Axions: A Survey From Neutrinos to Cosmology

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Abstract

The existence of neutral pseudo-scalar bosons, that is the axions, has been proposed long ago by Peccei and Quinn to explain the suppression of the electric dipole moment of the neutron. The associated U(1) symmetry breaks at very high energy, and it guarantees that the interaction of other particles with axions is very weak. We shall review the axion properties in connection with apparently very different contexts, like neutrino physics, dark matter and cosmology. We shall explore the case of neutrinos by allowing interactions with axions as a mass mechanism, then proceed to discuss our results for neutrino-dark matter interactions and finally discuss some cosmological scenarios related to axions

Contents of the talk

- The theta angle problem in QCD
- The axion:Peccei-Qinn, Weinberg, Wilczek
- Axion-pion couplings: the mass and lifetime of the axion
- Dark matter and the BEC mechanism
- Neutrino-axion couplings and the neutrino mass
- Conclusions

The theta angle problem in QCD

To the Lagrangian

$$L = -(1/2) g_{\alpha\beta} F^{\alpha}_{\mu\nu} F^{\beta,\mu\nu}$$

where $F^{\alpha}_{\mu\nu}$ is the gauge field tensor, α, β are structure constant indexes,and $g_{\alpha\beta}$ is a constant matrix, if the CP and T invariances are not assumed, one may add the term

$$L' = -(1/2) \Theta_{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} F^{\alpha}_{\mu\nu} F^{\beta}_{\rho\sigma}$$

where $\Theta_{\alpha\beta}$ is another constant matrix. It induces a neutron electric dipole moment $d_n \approx abs(\Theta) e \frac{m_\pi^2}{m_N^2}$. Then if $d_n \leq 10^{-25} ecm$ it implies $\Theta \leq 10^{-9} \rightarrow 10^{-11}$

The axion:Peccei-Quinn

R. Peccei and H. Quinn (1977) proposed the introduction of a pseudo scalar field $a(x,t)$ such that

$$\Theta \rightarrow \Theta + \frac{a(x,t)}{f}$$

(f being a strength constant) such that the non-vanishing vacuum expectation value of $a(x,t)$ causes $\Theta \rightarrow 0$.

This assumption was later extended, separately, by S. Weinberg and F. Wilczeck (1978), by the Lagrangian

$$\begin{aligned} L &= -(1/2)\partial_\mu\phi\partial^\mu\phi \\ &+ \frac{1}{64\pi^2}(\Theta + \frac{\phi}{f})\epsilon^{\mu\nu\rho\sigma}F^\alpha_{\mu\nu}F^\beta_{\rho\sigma} \\ &- i\frac{f_\mu}{f}\partial_\mu\phi\bar{u}\gamma_5\gamma^\mu u \\ &- i\frac{f_d}{f}\partial_\mu\phi\bar{d}\gamma_5\gamma^\mu d \end{aligned}$$

where $\phi(x,t)$ is a scalar boson field

The axion:Weinberg, Wilczek

The departure respect to the Peccei-Quinn axion is just the transformation of this Lagrangian to a pion-axion basis just that the axion mass will naturally arise from the triangular vertices axion-two pions mediated by quarks. It looks like

$$\begin{aligned} L_{\pi-\phi} &= -(1/2)\partial_\mu \pi^0 \partial^\mu \pi^0 \\ &- (1/2)\partial_\mu \phi \partial^\mu \phi \\ &- (1/2)\rho^T M_0^2 \rho \end{aligned}$$

where M_0 is a 2x2 mass-matrix and $\rho^T = (\pi^0, \phi)$. The eigenvalues of M_0^2 are m_π^2 and m_ϕ^2. The axion mass becomes then

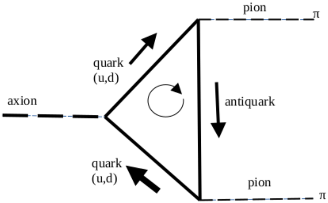
$$m_\phi^2 = \frac{\langle \bar{u} u \rangle \, m_u m_d}{f^2(m_u + m_d)}$$

or

$$m_\phi = \frac{m_\pi f_\pi \sqrt{m_d m_u}}{f(m_u + m_d)}$$

Axion-Quarks to $2\pi^0$:Mass Diagram

Diagram allowed by the effective $L_{\phi-\pi}$ Lagrangian. It gives mass to the axion.



Axion-pion couplings: the mass and lifetime of the axion

The actual value of m_ϕ becomes

$$m_\phi \leq 6 \times 10^{-6} eV \left[\frac{10^{12} GeV}{f} \right]$$

- Symmetry argument: The axion is the Goldstone boson of the U(1) symmetry introduced by Peccei and Quinn which breaks at $f \geq 4 \times 10^8$ GeV, and in the Weinberg-Wilczeck formulation it results from the condition $\left\langle \Theta + \frac{\phi}{f} \right\rangle = 0$

With values of the mass smaller than fractions of meV the lifetime is of the order of 10^{24} sec, which is more than enough to travel cosmological distances without decaying.

Something else about axions

- The scale factor f has lower limits which vary from 10^8 GeV(symmetry arguments) to 10^{11} GeV (typical Peccei-Quinn scale)
- Effective Lagrangian
  $L = -(1/2)\partial_\mu \phi \partial^\mu \phi - (1/2) m_\phi^2 \phi^2 + terms(\phi^4)$

Our proposal consists in:

- Adding to the effective Lagrangian an axion-neutrino term
- Treating explicitly the terms with (ϕ^4) to replace gravitational long range interactions in the axion BEC in dark matter

Axion-Dark Matter and the BEC mechanism

Motivations:Most of the matter in the Universe is "dark", its existence is manifest from astronomical evidences. Basically:

- It is non-baryonic
- It is collision-less
- Composition is unknown. Among the candidates: WIMPS, Sterile neutrinos, axions,

Among the experiments devoted to the detection of dark matter particle, ADMeX (Axion Dark Matter electron-X) aims at the detection of axions by the measurement of the process:

$$axion \rightarrow electron - positron\, loop \rightarrow 2\,\gamma\, rays.$$

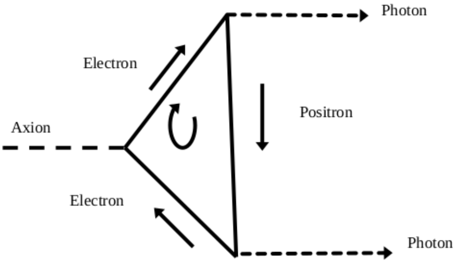
It is the equivalent of the process

$$axion \rightarrow quark - antiquark\, loop \rightarrow 2\,\pi$$

of the Weinberg-Wilczeck effective Lagrangian

Axion-electrons to 2γ:

Diagram allowed by the effective $L_{\phi-\gamma}$ Lagrangian. (Axion Dark Matter electron-X)



Axion-Bose Einstein Condensation (BEC): required conditions

The possibility that axions may be in a BEC phase was suggested years ago, among others by P. Sikivie and Q. Yang. It requires

- (i)large phase space density
- (ii)thermal equilibrium

While (*i*) has been demonstrated, the condition (*ii*) may be questioned if axions are weakly interacting. However, as advocated by Sikivie and Yang, the axion may reach thermal equilibrium by their gravitational interaction.

- Remark: We are suggesting another possibility, which is based on the treatment of pairing-like interaction between axions.

Axion-Bose Einstein Condensation: basic notions

The conventional statistical mechanics treatment, for massive bosons in the BEC regime, gives for the density ρ and critical temperature T_c the expressions

$$\rho = (\frac{2mc^2 kT}{8\pi\hbar^2 c^2})^{\frac{3}{2}} \sum_n \frac{\eta^{n+1}}{(n+1)^{\frac{3}{2}}}$$
$$kT_c = 2\pi \frac{\hbar^2 c^2}{mc^2} (\rho)^{\frac{2}{3}}$$

where $\eta = e^{\beta\mu} \to 1$ as $\mu \to 0^-$

- Remark: the density of axions should be determined independently, in order to calculate the critical temperature T_c

Axion-Bose Einstein Condensation: the critical temperature

There is a discrepancy about the abundance of axions. I shall quote here two of the estimates, namely:

$$\Omega \approx 0.2(\frac{f}{10^{12}\,GeV})^{\frac{8}{3}} \; (Yamaguchi)$$

$$\Omega \approx (\frac{f}{10^{11-12}\,GeV})^{\frac{7}{6}} \; (Guth\, et\, al)$$

We shall proceed by calculating the density of dark matter as $\rho = \alpha\rho_{visible}$ where α is the ratio between dark and visible matter (extracted from astronomical evidences)

Axion-Bose Einstein Condensation: the critical temperature

From the previously introduced equations we have obtained the following results:

$\rho_{visible}(grs/cm^3)$	fraction α	f (GeV)
9.9×10^{-30}	5.217	$10^{11}\rightarrow10^{14}$

mc^2 (eV)	critical BEC temperature (degrees K)
$10^{-5}\rightarrow10^{-3}$	$10^{-3}\rightarrow10^{-0}$

- T_c (axion BEC) $\approx 10.69\times(10^{-4}\rightarrow10^{-1})$ (degrees Kelvin)
- Remark:The axion component of dark matter may still have to evolve to the BEC phase because T_c is smaller but not much smaller than the temperature of the Universe which is about a couple of degrees Kelvin, contrary to values based on other assumptions, like pre-hadronic formation, etc.

Axion-Bose Einstein Condensation: the effective Lagrangian

As we have seen, the effective Lagrangian for massive axions is written

$$L = -(1/2)\partial_\mu \phi \partial^\mu \phi - (1/2) m^2 \phi^2 + terms(\phi^4)$$

Writing for the axion field $\phi = \sqrt{\frac{1}{2m}}(e^{-imt}\psi(x,t) + e^{imt}\psi^*(x,t))$ the Lagrangian becomes

$$\begin{aligned} L &= i(1/2)(\dot{\psi}(x,t)\psi^*(x,t) - \psi(x,t)\dot{\psi}^*(x,t)) \\ &- \frac{1}{2m}\nabla\psi(x,t)\nabla\psi^*(x,t) - \frac{g}{2m^2}(\psi(x,t)\psi^*(x,t))^2 \end{aligned}$$

- Remark: Notice the pairing-like structure of the last term of the effective Lagrangian. It may cause a coherent zero momentum state(like a BEC).

Neutrino-axion couplings and the neutrino mass

In addition to their role in cosmology, axions may play a role in neutrino physics, because the coupling of neutrinos with axions could provide a mechanism to explain for non-zero neutrino masses We start from the Lagrangian

$$\mathcal{L}_{int} = ig_{a\nu}\bar{\nu}\gamma^{\mu}\gamma^{5}\nu\partial_{\mu}\phi$$

which describes the derivative coupling between neutrinos (ψ) and axions (ϕ).
By separating spatial and temporal derivatives, the Lagrangian is split up in the following terms:

$$\mathcal{L}_{int} = ig_{a\nu}\nu^{\dagger}\vec{\sigma}\nu.\vec{\nabla}\phi + ig_{a\nu}\nu^{\dagger}\gamma^{5}\nu\partial_{0}\phi.$$

Neutrino-axion couplings:U(1) symmetry breaking

The breaking of the U(1) symmetry implicit in the potential

$$V(\phi) = -\frac{\mu^2}{2}(|\phi|^2 - \frac{1}{f^2}|\phi|^4).$$

leads to

$$\langle\phi\rangle_0 = 0 \ \ (\mathrm{unstable\,point}),$$

and

$$\langle\phi\rangle_0 = \frac{f}{\sqrt{2}}.$$

Thus the Lagrangian, written in natural units looks like:

$$L = g_a\langle\phi\rangle_0\psi^\dagger\psi + g_{a\nu}(\psi^\dagger\vec{\sigma}\psi).\vec{p}$$

In consequence

$$m_\nu = \frac{g_a f}{\sqrt{2}}$$

Neutrino-axion couplings: the amplitudes

To calculate the contributions to the neutrino mass coming from the spin-dependent term of the Lagrangian, we write, for the transition amplitude

$$\mathcal{A}_{i\to f} = \langle f|\mathrm{T}\left\{(-i)\int d^4x\hat{\mathcal{H}}_{int}(x)\right\}|i\rangle = -ig_{a\nu}\int d^4x\,\langle f|\vec{\nabla}\Phi.\vec{\mathbf{S}}|i\rangle,$$

where $\vec{\mathbf{S}}$ is acting on the neutrino sector.

Neutrino-axion couplings:spin dependent terms

For spin-up neutrino states we get:

$$\langle f|\vec{\nabla}\Phi.\vec{\mathbf{S}}|i\rangle = i\mathcal{N}_i\mathcal{N}_f\left[\left(1+\frac{(p'_z p_z - p'_- p_+)}{(E+m)(E'+m)}\right)\frac{\partial\Phi}{\partial z} + \frac{(p'_- p_z + p'_z p_+)}{(E+m)(E'+m)}\frac{\partial\Phi}{\partial x} + i\frac{(-p'_z p_+ + p'_- p_z)}{(E+m)(E'+m)}\frac{\partial\Phi}{\partial y}\right]$$

and for spin down states

$$\langle f|\vec{\nabla}\Phi.\vec{\mathbf{S}}|i\rangle = -i\mathcal{N}_i\mathcal{N}_f\left[\left(1+\frac{(p'_z p_z - p'_+ p_-)}{(E+m)(E'+m)}\right)\frac{\partial\Phi}{\partial z} + \frac{(p'_+ p_z + p'_z p_-)}{(E+m)(E'+m)}\frac{\partial\Phi}{\partial x} + i\frac{(p'_+ p_z - p'_z p_-)}{(E+m)(E'+m)}\frac{\partial\Phi}{\partial y}\right],$$

Neutrino-axion couplings: One loop corrections

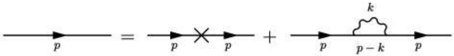


Figure: Zero-order and one-loop corrections to the neutrino propagator. The incoming neutrino with momentum p (solid line) is coupled to the axion field with momentum k (wavy line). The zero-order value is indicated by a cross on the solid line and its correction by the loop at the $p - k$ line .

The one-loop neutrino propagator is defined by the expression

$$\delta S(p) \;=\; S(p) + S(p)\left(\Sigma\left(p\right)\right)S(p)\;,$$

where

$$\Sigma(p) \;=\; \frac{g^2}{16\pi^2}\Gamma(\frac{\epsilon}{2})\int_0^1 \mathrm{d}x\,[(\epsilon-2)p(1-x)+(4-\epsilon)m] \times\left[\frac{(m^2-m_a^2)x+m_a^2-p^2x(1-x)}{4\pi\xi^2}\right]^{-\epsilon/2}\;.$$

p is the neutrino 4-momenta, m and m_a are the neutrino and the axion mass. We work in $d=4+\epsilon$ dimensions and included a parameter ξ which has dimension of mass.

After evaluating $\Sigma(p)$ on shell, that is by taking $p^2 = m^2$, and integrating in the variable x, we have finally obtained the 1-loop correction to the neutrino mass due to the interaction with axions.

$$\Sigma(p) \;=\; \frac{g^2}{8\pi^2}(p\Sigma_p + m\Sigma_m) \;,$$

where

$$\begin{aligned}\Sigma_p \;&=\; -\frac{1}{\epsilon} + \frac{\gamma}{2} - 1 + \frac{1}{2}\frac{m_a^2}{m^2} + \frac{1}{2}\ln(\frac{m^2}{4\pi\xi^2}) + \frac{1}{4}\frac{m_a^2}{m^2}\beta\ln(\frac{m^2}{m_a^2}) \\ &+\; \zeta\sqrt{\beta}\frac{m_a}{m}\left[\mathrm{Arctg}(\frac{m}{m_a}\sqrt{\beta}) + \mathrm{Arctg}(\frac{m_a}{m\sqrt{\beta}})\right]\end{aligned}$$

and

$$\begin{aligned}\Sigma_m \;&=\; \frac{4}{\epsilon} - 2\gamma + 3 - 2\ln(\frac{m^2}{4\pi\xi^2}) + \frac{m_a^2}{m^2}\ln(\frac{m^2}{m_a^2}) \\ &-\; 2\frac{m_a}{m}\sqrt{\beta}\left[\mathrm{Arctg}(\frac{m}{m_a}\frac{\zeta}{\sqrt{\beta}}) + \mathrm{Arctg}(\frac{m_a}{m\sqrt{\beta}})\right]\end{aligned}$$

with $\beta = \frac{4m^2 - m_a^2}{m^2}$, $\zeta = \frac{2m^2 - m_a^2}{m^2}$ and $\Gamma(\frac{\epsilon}{2}) = \frac{2}{\epsilon} - \gamma$

The one-loop neutrino propagator is then

$$\begin{aligned} \delta S &= \frac{1}{\not{p} - m - \Sigma(p)} \\ &= \frac{1}{\not{p} - m - \Sigma(p)\big|_{p^2=m^2}} \left(1 - \frac{\partial \Sigma(p)}{\partial \not{p}}\big|_{p^2=m^2}\right)^{-1} , \end{aligned}$$

therefore, the physical mass of the neutrino can be computed as

$$m_\nu = m + \Sigma(p)\big|_{p^2=m^2} .$$

To eliminate divergencies, we defined the mass

$$\tilde{m}_\nu = m\left[1 + \frac{g^2}{8\pi^2}\frac{3}{\epsilon}\right] ,$$

Neutrino-axion couplings: One loop corrections

The effective neutrino mass is finally written as

$$\frac{m_\nu}{\tilde{m}_\nu} - 1 = \frac{g^2}{8\pi^2}\Bigg[-\frac{3}{2}\gamma + 2 + \frac{1}{2}\frac{m^2}{m_\nu^2} - \frac{3}{2}\ln(\frac{m_\nu^2}{4\pi\xi^2}) + \frac{1}{4}\frac{m^4}{m_\nu^4}\ln(\frac{m_\nu^2}{m^2})$$
$$- 2\frac{m}{m_\nu}\sqrt{\beta}(\frac{m^2}{2m_\nu^2}\mathrm{Arctg}(\frac{m}{m_\nu}\sqrt{\beta}) + \mathrm{Arctg}(\frac{m_\nu}{m}\frac{\zeta}{\sqrt{\beta}})$$
$$- \frac{\zeta}{2}\mathrm{Arctg}(\frac{m_\nu}{m}\sqrt{\beta}))\Bigg] \; .$$

The derivation of the previous equations involved the ordering of higher order corrections to the propagator, as well as the strength of the coupling g for each mass scale m of the axion.

Neutrino-Axions coupling: some final words

- The breaking of the U(1) symmetry at the level of the Lagrangian which describes the interaction between the axion and the neutrino, at zeroth order, gives mass to the neutrino. That mass is dependent upon the coupling constant of the Lagrangian (g) and of the constant (f) which determines the mass of the axion
- The one loop corrections to the zeroth order mass are also dependent upon these constants but they are non-divergent.
- In order to complete the scheme one has to take into account the square-mass differences between the three light-mass eigenstates $\Delta m_{ij}{}^2$ (both in the normal and inverse ordering), and the amplitudes U_{ij} relating the mass and flavor states

We shall show some results in the next slides.

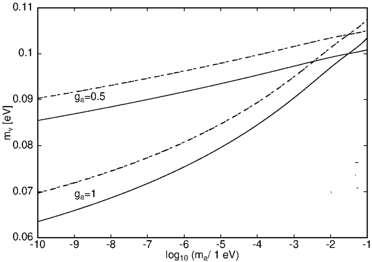


Figure: The effective neutrino mass m_ν, as a function of m_a and of the scaled coupling g_a. Solid line:$\tilde{m}_\nu$ fixed at the zeroth order neutrino mass, dashed line: $\tilde{m}_\nu = 1 meV$

Neutrino-axion couplings: the neutrino mass

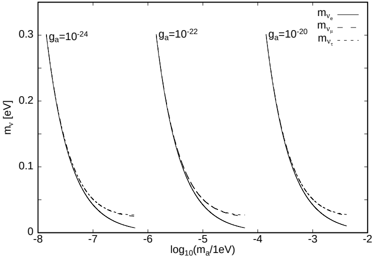


Figure: The effective neutrino mass m_ν, as a function of m_a and g_a. The curves show the results for the three neutrino flavors

Neutrino-axion couplings: the neutrino mass

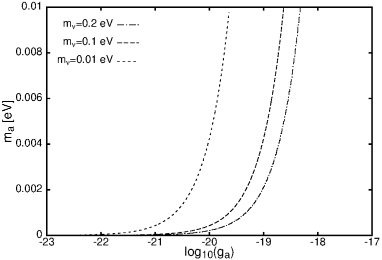


Figure: The effective neutrino mass m_ν, as a function of m and g. The curves show the domains determined by the one loop corrections

Neutrino-axion couplings: the neutrino mass

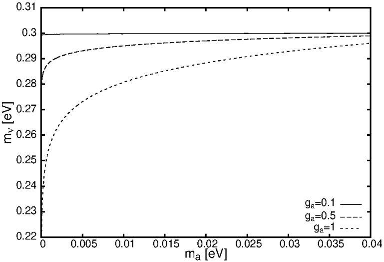


Figure: The effective neutrino mass m_ν, as a function of m_a and g_a. The curves show the domains determined by the one loop corrections as a function of the scaled coupling.

Figure: Allowed region for $<\lambda>$, $<\eta>$ and m_ν when all the experimental limits are taken into account in the analysis.

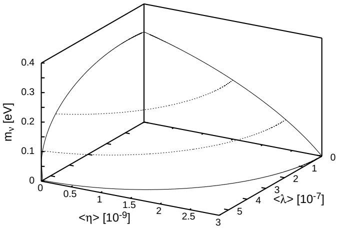
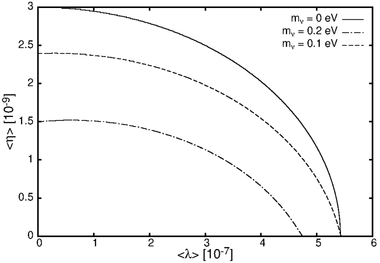


Figure: Neutrino mass m_ν as function of $< \lambda >$, and $< \eta >$.The curve denoted $m_\nu = 0$ corresponds to the mass independent terms of the half-life



Conclusions

- Axions may be a dominant component of non-baryonic dark matter of the Universe, as postulated in the literature. In addition to their role in solving the strong CP problem they exhibit interesting properties in connection with Cosmology and extensions of the Standard Model of Electroweak Interactions.

Our results suggest that:

- i).The gravitational thermalization, assumed to be the only mechanism needed to allow for BEC, may be replaced by pairing-like self-interactions
- ii)The critical temperature for axion-BEC may not be so small as previously thought
- iii)Neutrino-axion couplings may explain for non-zero values of the neutrino mass

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