Nuclear decay rates at elevated temperatures

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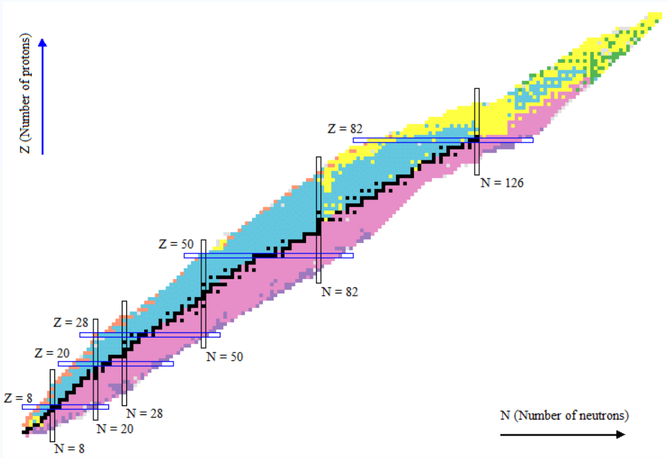

Collaborators:

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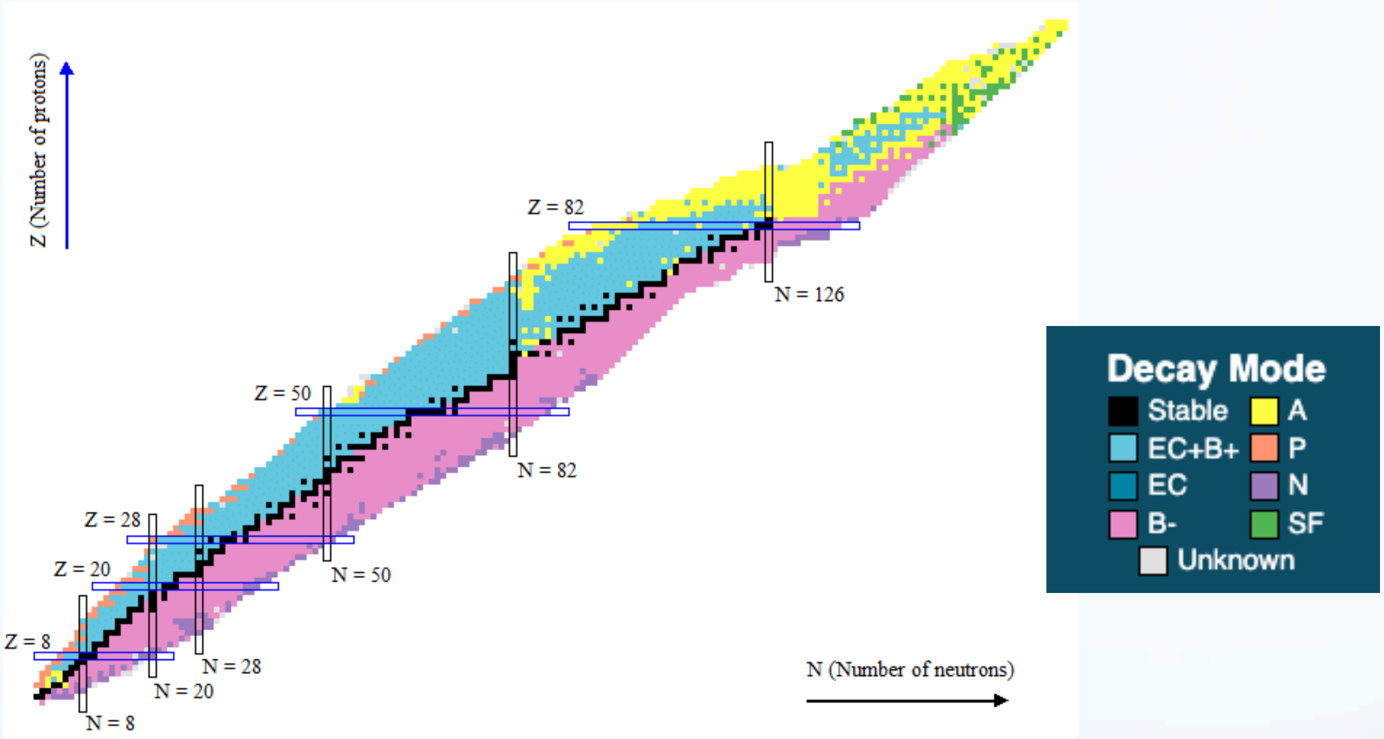
Olga Liliana Caballero (University of Guelph, Canada)

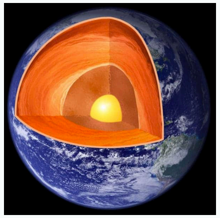
RADIOACTIVE DECAY



Taken from: www.nndc.bnl.gov

IMPORTANT AND UBIQUITOUS!

1 Radioactive decay accounts for half of Earth's heat



The current total heat flux from the Earth to space is 44.2 TW

- Precise measurements of the geoneutrino flux from the Kamioka Liquid Scintillator Antineutrino Detector, Japan, with existing measurements from the Borexino detector, Italy find that decay of uranium-238 and thorium-232 together contribute about 20 TW to Earth's heat flux.
- The neutrinos emitted from the decay of potassium-40 are known to contribute 4 TW.
- Taken together, observations indicate that heat from radioactive decay contributes to about half of Earth's total heat flux!

The KamLAND Collaboration, Nature Geosci 4, 647–651 (2011)

2 Radioactive dating

Knowing the decay rate of radioactive elements can help date ancient fossils and other artifacts

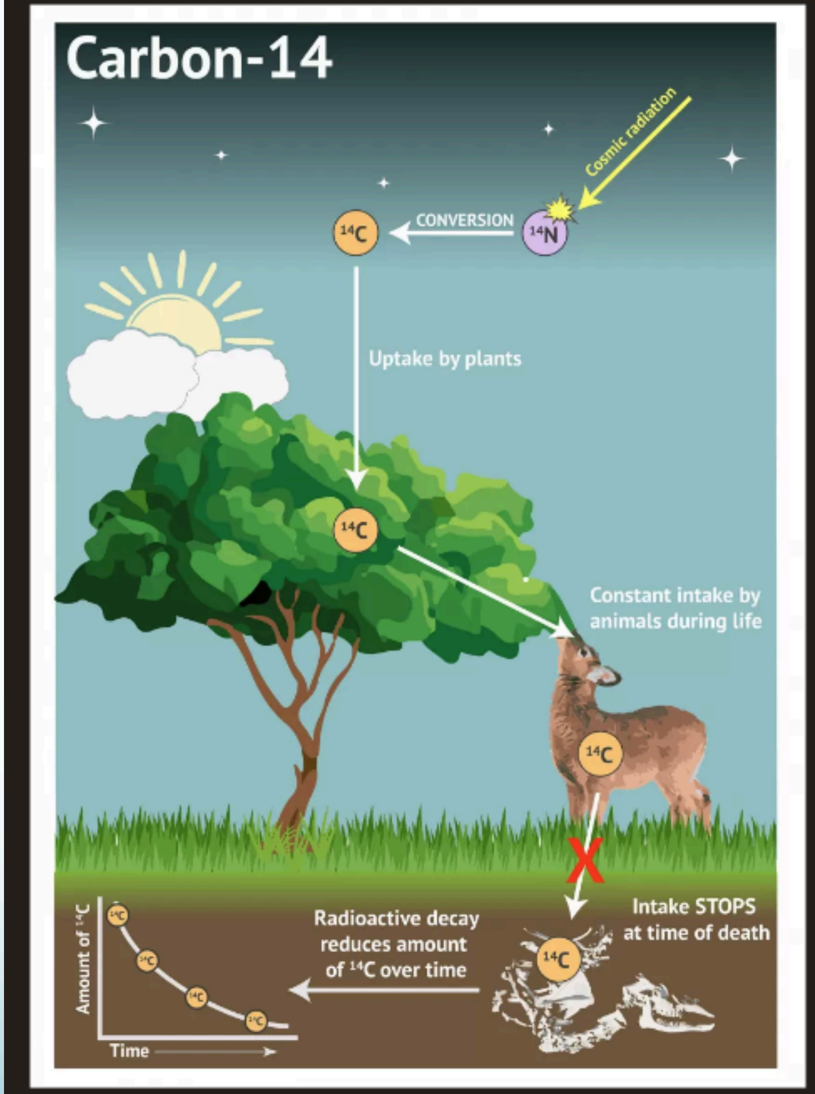
$n + {}^{14}N \rightarrow {}^{14}C + p$

${}^{14}C + O_2 \rightarrow {}^{14}CO + O$

${}^{14}C + OH \rightarrow {}^{14}CO_2 + p$

Ratio: 14C/12C = 1.25

${}^{14}C \rightarrow {}^{14}N + e + \text{antineutrino}$



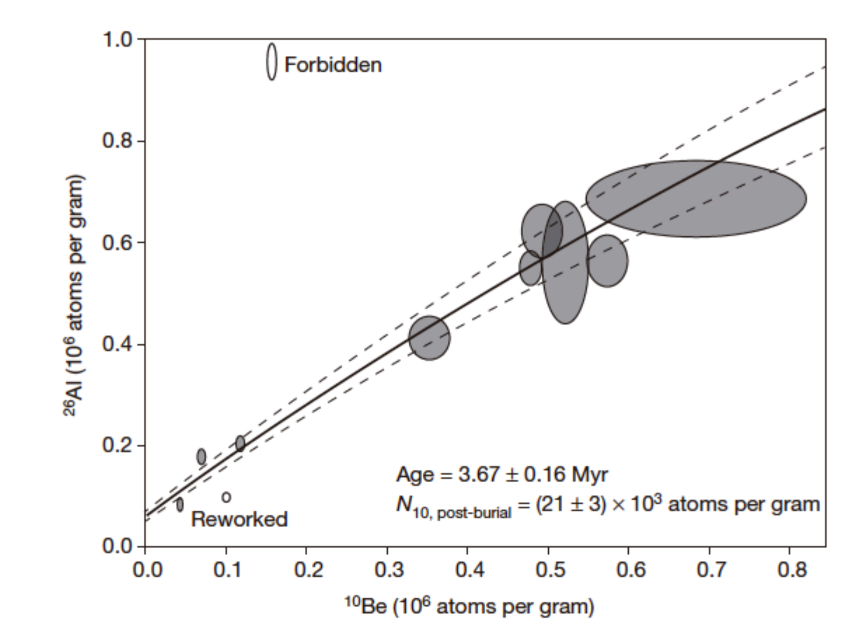
(Human Origins Program, NMNH, Smithsonian Institution)

The Little Foot skull (STW 573). (Photo courtesy of the University of the Witwatersrand)

Little Foot (dated by isochron burial dating at 3.67 million years old) is an older relative of Lucy, a famous Australopithecus skeleton dated by argon-argon dating at 3.18 million years old

Reconstruction of Lucy at the National Museum of Anthropology in Mexico

Burial dating is based on the radioactive decay of ^{26}Al and ^{10}Be in quartz. These nuclides build up by exposure to secondary cosmic radiation near the ground surface, and subsequently decay when sediment is buried and cosmogenic nuclide production is attenuated. Because ^{26}Al decays faster than ^{10}Be, the ratio $^{26}Al/^{10}Be$ decreases over time.



Granger, D., Gibbon, R., Kuman, K. et al. New cosmogenic burial ages for Sterkfontein Member 2 Australopithecus and Member 5 Oldowan.
Nature 522, 85–88 (2015).

Nuclear medicine uses radioactive materials and their emitted radiation from the body to diagnose and treat disease

- Nuclear medicine uses radiation to provide diagnostic information about the functioning of a person's specific organs, or to treat them. Diagnostic procedures using radioisotopes are now routine.
- Radiotherapy can be used to treat some medical conditions, especially cancer, using radiation to weaken or destroy particular targeted cells.
- Over 50 million nuclear medicine procedures are performed each year, and demand for radioisotopes is increasing.
- Sterilization of medical equipment is also an important use of radioisotopes.

Nuclear cosmochronology

closely linked to nucleosynthesis theories

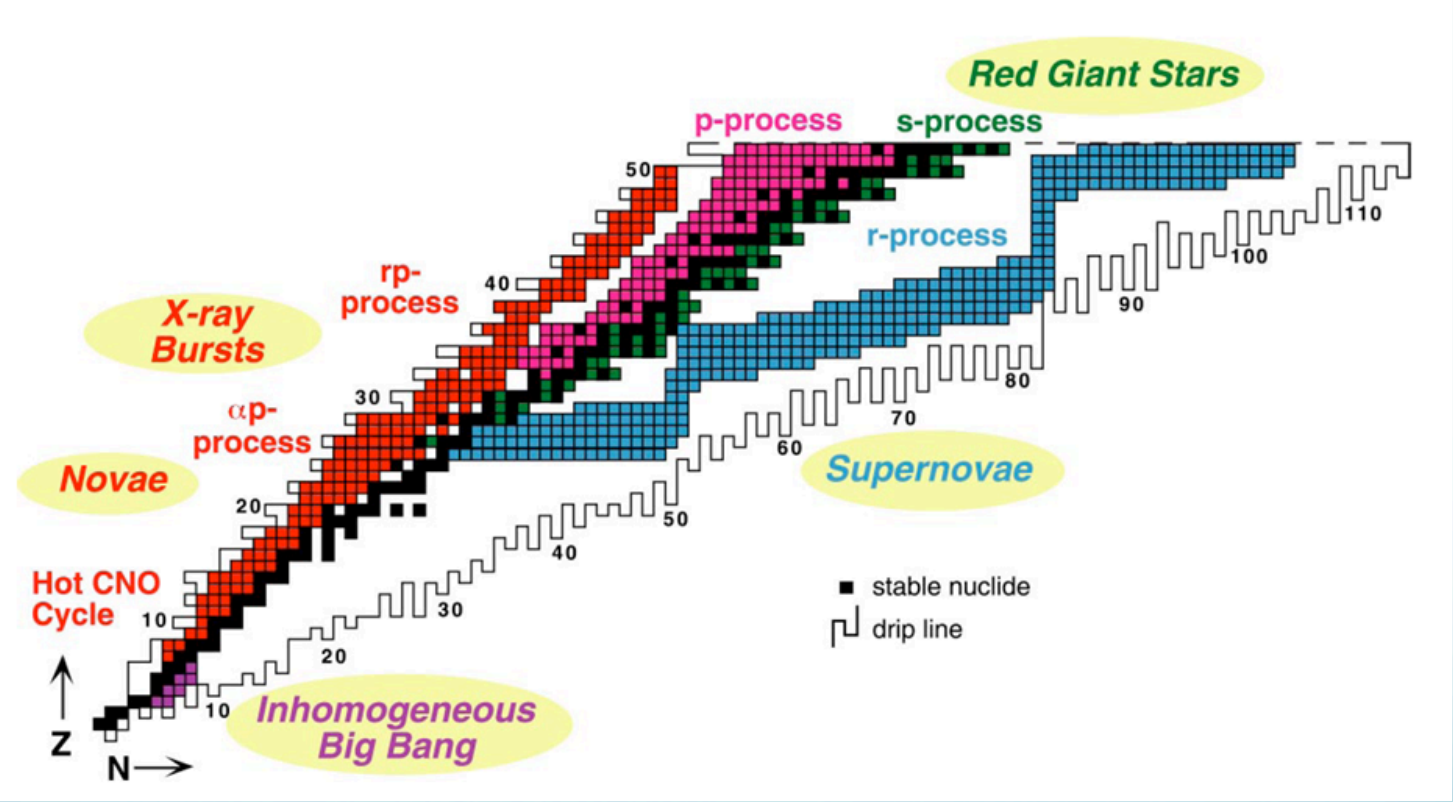
Nuclear chronometers, which predict the ages of the oldest stars by comparing the present and initial abundances of long-lived radioactive nuclides, provide an independent dating technique for the cosmos.

High precision nuclear chronometer for the cosmos,
X. H. Wu et al., Ap. J. 941 (2022) 152.

The production history of the 232Th/238U and 235U/238U chronometers produced by the r-process should trace the rate of star formation activity in the Galaxy. This implies that 232Th/238U and 235U/238U chronometer dating should therefore provide an excellent measure of the age of the Galaxy.

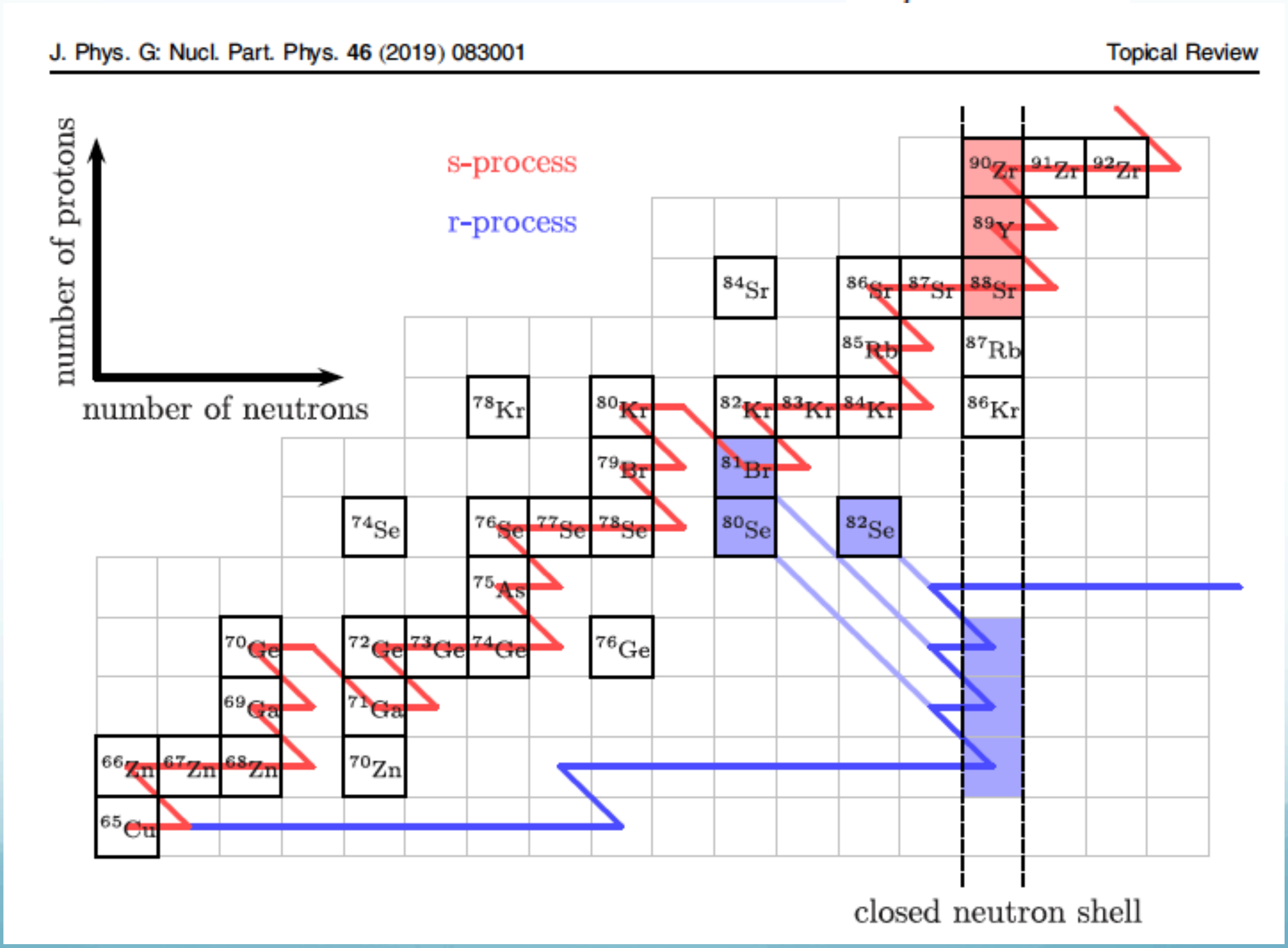
The age of the universe from nuclear chronometers
J. W. Truran, PNAS 95 (1998) 18.

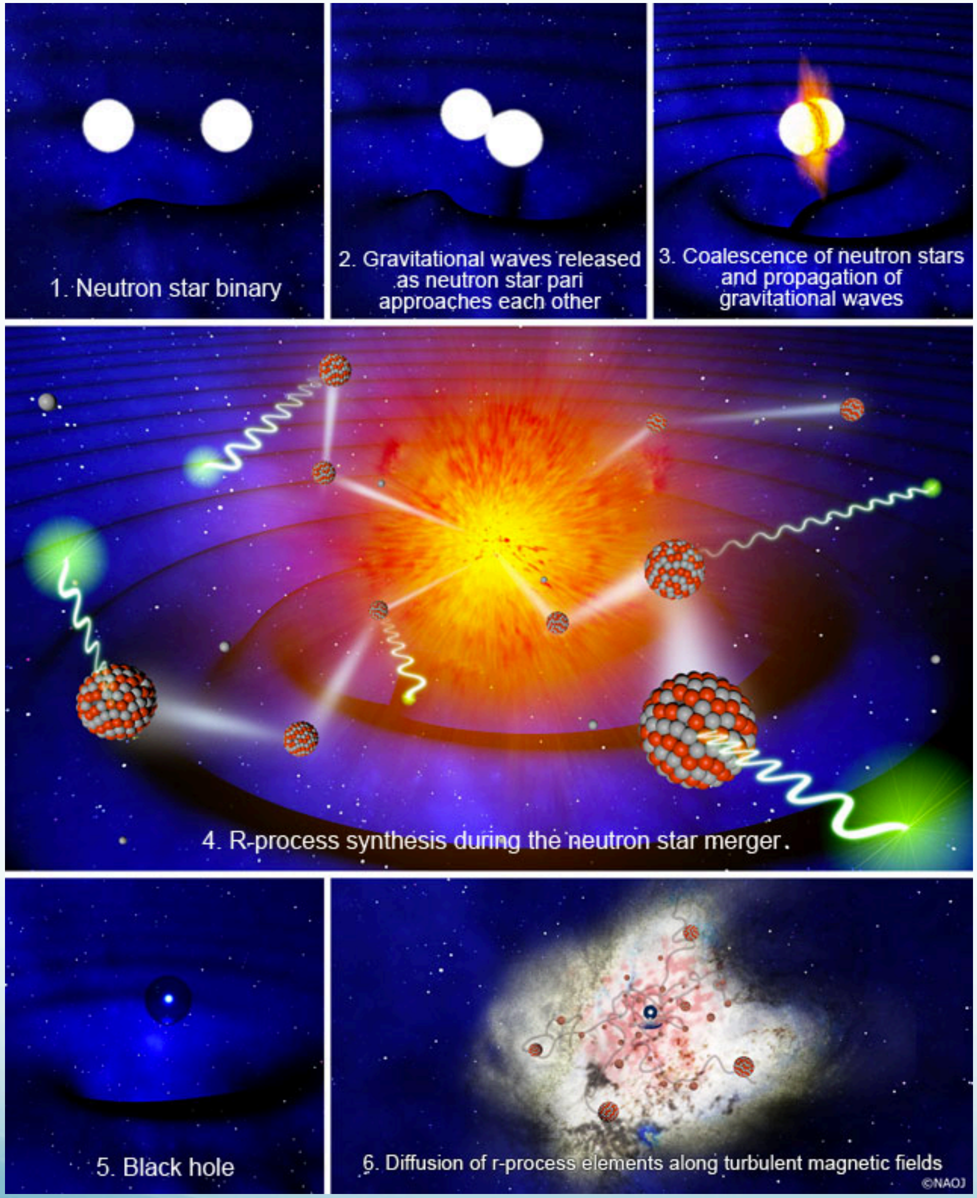
Nucleosynthesis calculations and kilonova modelling



M. S. Smith and K. E. Rehm, Annu. Rev. Nucl. Part. Sci. 2001. 51:91–130 s - process $\tau_\beta \ll \tau_{n\gamma}, \quad (\lambda_\beta \gg \lambda_{n\gamma}).$

r - process $\tau_\beta \gg \tau_{n\gamma}, \quad (\lambda_\beta \ll \lambda_{n\gamma}).$





As binary neutron stars circle around each other, they lose energy through gravitational waves and slowly fall towards one another, taking at least 10 million years to finally collide and merge into a single object. One merger event can eject an amount of gold 70 times the mass of the earth.

T. Tsujimoto and T. Shigeyama, A&A 565, L5 (2014)
Enrichment history of r-process elements shaped by a merger of neutron star pairs

Digging deeper

- In addition to producing kilohertz gravitational waves detectable by ground-based interferometers, compact object mergers involving a neutron star (NS) are likely to emit a variety of electromagnetic signals.

- Immediately postmerger, the accretion of disrupted NS material onto the central black hole or hypermassive NS may drive a short gamma ray burst

- Mergers may also produce optical/infrared transients powered by the radioactive decay of heavy elements synthesized via rapid neutron capture (the r - process)

- The r -process is expected to operate in material ejected from the system dynamically

- Among the electromagnetic signals emitted, radioactive transients known as KILONOVAE peak on the scale of days to weeks after merger

Any rigorous kilonova model must address the following aspects of radioactivity:

i. the total amount of radioactive energy released;

ii. the decay channels that dominate the energy production during different phases of kilonova evolution; and

iii. the efficiency with which suprathermal radio-active decay products - β particles, α-particles, γ-rays, and fission fragments - transfer their energy to the thermal back-ground. Once thermalized, the energy is re-radiated as thermal emission, powering the kilonova light curve.

THE ASTROPHYSICAL JOURNAL, 829:110 (20pp), 2016 October 1
doi:10.3847/0004-637X/829/2/110

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RADIOACTIVITY AND THERMALIZATION IN THE EJECTA OF COMPACT OBJECT MERGERS AND THEIR IMPACT ON KILONOVA LIGHT CURVES

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- The r-process nucleosynthesis path is along very unstable, exotic and neutron-rich nuclei

- The explosive conditions in r-process sites with high temperatures could result in nuclei existing in excited states

- For high ambient temperatures, the population factor for excited energy levels of a nucleus is large

- Possible effects of the nuclear thermal excitations are taken into account in the forward and reverse reaction rates

BUT

- not in the case of alpha decays!

(as found in libraries publicly available for the scientific community)

Stellar decay rates

When thermal equilibrium has been reached between all nuclear states, the decay rate at a temperature *T* can be given by a rather simple formula

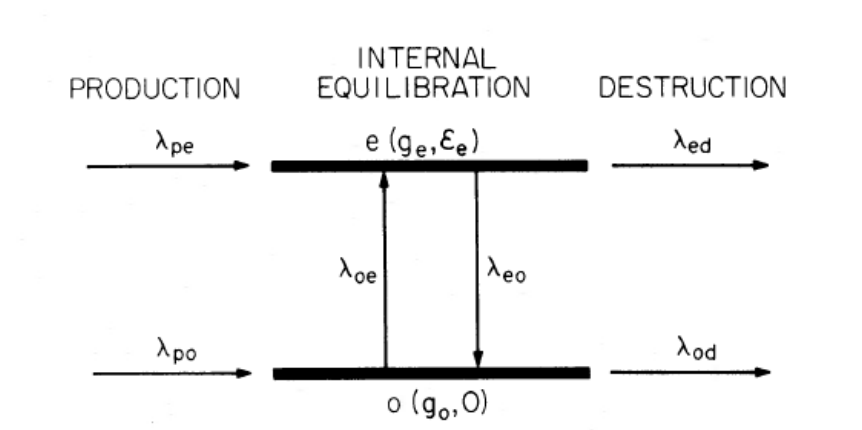
R. A. Ward and W. A. Fowler, ApJ 238 (1980) 266

$$\lambda_d^{\text{eff}} = \frac{\sum_i g_i \lambda_{id} \exp\left(-\mathscr{E}_i/kT\right)}{\sum_i g_i \exp\left(-\mathscr{E}_i/kT\right)} .$$

$\mathscr{E}_i$ → Energy of the i[th] excited state

$\exp\left(-\mathscr{E}_i/kT\right)$ → Population probability

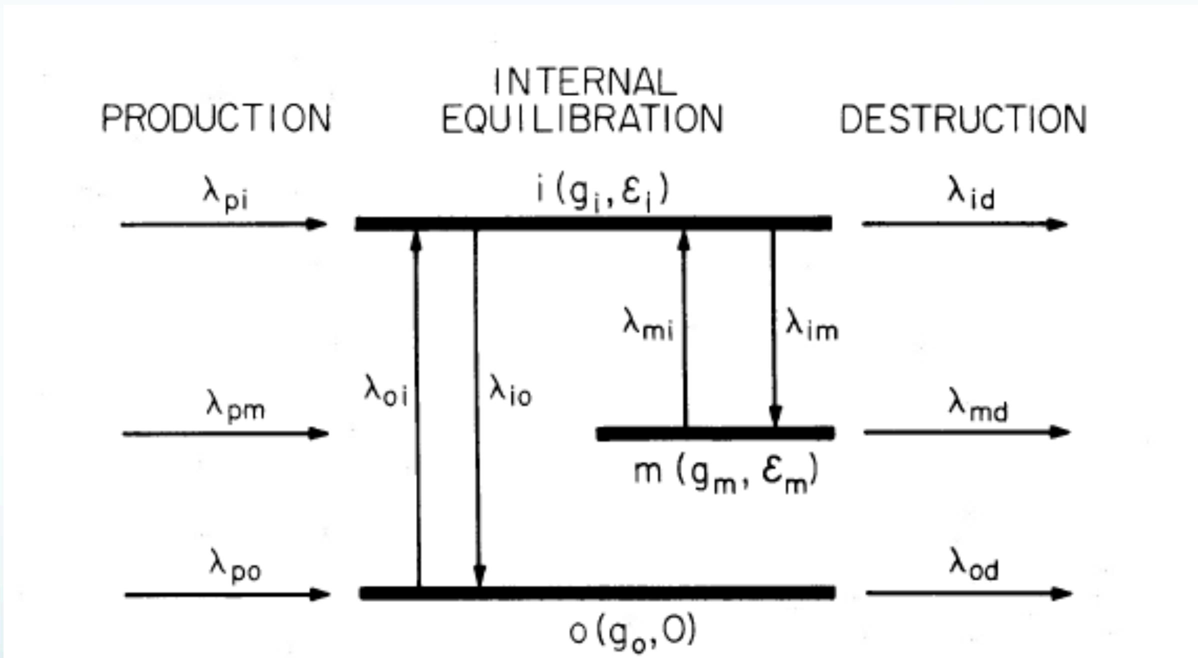
λ_{id} →decay rate of the i[th] excited state



Internal equilibration of a single excited state and ground state of a nucleus

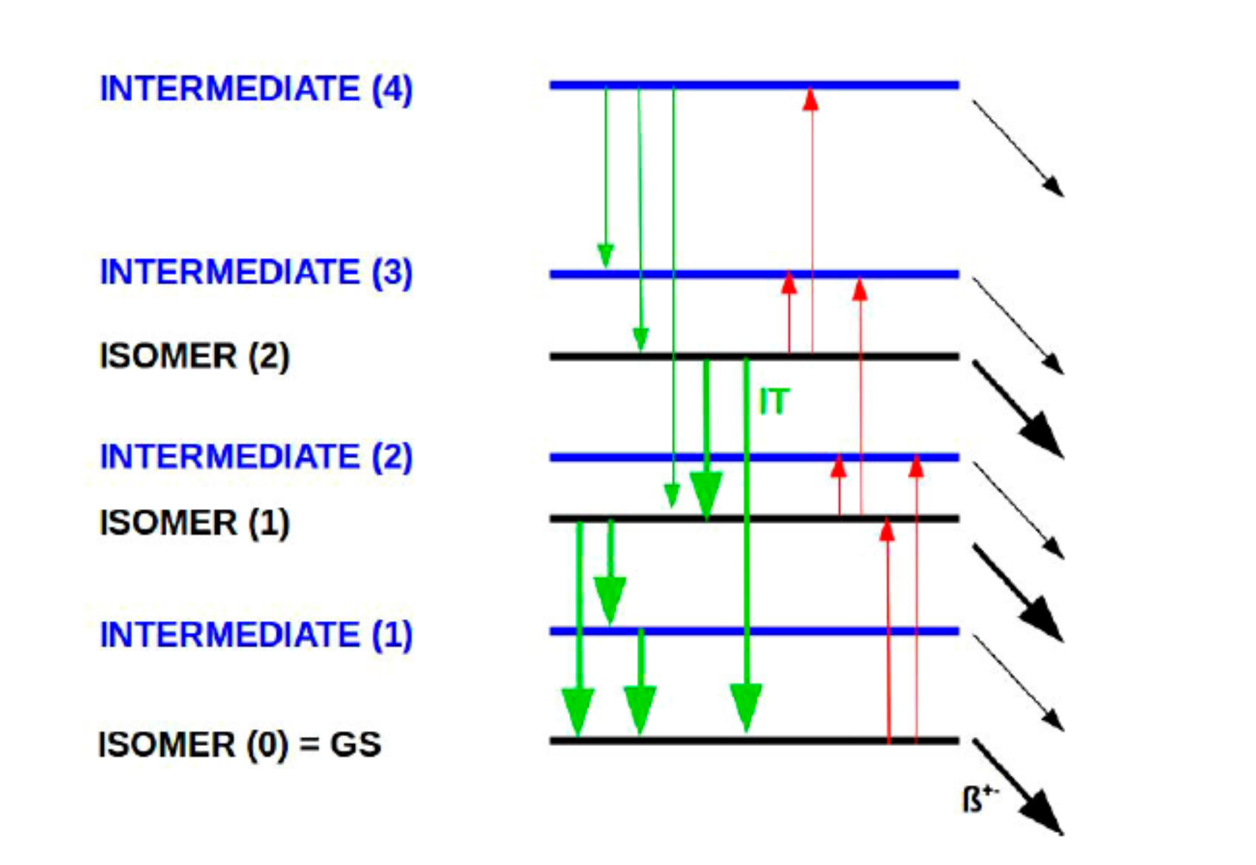
What happens when we have isomeric states?

Ward and Fowler showed that there exist criteria for limiting temperatures below which the isomeric states are not in equilibrium with the ground state and should be treated as a separate species of nuclei!



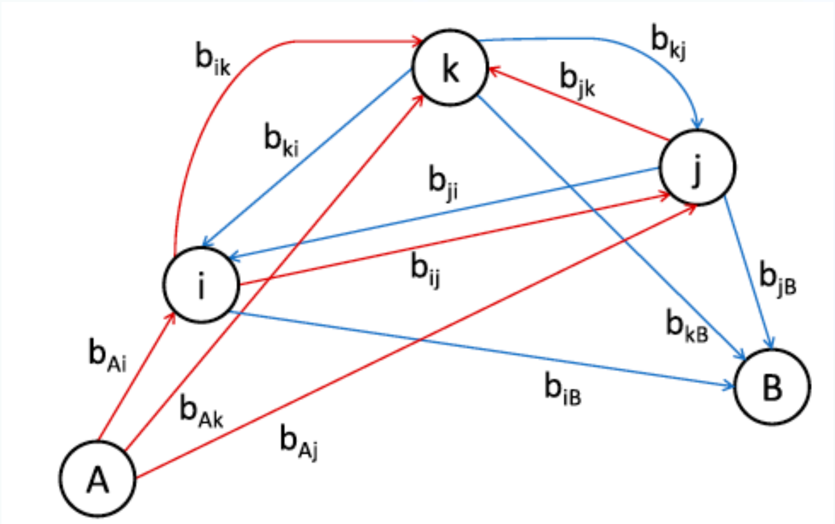
Three level nuclear system with no direct transitions between the first two states

In a more realistic scenario …



R. Reifarth et al., Int. J Mod. Phys. A 33 (2018) 1843011

Extensive literature exists on the beta decay rates at elevated temperatures



Schematic of transitions from state A to state B when A and B have little or no direct coupling

Calculations not trivial!!

THE ASTROPHYSICAL JOURNAL SUPPLEMENT SERIES, 252:2 (17pp), 2021 January
https://doi.org/10.3847/1538-4365/abc41d

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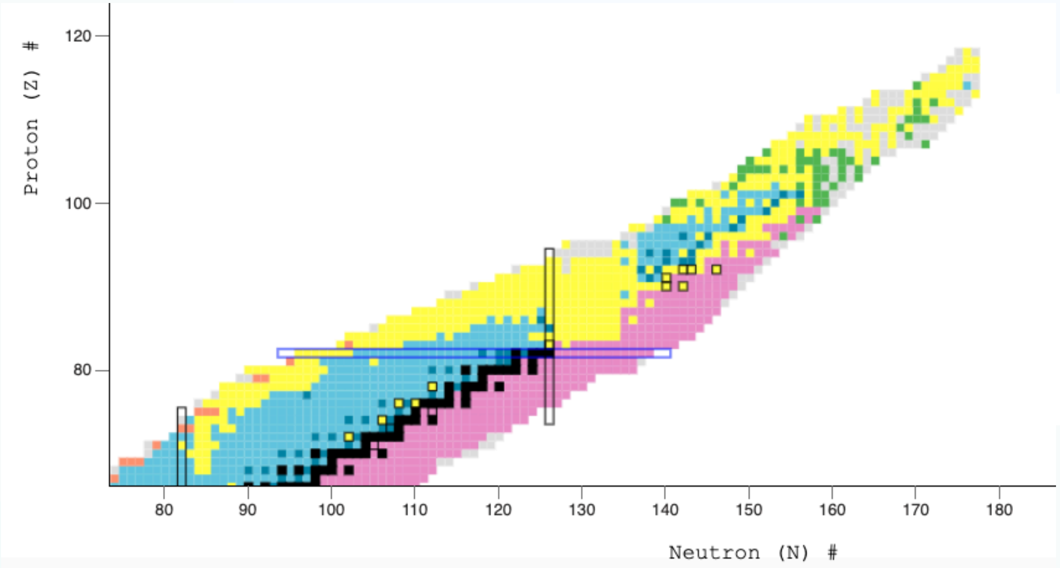
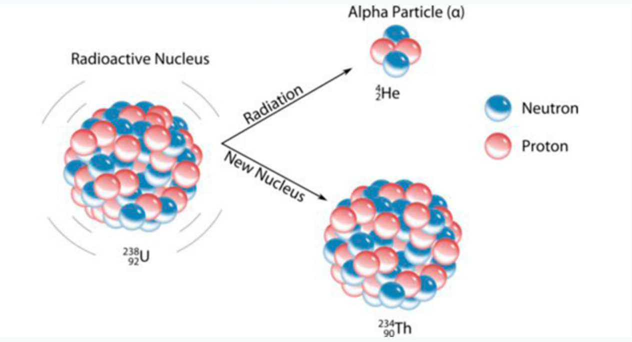
Astromers: Nuclear Isomers in Astrophysics*

G. Wendell Misch[1,2,3,4], Surja K. Ghorui[5,3], Projjwal Banerjee[6], Yang Sun[3,7,8], and Matthew R. Mumpower[1,2,4]

Alpha Decay at Elevated Temperatures





Essential:

1. Data on the excited levels of nuclei and their alpha decay half-lives

2. A good decay law in the absence of data

Some semi-empirical formulae for ground state half-lives of alpha decay

Geiger-Nuttal law (1911)

$$\log_{10} t_{1/2} = a(Z)Q^{-1/2} + b(Z)$$

Hundred years later … New Geiger-Nuttal law

Y. Ren and Z. Ren, Phys. Rev. C 85 (2012) 044608

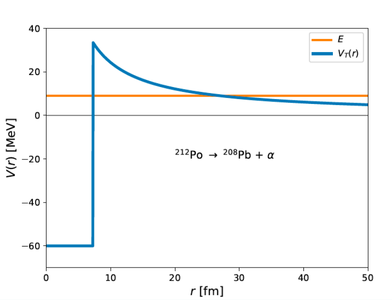
$$lgT_{1/2} = a\sqrt{\mu}\, Z_c Z_d/\sqrt{Q} + b\sqrt{\mu}\sqrt{Z_c Z_d} + c + S + P\, l(l+1),$$

More recently … Universal decay laws for emission of charged particles and nuclei (in their ground states)

Decay law for the alpha decay of excited nuclei

Preformed Cluster Model

$$t_{1/2} = \frac{\ln(2)}{P_\alpha \, \nu \, P}$$



$$\log\left(\nu \, t_{1/2}\right) = -\log P - \log P_\alpha + \text{constant}\,.$$

$$k\left(r\right) = \sqrt{\tfrac{2\mu}{\hbar^2}\left|V\left(r\right) - E\right|}$$

$$P = \exp\left[-2\int_{R_0}^{R} k(r)\,\mathrm{dr}\right]$$

$$V\left(r\right) = Z_\alpha Z_d \frac{e^2}{r} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}\,.$$

$$\int_{R_0}^{R} k(r)\,dr = \sqrt{\frac{2\mu E}{\hbar^2}} R\left[\frac{\pi}{2} - 2\left(\frac{R_0}{R}\right)^{1/2}\right] + \sigma\sqrt{\frac{2\mu E}{\hbar^2}}\frac{1}{\sqrt{R_0 R}}$$

$$\sigma = \frac{\hbar^2}{2\mu E}\ell(\ell+1)$$

Effective Q-value

Energy conservation implies

$$m_p + E_p^* = m_\alpha + m_d + E_d^* + E_\alpha + E_{recoil}$$

$$E_\alpha = Q_\alpha + E_p^* - E_d^*$$

Since the Q-value is usually taken to be the tunneling energy in ground state decays, here we define an effective Q-value

$$Q_{eff} = Q_\alpha + E_p^* - E_d^* = Q_\alpha + \Delta E^*$$

which leads to the penetration probability as

$$\log P = -\left(\beta_1 - \beta_2 \frac{\Delta E^*}{Q_\alpha}\right) \chi' - \beta_3 \rho' - \beta_4 \frac{\ell(\ell+1)}{\rho'}$$

$$\beta_1 = \frac{\pi e^2 \sqrt{2m_0}}{\hbar \ln 10}$$

$$\beta_2 = \frac{\beta_1}{2}$$

$$\beta_3 = -\frac{4e\sqrt{2m_0 r_0}}{\hbar \ln 10}$$

$$\beta_4 = -\frac{2\hbar}{\ln 10 e\sqrt{2m_0 r_0}}$$

$$\chi' = Z_\alpha Z_d \sqrt{\frac{A_\alpha A_d}{A_p Q_\alpha}}$$

$$\rho' = \sqrt{Z_\alpha Z_d \frac{A_\alpha A_d}{A_p} \left(A_\alpha^{1/3} + A_d^{1/3}\right)}$$

Excitation energy dependent decay law

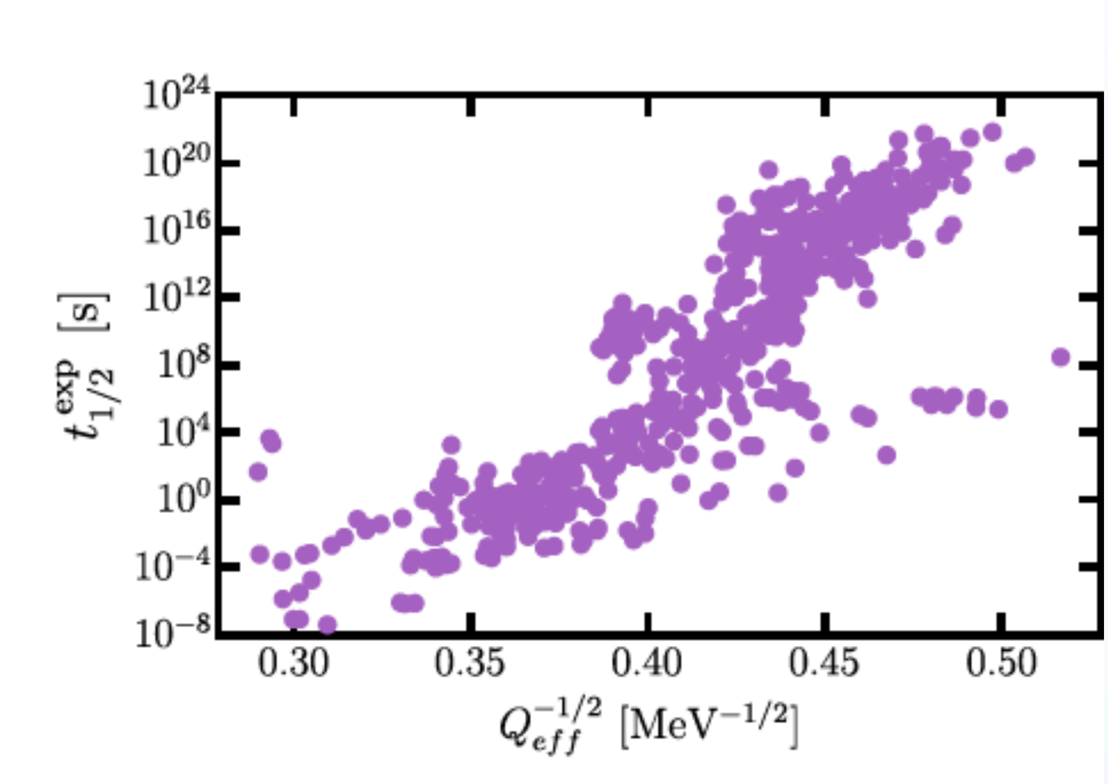
$$\log t_{1/2}^{\mathrm{exc}} = \beta_1\,\chi' + \beta_2\,\chi'\,\frac{\Delta E^*}{Q_\alpha} + \beta_3\,\rho' + \beta_4\,\frac{\ell(\ell+1)}{\rho'} + \beta_5\,\delta_{oe}$$

All constants defined earlier will be treated as free parameters and fitted to data on alpha decay of excited parent nuclei to the ground and excited states of the daughter nuclei

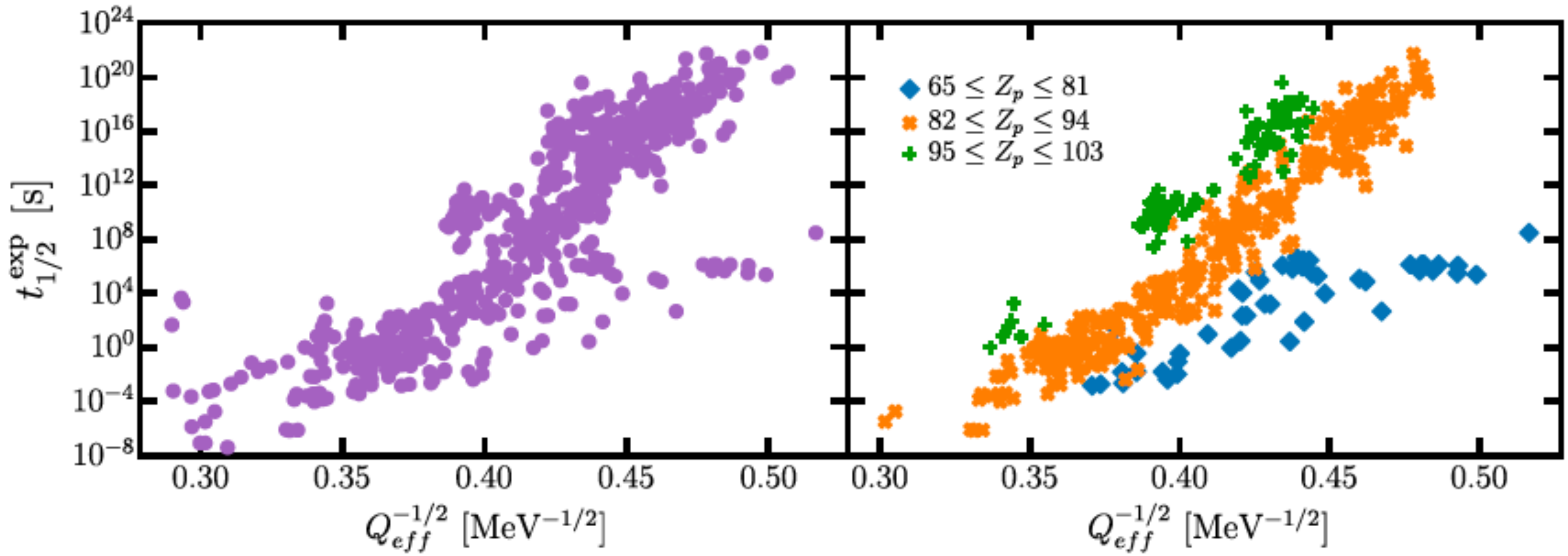
Similar model has been used earlier for cluster decay

D. F. Rojas-Gamboa, N. G. Kelkar and O. L. Caballero, Nucl. Phys. A 1028 (2022) 122524

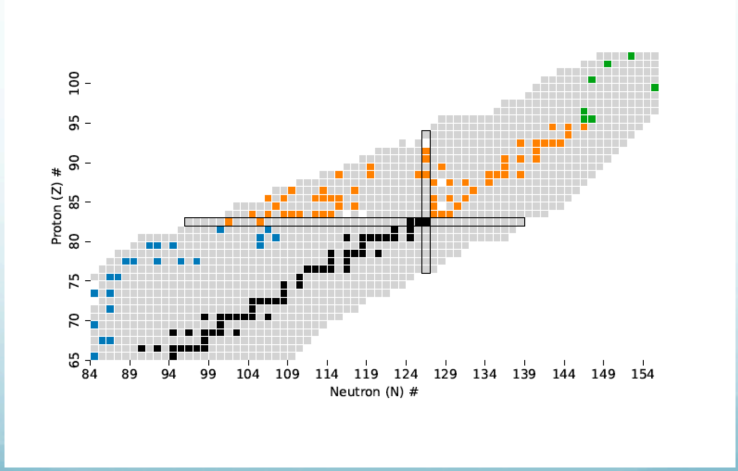
Choice of data to obtain a semi-empirical formula for half-lives



592 alpha decays with ground state parent to excited state daughters, excited parent nuclei to ground state and excited daughters



$t_{1/2}^{\exp}$ [s]
10^{24}
10^{20}
10^{16}
10^{12}
10^{8}
10^{4}
10^{0}
10^{-4}
10^{-8}
0.30 0.35 0.40 0.45 0.50
$Q_{eff}^{-1/2}$ [MeV$^{-1/2}$]
$65 \leq Z_p \leq 81$
$82 \leq Z_p \leq 94$
$95 \leq Z_p \leq 103$
0.30 0.35 0.40 0.45 0.50
$Q_{eff}^{-1/2}$ [MeV$^{-1/2}$]



Proton (Z) #
65 70 75 80 85 90 95 100
84 89 94 99 104 109 114 119 124 129 134 139 144 149 154
Neutron (N) #

Modified decay laws for comparison

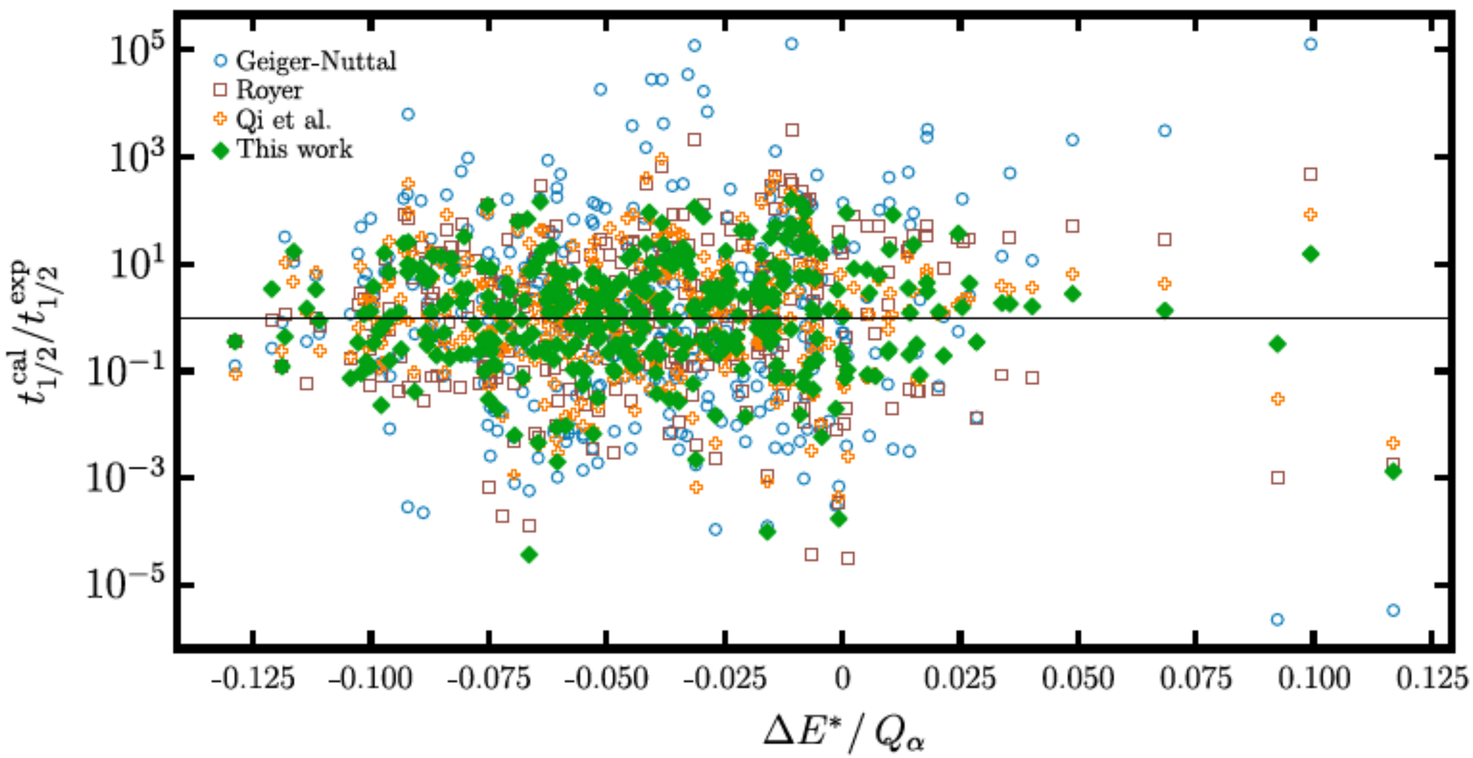
$$\log t_{1/2}^{(\mathrm{GN})} = a\, Q_{eff}^{-1/2} + b\,,$$

$$t_{1/2}^{(\mathrm{Royer})} = a\, Z_p Q_{eff}^{-1/2} + b\, A_p^{1/6} Z_p^{1/2} + c\,.$$

G. Royer, J. Phys. G 26 (2000) 1149

$$\log t_{1/2}^{(\mathrm{Qi})} = a\, \chi'_{eff} + b\, \rho' + c + d\, \rho' \sqrt{\ell(\ell+1)} + e \sqrt{I_p(I_p+1)} + f\, A_p \left[1 - (-1)^{\ell}\right]$$

A. Soylu and C. Qi, Nucl. Phys. A 1013 (2021) 122221



Model	R^2	MSE
Geiger-Nuttal law	0.939	3.14
Royer formula	0.966	1.75
Qi *et al.* UDL	0.976	1.23
This work	0.980	1.04

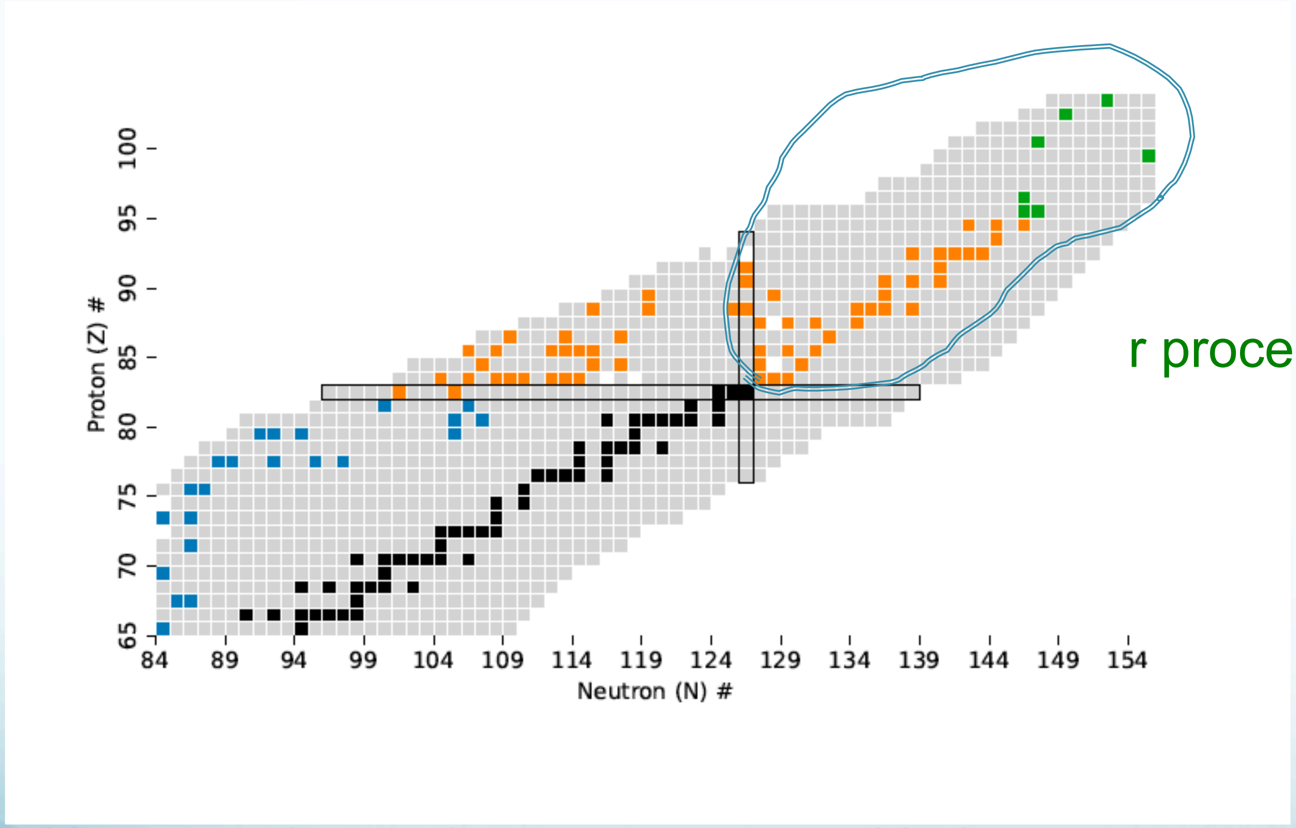
$$R^2 = \frac{\mathrm{TSS} - \mathrm{RSS}}{\mathrm{TSS}} = 1 - \frac{\mathrm{RSS}}{\mathrm{TSS}}$$

$$\mathrm{TSS} = \sum (y_i - \bar{y})^2$$

$$\mathrm{RSS} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{f}(x_i))^2$$

Alpha decay in different astrophysical scenarios



$N \geq 126$

$Z \geq 82$

r process aftermath

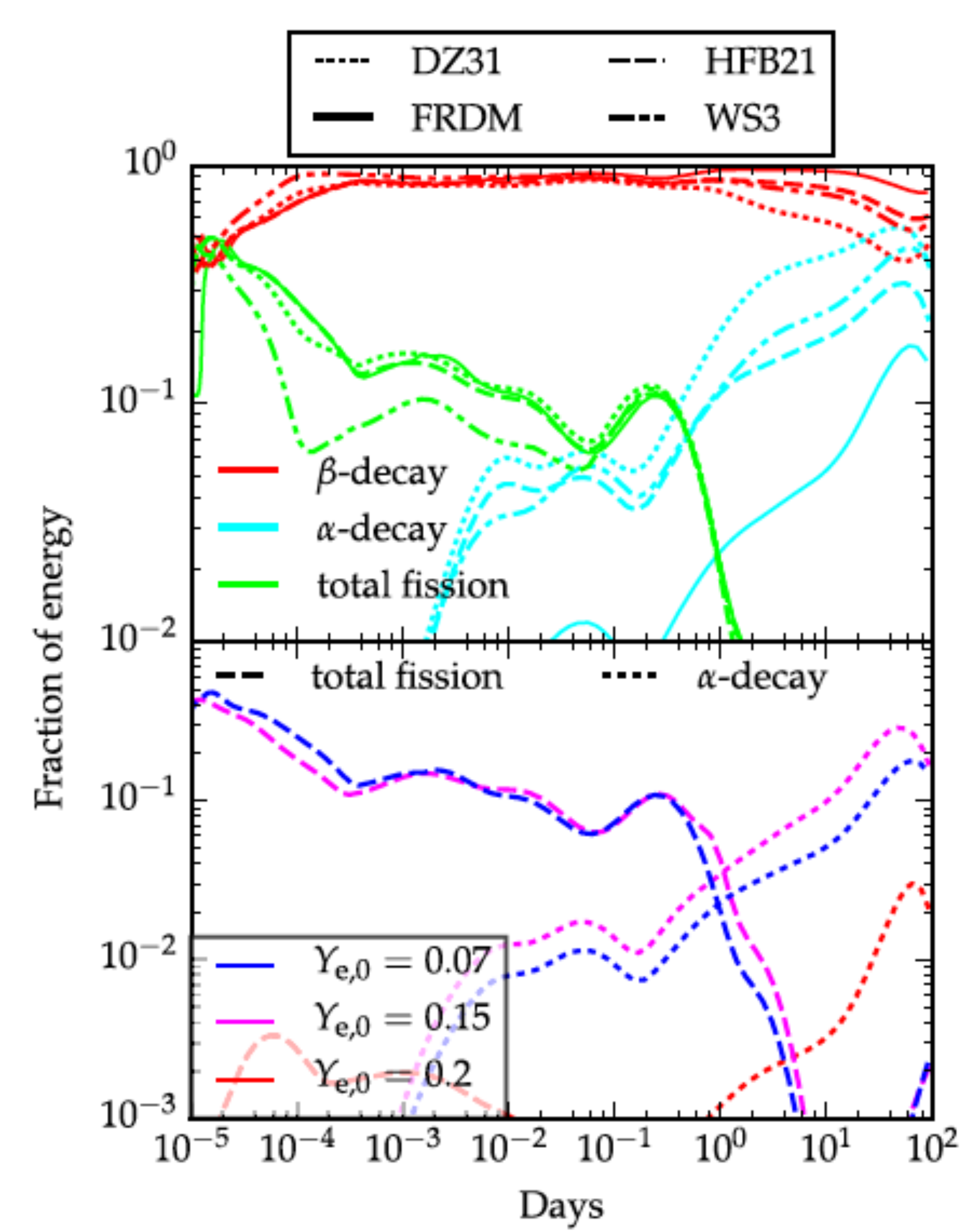
Heavy and superheavy elements with these neutron and proton numbers will decay to stability by the decay mode available such as alpha, beta decay or fission

At early stages of the r-process which can start from 10 GK and reach 3 GK, there is competition between neutron capture, photodissociation, beta decay under certain conditions, following the well known r-process path below the line of stability

At later stages, neutrons are exhausted – neutron freeze out, very unstable neutron rich nuclei including actinides will decay to stability

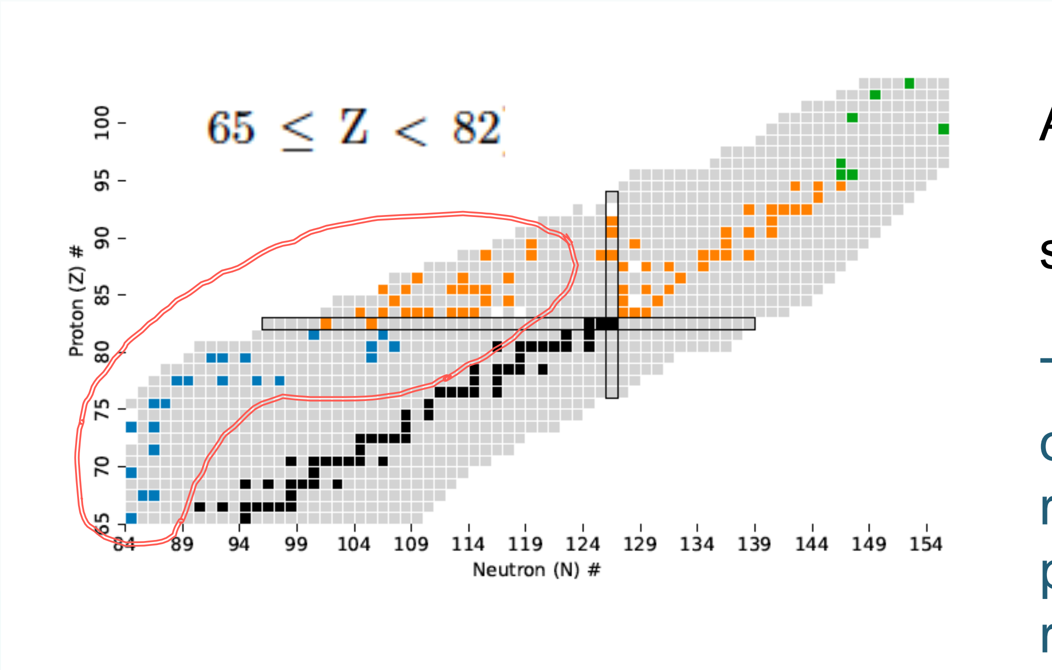
Simulations identify alpha decays and especially the decay chains originating from actinide nuclei as important contributors to emitted light curves expected from r-process events

J. Barnes et al., ApJ 829 (2016) 110



J. Barnes et al., The Astrophys. J. 829:110 (2016)

- Alpha decay plays a role in competition with beta decay and fission, in powering and shaping the light curves of kilonovae.



Abundances of p-nuclei

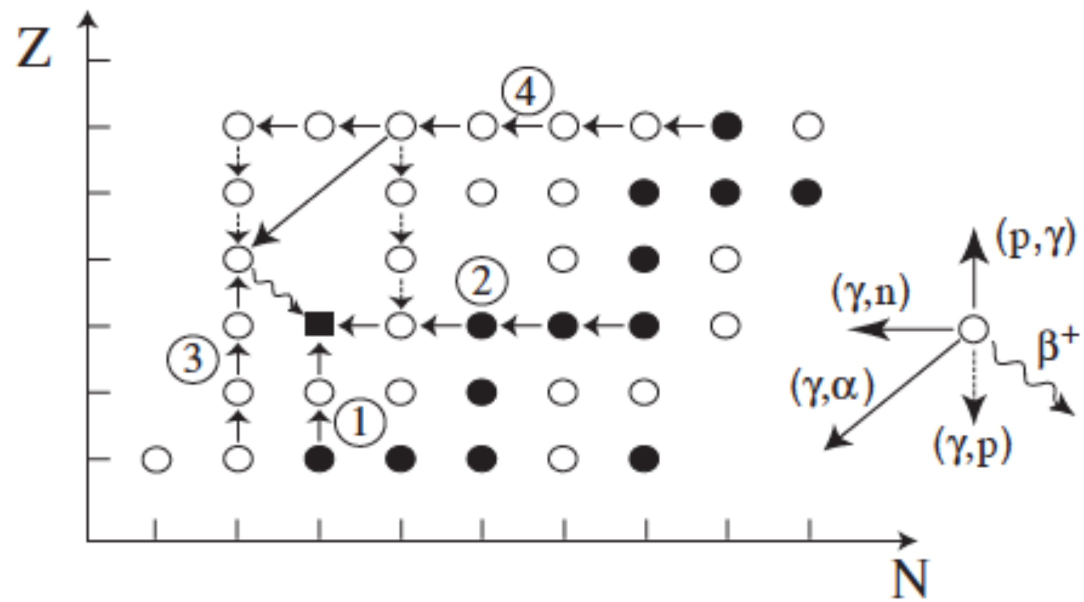
$34 \leq Z \leq 80$

seem to be poorly understood

The p-process or γ process which occurs by dissociation of neutron rich nuclei via reactions with photons is supposed to be responsible for the production of p-nuclei
2 - 3 GK

During the photodisintegration period, neutron, proton and alpha emission channels compete with each other and with beta decays further away from stability

M. Arnould, S. Goriely / Physics Reports 384 (2003) 1–84



Possible nuclear routes through which seed s- or r- nuclides (black dots) can be transformed into p-nuclide (black square).

Open dots correspond to unstable nuclei

- The p-process moves matter from nuclei previously produced by the s- and r-process to the proton rich side of stability.
- Typical temperatures are of the order of 2 – 4 GK
- Path of the p-process is still unclear due to uncertainties in nuclear physics and astrophysical modeling
- Alpha emission from neutron deficient nuclei competes with other channels including beta decay

D. L. Lambert, The Astron. Astrophys. Rev. 3 (1992) 201

A. Choplin et al. A & A 661 (2022) A86

M. Arnould and S. Goreily, Phys. Rep. 384 (2003) 1

PHYSICAL REVIEW LETTERS **132**, 192701 (2024)

Editors' Suggestion | Featured in Physics

Production of p Nuclei from r-Process Seeds: The νr Process

Zewei Xiong,[1,*] Gabriel Martínez-Pinedo,[1,2,3] Oliver Just,[1,4] and Andre Sieverding[5]

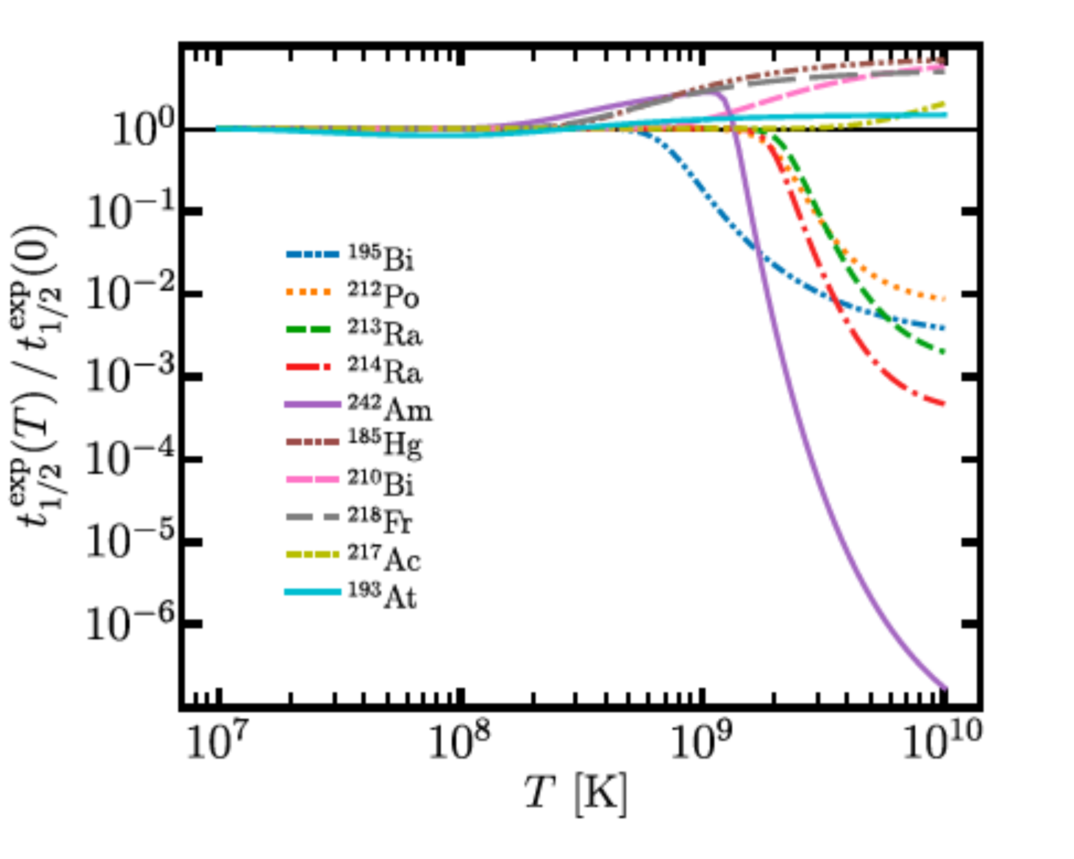
During the explosive conditions of an r-process, neutron-rich nuclei can increase their proton number by the capture of neutrinos in the above proposed

"νr-process"

The large neutrino flux moves matter from the neutron-rich side to the proton-rich side

Unstable nuclei in the vr – process path will also decay to stability by emitting an alpha

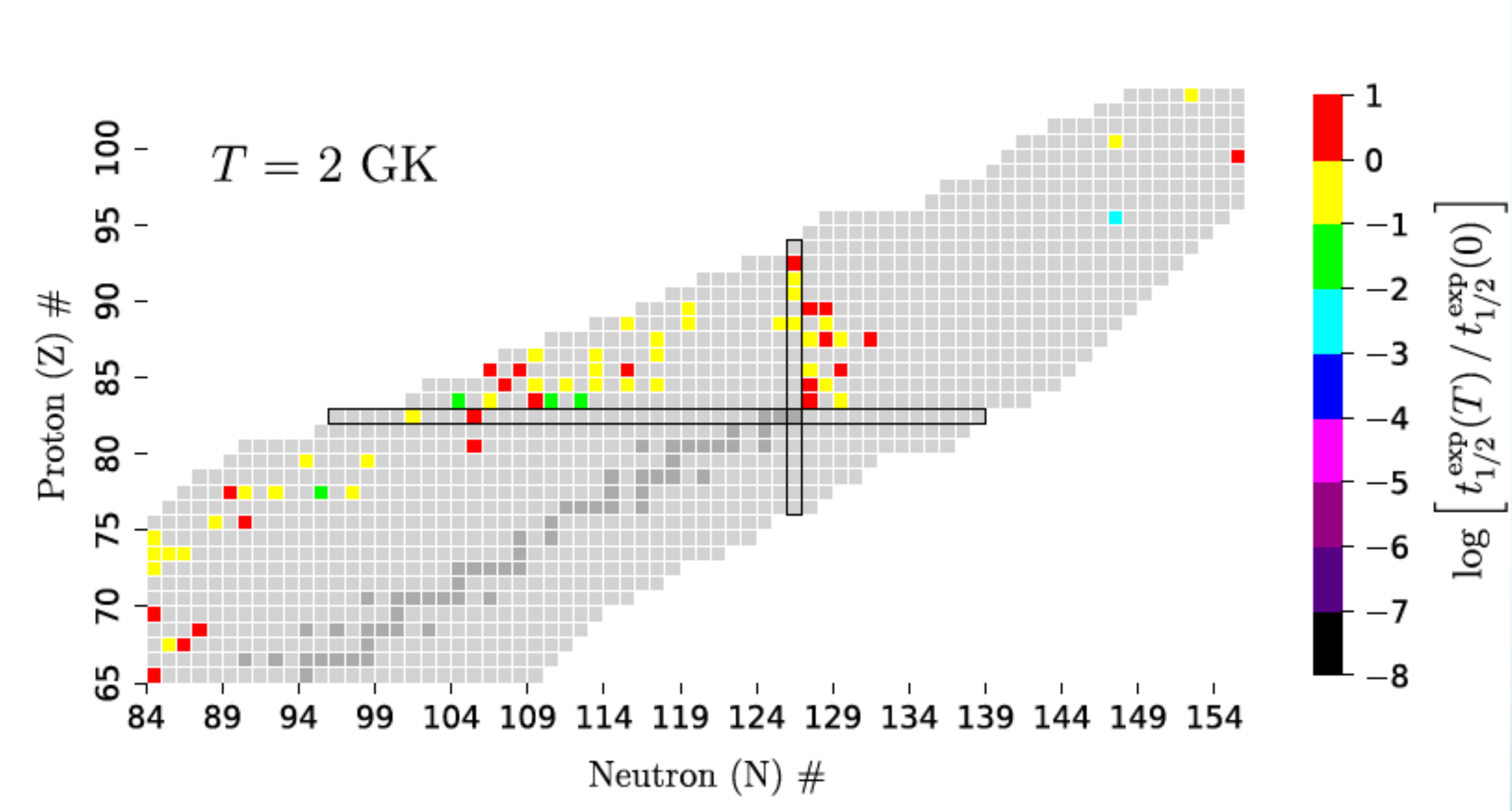
Stellar alpha decay half-life



$$t_{1/2}(T) = \left[\frac{1}{\mathcal{G}} \sum_i \frac{g_{p_i} \exp\left(-E^*_{p_i}/k_B T\right)}{t_{1/2}\left(E^*_{p_i}\right)} \right]^{-1}$$

$$\mathcal{G} = \sum_i g_{p_i} \exp\left(-E^*_{p_i}/k_B T\right)$$

$$g_{p_i} = (2J_{p_i} + 1)$$



$T = 2$ GK
Proton (Z) #
Neutron (N) #
$\log\left[t_{1/2}^{\exp}(T)\,/\,t_{1/2}^{\exp}(0)\right]$
65
70
75
80
85
90
95
100
84
89
94
99
104
109
114
119
124
129
134
139
144
149
154
1
0
−1
−2
−3
−4
−5
−6
−7
−8

Isotope	Isomer J^{π}	total $t_{1/2}^{exp}$	$\%\alpha$
^{149}Tb	$11/2^-$	4.17(5) min	0.02
^{153}Ho*	$1/2^+$	9.3(5) min	0.18
^{153}Tm*	$1/2^+$	2.5(2) s	92
^{155}Lu*	$1/2^+$	138(8) ms	76
	$(25/2^-)$	2.69(3) ms	100
^{165}Re*	$(11/2)^-$	1.74(6) s	13
^{166}Ir	(9^+)	15.1(9) ms	98.2
^{185}Hg*	$13/2^+$	21.6(15) s	0.03
^{187}Pb*	$(13/2^+)$	18.3(2) s	9.5
^{191}Po	$(13/2^+)$	93(3) ms	96
^{191}At	$(7/2^-)$	2.1(+4-3) ms	100
^{192}Bi*	(10^-)	39.6(4) s	10
^{193}At	$(7/2^-)$	21(5) ms	100
	$(13/2^+)$	27(+4-3) ms	24
^{210}Bi	9^-	$3.04 \times 10^6(6)$ y	100
^{211}Po	$(25/2^+)$	25.2(6) s	99.98
^{214}At	9^-	760(15) ns	100
^{216}Ac	(9^-)	441(7) μs	100
^{218}Fr	$(8^-, 9^-)$	22.0(5) ms	100
^{217}Ac	$(29/2^+)$	740(40) ns	4.51
^{218}U	(8^+)	0.56(+26-14) ms	100
^{242}Am	5^-	141(2) y	0.45
	$(2^+, 3^-)$	14.0(10) ms	< 0.005
^{254}Es*	2^+	39.3(2) h	0.32

List of isomeric states responsible for increased half-lives of nuclei

Summary and Outlook

- Majority of nuclei studied show a decrease in the half-lives with the temperature (T) with many cases showing about an order of magnitude decrease at T=2 GK.
- Exceptional cases where half-life increases at the most by an order of magnitude are identified.
- The increase is associated with the presence of isomeric states.
- Excitation energy dependent alpha decay law can be used for kilonova studies as well as nucleosynthesis calculations
- Better estimates of stellar decay rates based on proper consideration of thermal equilibration of isomeric states via intermediate states remains to be done
- Need for more data on alpha decay of excited nuclei!

THANK YOU FOR LISTENING!