inversion around $N = 8$
- Appearance of halo nuclei: ^{11}Be, ^{14}Be
- Alpha cluster structure
- Highly deformed states and shape coexistence

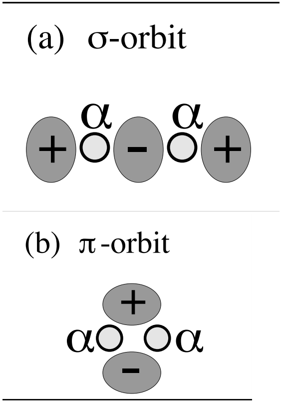
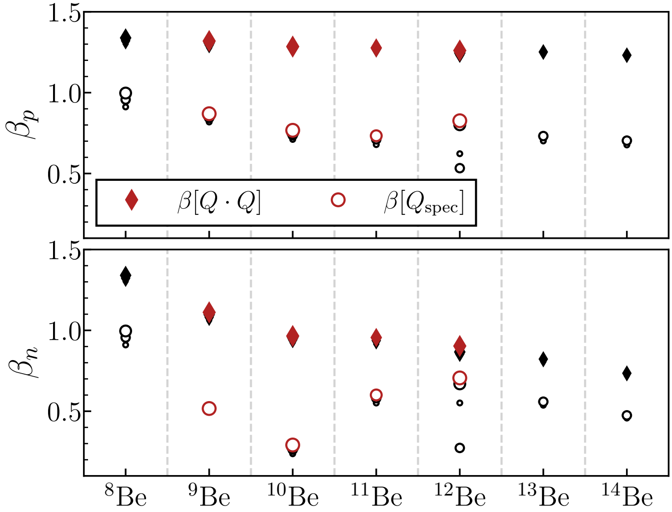


Figure: Y. Kanada-En'yo and H. Horiuchi. Phys. Rev. C **68** (2003) 014319.

Quadrupole deformation



$$\beta_p \propto \frac{Q_{0,p}}{\sqrt{Z}\langle r_p^2 \rangle} \qquad \beta_p \propto \frac{Q_{0,n}}{\sqrt{N}\langle r_n^2 \rangle}$$

From q-invariant (*dynamic deformation*)

$$\langle Q \cdot Q \rangle = \langle Q_0 \cdot Q_0 \rangle$$

If symmetric rotor (*static deformation*)

$$Q(J) = \frac{\hat{J}}{(1+\delta_{K,0})}(JK20|JK)Q_0$$

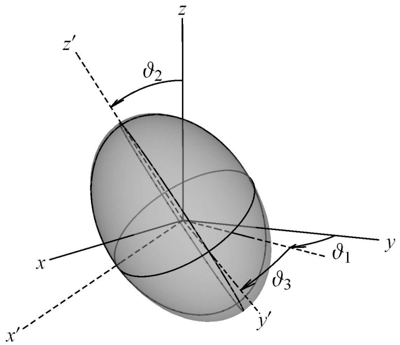
D. J. Rowe. Rep. Prog. Phys. **48**(1985) 1419.
D. J. Rowe, Nuclear Collective Motion: Models and Theory (2010).

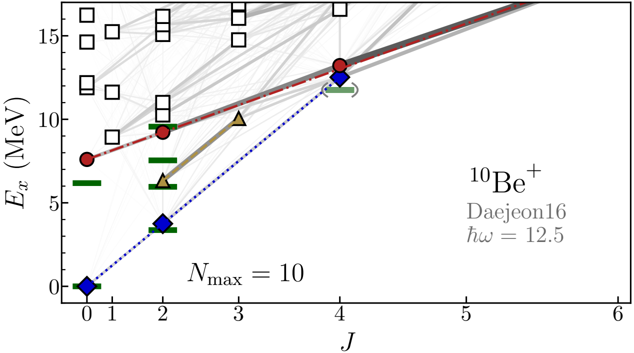
Nuclear rotations

Characterized by rotation of intrinsic state $|\phi_K\rangle$ by Euler angles ϑ $\quad (J = K, K+1, \ldots)$

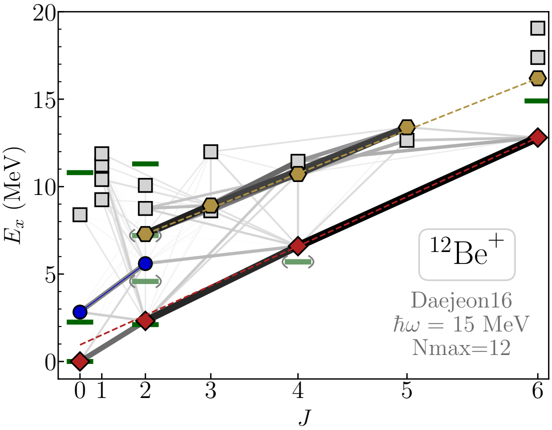
$$|\psi_{JKM}\rangle \propto \int d\vartheta \left[\mathscr{D}^J_{MK}(\vartheta)|\phi_K;\vartheta\rangle + (-)^{J+K}\mathscr{D}^J_{M-K}(\vartheta)|\phi_{\bar{K}};\vartheta\rangle \right]$$

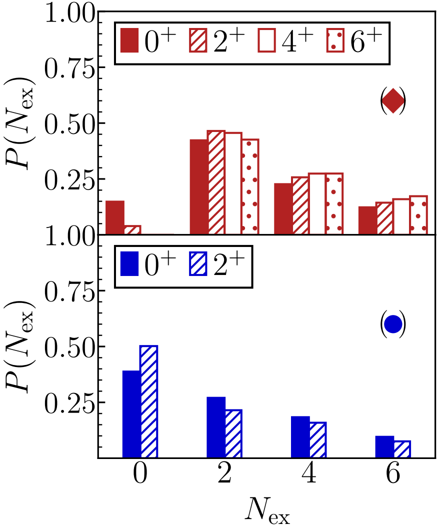
Rotational energy: $E(J) = E_0 + A[J(J+1)]$





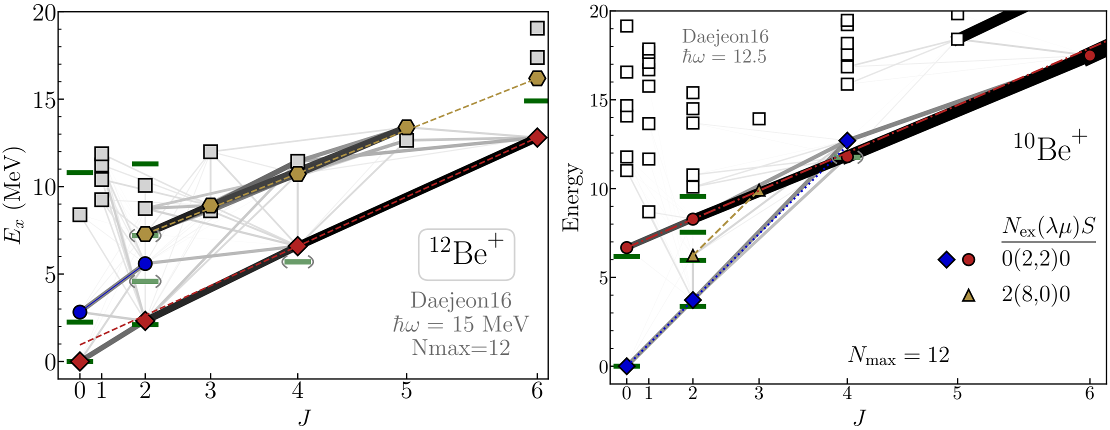
^{12}Be rotational bands





A. E. McCoy, M. A. Caprio, P. Maris, P. J. Fasano arXiv:2402.12606.

^{10}Be and ^{12}Be intruder bands



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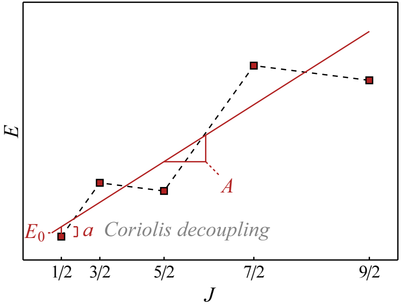
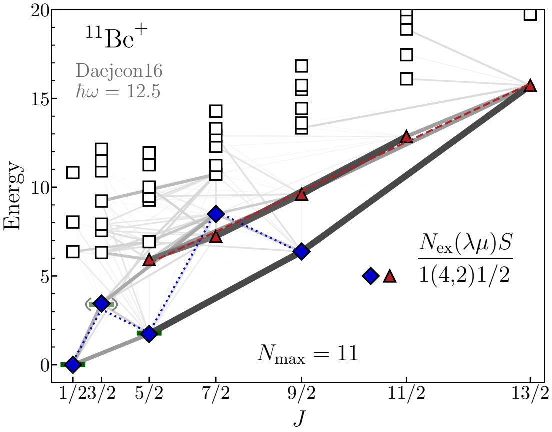
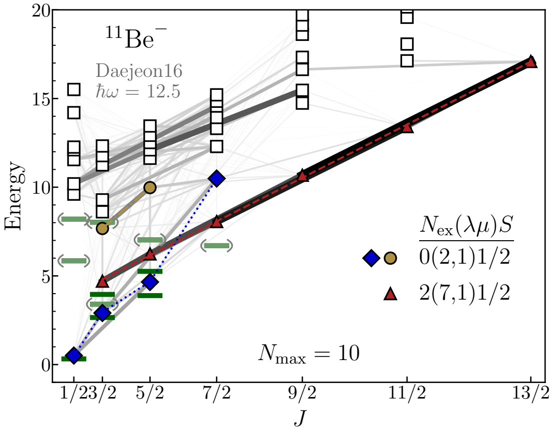
Rotational energy: $E(J) = E_0 + A[J(J+1)] + \underbrace{a(-)^{J+1/2}(J+\frac{1}{2})}_{\text{Coriolis}\,(K=1/2)}$

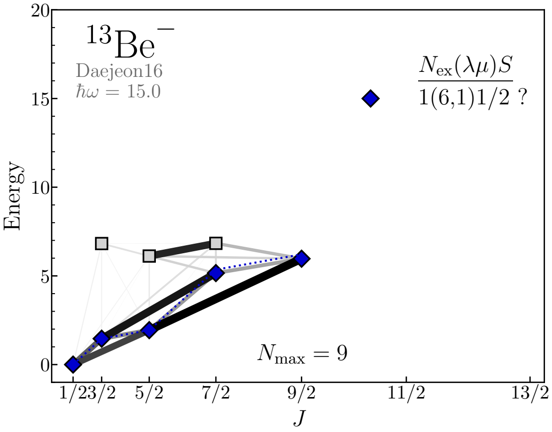


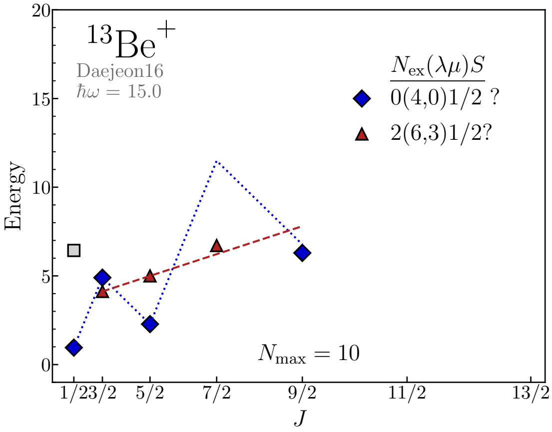
^{11}Be rotational bands





^{13}Be rotational bands





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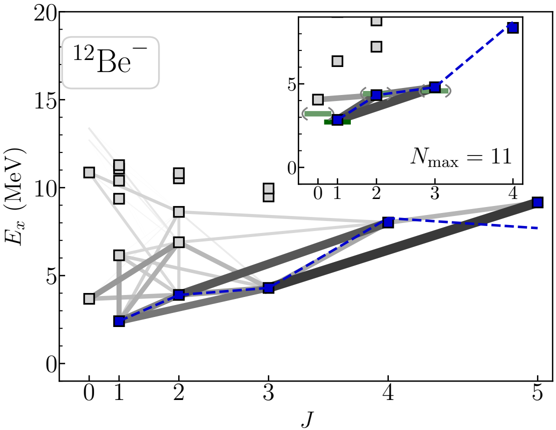
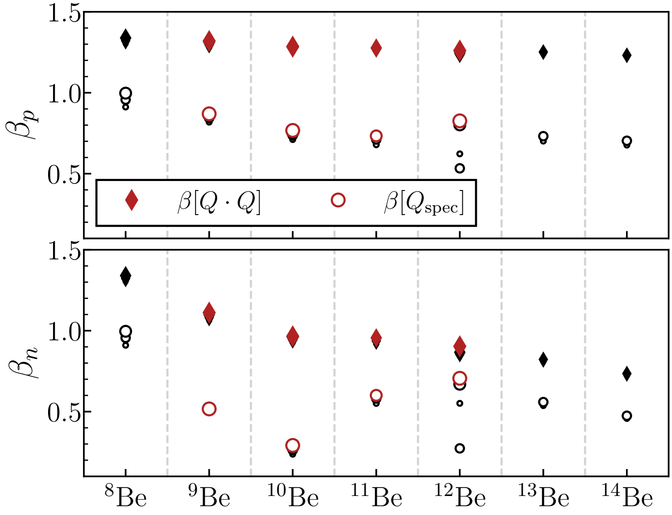


Bohr and Mottelson Vol 2

Quadrupole deformation



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D. J. Rowe, Nuclear Collective Motion: Models and Theory (2010).

Elliott SU(3)

Labels (λ,μ) associated with deformation parameters β and γ

O. Castanos, J. P. Draayer, Y. Leschber, Z. Phys. A **329** *(1988) 3.*

Lowest energies correspond to most deformed state

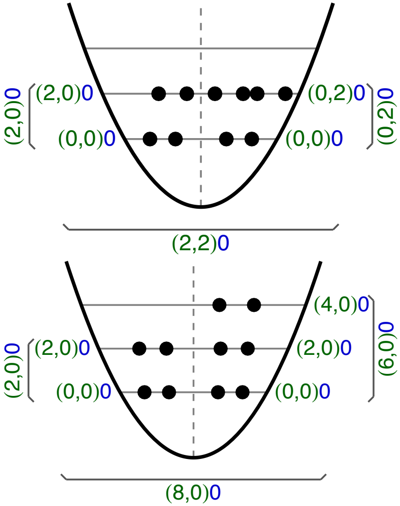
D. J. Rowe, G. Thiamova, and J. L. Wood. Phys. Rev. Lett. **97** *(2006) 202501.*

$$\beta^2 \propto \langle Q\cdot Q\rangle / \langle r^2\rangle^2$$

$$H = \underbrace{H_0}_{\text{shell}} - \underbrace{\kappa Q\cdot Q}_{\text{correlations}} + L\cdot S$$

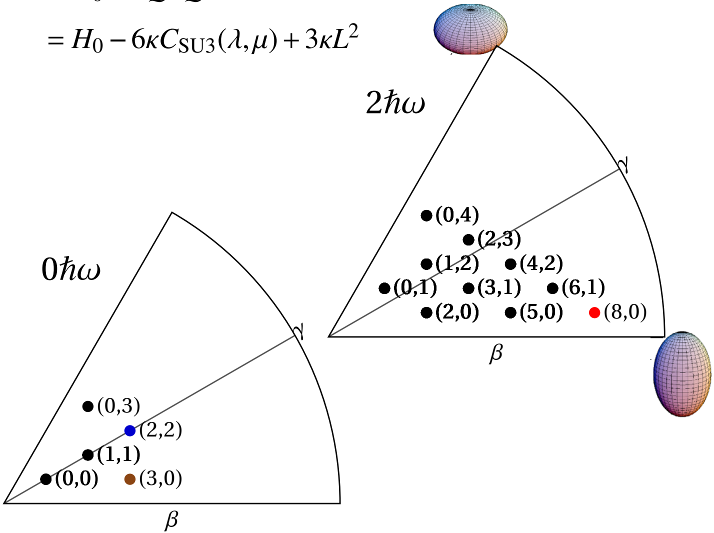
SU(3) symmetry of a configuration

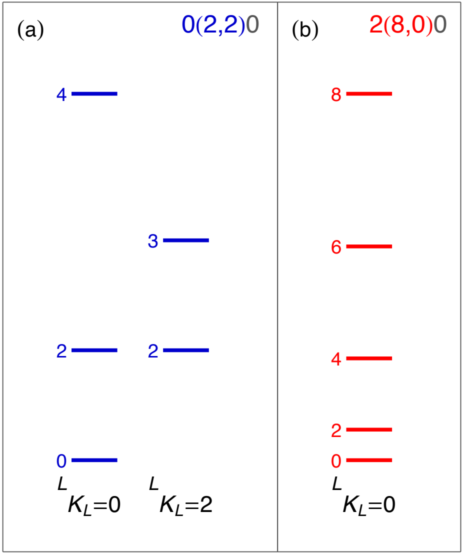
- Each particle has SU(3) symmetry $(N,0)$, $N = 2n+\ell$
- Allowed spins dictated by antisymmetry constraints
- Final quantum numbers are $N_{\text{ex}}(\lambda\mu)S$.



Elliott rotational bands: ^{10}Be

$$H = H_0 - \kappa Q \cdot Q$$
$$= H_0 - 6\kappa C_{SU3}(\lambda,\mu) + 3\kappa L^2$$





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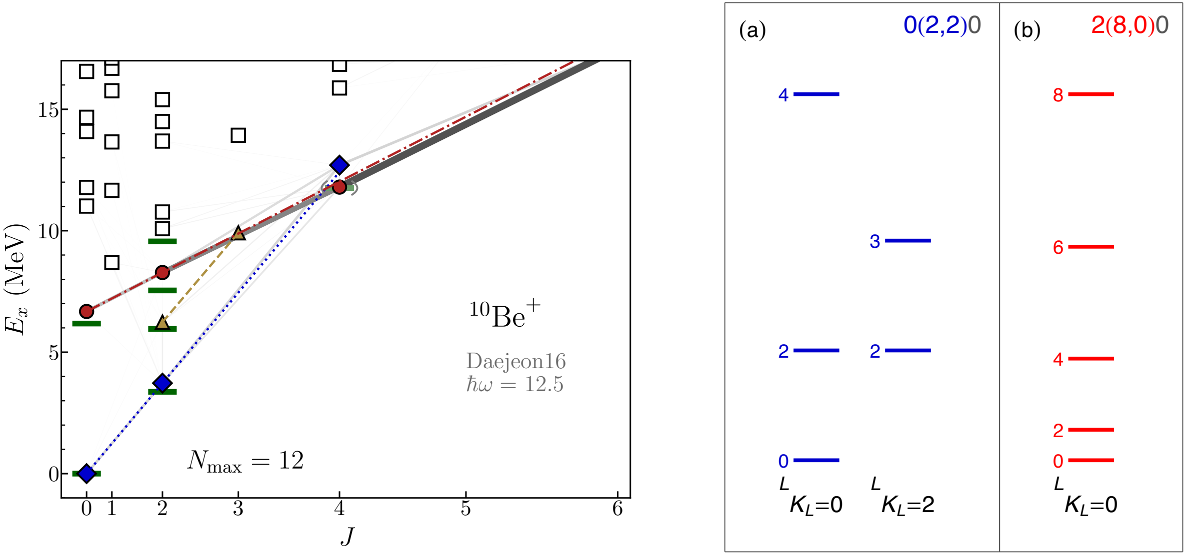


Beryllium isotopes

- Island of inversion around $N = 8$
- Appearance of halo nuclei: ^{11}Be, ^{14}Be
- Alpha cluster structure
- Highly deformed states and shape coexistence
- Emergence of rotational dynamics
- Emergence of Elliott SU(3) dynamical symmetry.

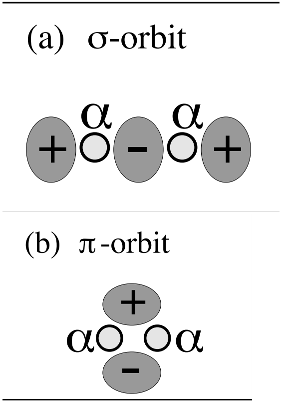


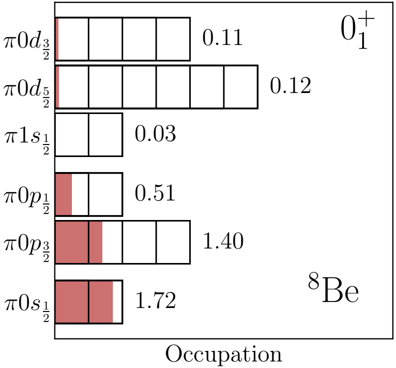
Figure: Y. Kanada-En'yo and H. Horiuchi. Phys. Rev. C **68** (2003) 014319.

Effective single particle picture

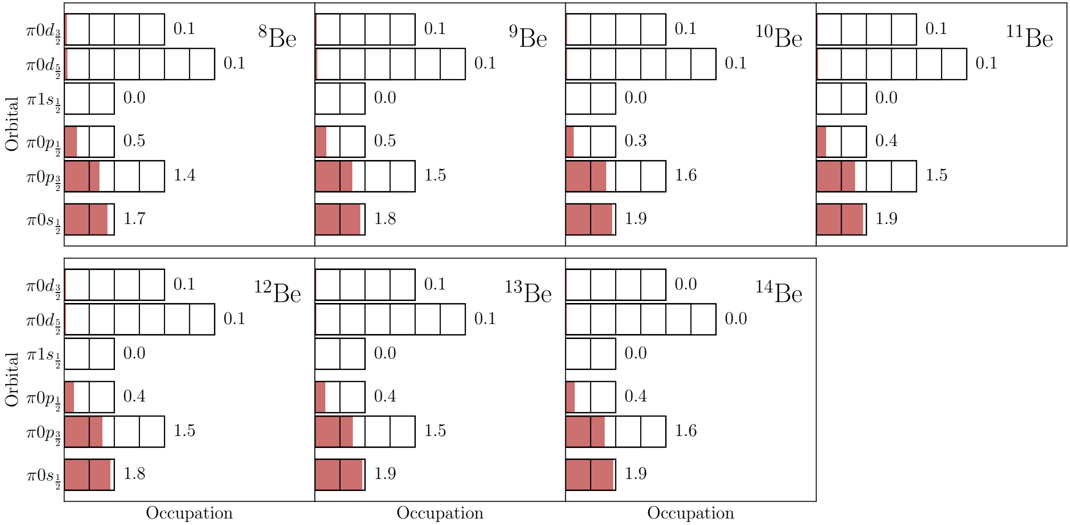
- Many different ways to choose single particle basis
- Natural orbitals obtained by diagonalizing the density matrix

$$\hat{\rho} = \sum_{\alpha\beta} |\alpha\rangle \langle \Psi | a_\alpha^\dagger a_\beta | \Psi \rangle \langle \beta |$$

- Maximize occupation number of lowest orbitals



Natural Orbital Occupations



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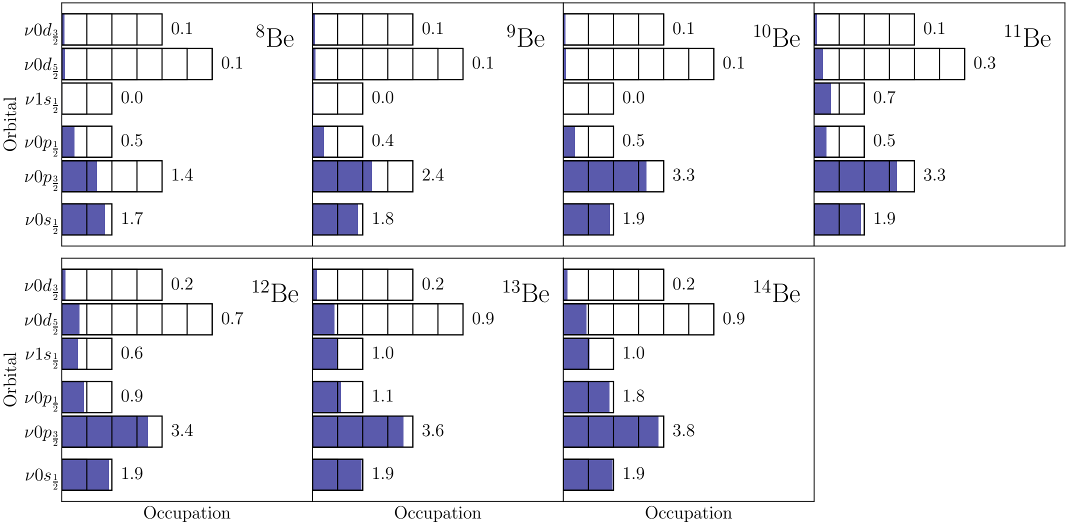
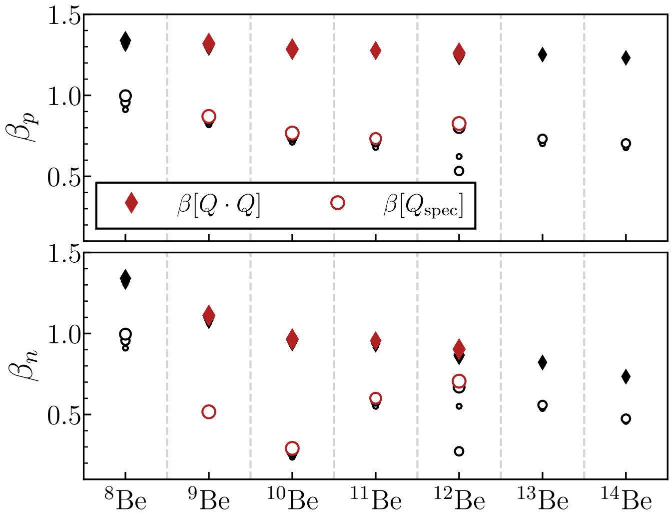


Quadrupole deformation



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D. J. Rowe, Nuclear Collective Motion: Models and Theory (2010).

Nilsson Model

Wood Saxon potential

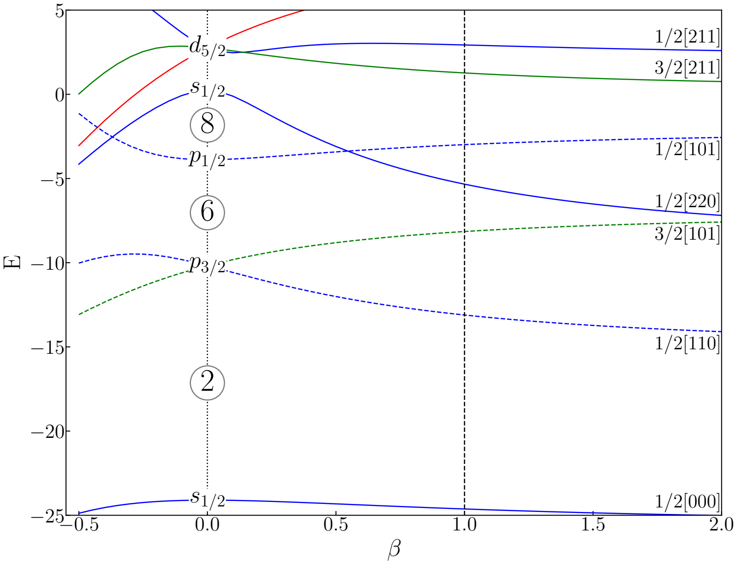
$$V_{\mathrm{WS}} = \frac{-V_0}{1+e^{(r-R)/a}},$$

$a = 0.67\,\mathrm{fm},\ V_0 = 57\,\mathrm{MeV},$
$R = 1.27(A = 8)^{1/3}\,\mathrm{fm}.$

Nilsson Hamiltonian

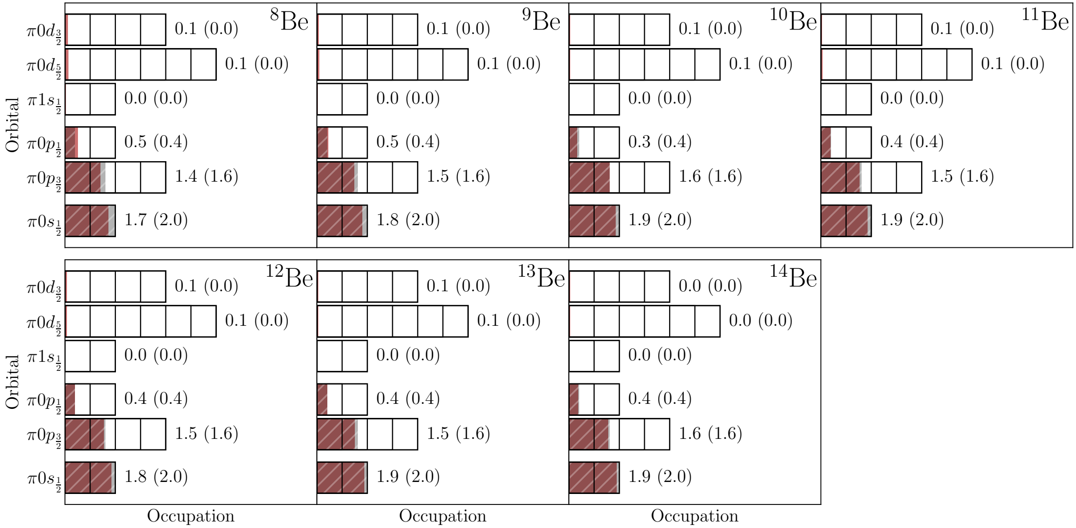
$$H = V_{\mathrm{WS}} + \beta\hbar\omega r^2 Y_{20}$$

$\hbar\omega = 12.5, \beta = 1$

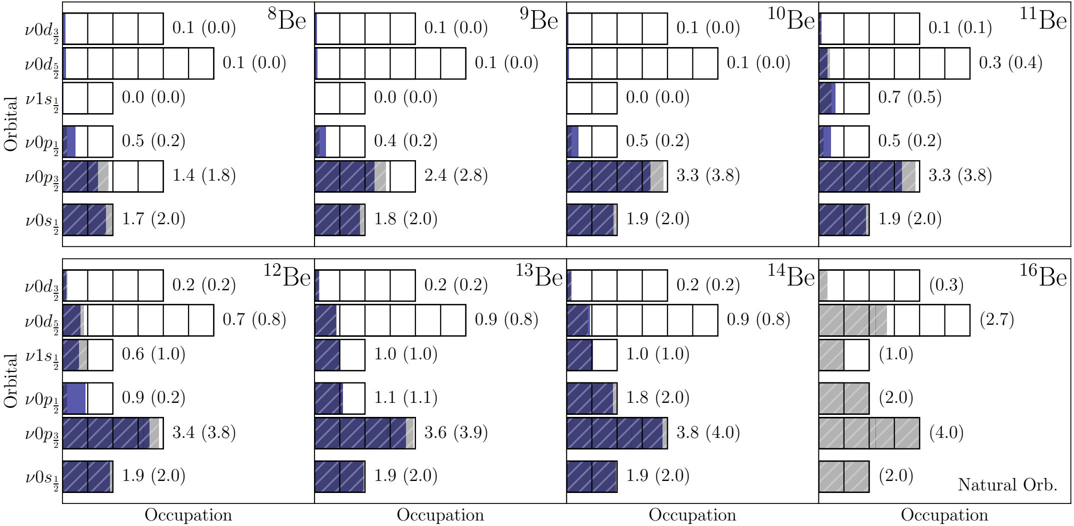


Wood Saxon parameters: J. Suhonen. From Nucleons to Nucl
Concepts of Microscopic Nuclear Theory, Chapter 3.

Natural Orbital Occupations



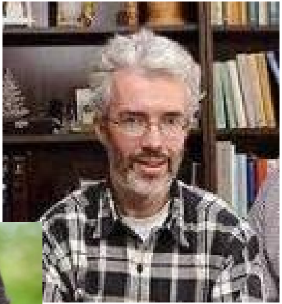
Natural Orbital Occupations



Acknowledgements

In collaboration with...

Patrick Fasano *ANL*
Mark Caprio *Univ. Notre Dame*
Pieter Maris *Iowa State Univ.*



Beryllium isotopes

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- Alpha cluster structure
- Highly deformed states and shape coexistence
- Simple effective single particle picture: Nilsson model

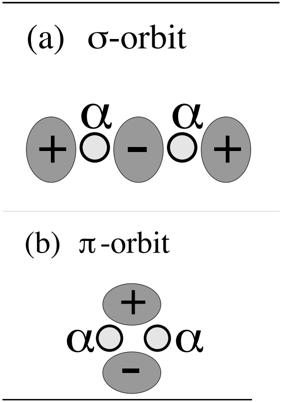
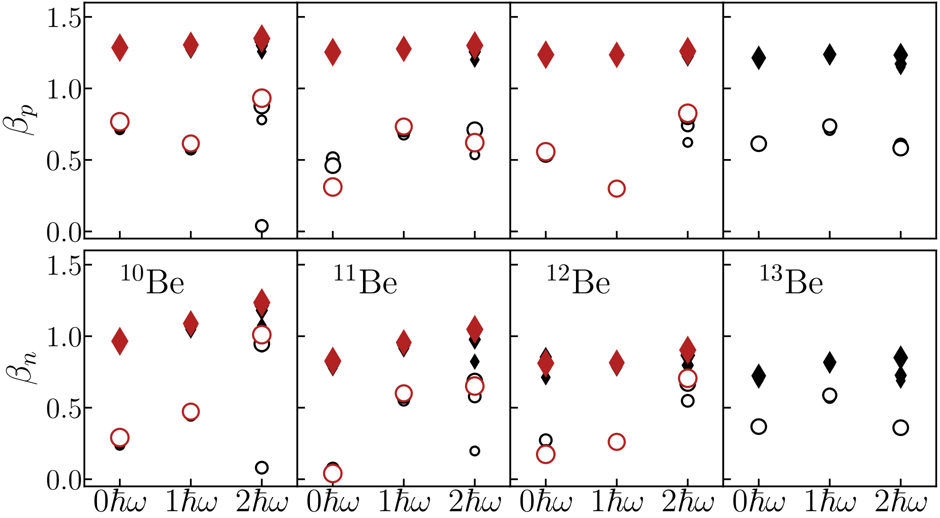
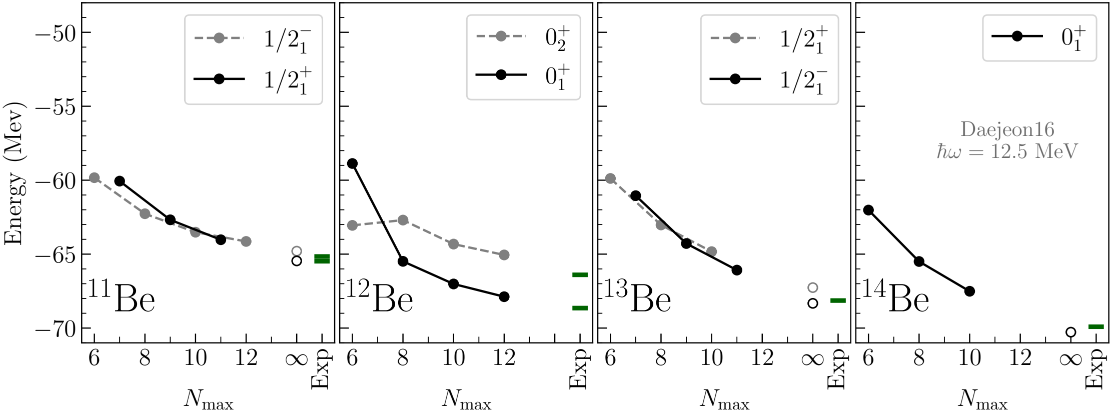


Figure: Y. Kanada-En'yo and H. Horiuchi. Phys. Rev. C **68** (2003) 014319.

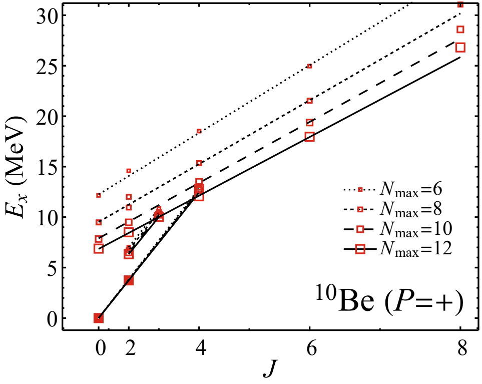
Quadrupole deformation

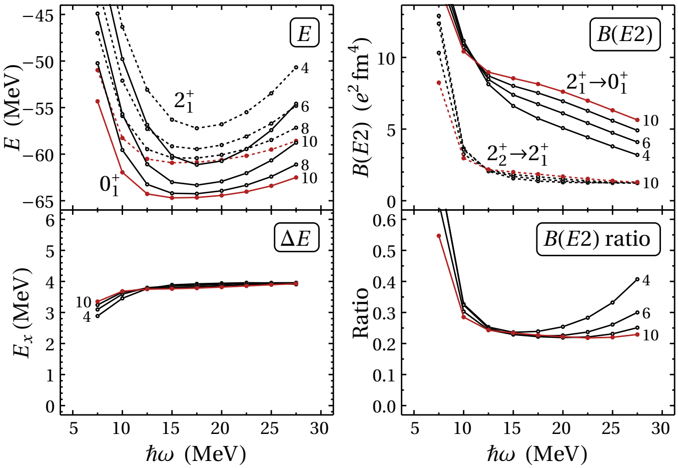


Binding energies



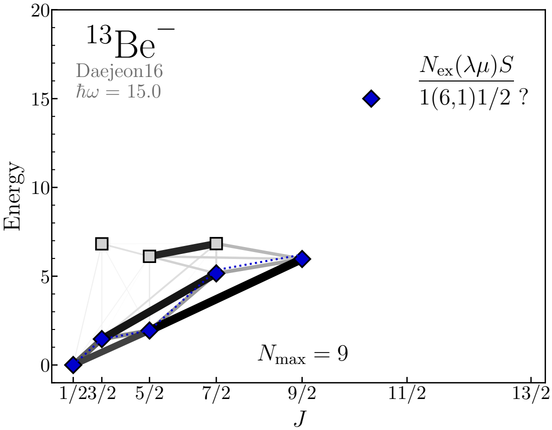
Robustness of band properties

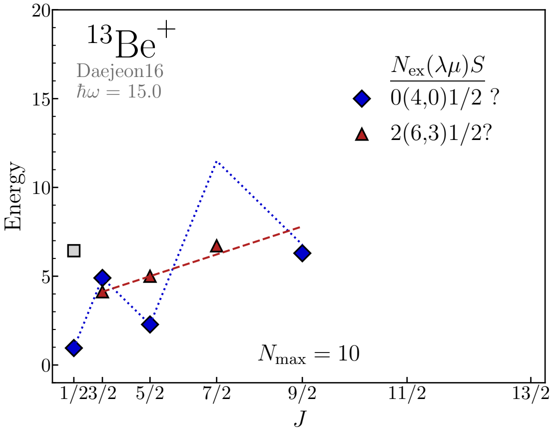




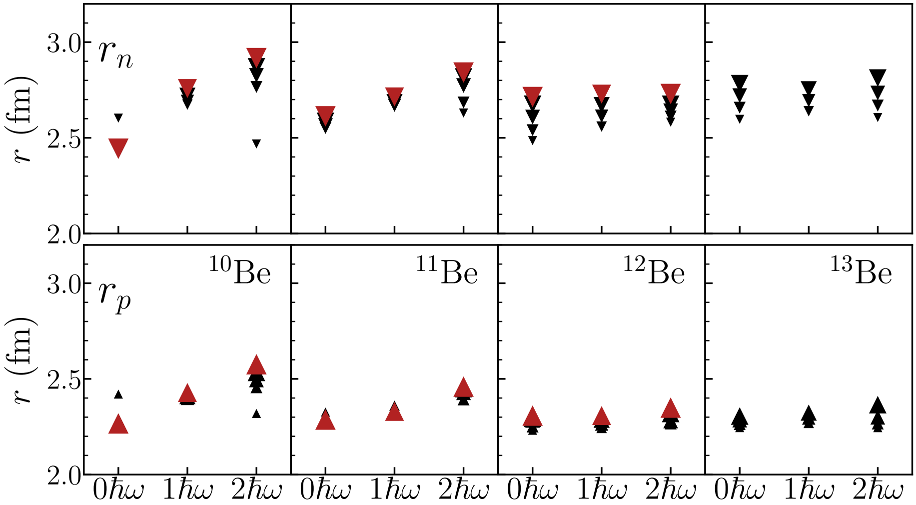
M. A. Caprio, P. J. Fasano, A. E. McCoy, P. Maris, J. P. Vary, Bulg. J. Phys. **46**, 455 (2019).

^{13}Be rotational bands



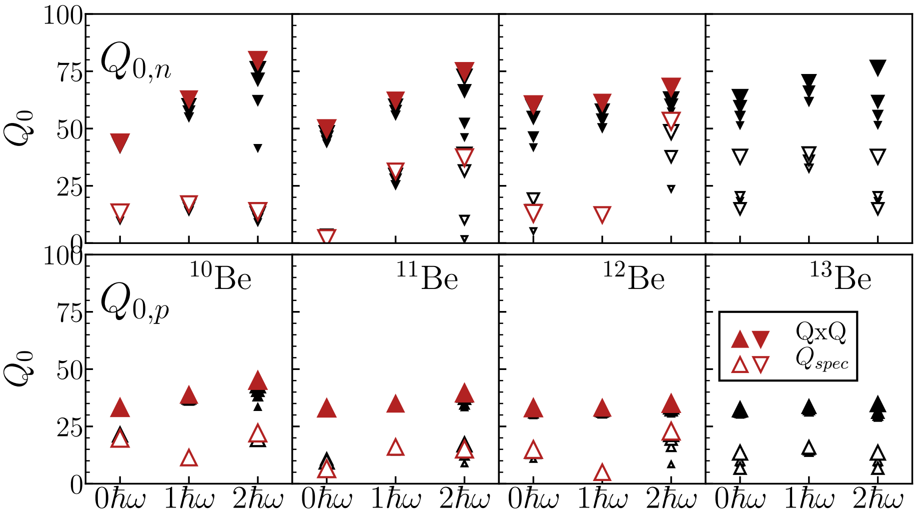


Radii



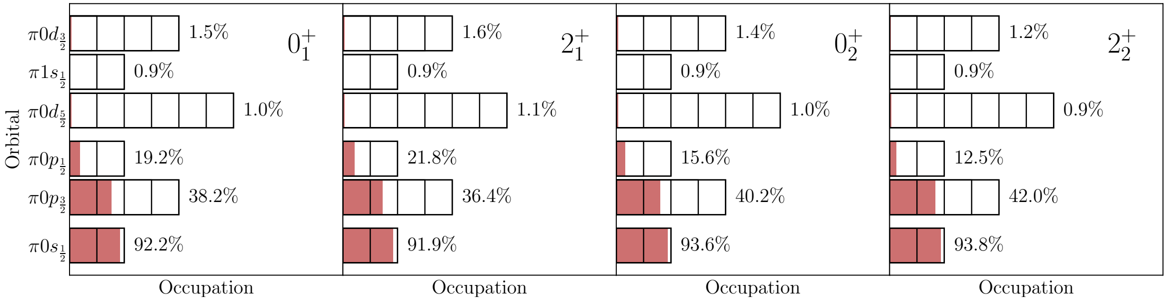
r_n
r_p
r (fm)
r (fm)
3.0
2.5
2.0
^{10}Be
^{11}Be
^{12}Be
^{13}Be
$0\hbar\omega$ $1\hbar\omega$ $2\hbar\omega$

Intrinsic quadrupole moment



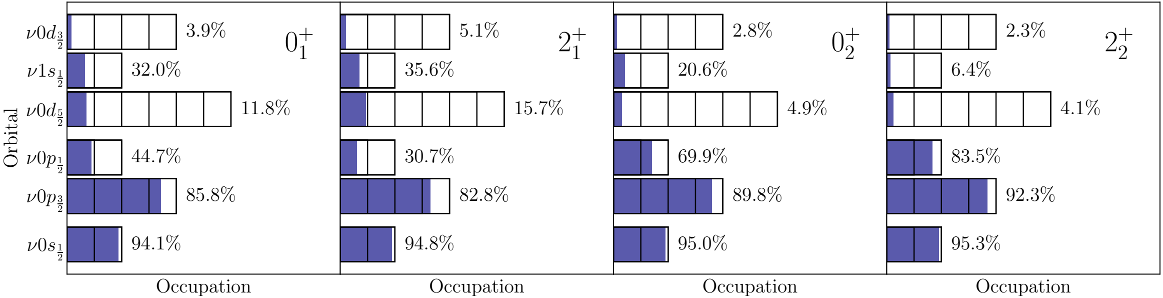
Natural orbital occupations

Protons



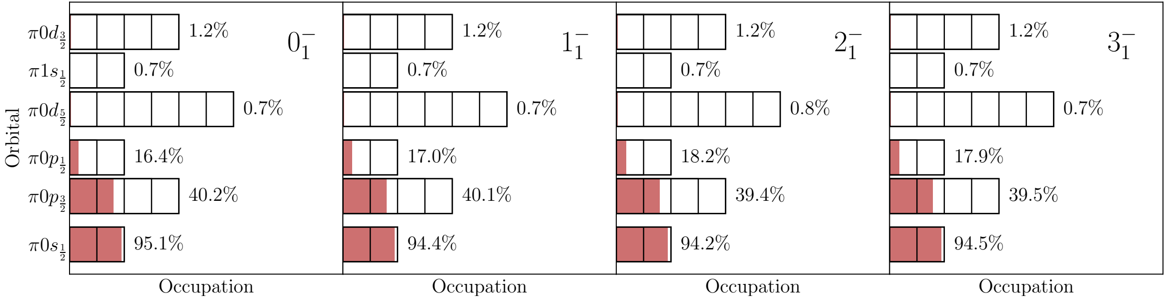
Natural orbital occupations

Neutrons



Natural orbital occupations

Protons



Natural orbital occupations

Neutrons