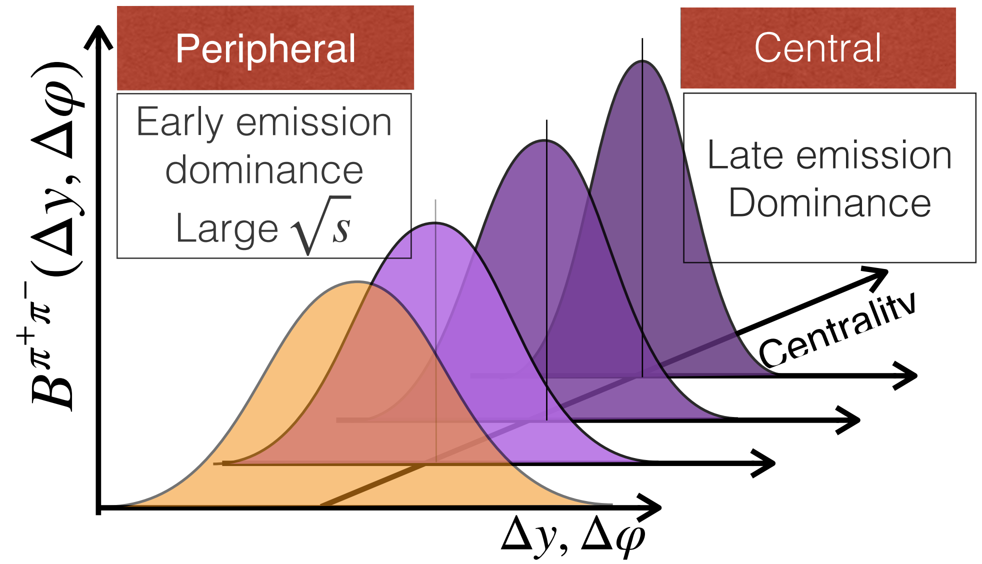
Working with Balance Functions

Claude A. Pruneau, Wayne State University
with S. Basu, A. Dobrin, V. Gonzalez, B. Hanley, A. Manea, A. Marin, J. Pan & ALICE Collaboration

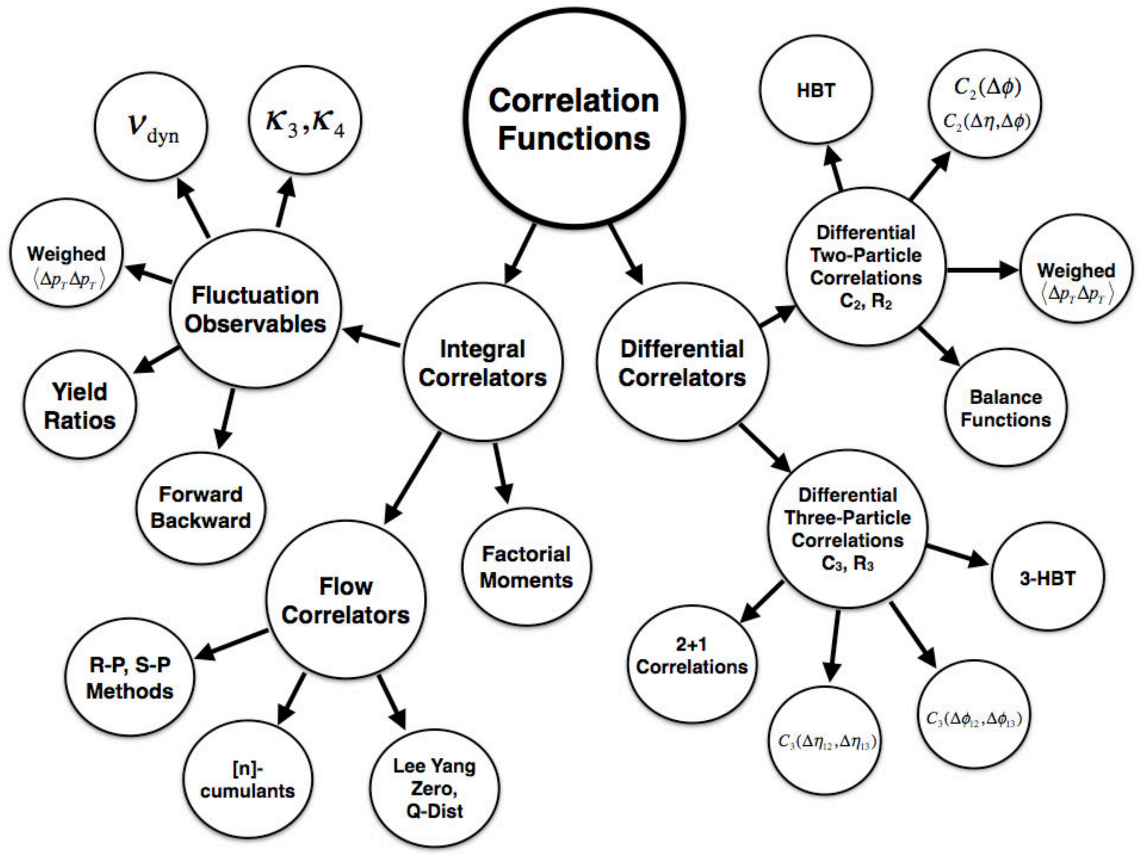
- **Recent Works on Balance Functions**
  - *Flavor Balancing*, In progress
  - *Mixed Species Charge and Baryon Balance Functions Studies with PYTHIA*, PhysRevC.109.064913,
  - *Multi-particle Integral and Differential Correlation Functions*, PRC 109 (2024) 4, 044904,
  - *Accounting for non vanishing net-charge with unified balance functions*, PRC 107 (2023) 1, 014902,
  - *Effects of non vanishing net charge on balance functions and their integrals*, PRC 107 (2023) 5, 054915
  - General balance functions of identified charged hadron pairs of π, K, $p\bar{p}$ in PbPb Collisions…, PLB 833 (2022) 137338
  - *Role of baryon number conservation in measurements of fluctuations*, PRC 100 (2019) 3, 034905

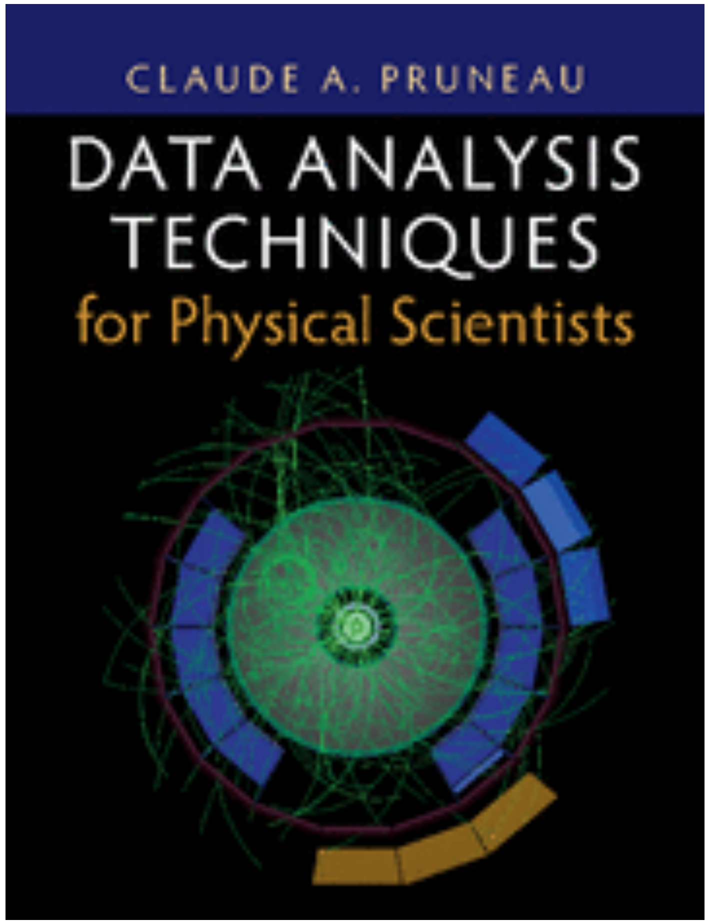


"Good Bedtime Reading"

Available at amazon.com

Data Analysis Techniques





Published in 2017

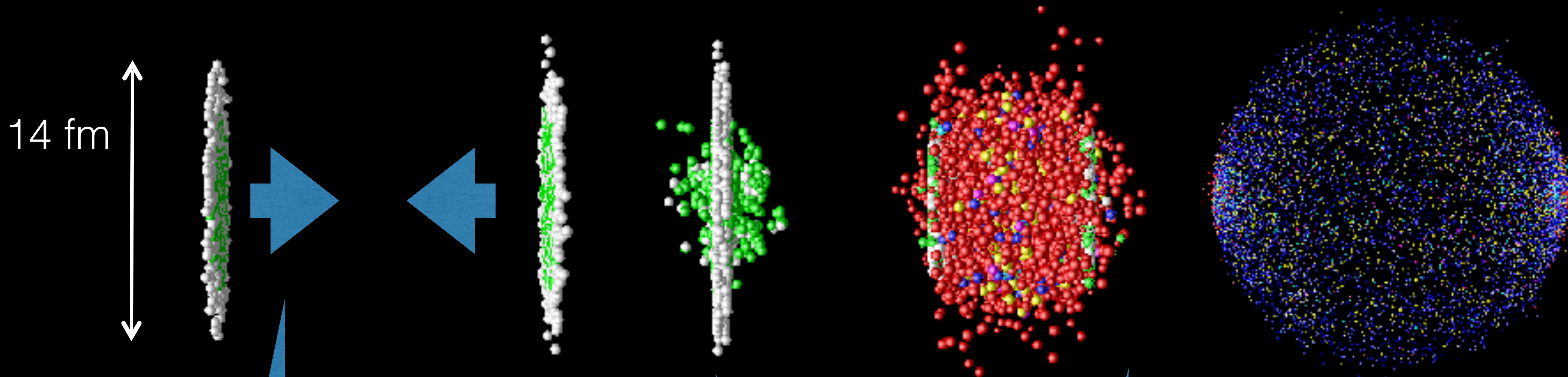
"This ambitious book provides a comprehensive, rigorous, and accessible introduction to data analysis for nuclear and particle physicists working on collider experiments, and outlines the concepts and techniques needed to carry out forefront research with modern collider data in a clear and pedagogical way. The topic of particle correlation functions, a seemingly straightforward topic with conceptual pitfalls awaiting the unaware, receives two full chapters. Professor Pruneau presents these concepts carefully and systematically, with precise definitions and extensive discussion of interpretation. These chapters should be required reading for all practitioners working in this area."
Dr. Peter Jacobs, Lawrence Berkeley National Laboratory

"*Data Analysis Techniques for Physical Scientists* is both monumental and accessible. While targeted towards data analysis methods in nuclear and particle physics, its breadth and depth insure that it will be of interest to a much broader audience across the physical sciences. Designed as a textbook, with ample problems and expository text, this wonderful new addition to the literature is also suitable for self-study and as a reference. As such, it is the book that I will first recommend to my students, be they undergraduates or graduate students."
Professor W. A. Zajc, Columbia University

Topics	Chapters
Classical Statistics	5
Bayesian Statistics	1
Data Reconstruction/ Analysis Methods	2
Correlation Functions	2
Data Correction/Unfolding	1
Basic Monte Carlo Techniques	2

704 pages

Relativistic Nucleus-Nucleus Collisions



Before Collisions at
$\sqrt{s_{\mathrm{NN}}}$ = 0.2 TeV @ RHIC
= 13 TeV @ LHC

Spatial Contraction:
$\gamma \approx 100 - 10000$

De Broglie Wavelength:
$$\lambda = \frac{\hbar c}{p} = \frac{197.32\,\mathrm{MeV\,fm}}{13\mathrm{TeV}}$$

Collision
Duration: $\ll 1$ fm/c

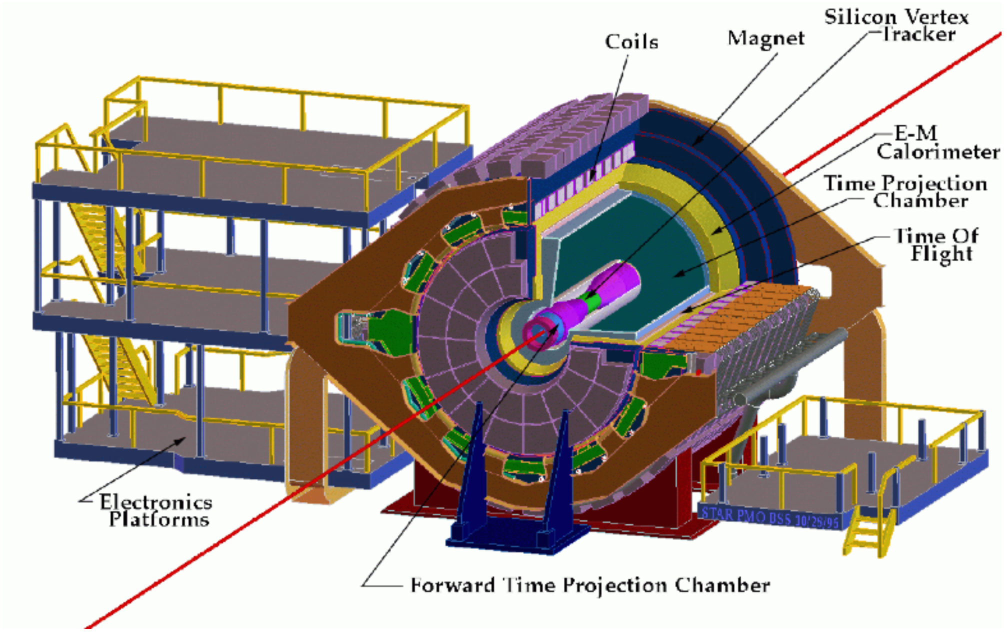
Medium: Quark Gluon Plama (QGP) Formation & Expansion
Life time: ≈ 10 fm/c

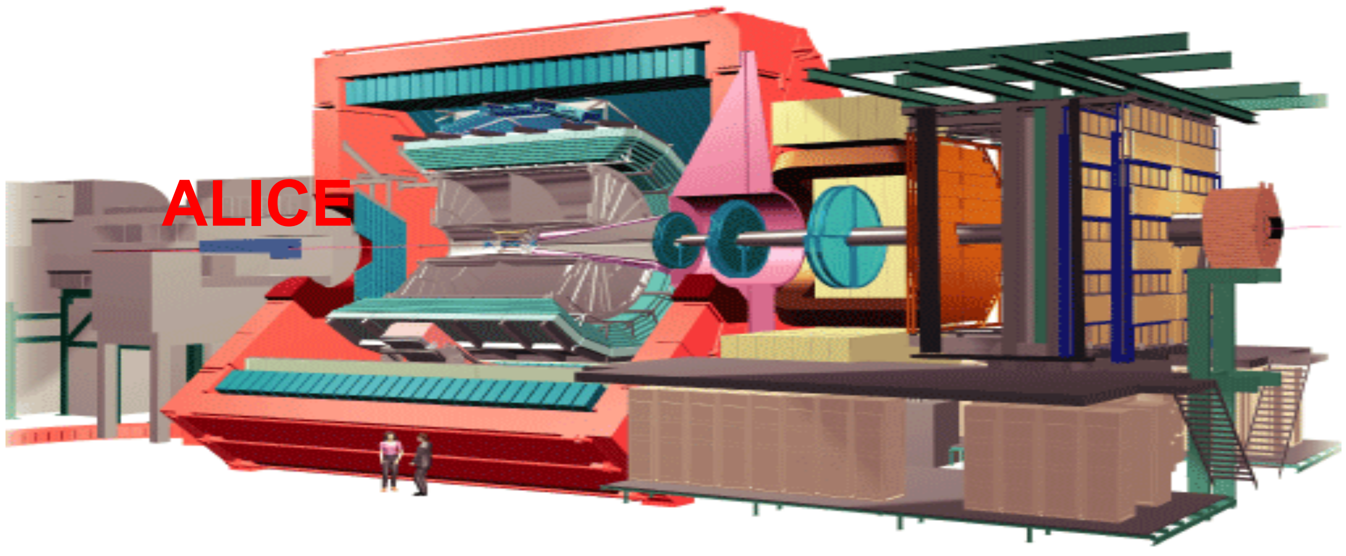
Hadronization
Particle Free Streaming…

Up to ~10000 particles produced per Pb-Pb collision at LHC!!!

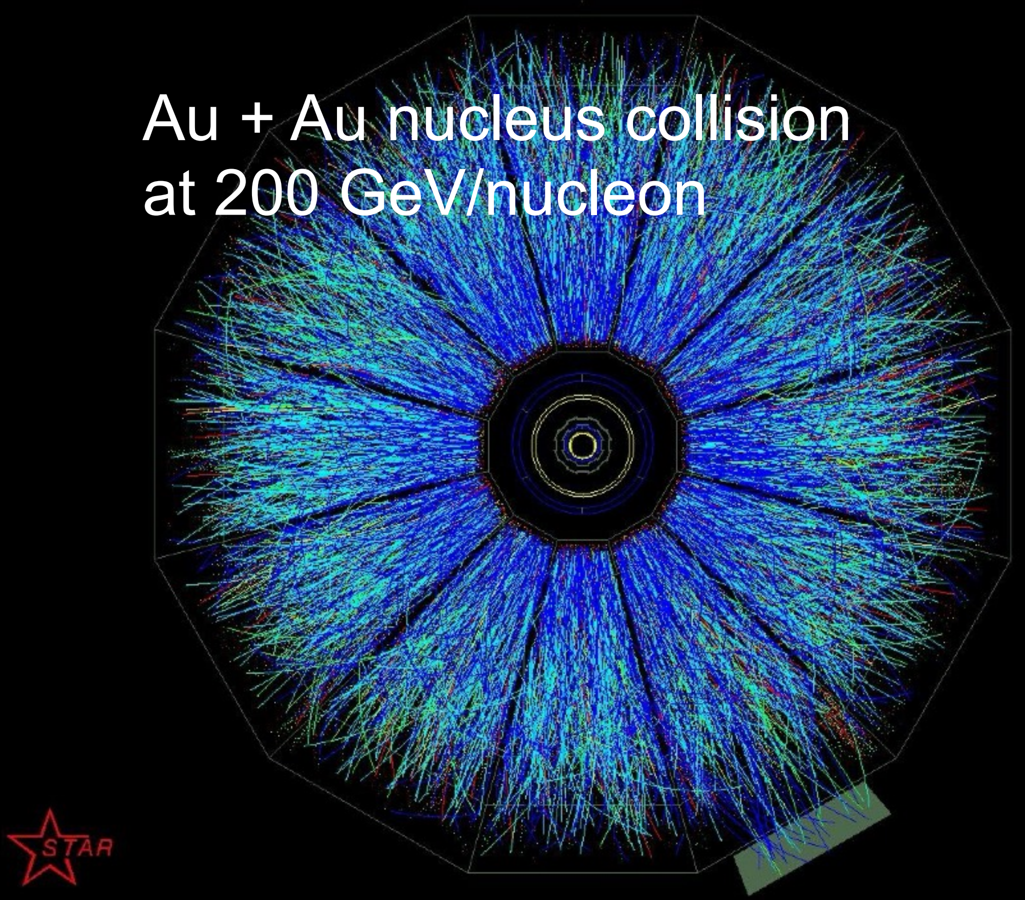
Note: 1 fm = 10^{-15} m; 1 fm/c = 10^{-15} m/3×10^{8} m/s = 0.33×10^{-23} s

Collision Snapshots

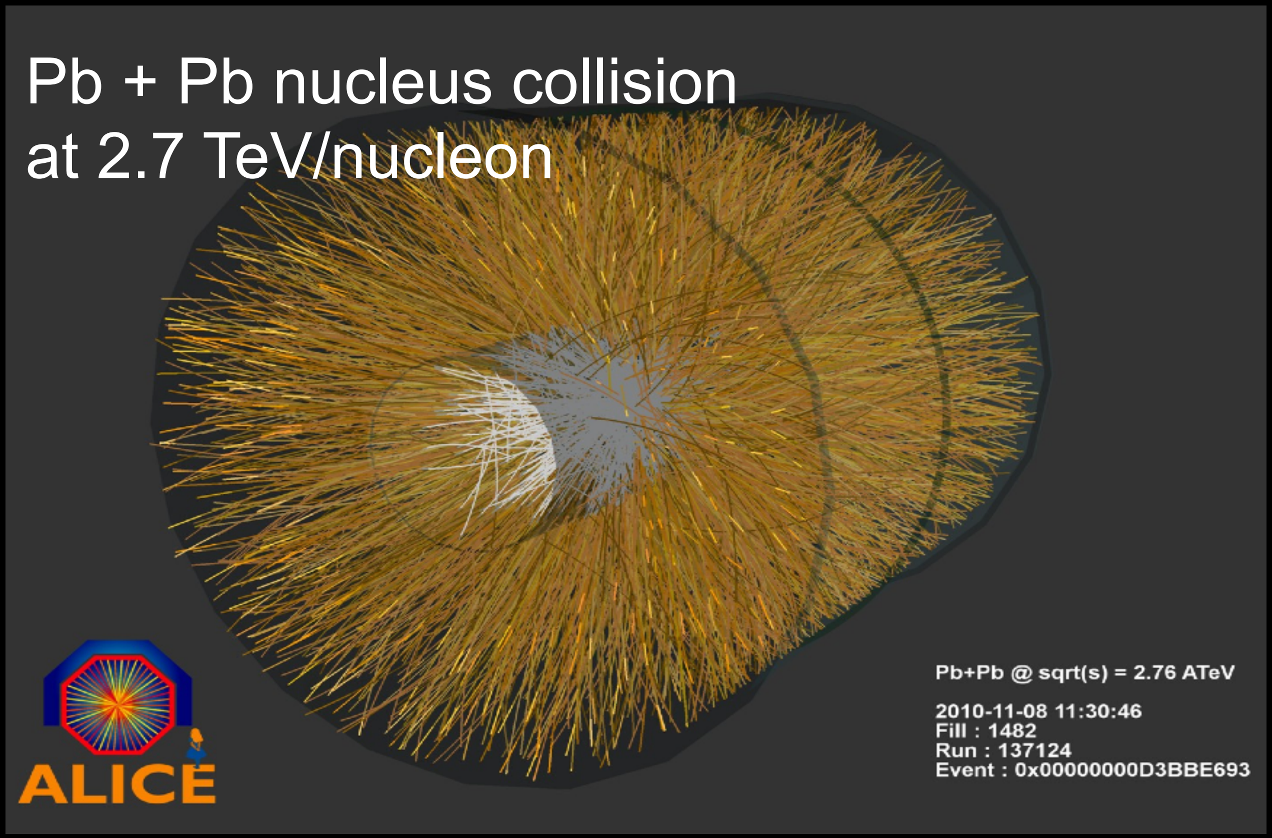




Au + Au nucleus collision
at 200 GeV/nucleon



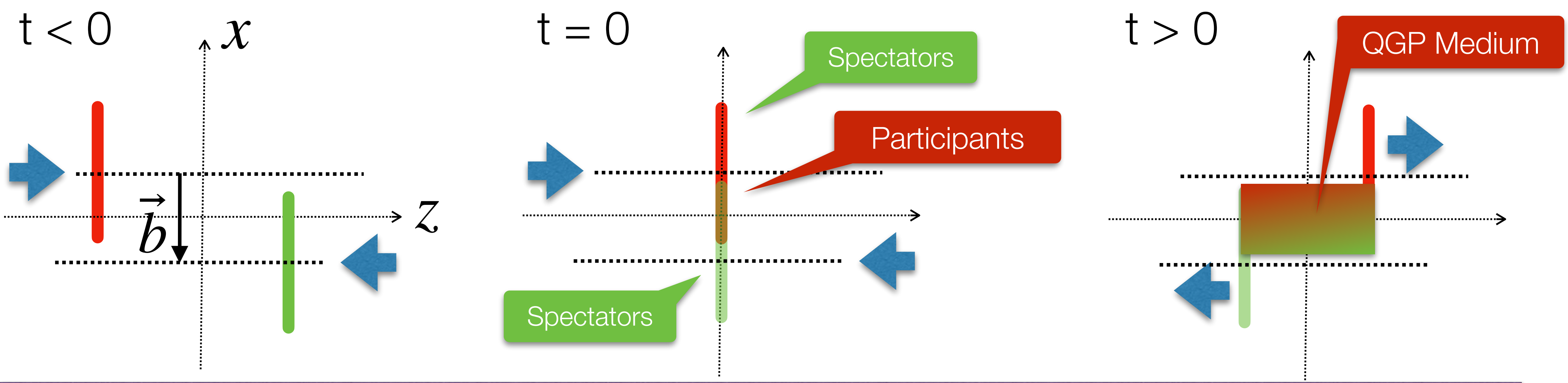
Pb + Pb nucleus collision
at 2.7 TeV/nucleon



Formation of Dense Nuclear Matter

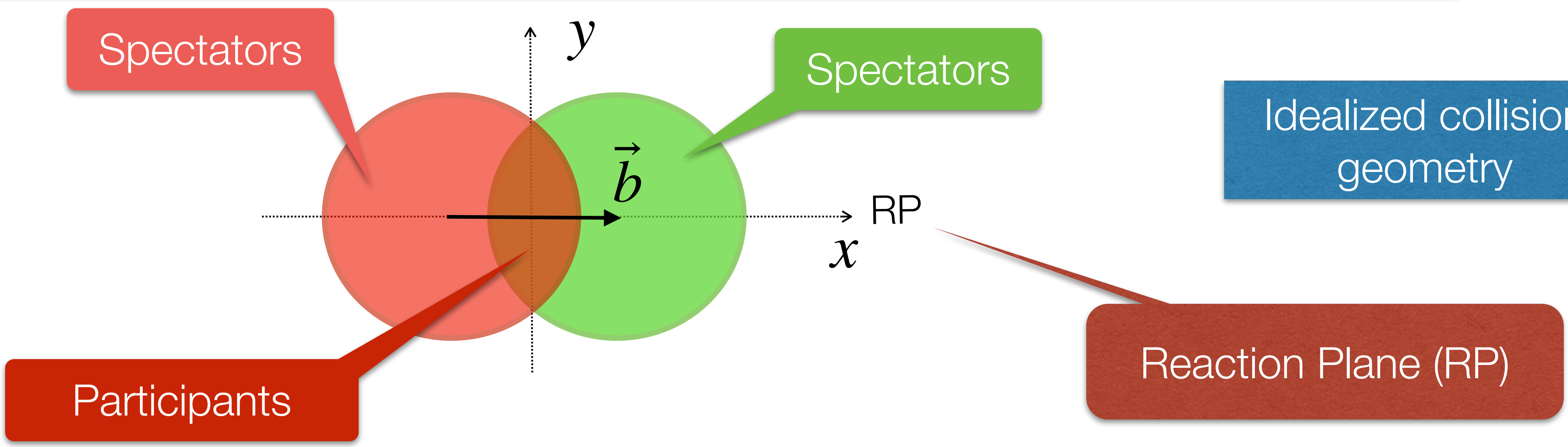
Quark Gluon Plasma Formation in AA Collisions

Longitudinal Direction



Traversing time $< 10^{-3}$ fm/c; Thermalization (?) < 1 fm/c; Fireball lifetime (?) ~ 10 fm/c

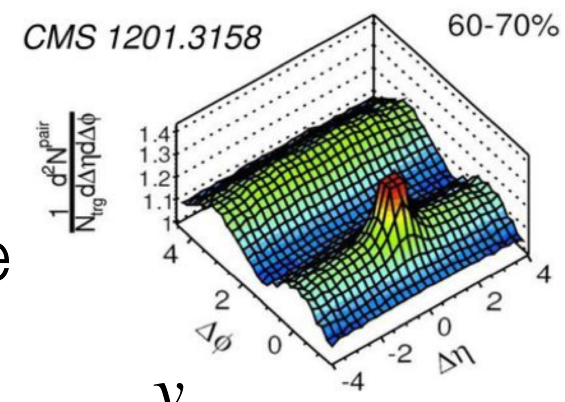
Transverse plane



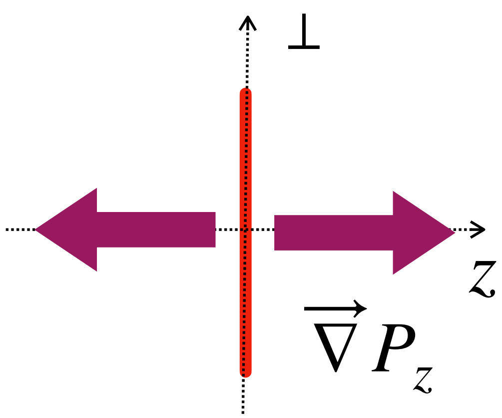
Idealized collision geometry

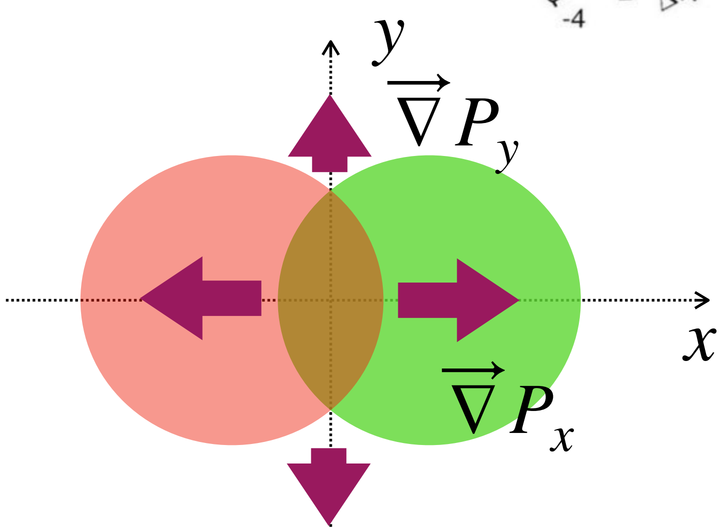
QGP Expansion & Collective

- **Anisotropic energy density profiles** produce
  - **Large & asymmetric pressure gradients**,
  - Drive rapid **outward expansion of the system** in both the **longitudinal** and **transverse** directions.



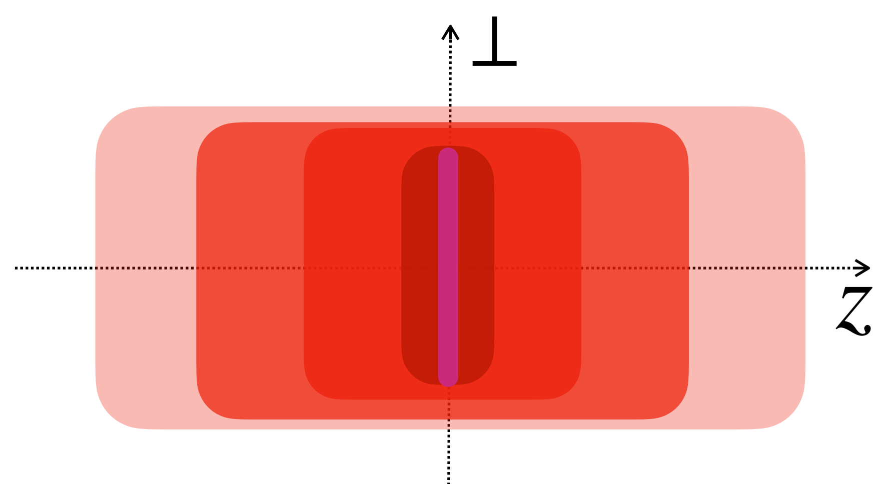
$$\vec{\nabla} P_z \gg \vec{\nabla} P_x \gg \vec{\nabla} P_y$$

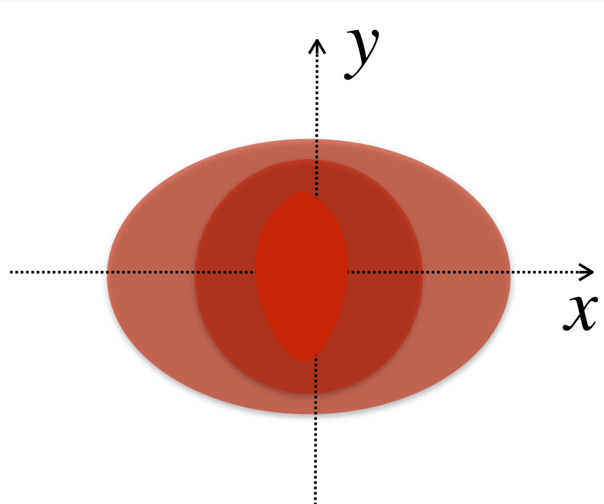




Longitudinal > In-plane > Out of plane Expansion:

- **Longitudinal/Isentropic Expansion**
- **Anisotropic Transverse Expansion**
  - **Anisotropic Flow**



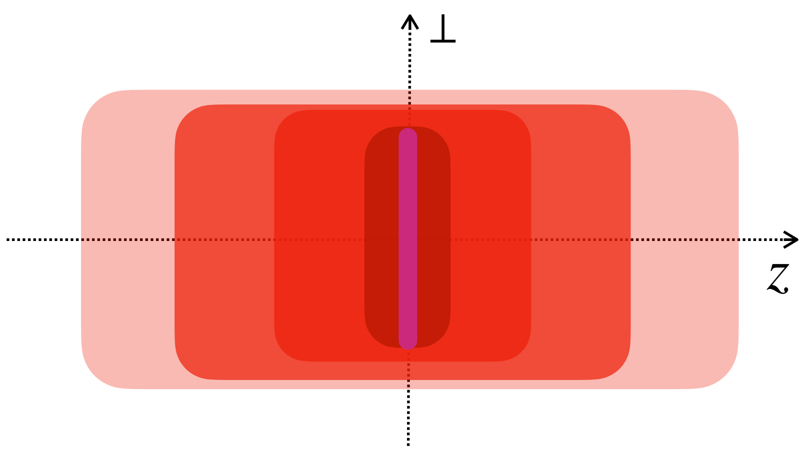


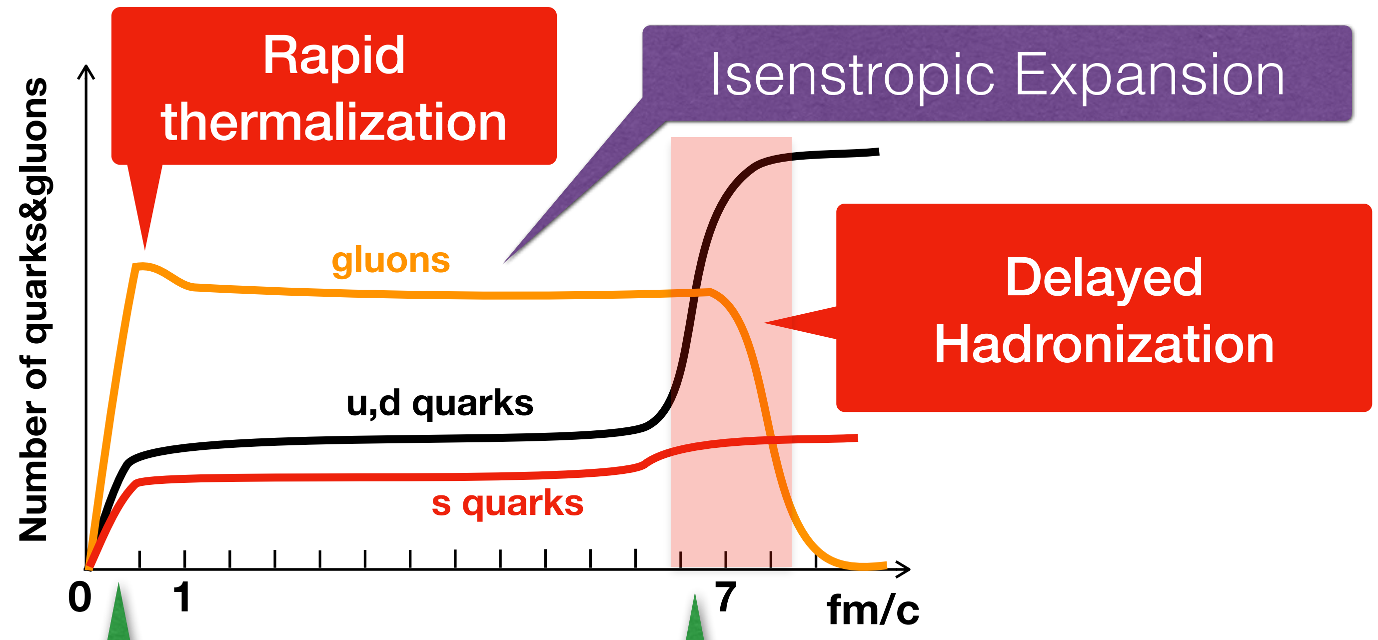
QGP Hypothesis, Thermalization, *Isentropic Expansion*

Anisotropic Pressure Gradients

$$\vec{\nabla} P_z \gg \vec{\nabla} P_x \gg \vec{\nabla} P_y$$

- **Longitudinal/Isentropic Expansion**
- Anisotropic Transverse Expansion





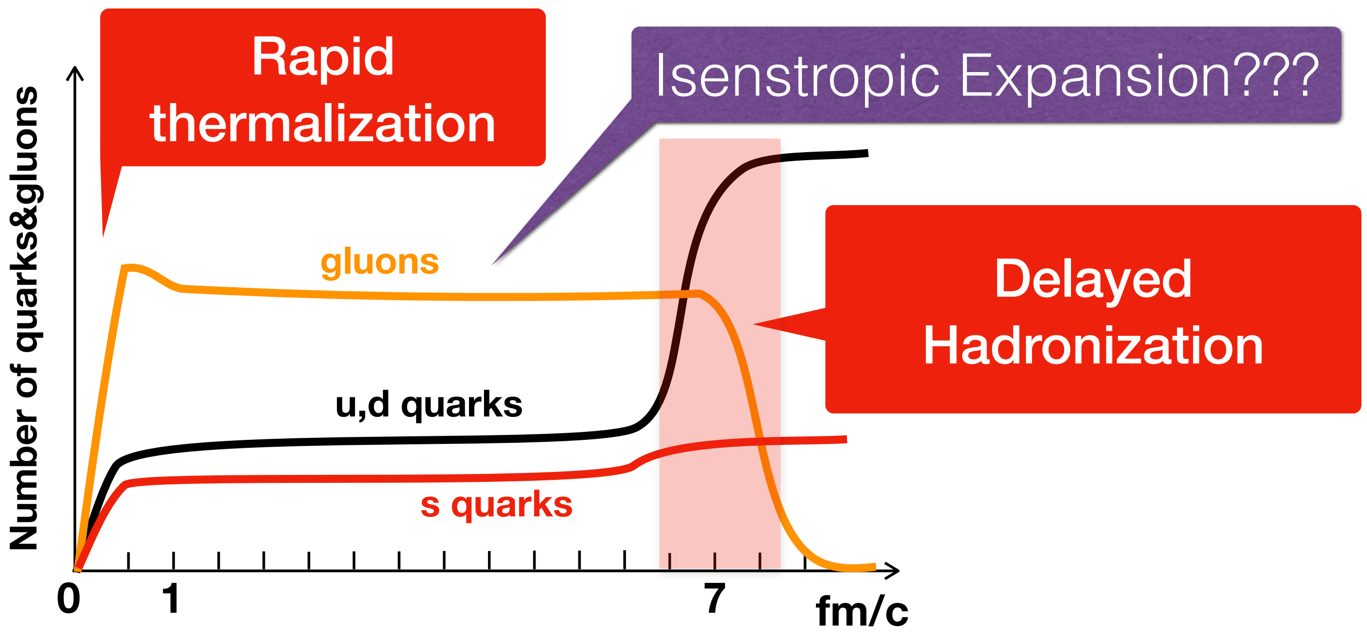
Two stages of quark production + Delayed hadronization

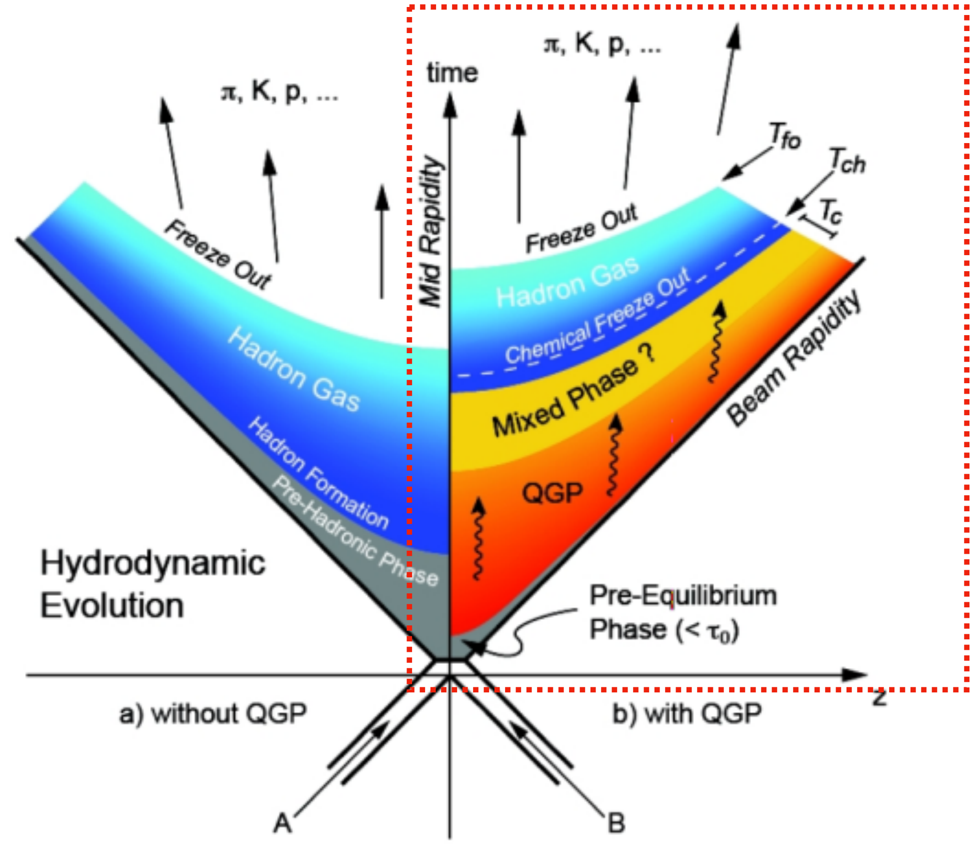
Bass, Danielewicz, Pratt, PRL 85 (2000) 2689

Have we fully vetted this ?

QGP Hypothesis, Thermalization, *Isentropic Expansion*

Bass, Danielewicz, Pratt, PRL 85 (2000) 2689





Two Stage Quark production:

- **Early Emission**:
  - High $\sqrt{s}$ processes; Long time for expansion: Large quark separation in rapidity: **broad charge balance functions**
- **Late Emission**:
  - Low temperature, low $\sqrt{s}$ processes, Short time for expansion; Narrow quark separation in rapidity: **narrow charge balance functions**
- **Narrowing of Balance Functions for pions** but not for kaons - from peripheral to central impact parameter collisions…

Unified Balance Functions

Conditional Density

$$\rho_2^{\alpha|\beta}(y_1\,|\,y_2) = \frac{\rho_2^{\alpha\beta}(y_1, y_2)}{\rho_1^{\beta}(y_2)}$$

Pratt’s Balance Functions

$$B^{\alpha|\bar{\beta}}(y_1\,|\,y_2) = \rho_2^{\alpha|\bar{\beta}}(y_1\,|\,y_2) - \rho_2^{\bar{\alpha}|\bar{\beta}}(y_1\,|\,y_2)$$

Unified Balance Functions

$$B^{+-}(y_1, y_2\,|\,y_0) = \frac{1}{\langle N_1^- \rangle}\left[\rho_2^{+-}(y_1, y_2) - \rho_2^{--}(y_1, y_2) - \rho_1^{+}(y_1)\rho_1^{-}(y_2) + \rho_1^{-}(y_1)\rho_1^{-}(y_2)\right]$$

$$B^{+-}(y_1, y_2\,|\,y_0) = \frac{1}{\langle N_1^- \rangle}\left[C_2^{+-}(y_1, y_2) - C_2^{--}(y_1, y_2)\right]$$

2-Cumulants

- *Accounting for non vanishing net-charge with unified balance functions*, C.P. et al, PRC 107 (2023) 1, 014902,
- *Effects of non vanishing net charge on balance functions and their integrals*, C.P. et al, PRC 107 (2023) 5, 054915
- General balance functions of identified charged hadron pairs of π, K, $p\bar{p}$ in PbPb Collisions…, C.P. et al, PLB 833 (2022) 137338

ALICE, PLB 723 (2013) 267.
ALICE, Eur. Phys. J. C (2016) 76

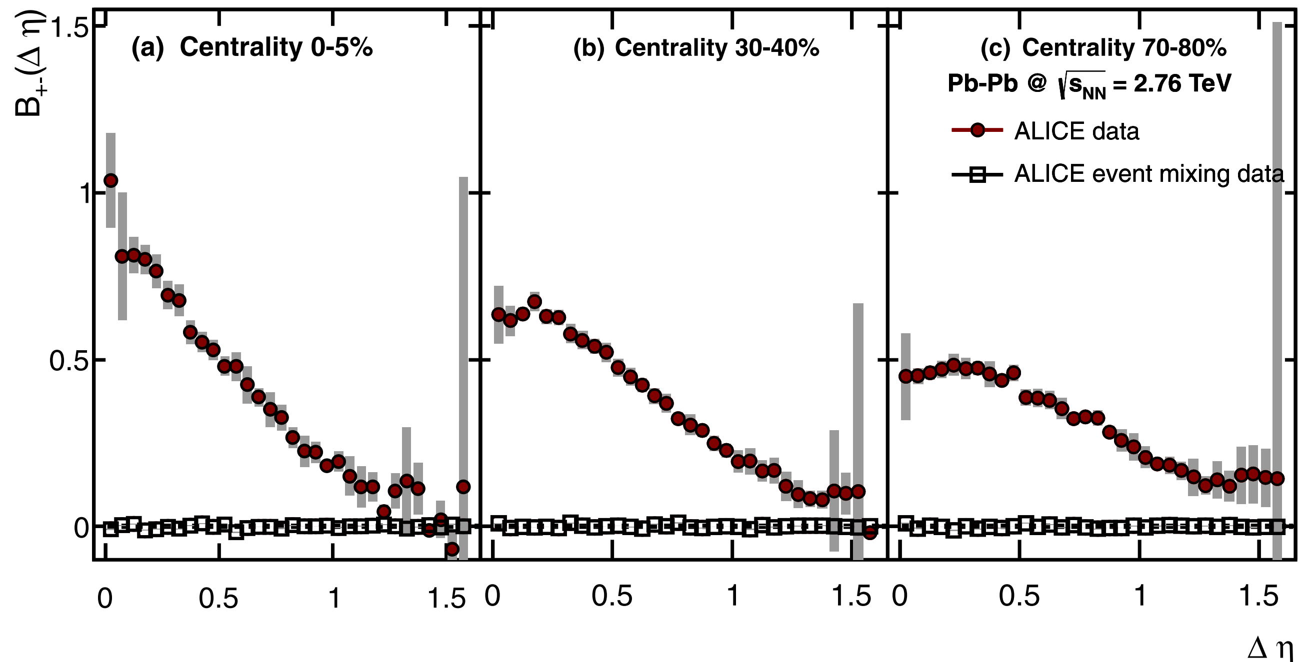
Charge Balance Functions at ALICE

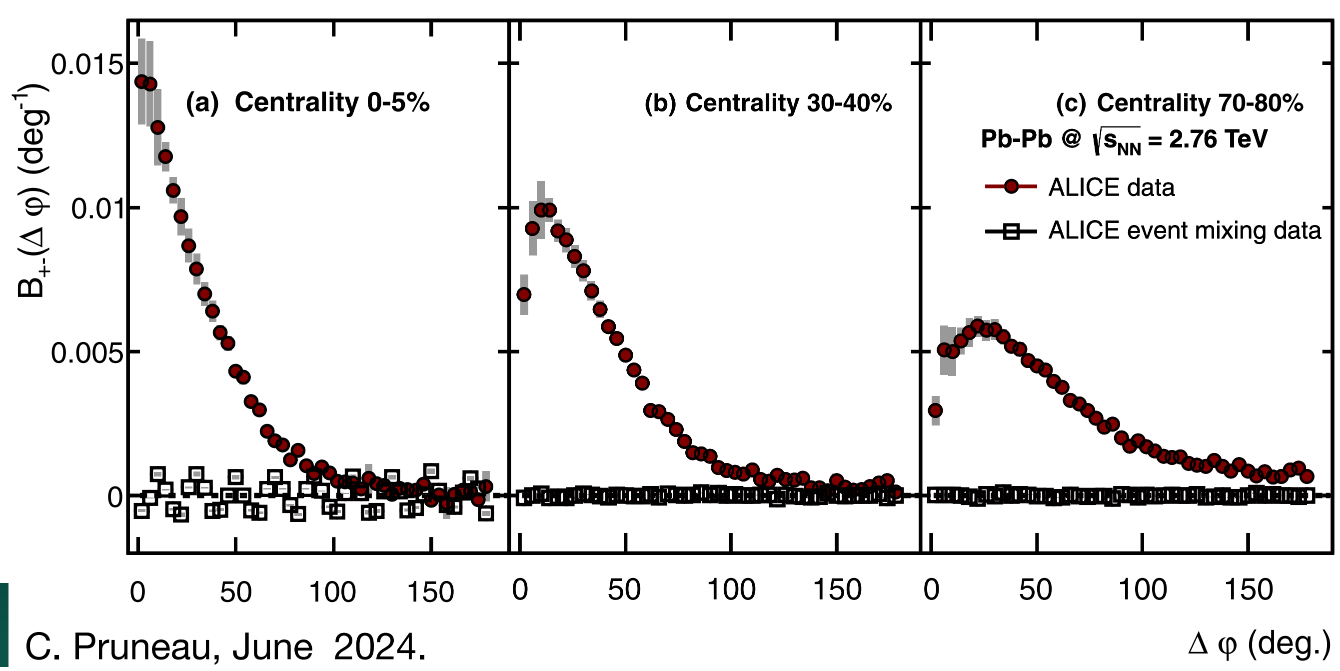
Similar results from STAR: PRL 90 (2003)172301; PRC 82 (2010); PRC 94 (2016) 024909

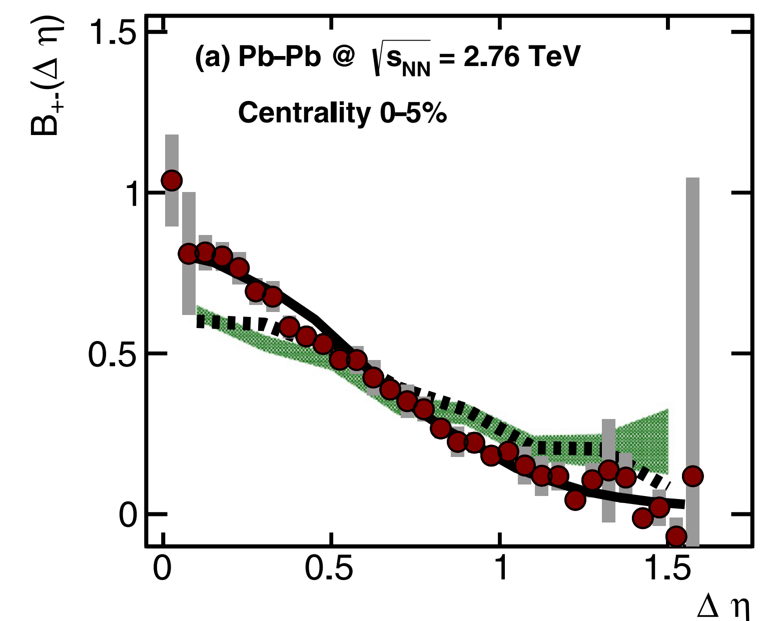
Balance Function: $$B_{+-}(\Delta\eta) = \frac{1}{2}\Big(C_{+-}(\Delta\eta) + C_{-+}(\Delta\eta) - C_{--}(\Delta\eta) - C_{++}(\Delta\eta)\Big).$$

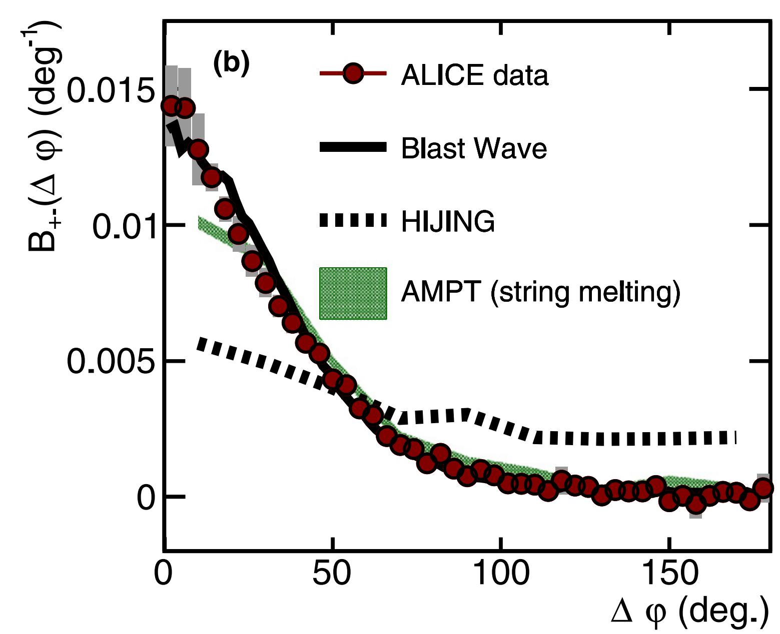
$$C_{ab} = (N_{ab}/N_b)/f_{ab}$$

BF Narrowes vs. Centrality









Charge Balance Functions at ALICE & STAR

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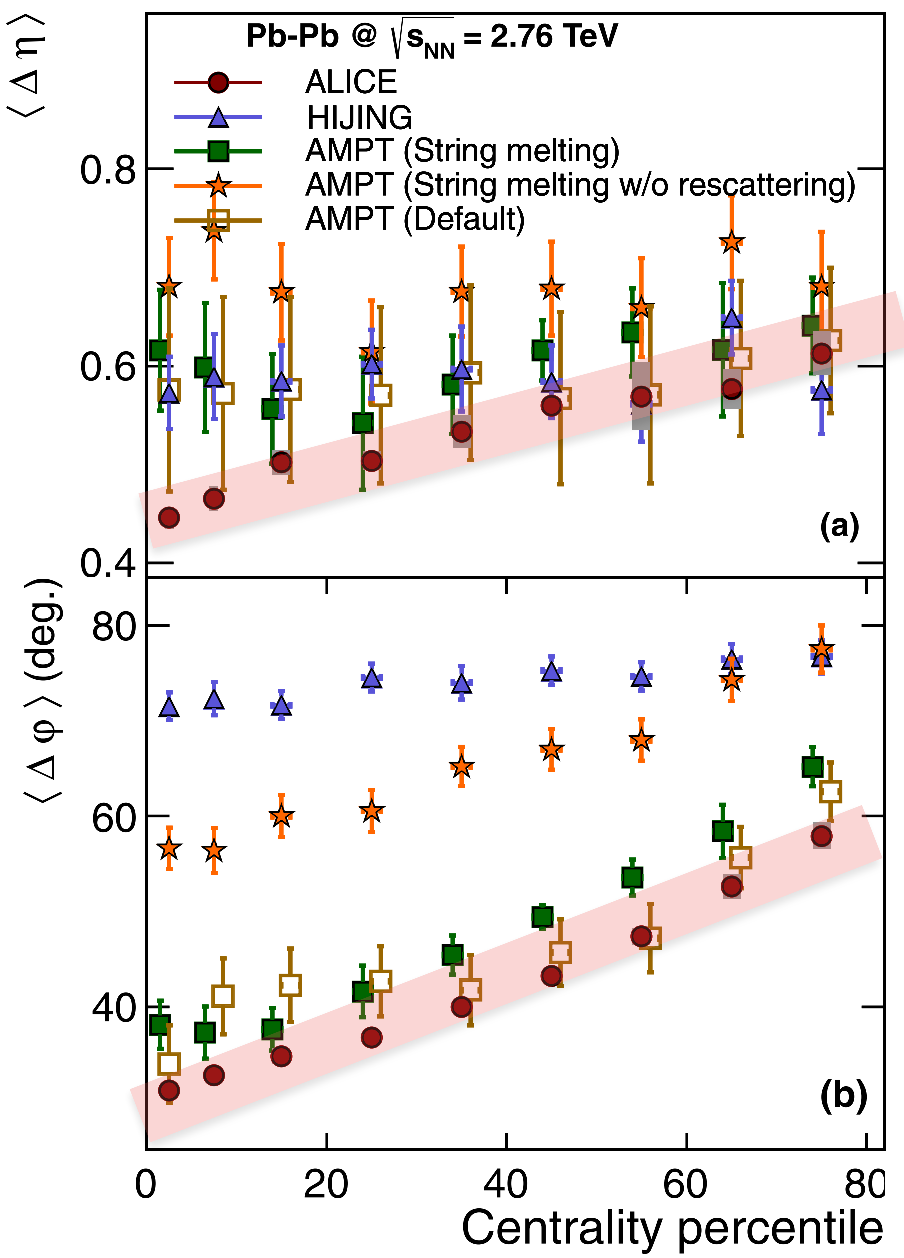
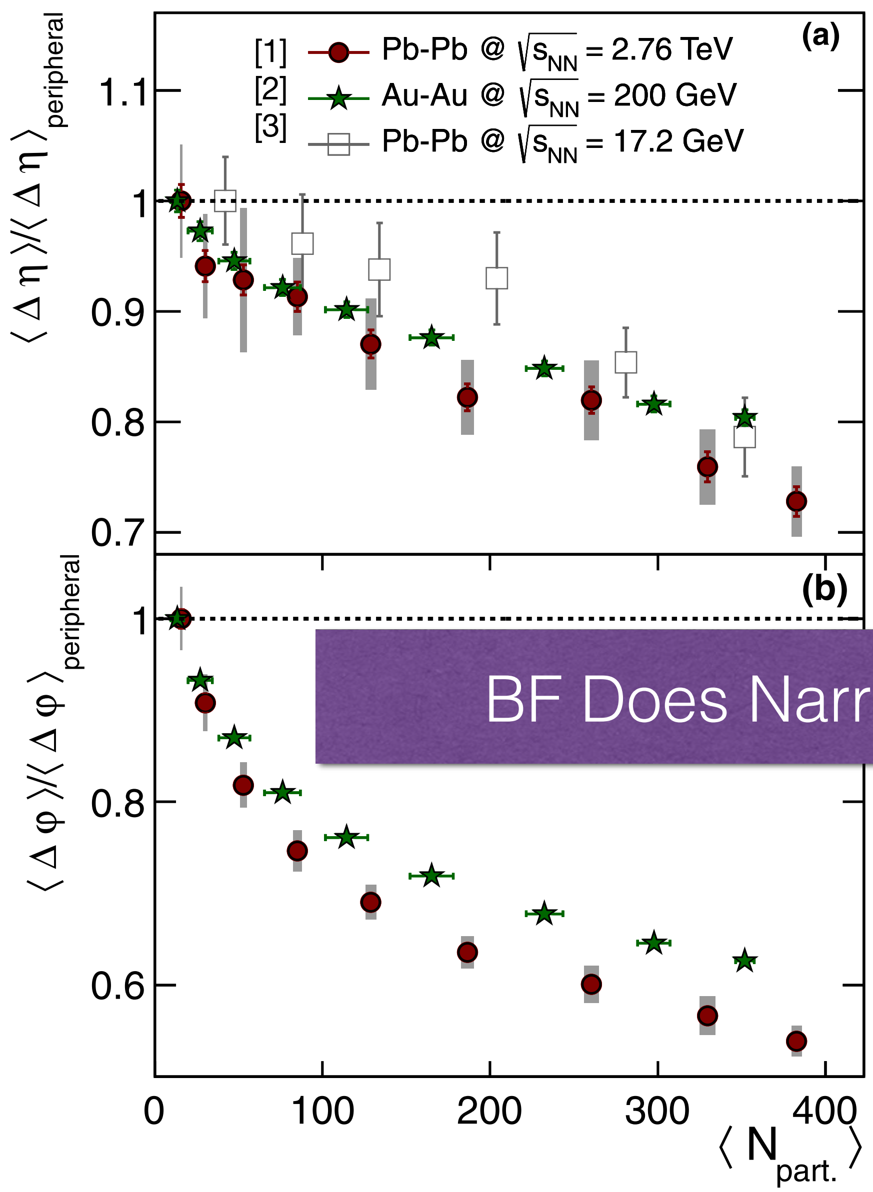
$$C_{ab} = (N_{ab}/N_b)/f_{ab}$$





BF Does Narrow vs. Centrality

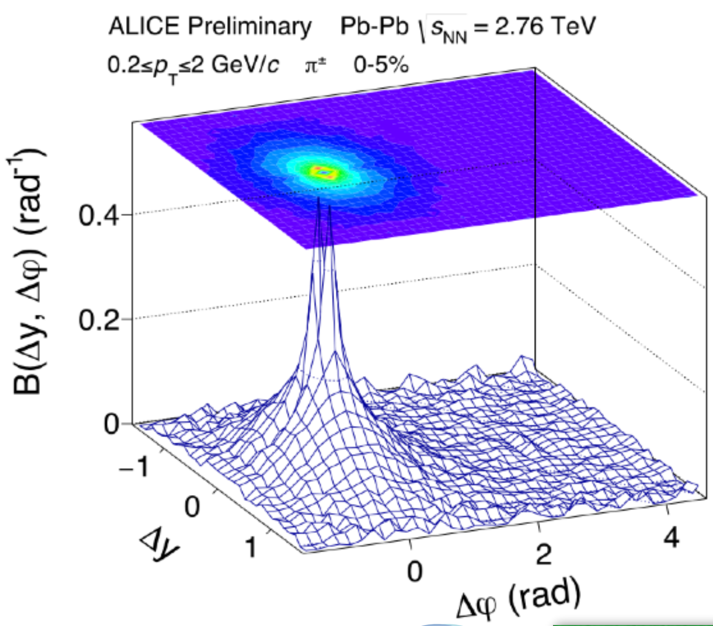
[1] ALICE, PLB 723 (2013) 267.
[2] STAR, Phys. Rev. C **82**, 024905
[3] NA49, Phys. Rev. C **76**, 024914

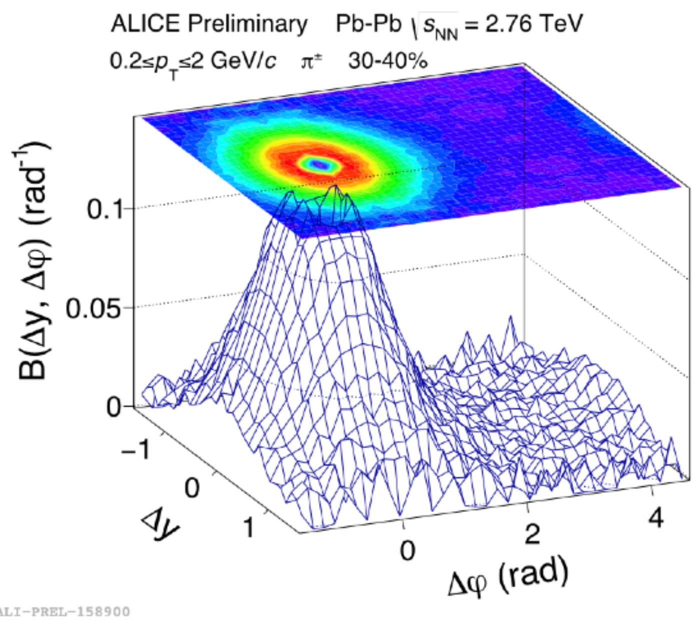
J. Pan, PhD, Wayne State (2019)
ALICE PLB 833 (2022) 137338

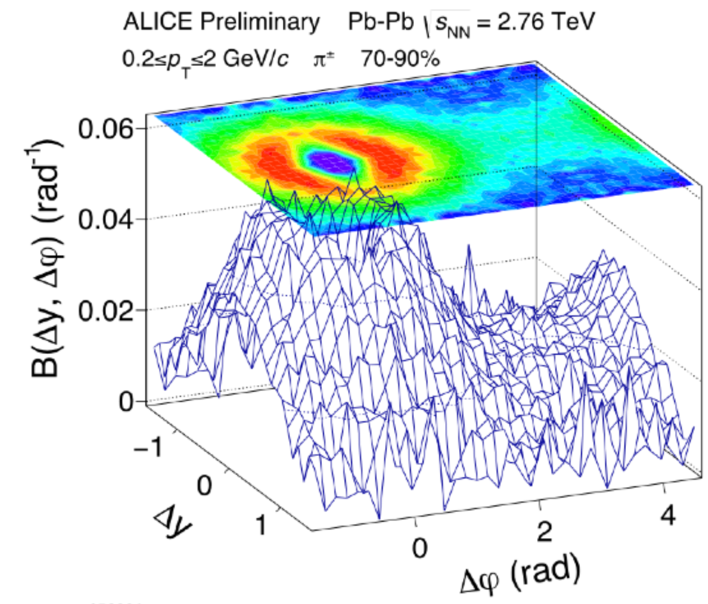
PID Balance Functions in Pb—Pb

Pion, Kaon Balance Functions

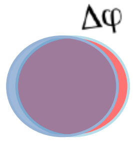
$B^{\pi^{\pm}\pi^{\mp}}$



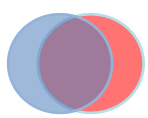


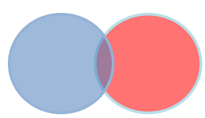


Considerable shape dependence on collision centrality



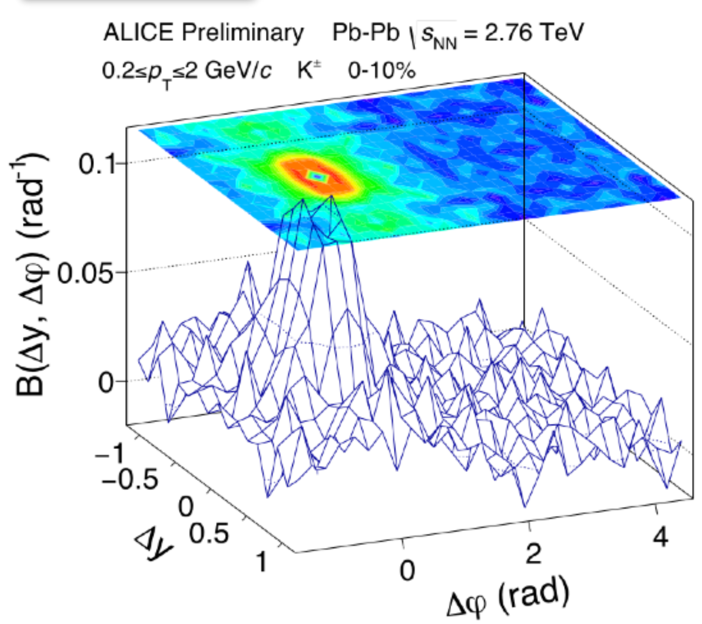
Central

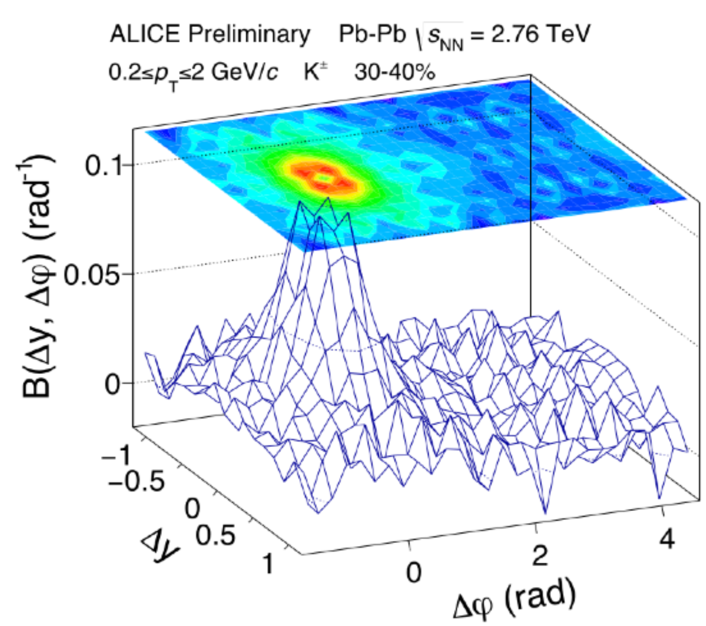


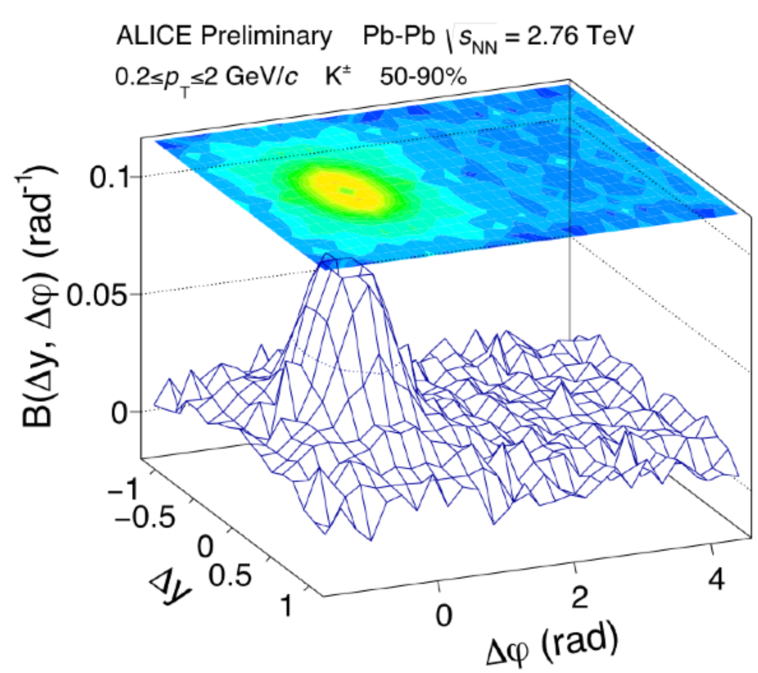


Peripheral

$B^{K^{\pm}K^{\mp}}$







Very modest shape dependence on collision centrality

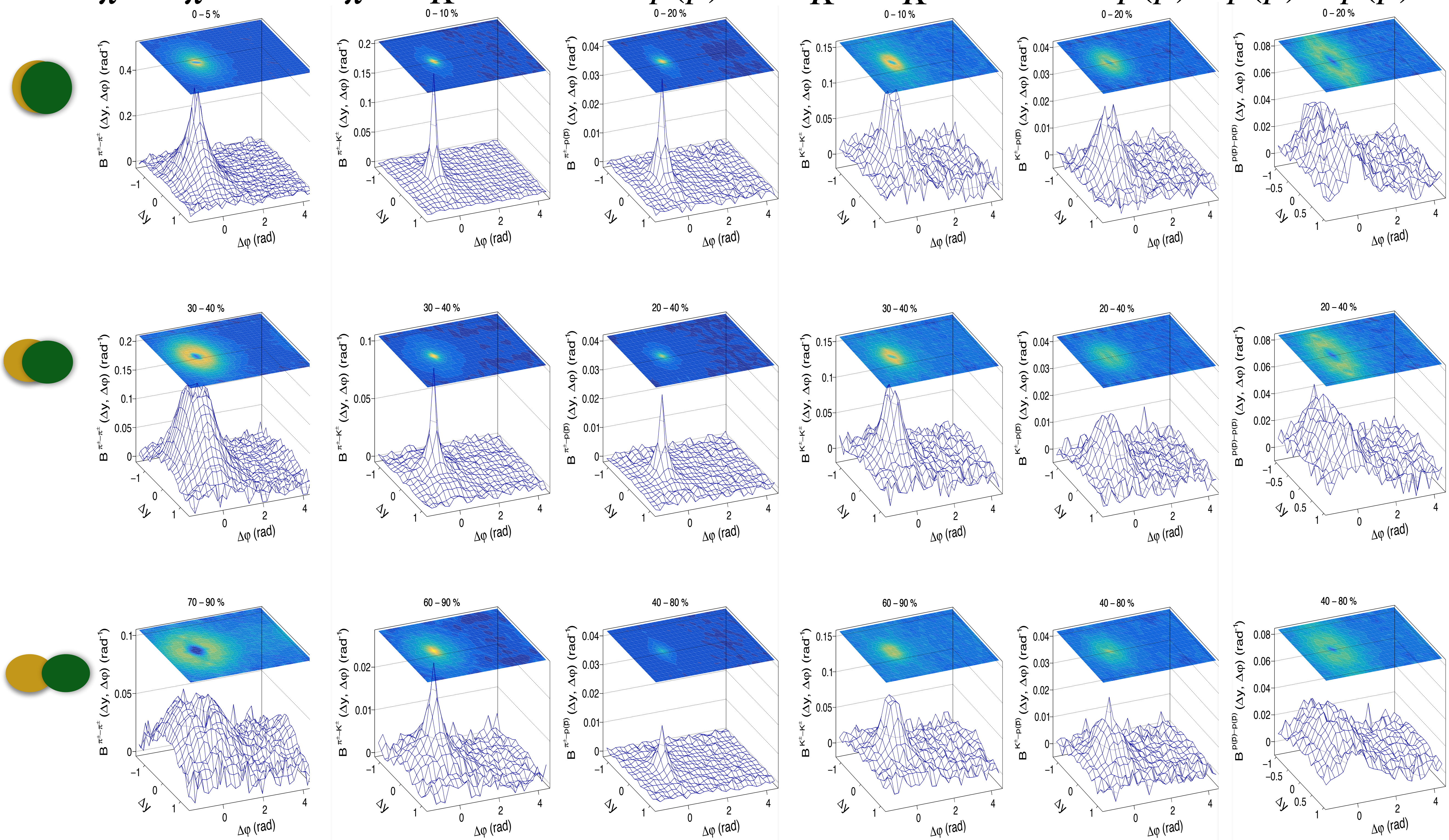
General Balance Functions

J. Pan, PhD, Wayne State (2019)
ALICE PLB 833 (2022) 137338

ALICE

$(\pi, K, p) \otimes (\pi, K, p)$ Pb+Pb @ 2.76 TeV

$\pi^{\pm} - \pi^{\pm}$ | $\pi^{\pm} - K^{\pm}$ | $\pi^{\pm} - p(\bar{p})$ | $K^{\pm} - K^{\pm}$ | $K^{\pm} - p(\bar{p})$ | $p(\bar{p}) - p(\bar{p})$



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PID Balance Functions in Pb—Pb

J. Pan, PhD, Wayne State (2019)
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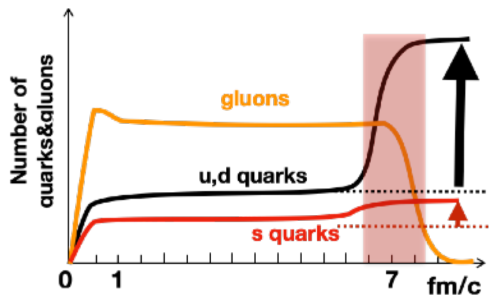
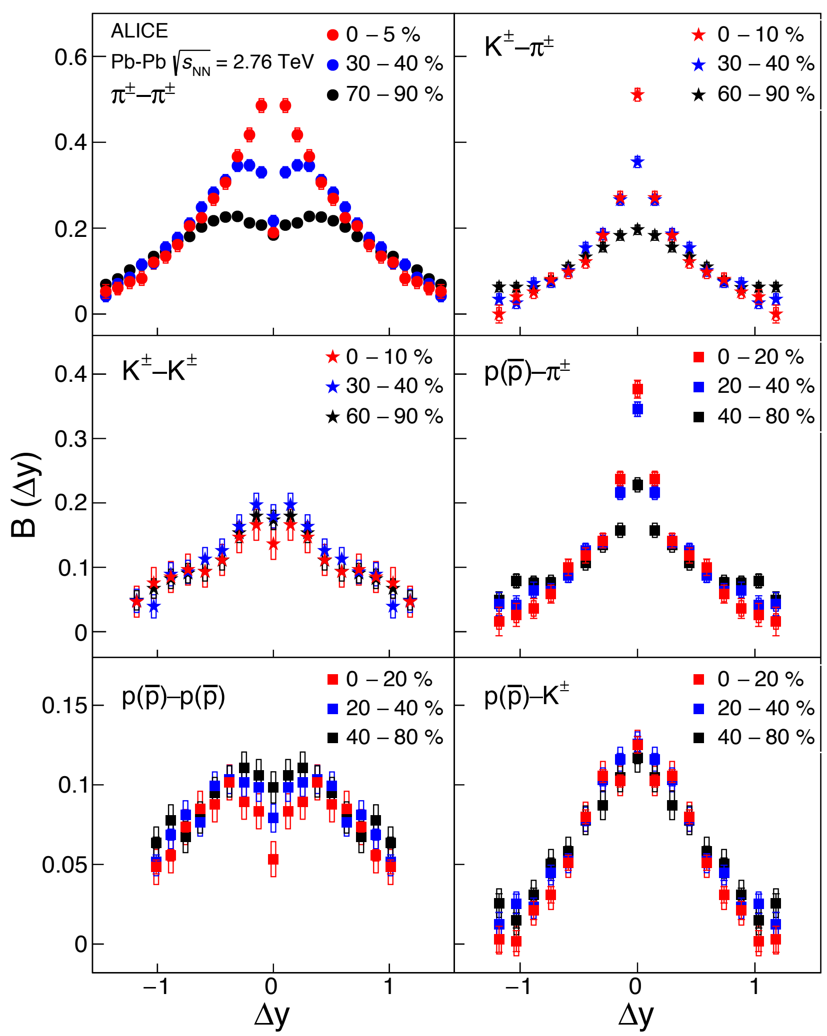
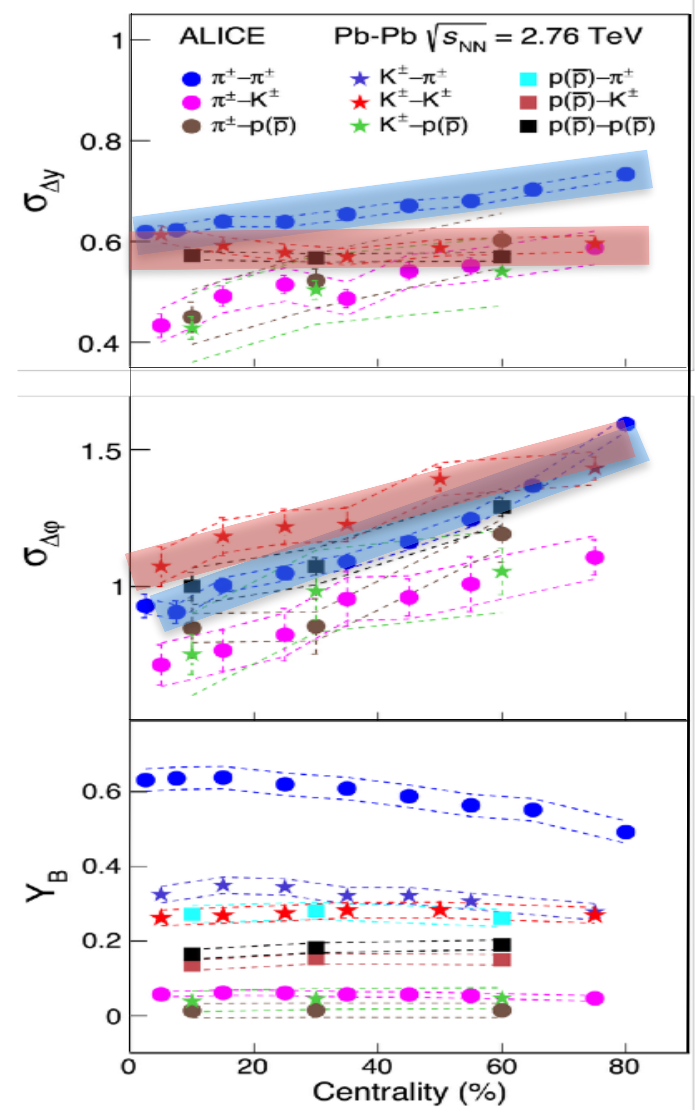
$(\pi, K, p) \otimes (\pi, K, p)$



S. Pratt, NPA 698 (2002) 531c: Pions and Kaons BFs to have different collision centrality dependence.





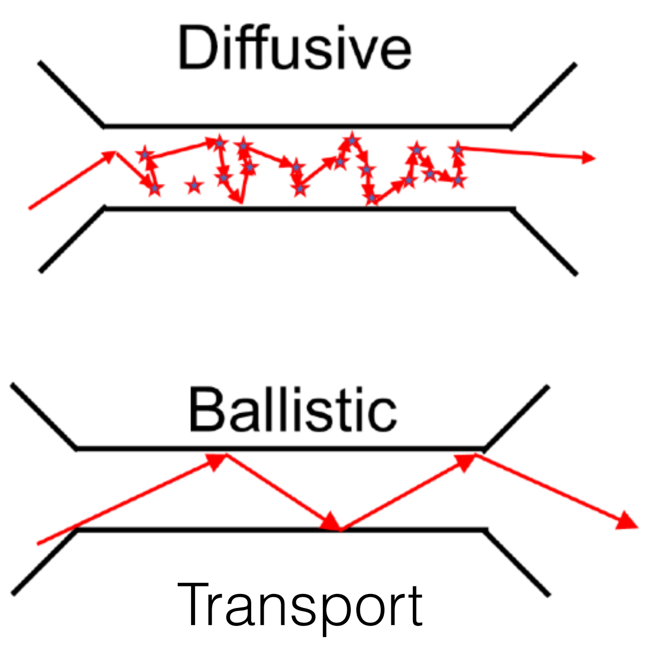
- **1st BF measurement of "full" cross-species matrix** of $\pi^{\pm}$, $K^{\pm}$ and $p(\bar{p})$
  - Differential B(Δy) profile.
  - Better constraints on models.
- $B_{\pi^{\pm}\pi^{\pm}}$ **: clear centrality dependence,**
- $B_{K^{\pm}K^{\pm}}$ **: no centrality dependence,**
- $B_{p\bar{p}}$ and cross-species pairs : moderate centrality dependence.
- Differences in BF shape, magnitude, centrality evolution between different species pairs
- **Different pair production mechanisms/times** for $\pi^{\pm}$ (up/down quark meson), $K^{\pm}$ (strangeness meson) and $p(\bar{p})$ (baryon).

Corroborates/Qualitative Agreement with K BFs by STAR PRC 82 (2010) 024905.

Supports two stage emission scenario (delayed hadronization)

Role of Transport & Diffusion

- **Persistence of correlations**
  - Elastic/Quasi-elastic scatterings of qq or qg, annihilation of $q\bar{q}$ do not eliminate correlations.
- **Partial/local thermalization does not eliminate (long range) correlations.**
  - **Long range charge correlations are “frozen in.”**
- Transport (longitudinal/transverse) **modifies** correlations, i.e., $B(\Delta\vec{p})$
  - Same “logic” valid for **quark level and hadron level correlations.**
- **Full thermalization would be achieved** when balance function $B(\Delta\vec{p})$ is uniform, i.e., when initial correlations appear to have vanished.
  - $\rightarrow$ **THIS IS NOT OBSERVED**



- **Integral correlators (measured within a specific fiducial volume) average out the strength of correlations** over phase space and are thus **much less sensitive to the $\Delta\vec{p}$ dependence of these correlations**
- By contrast, differential correlations, e.g., balance functions, provide a detailed account of the evolution of $q-\bar{q}$ correlations vs. collision centrality.
  - Sensitivity to Initial Correlations
  - Sensitivity to transport

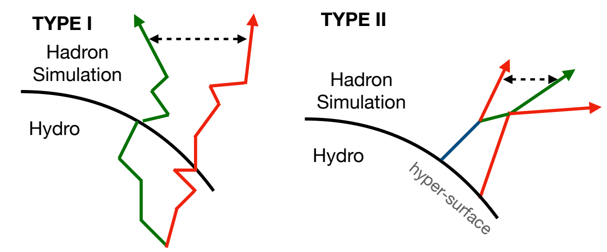
“New” Works/Ideas

J. Pan, PhD, Wayne State (2019)
ALICE PLB 833 (2022) 137338

Light Quark Diffusivity

Pratt & Plumberg, PRC 104 (2021) 014906

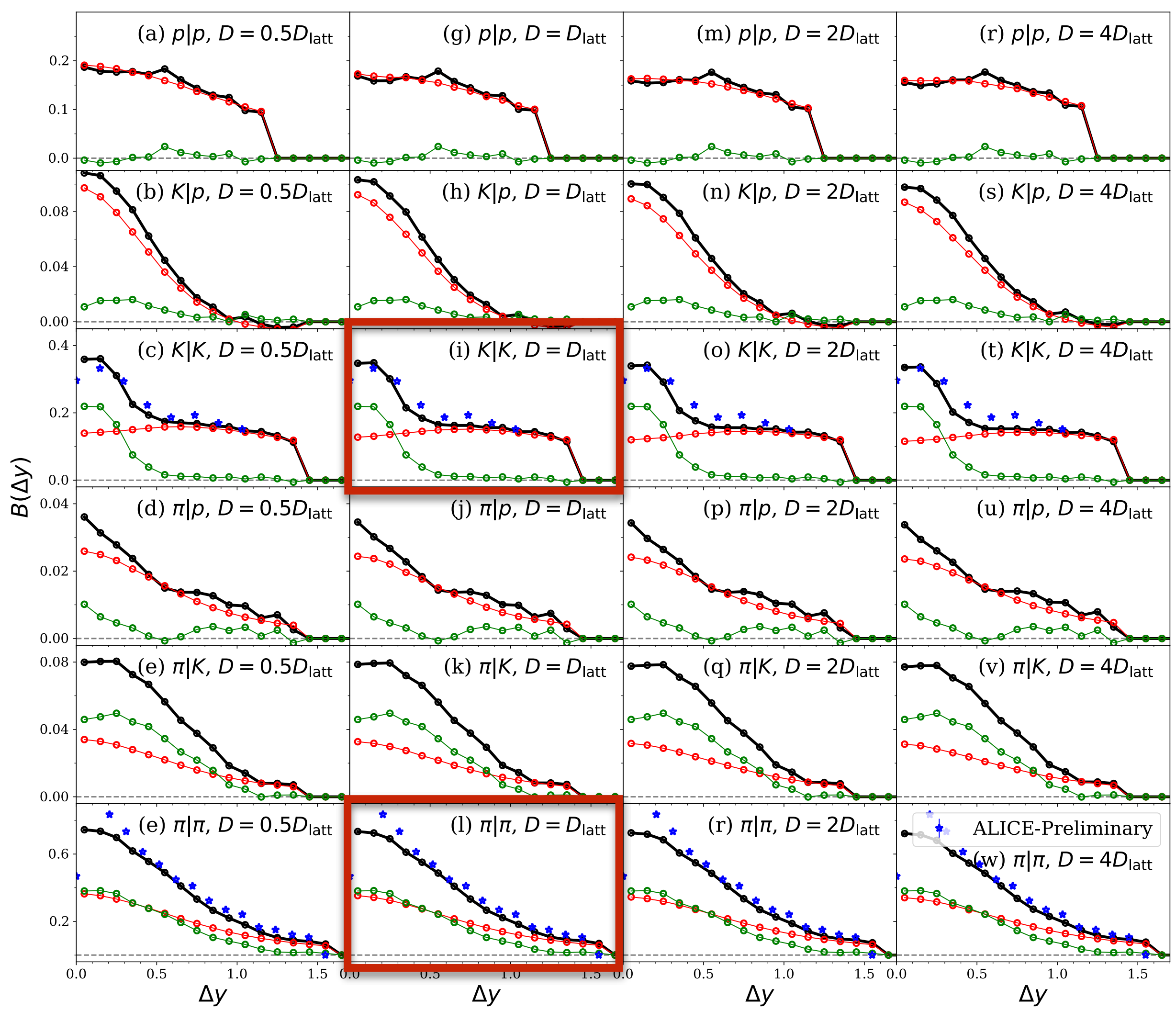
Comparison of theoretical predictions with ALICE data.
QGP evolution with diffusion and lattice susceptibilities, hadronic simulations, resonance cocktail…

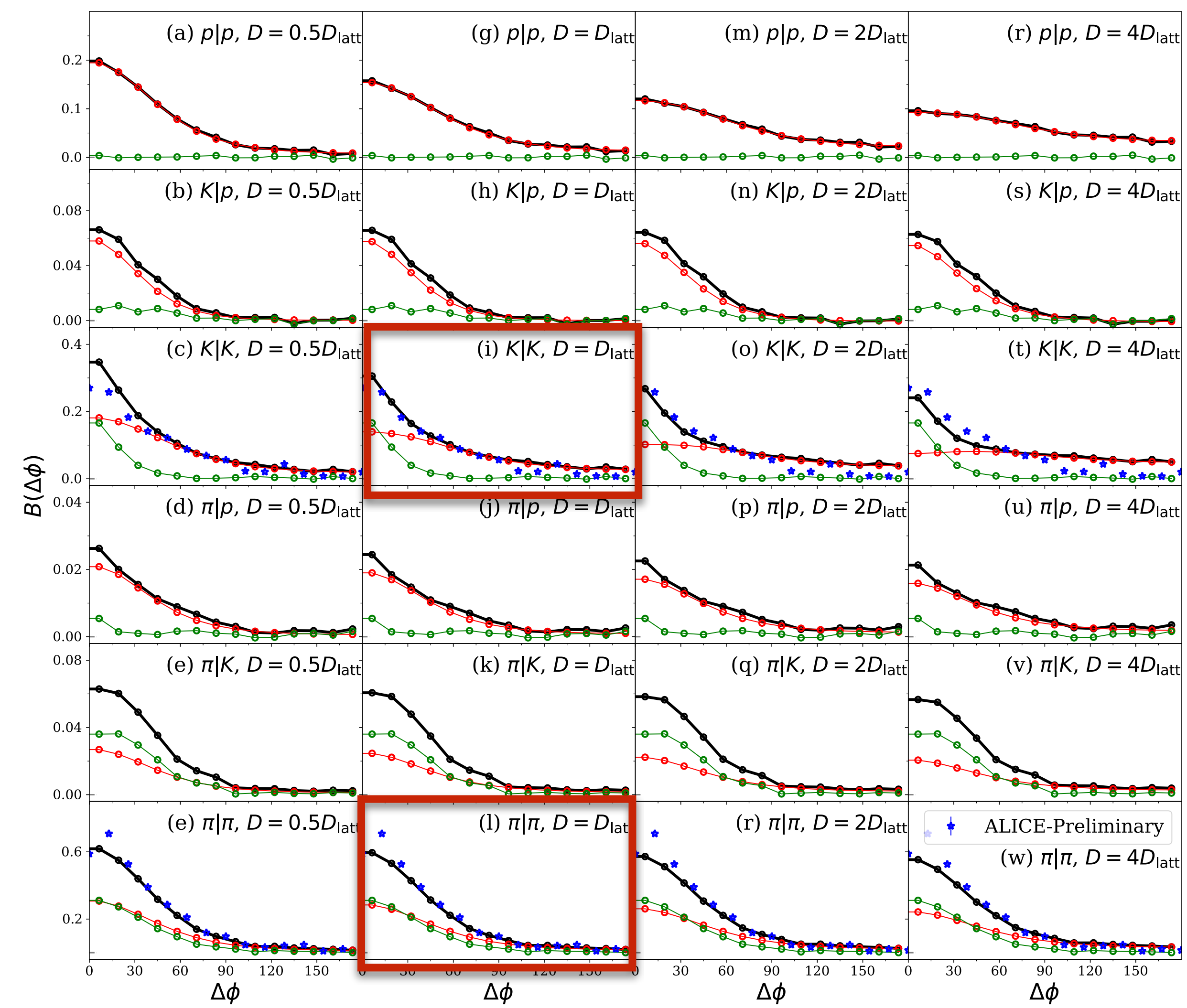


$$\sigma_y^2 = 2\sigma_0^2 + 2T_b/M_\perp + 4\beta \ln(\tau_f/\tau_0).$$

$$D = \beta\tau$$ from lattice calculations

Type-I Contributions   Type-II Contributions   Total   ALICE





ALICE BFs sensitive to light quark diffusivity
ALICE results favor LQCD values!

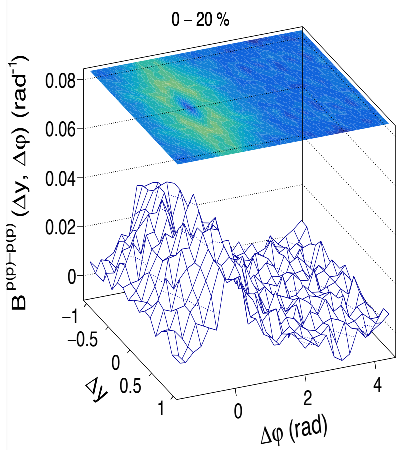
CP et al., 2310.07618 [hep-ex]

Suppressing Hadron (Strong) Decays

Role of decays depends on types of BF considered:

Essentially no particles decay into a baryon and anti-baryon

$p\bar{p}$



Correlation not from decays!*

*but decays may contribute e.g., $\Delta^+ \rightarrow p + X$

Longitudinally broad
Azimuthally narrow.

A Strong Case for ALICE 3

Baryon Transport
What Carries the Baryon Number?

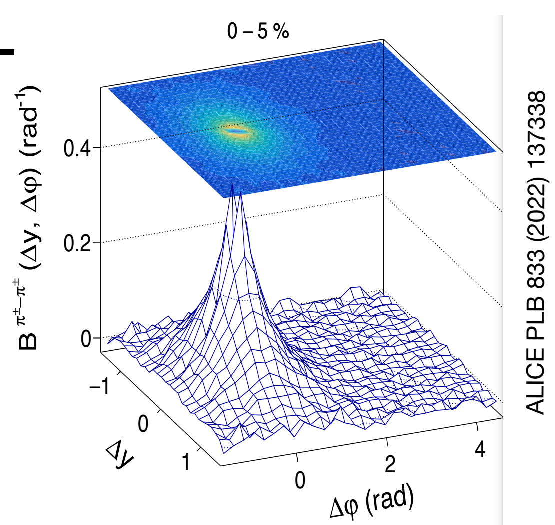
But resonances can decay into (Examples)

+Q & -Q: $\rho^0 \rightarrow \pi^+ + \pi^-$

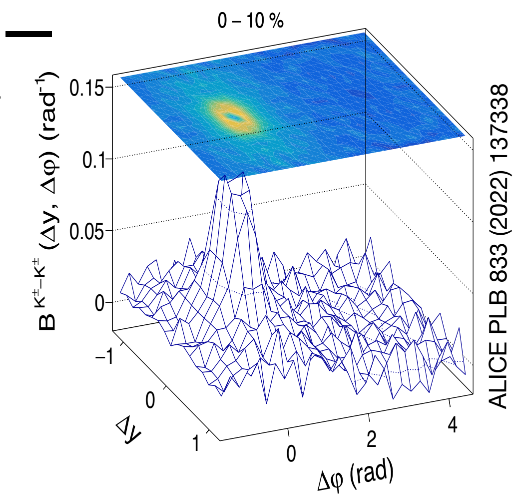
+S & -S: $\phi^0 \rightarrow K^+ + K^-$

Baryon + Meson: $N^* \rightarrow p + \pi^-$

$\pi^+\pi^-$



K^+K^-



Longitudinally & Azimuthally narrow.

Can we suppress or eliminate correlations originating from decays?

YES: Use N>2 Balance Functions

A. Andronic, et al., PLB 792 (2019) 304

Thermal Hadron Production

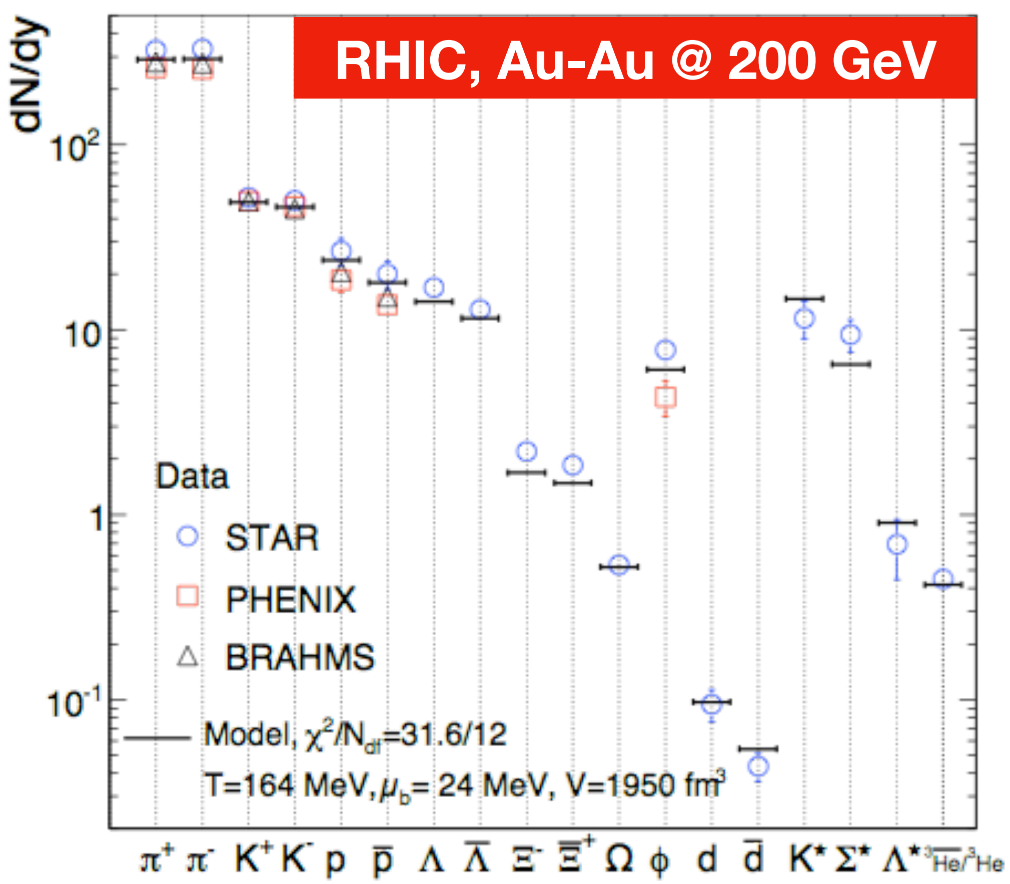
GCE Partition Function:

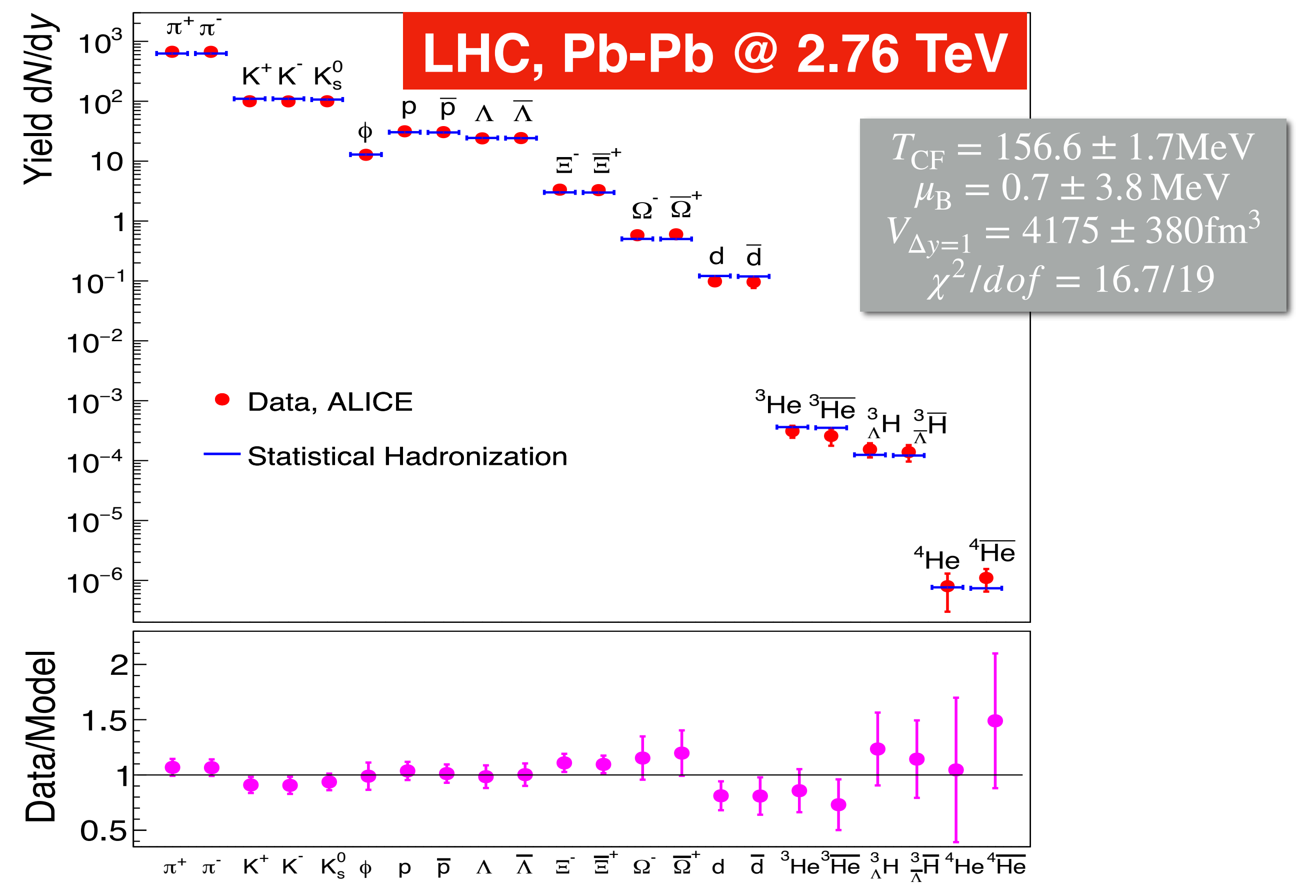
$$Z(V,T,\mu_B,\mu_Q,\mu_S) = \mathrm{Tr}\left[e^{-\beta\left(H-\sum_i \mu_i N_i\right)}\right]$$

$\beta = 1/T$, w/ T : System Temperature
H : Hamiltonian
μ_i: Chemical potentials
N_i: Conserved number operators

Particle density

$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}$$





HRG Model w/ parameters T, μ_B, V
W/ "feed-downs": E&M, Strong Decays: e.g., $\Delta \rightarrow p(n)+\pi$, $\rho \rightarrow \pi+\pi$, etc…
Fit to ratios: Volume V cancels out

Thermal HG models predict observed abundances with spectacular precision!

Can we explicitly explore chemistry/contributions from particlization vs. feed-downs?

Thermalized Systems!?!

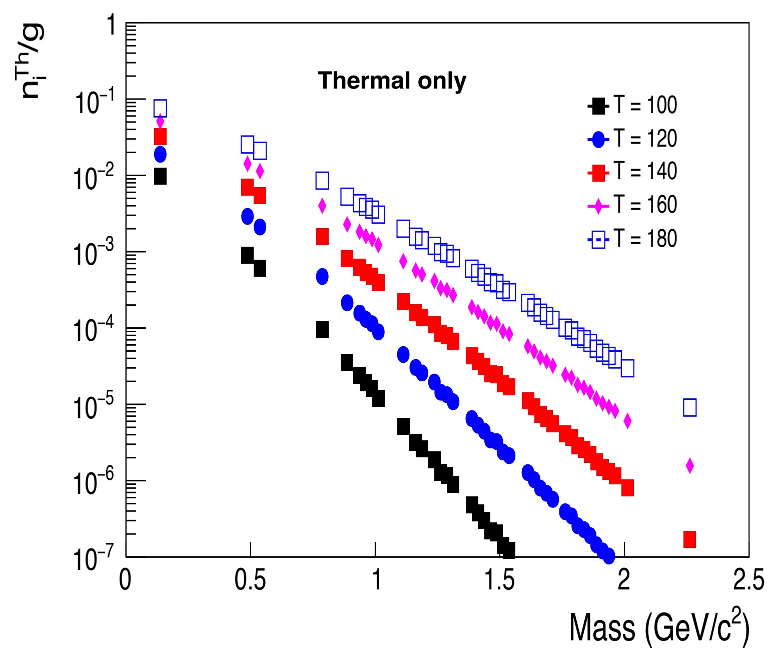
CP et al.,In preparation

Hadron Resonance Gas Model(s) & BFs…

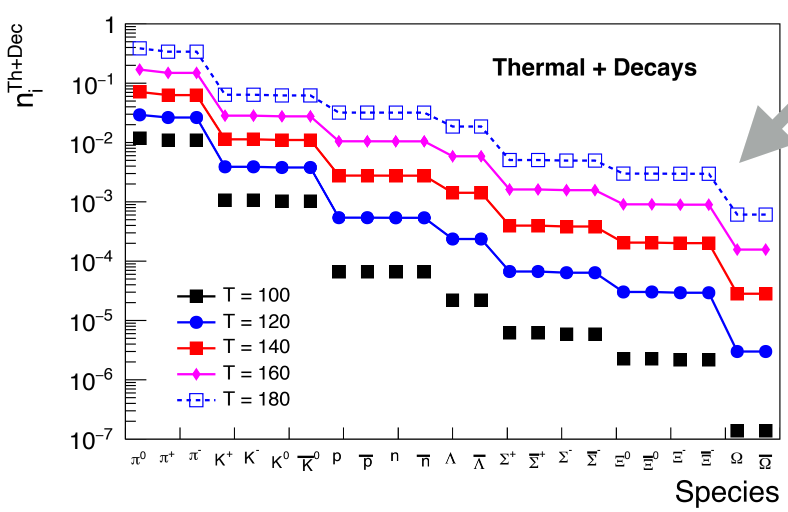
Hadrons feed down into pairs: measure their BFs

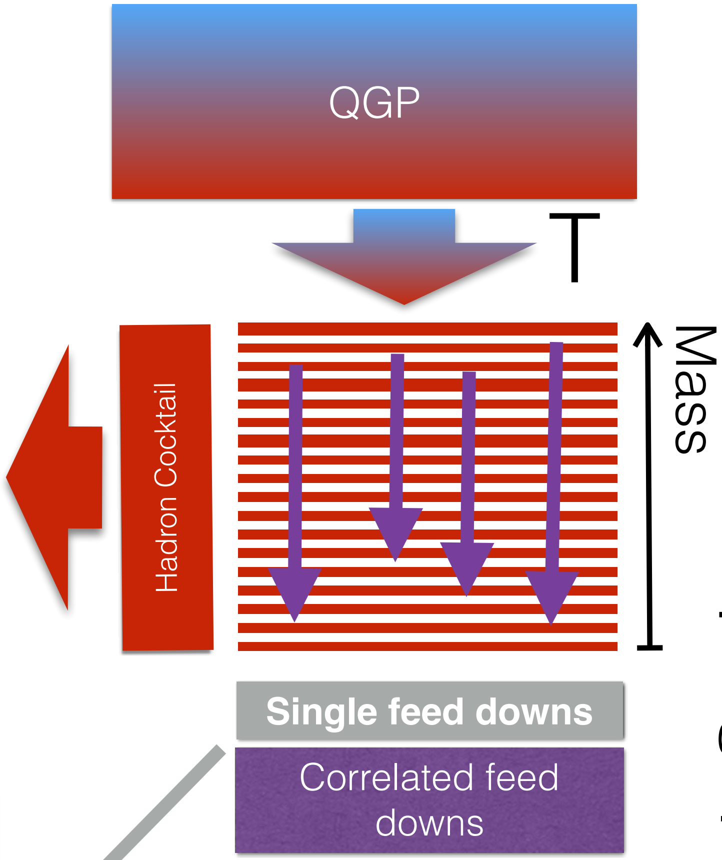
Thanks to Chun Shen

Single Spectra: Thermal Only



Single Spectra: Thermal + Feed Down





Examples:

$$\rho^0 \to \pi^+ + \pi^-$$

…

$$\Delta^0 \to p^+ + \pi^-$$

…

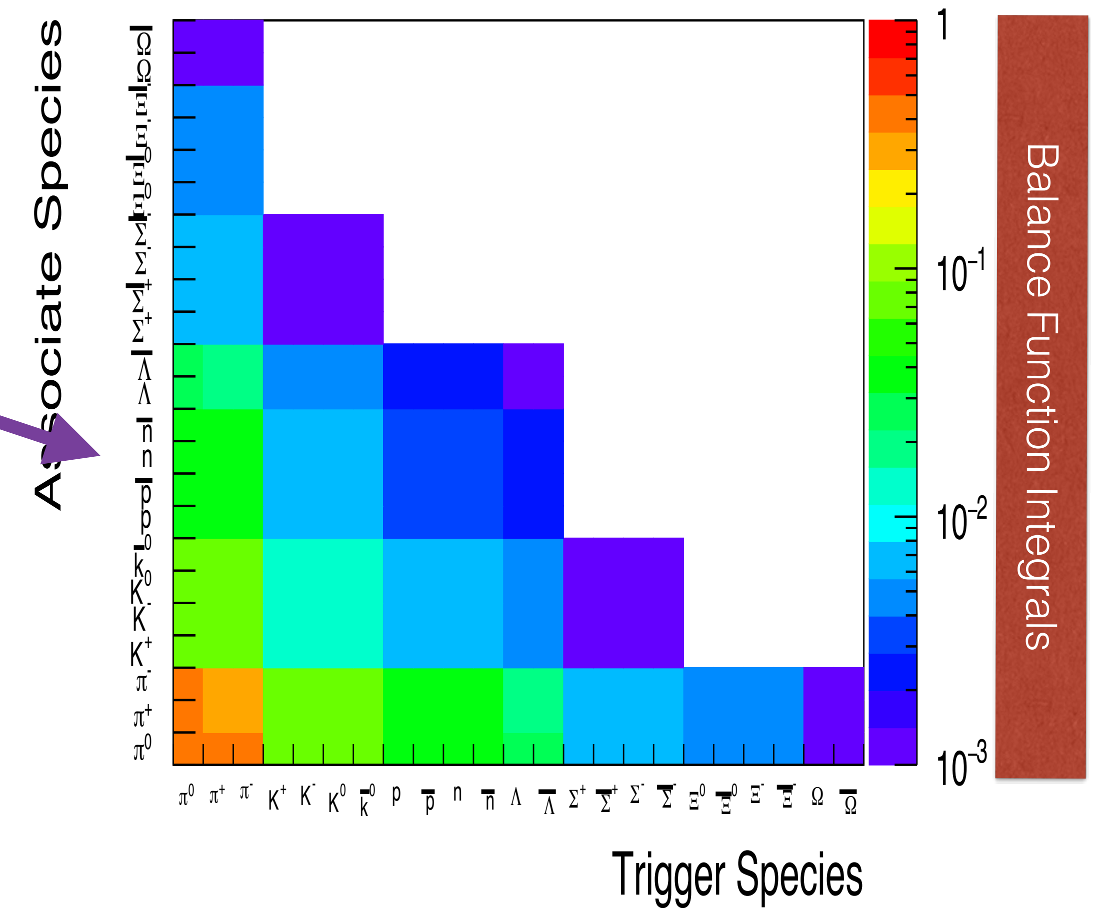
Integrals of BFs

$$I^{\alpha\beta}(\Omega) = \int_\Omega d\vec{p}_1 d\vec{p}_2 B^{\alpha\beta}(\vec{p}_1,\vec{p}_2)$$

function of centrality ???

Fractional balance function integrals:

$$f^{\alpha\beta}(\Omega) = \frac{I^{\alpha\beta}(\Omega)}{\sum_\beta I^{\alpha\beta}(\Omega)}$$



Unified Balance Functions

Conditional Density

$$\rho_2^{\alpha|\beta}(y_1\,|\,y_2) = \frac{\rho_2^{\alpha\beta}(y_1, y_2)}{\rho_1^{\beta}(y_2)}$$

Pratt’s Balance Functions

$$B^{\alpha|\bar{\beta}}(y_1\,|\,y_2) = \rho_2^{\alpha|\bar{\beta}}(y_1\,|\,y_2) - \rho_2^{\bar{\alpha}|\bar{\beta}}(y_1\,|\,y_2)$$

Unified Balance Functions

$$B^{+-}(y_1, y_2\,|\,y_0) = \frac{1}{\langle N_1^- \rangle}\left[\rho_2^{+-}(y_1, y_2) - \rho_2^{--}(y_1, y_2) - \rho_1^{+}(y_1)\rho_1^{-}(y_2) + \rho_1^{-}(y_1)\rho_1^{-}(y_2)\right]$$

$$B^{+-}(y_1, y_2\,|\,y_0) = \frac{1}{\langle N_1^- \rangle}\left[C_2^{+-}(y_1, y_2) - C_2^{--}(y_1, y_2)\right]$$

2-Cumulants

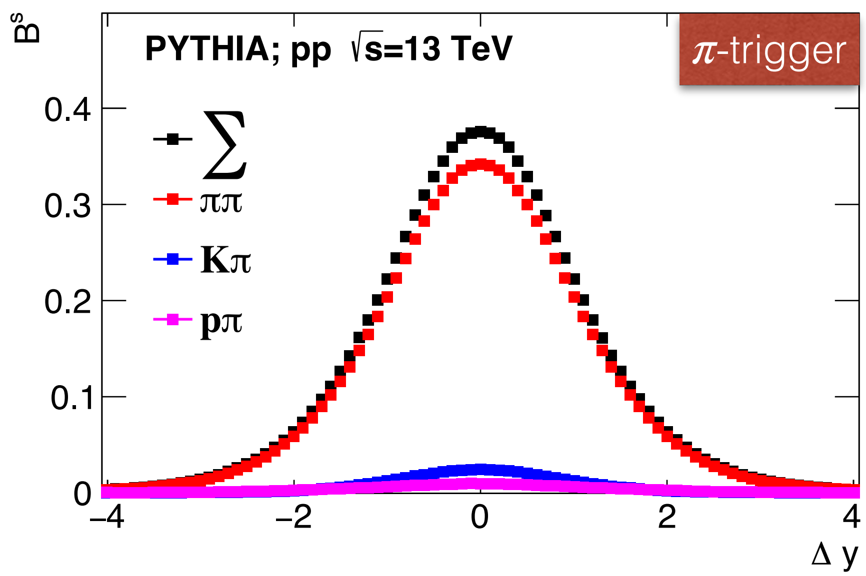
- *Accounting for non vanishing net-charge with unified balance functions*, C.P. et al, PRC 107 (2023) 1, 014902,
- *Effects of non vanishing net charge on balance functions and their integrals*, C.P. et al, PRC 107 (2023) 5, 054915
- General balance functions of identified charged hadron pairs of π, K, $p\bar{p}$ in PbPb Collisions…, C.P. et al, PLB 833 (2022) 137338

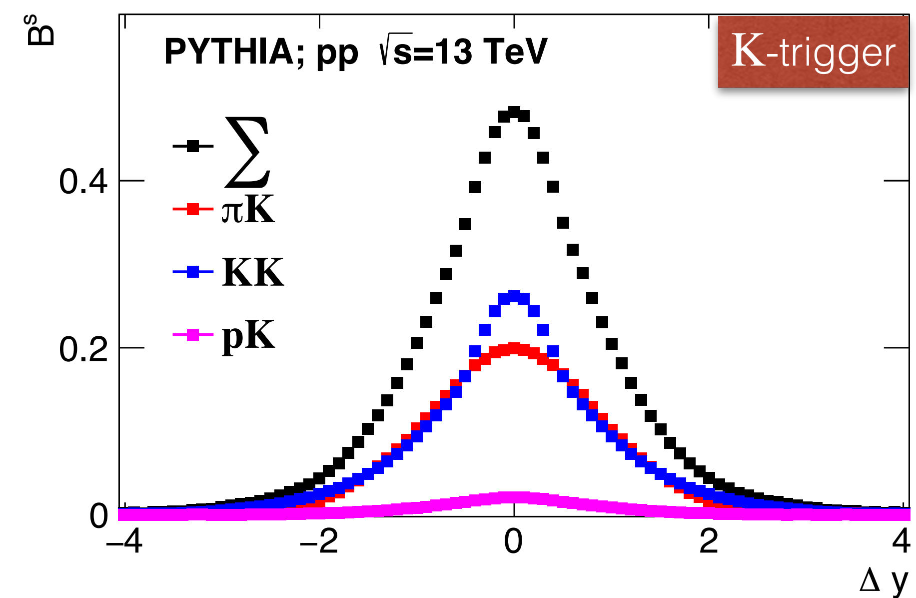
CP et al., e-Print: 2403.13007 [hep-ph]

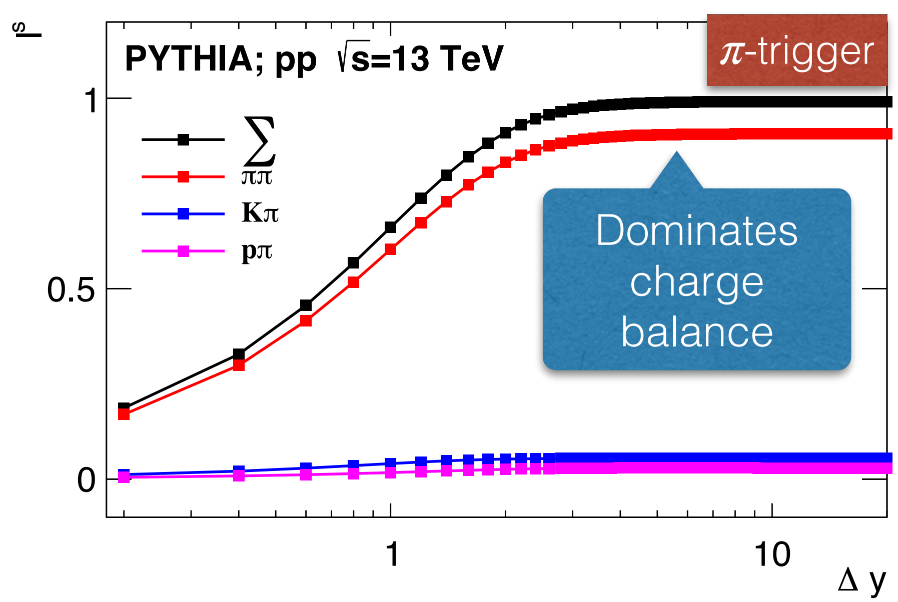
Charge BF w/ Mixed Species: $\pi^{\pm}$, $\mathrm{K}^{\pm}$, $\mathrm{p}(\bar{\mathrm{p}})$

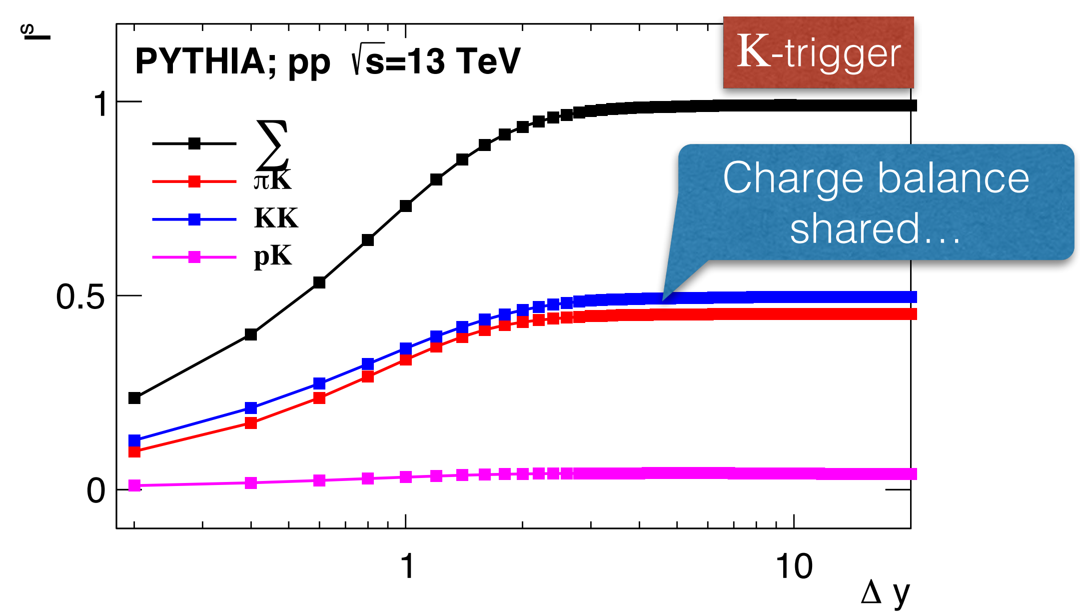
$$B^{\alpha\bar{\beta},s}(\Delta y) = \frac{1}{\langle N_1^{\bar{\beta}}\rangle}\left[C_2^{\alpha\bar{\beta}}(\Delta y) - C_2^{\bar{\alpha}\bar{\beta}}(\Delta y)\right] + \frac{1}{\langle N_1^{\beta}\rangle}\left[C_2^{\bar{\alpha}\beta}(\Delta y) - C_2^{\alpha\beta}(\Delta y)\right]$$

Simulations
PYTHIA8
pp @ $\sqrt{s} = 13$ TeV
$\pi^{\pm}$, $\mathrm{K}^{\pm}$, $\mathrm{p}(\bar{\mathrm{p}})$
pT>0, |y|<10









Charge balancing determined by particle production dynamics

CP et al., e-Print: 2403.13007 [hep-ph]

Baryon Balance Functions

Simulations w/ PYTHIA8

pp @ $\sqrt{s} = 2.76, 5.02, 13$ TeV

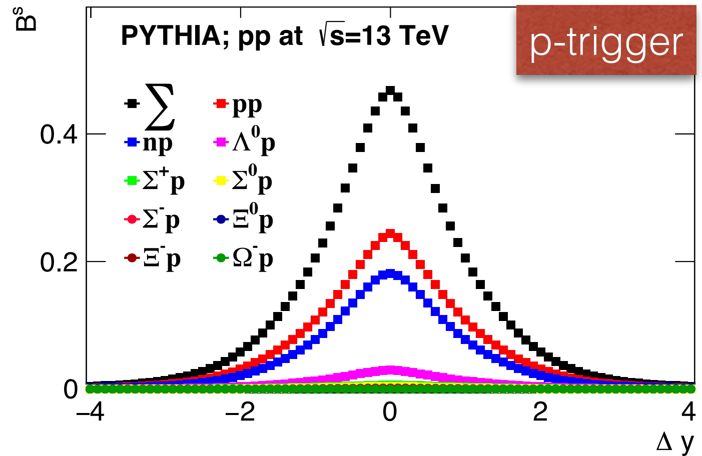
Species	$c\tau$ (m)	Observation Method
p	long lived	spectrometer
n	$\tau = 877.8$ s	hadronic calorimeter
Λ^0	0.079	$\Lambda^0 \to p + \pi^-$
Σ^-	0.045	$\Sigma^- \to n + \pi^-$
Σ^0	0.022 nm	$\Sigma^0 \to \Lambda^0 + \gamma$
Σ^+	0.024	$\Sigma^+ \to p + \pi^0$
Ξ^-	0.049	$\Xi^- \to \Lambda^0 + \pi^-$
Ξ^0	0.087	$\Xi^0 \to \Lambda^0 + \pi^0$
Ω^-	0.024	$\Omega^- \to \Lambda^0 + K^-$

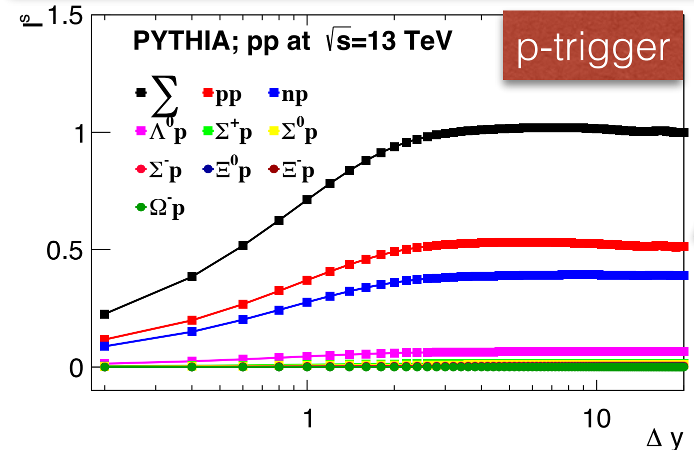
Charge balancing determined by particle production dynamics:

BF & Integral have great potential to constrain models…

In pp & AA collisions

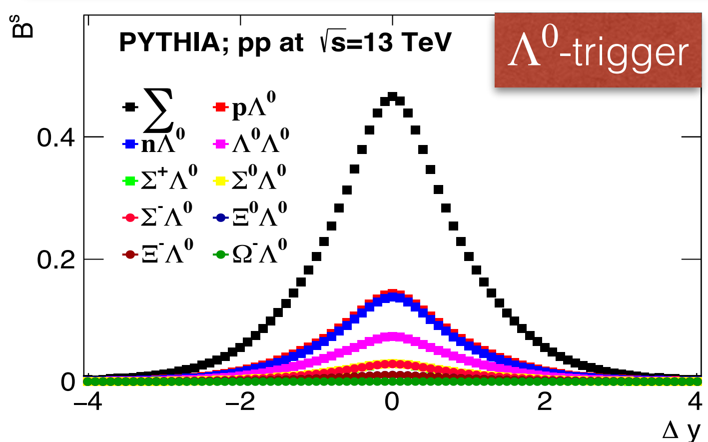
Cumulative Integrals

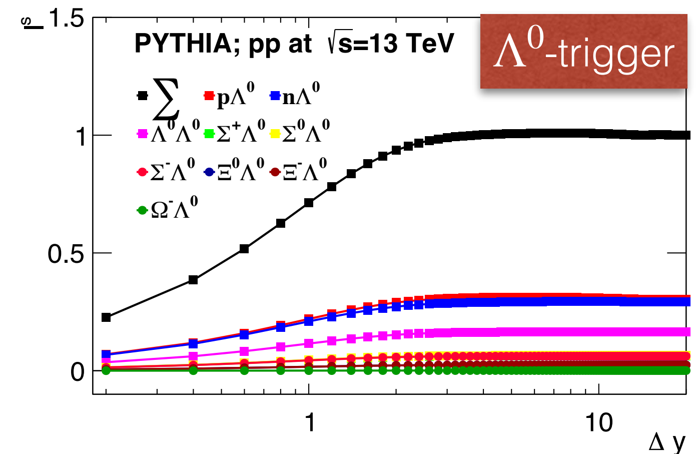




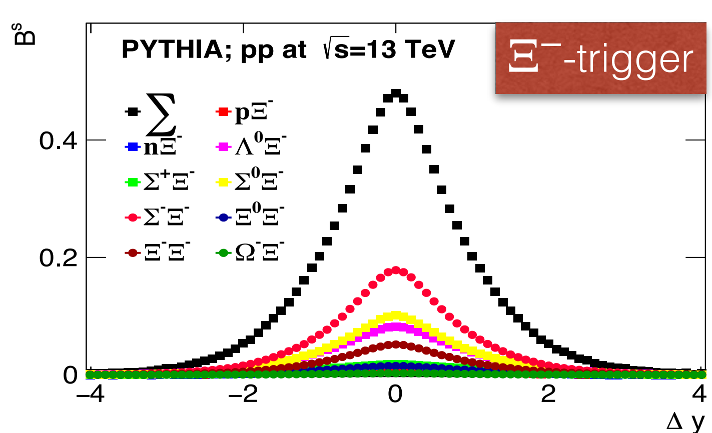
Dominates baryon balance

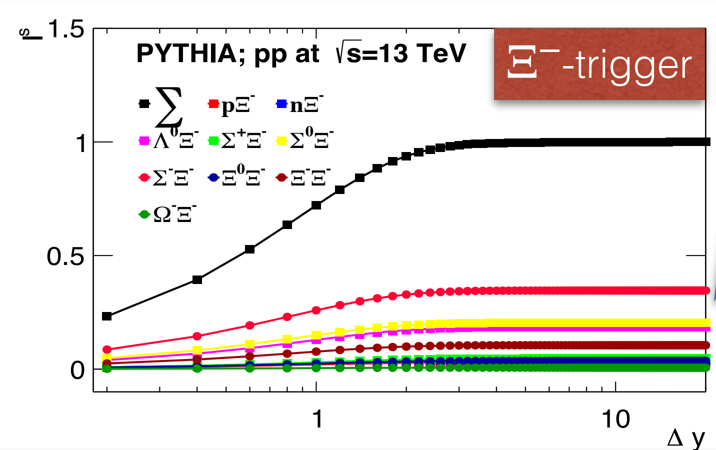
$\bar{\mathrm{p}}$, $\bar{\mathrm{n}}$ balance p





$\bar{\mathrm{p}}$, $\bar{\mathrm{n}}$, $\bar{\Lambda}$ balance Λ





Shared baryon balance

$\bar{\Sigma}^+$, $\bar{\Sigma}^0$, $\bar{\Lambda}^0$ balance Ξ^-

C.P., Phys.Rev.C 100 (2019) 3, 034905

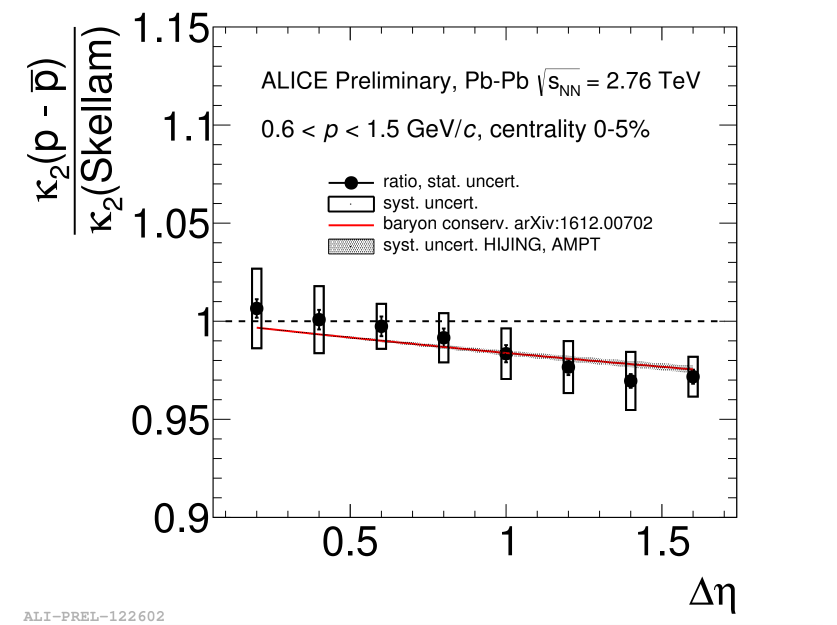
Baryon Number Conservation

Second order cumulant : $\kappa_2(\Delta N_p) = F_1^{\bar{p}} + F_1^{\bar{p}} + F_2^{p,p} + F_2^{\bar{p},\bar{p}} - 2F_2^{p,\bar{p}}$

Poisson limit (Skellam) : $\kappa_2^{Skellam}(\Delta N_p) = F_1^p + F_1^{\bar{p}} = \langle N_p \rangle + \langle N_{\bar{p}} \rangle$

Ratio of $\kappa_2(\Delta N_p)$ to Skellam :

$$r_{\Delta N_p} \equiv \frac{\kappa_2(\Delta N_p)}{\kappa_2^{\mathrm{Skellam}}(\Delta N_p)} = 1 + \frac{F_2^{p,p} + F_2^{\bar{p},\bar{p}} - 2F_2^{p,\bar{p}}}{F_1^p + F_1^{\bar{p}}}.$$

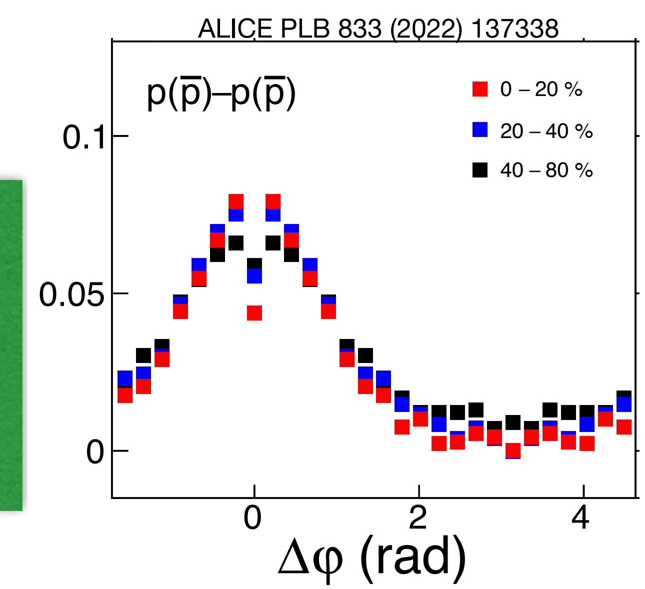


LHC: $\langle N_p \rangle \approx \langle N_{\bar{p}} \rangle$: $r_{\Delta N_p} = 1 + \frac{F_1^p}{2}\left[R_2^{p,p} + R_2^{\bar{p},\bar{p}} - 2R_2^{p,\bar{p}}\right],$

Proportional to Integral of Balance Function

Consequently : $r_{\Delta N_p} = 1 + \frac{1}{4}\frac{dN_T}{d\eta}\Delta\eta\nu_{\mathrm{dyn}}^{p,\bar{p}},$

(Local) Baryon Number conservation



Strong correlations exist: non Poisson behavior obtained from ν_{dyn} vs. $\Delta\eta$…

CP et al., 2310.07618 [hep-ex]

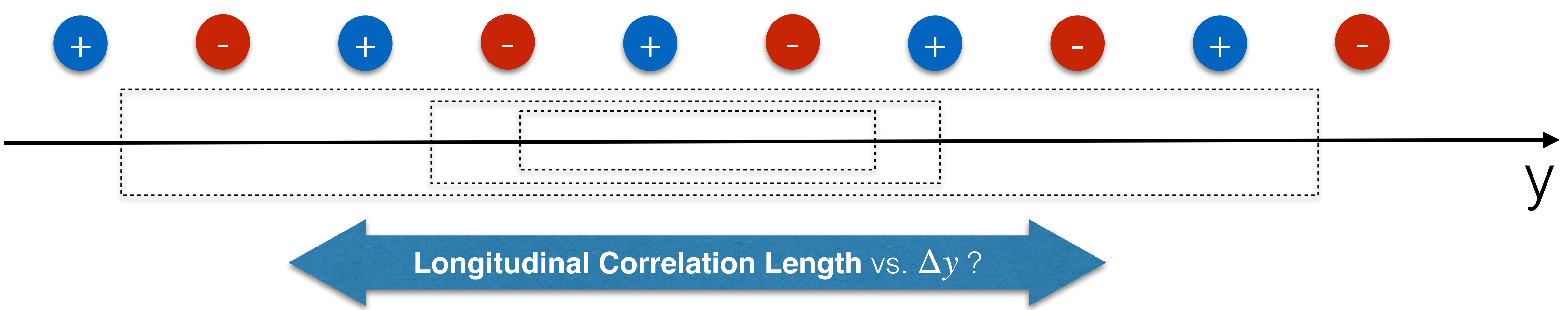
Suppressing Decays and Jets

Use n-cumulants w/ n>2 to suppress two prong decays

Use n-BFs w/ n>2 to suppress collective anisotropy

Use Rapidity Gap and Same Side to suppress jets

Example: string or quark production at different times and effective temperature? What is the correlation length?



CP et al., Phys.Rev.C 109 (2024) 4, 044904

Multi-particle Balance Functions

$$B_2^{+-}(\vec{p}_1,\vec{p}_2) = \frac{C_2^{+-}(\vec{p}_1,\vec{p}_2) - C_2^{--}(\vec{p}_1,\vec{p}_2)}{\langle N^- \rangle}$$

$$B_4^{+-}(\vec{p}_1,\ldots,\vec{p}_4) = \frac{1}{6} \times \frac{3C_4^{2+2-} - 4C_4^{1+3-} - C_4^{4-}}{\langle N^- (N^- - 1) \rangle}$$

$$B_6^{+-}(\vec{p}_1,\ldots,\vec{p}_6) = \frac{1}{60} \times \frac{10C_6^{3+3-} - 15C_4^{2+4-} + 6C_4^{1+5-} + C_4^{6-}}{\langle N^- (N^- - 1)(N^- - 2) \rangle}$$

$$B_8^{+-}(\vec{p}_1,\ldots,\vec{p}_8) = \frac{1}{840} \times \frac{35C_8^{4+4-} - 56C_8^{3+5-} + 25C_8^{2+6-} - 8C_8^{1+7-} + C_8^{8-}}{\langle N^- (N^- - 1)(N^- - 2)(N^- - 3)(N^- - 3) \rangle}$$

$$B_{10}^{+-}(\vec{p}_1,\ldots,\vec{p}_{10}) = \frac{1}{15120} \times \frac{126C_{10}^{5+5-} - 210C_{10}^{4+6-} + 120C_{10}^{3+7-} - 45C_{10}^{2+8-} + 10C_{10}^{1+9-} - C_{10}^{10-}}{\langle N^- (N^- - 1)(N^- - 2)(N^- - 3)(N^- - 3)(N^- - 4) \rangle}$$

Connection to QGP Susceptibilities??

CP et al., Phys.Rev.C 109 (2024) 4, 044904

n-Particle BFs vs. Net Charge Cumulants

RHIC BES: **Search for critical point**…
LHC/ALICE: **Study of QGP Susceptibilities**

$$\kappa_2^Q = F_1^+ + F_1^- + \underbrace{F_2^{++} - 2F_2^{+-} + F_2^{--}}_{B_2^{+-}},$$

$$\kappa_4^Q = F_1^+ + F_1^- + \cdots + \underbrace{F_4^{4+} - 4F_4^{3+1-} + 6F_4^{2+2-} - 4F_4^{1+3-} + F_4^{4-}}_{B_4^{+-}}$$

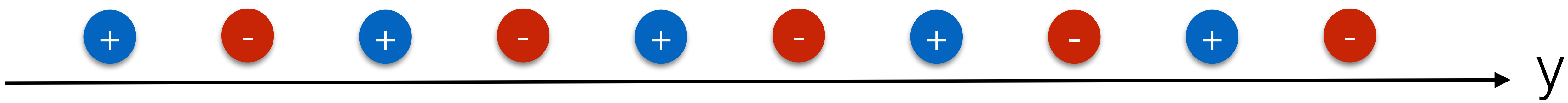
$$\kappa_6^Q = F_1^+ + F_1^- + \cdots + \underbrace{F_6^{6+} - 6F_6^{5+1-} + 15F_6^{4+2-} - 20F_6^{3+3-} + 15F_6^{2+4-} - 6F_6^{1+5-} + F_6^{6-}}_{B_6^{+-}}$$

Order "n" Net Charge Cumulants determined by order "n" balance functions!
What is the role of collision dynamics?
What is the role of susceptibilities?

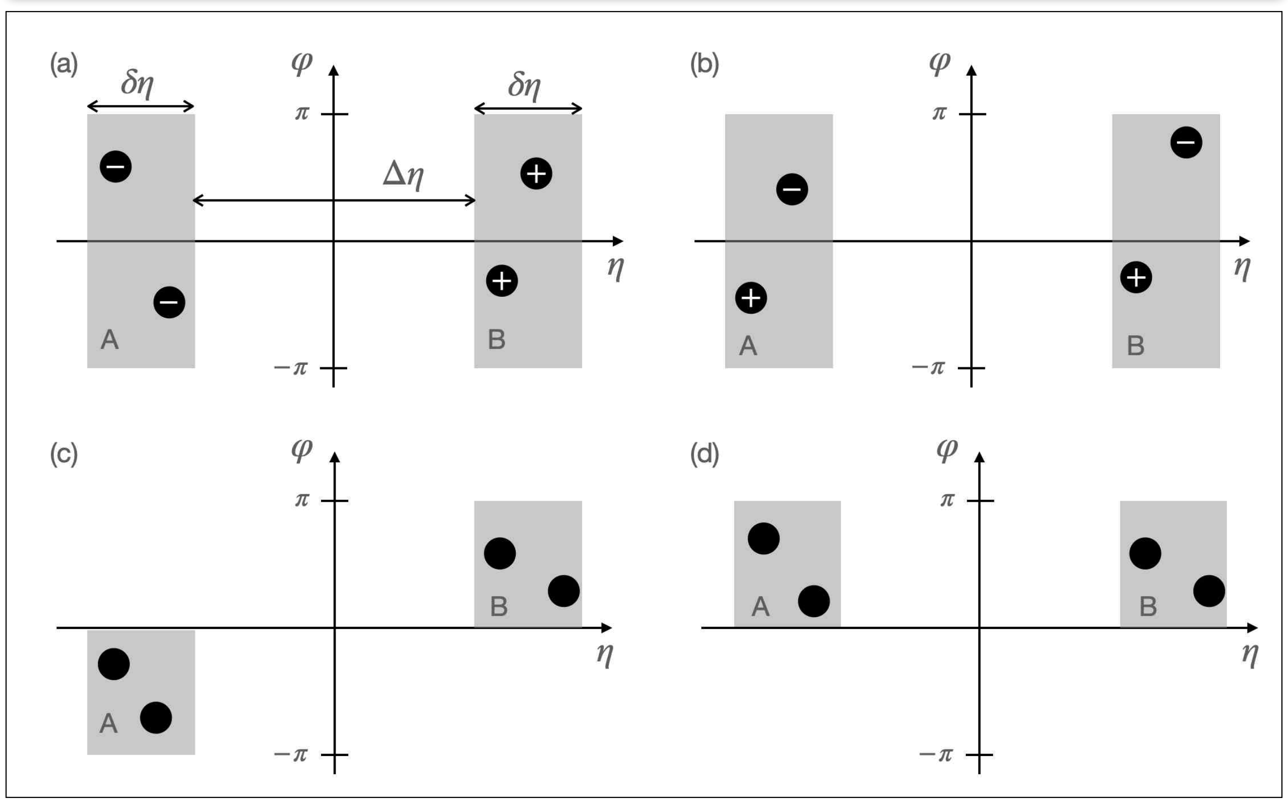
CP et al., Phys.Rev.C 109 (2024) 4, 044904

"Charge" Longitudinal Correlation Length

One string or quark production at different times and effective temperature?



Use cases/acceptance



- Measure B_2, B_4, … vs. the width of the rapidity acceptance.
- As the size of the acceptance increases, so should fraction of B_n relative to unity.
- But B_n with different values of n will have different magnitude and convergence rates towards unity.
- Convergence rate tells us about the **correlation length**.

- Caveats:
  - Statistics Hungry!!!
  - Do we currently have a meaningful acceptance for this measurements?
    - **ALICE 3 will!**
  - What will be the jets' contribution? Can be suppressed with eta-phi gaps…

Summary

- Some Prior Works on Delayed Hadronization
- Multiple new ideas for BFs measurements
  - Charge/Strangeness/Baryon 2-Balance Functions
    - Better understanding of particle production dynamics
    - Better constraints of production models (MC models)
    - “Calibration” of 2-cumulants
    - Identified particle 2-balance functions
  - Connection between Balance Functions and Net Charge Cumulants.
  - Charge/Strangeness/Baryon n-Balance Functions, with n>2
    - Evolution of longitudinal correlation vs. system size, collision centrality, etc.
  - Not discussed but important: charm/beauty BFs

Identified BFs w/ PYTHIA8

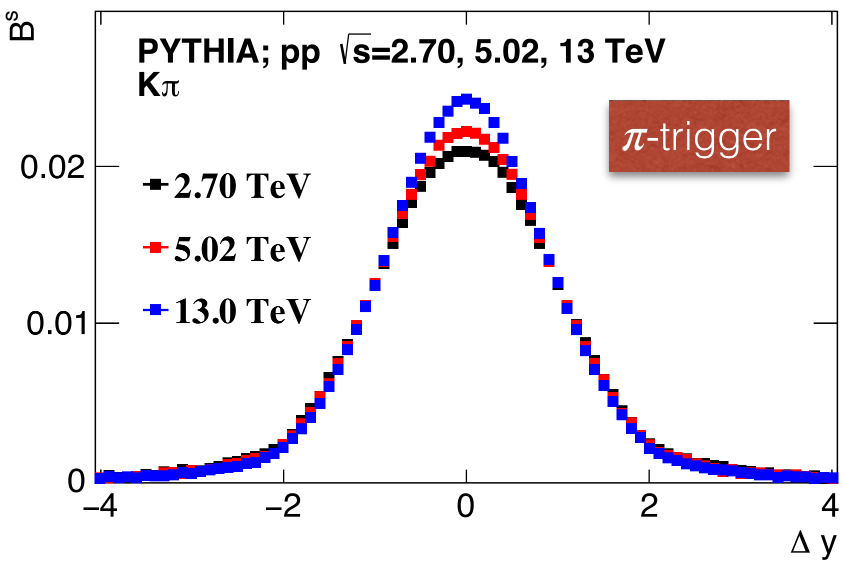
CP et al., e-Print: 2403.13007 [hep-ph]

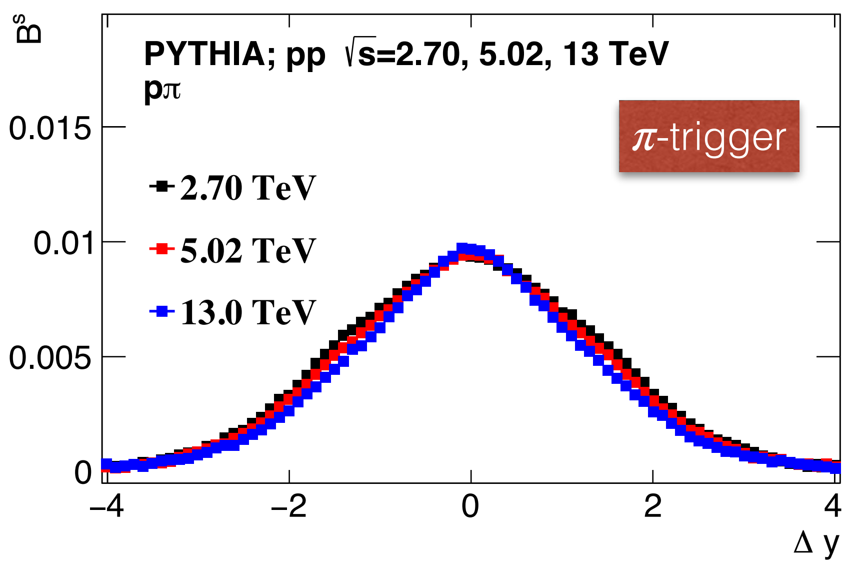
Charge BF w/ Mixed Species: $\pi^{\pm}$, $\mathrm{K}^{\pm}$, $\mathrm{p}(\bar{\mathrm{p}})$

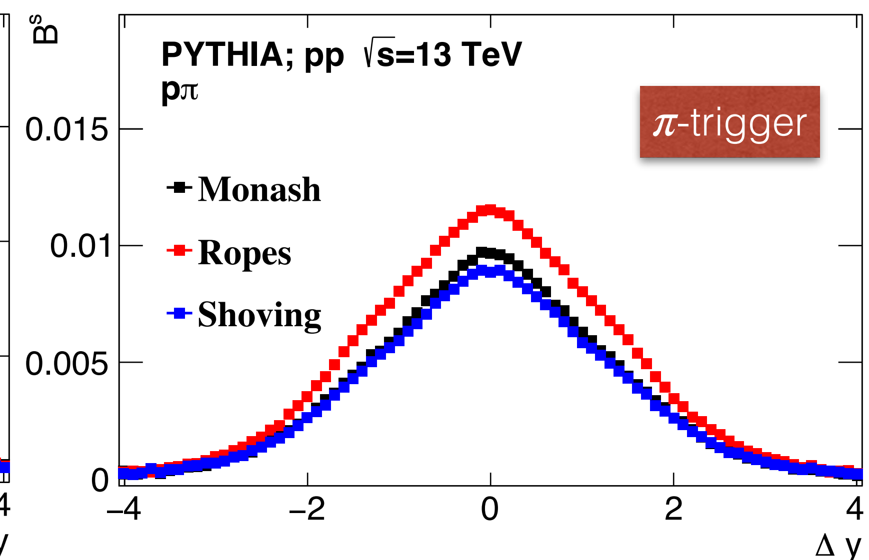
Simulations w/ PYTHIA8

pp @ $\sqrt{s} = 2.76, 5.02, 13$ TeV

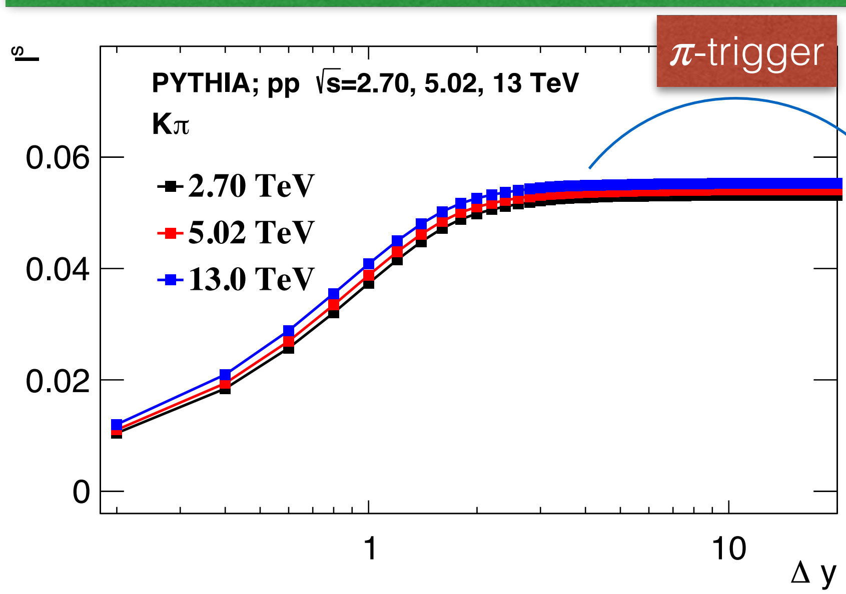
$\pi^{\pm}$, $\mathrm{K}^{\pm}$, $\mathrm{p}(\bar{\mathrm{p}})$ pT>0, |y|<10

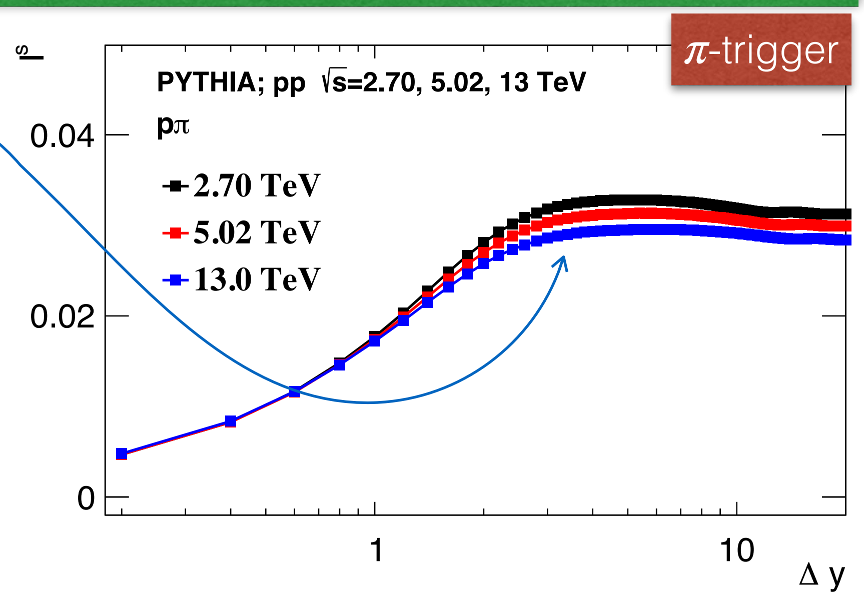






Cumulative Integrals





Charge balancing determined by particle production mechanisms:

- $\sqrt{s}$ dependence
- Model dependence
- BF have great potential to further constrain models in pp & AA collisions