Seniority isomers in nuclei

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A history of seniority

Relation with particle-hole conjugation

Examples of seniority isomers

Concluding remarks

LASNPA, Mexico, June 2024

Theory of complex spectra

In the 1940s Racah published a series of seminal papers on the application of group theory to atomic spectra. The third of the series (primarily concerned with coefficients of fractional parentage) contains the first mention of seniority.

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Theory of Complex Spectra. III

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(Received February 8, 1943)

The consideration of the phases of the fractional-parentage coefficients allows the extension of the matrix methods to configurations with more than two equivalent electrons. Tables are given for the parentages of the terms of p^n and d^n. Applications are made to the spin-orbit interaction of the d^n terms and to the electrostatic interaction between the configurations d^n, $d^{n-1}s$, and $d^{n-2}s^2$. Errata in Part II are indicated.

Racah's "seniority number"

In this section we shall classify the terms of the configuration l^n according to the eigenvalues of

$$Q=\sum_{i<j} q_{ij}, \qquad (34)$$

where q_{ij} is a scalar operator which operates on the two equivalent electrons i and j and is defined by the relation

$$(l^2LM|q_{ij}|l^2LM)=(2l+1)\delta(L,0). \qquad (35)$$

It will be shown that to every term of l^n with non-vanishing Q a term of the same kind corresponds in l^{n-2}, and this fact will allow us to assign to each term a "seniority number" according to the value of n for which the term appeared for the first time. Some useful relation between the fractional parentages of corresponding terms will be obtained and it will also be shown that the classification of the terms of l^{2l+1} according to the two possibilities of (76)II depends only on the seniority of the term.

We may thus assign to each term in the QSL scheme a "seniority number" v, which indicates the number of electrons of the first member of its chain; it follows immediately from (45) that Q depends only on n and v and that its values are given by

$$Q(n,v)=\tfrac{1}{4}(n-v)(4l+4-n-v). \qquad (50)$$

Confronting (41) and (50) we see that conjugate terms have the same seniority.

The seniority number suffices for distinguishing the different terms of the same kind in the configurations d^n but not in f^n, since there are in f^n terms of the same kind which have also the same seniority. For such configurations an unspecified parameter α must be maintained besides v; terms corresponding according to (49) will have the same values of v and of α.

Pairing

Definition of pairing interaction in a single-*j* shell:

$$\left\langle j^2;J\middle|\hat{V}_{\mathrm{pairing}}(1,2)\middle|j^2;J\right\rangle = -\tfrac{1}{2}(2j+1)\,g\,\delta_{J0}$$

Analytic solution of pairing hamiltonian for identical nucleons in a single-*j* shell:

$$\left\langle j^n \upsilon J\middle|\sum_{1\le k<l}^{n}\hat{V}_{\mathrm{pairing}}(k,l)\middle|j^n \upsilon J\right\rangle = -\tfrac{1}{4}g(n-\upsilon)(2j-n-\upsilon+3)$$

Seniority υ (number of nucleons not in pairs coupled to *J*=0) is a good quantum number.

Correlated ground-state solution (*cfr.* BCS).

G. Racah, *Theory of complex spectra III*, Phys. Rev. **63** (1943) 367

Conservation of seniority

Seniority υ is the number of particles not in pairs coupled to J=0 (Racah).

Conditions for the conservation of seniority can be derived for a general interaction.

Any two-body interaction between identical fermions with spin j conserves seniority if j≤7/2.

A. de-Shalit & I. Talmi, *Nuclear Shell Theory*
I. Talmi, *Simple Models of Complex Nuclei*

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Is seniority conserved in nuclei?

The interaction between nucleons is "short range".

A δ interaction is therefore a reasonable approximation to the nucleon two-body force.

The δ interaction between **identical** nucleons conserves seniority.

$\therefore$ In **semi-magic** nuclei seniority is conserved to a good approximation.

Particle-hole (ph) conjugation

A long history, cfr. Condon & Shortley (1935).

In atomic and nuclear physics: Racah and Bell.

In the language of second quantisation:

$$\hat{\Gamma}|0\rangle = a^+_{j,m=j} a^+_{j,m=j-1} \dots a^+_{j,m=-j} |0\rangle$$

$$\hat{\Gamma} a^+_{jm} \hat{\Gamma}^+ = (-)^{j+m} a_{j,-m} \equiv \tilde{a}_{jm}, \quad \hat{\Gamma} \tilde{a}_{jm} \hat{\Gamma}^+ = a^+_{jm}$$

E.U. Condon & G.H. Shortley, *The Theory of Atomic Spectra*

G. Racah, *Theory of complex spectra II*, Phys. Rev. **62** (1942) 438

J.S. Bell, *Particle-hole conjugation in the shell model*, Nucl. Phys. **12** (1959) 117

Seniority and ph conjugation

The particle-hole conjugation operator Γ transforms a problem of *n* fermions in a *j* shell into one with 2*j*+1−*n* fermions.

A representation of the ph transformation

$$\hat{\Gamma} = \exp\left[\tfrac{1}{2}\pi\left(\hat{S}_+ - \hat{S}_-\right)\right]$$

where $S_\pm$ are the quasi-spin operators

$$\hat{S}_+ = \tfrac{1}{2}\sqrt{2j+1}\left(a_j^+ a_j^+\right)_0^{(0)}, \quad \hat{S}_- = \left(\hat{S}_+\right)^+$$

A. Müller-Arnke, *A realization of the particle-hole conjugation operator in the shell model*, Nucl. Phys. A **215** (1973) 205

A geometric phase

The action of ph conjugation on a seniority state:

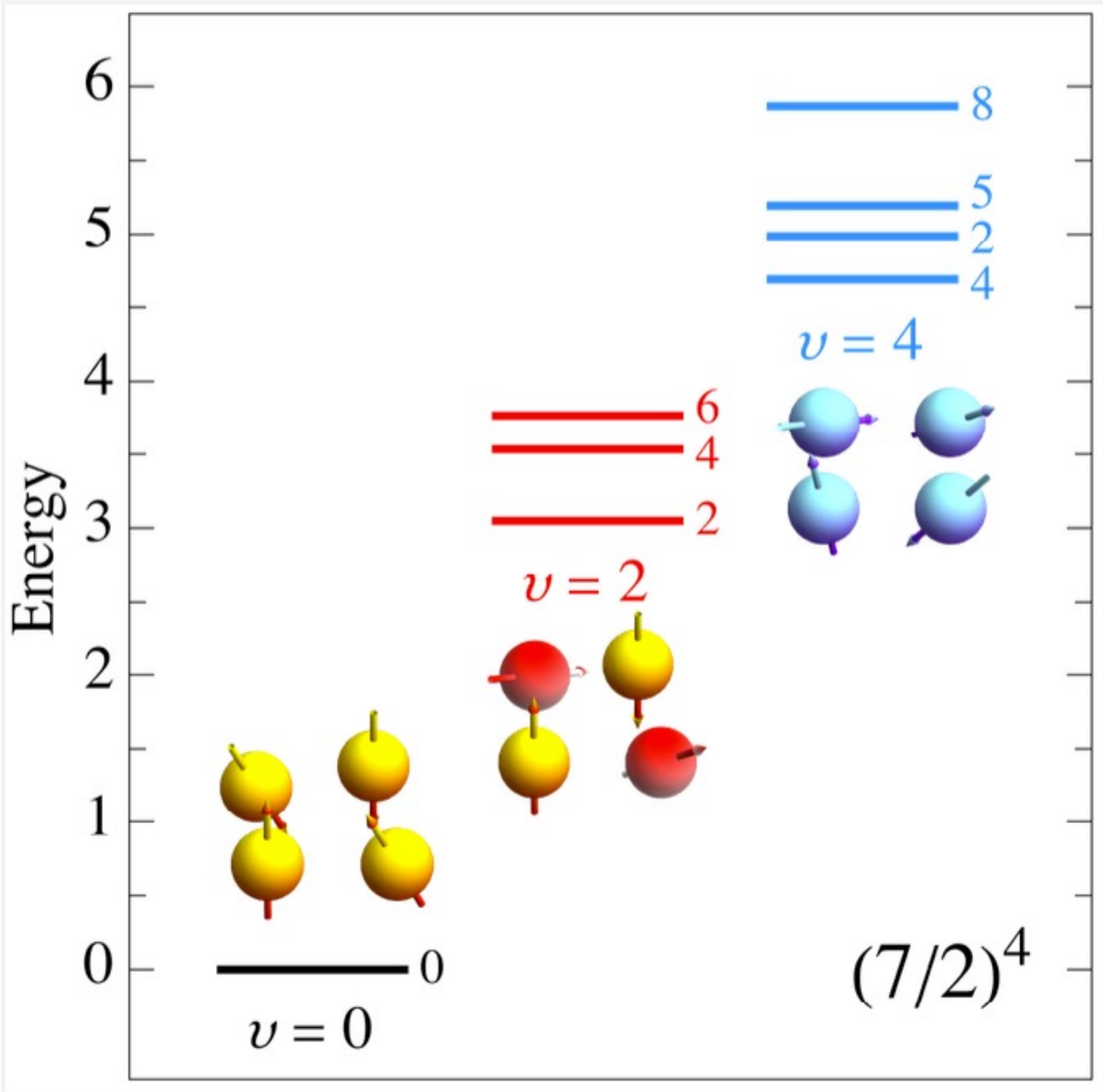
$$\hat{\Gamma}\left|j^n \upsilon J\right\rangle = (-)^{(n-\upsilon)/2}\left|j^{2j+1-n}\upsilon J\right\rangle$$

The sign is without any consequence **except** if the left and right states are the same, that is for a half-filled shell, $n=2j+1-n$.

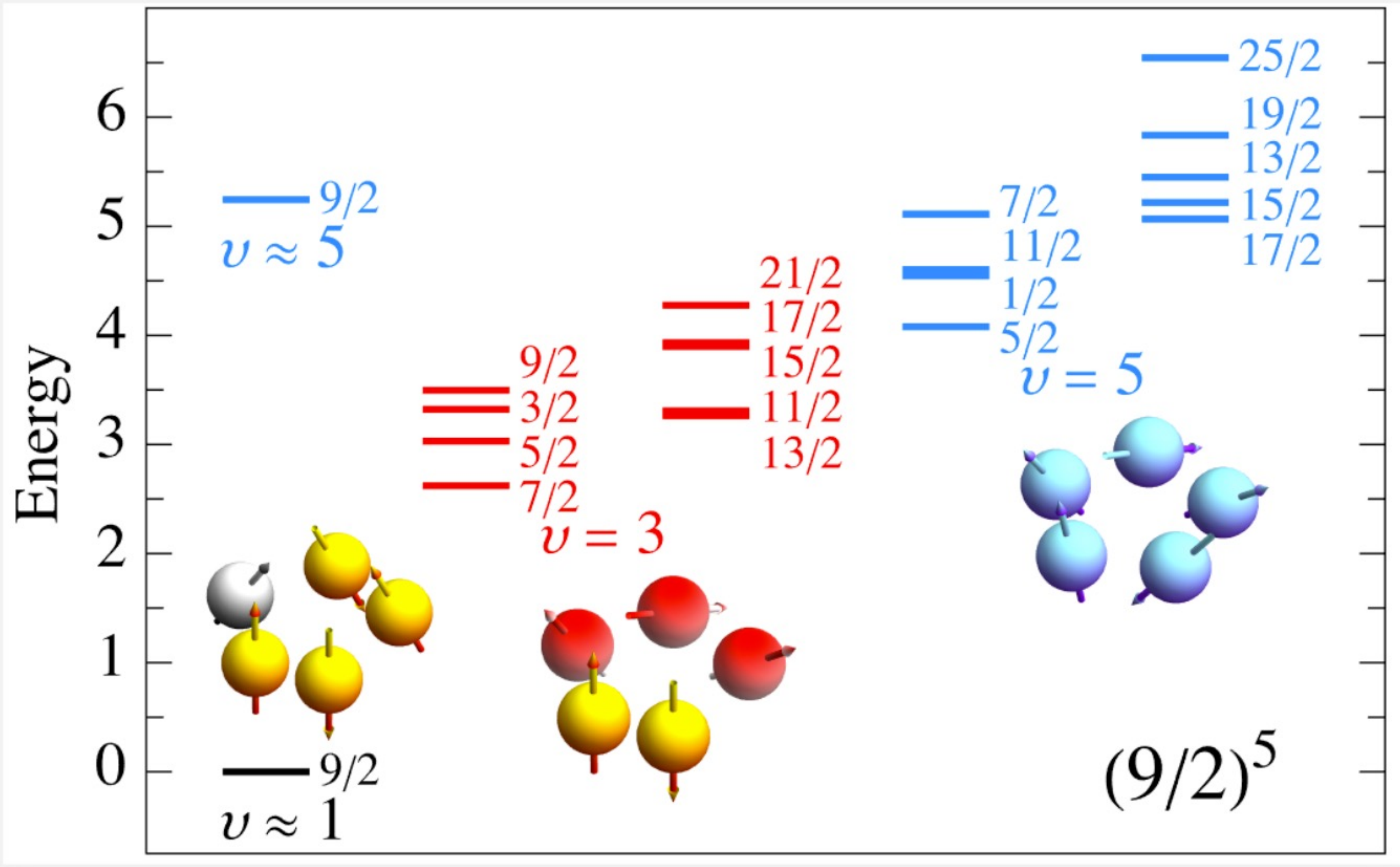
The observable consequence of this phase is that $\Delta\upsilon=\pm 2$ seniority mixing is forbidden if the interaction is at most **two-body**.

R.D. Lawson, *Theory of the Nuclear Shell Model*

Four nucleons in a j=7/2 shell



Five nucleons in a j=9/2 shell



E2 selection rules

Particle-hole conjugation implies

$$\left\langle j^n \upsilon J \left\| \hat{T}(E2) \right\| j^n \upsilon J' \right\rangle = \left\langle j^n \upsilon J \left\| \hat{\Gamma}^+ \hat{\Gamma} \hat{T}(E2) \hat{\Gamma}^+ \hat{\Gamma} \right\| j^n \upsilon J' \right\rangle$$

$$= -(-)^{n-\upsilon} \left\langle j^{2j+1-n} \upsilon J \left\| \hat{T}(E2) \right\| j^{2j+1-n} \upsilon J' \right\rangle$$

In mid-shell nuclei

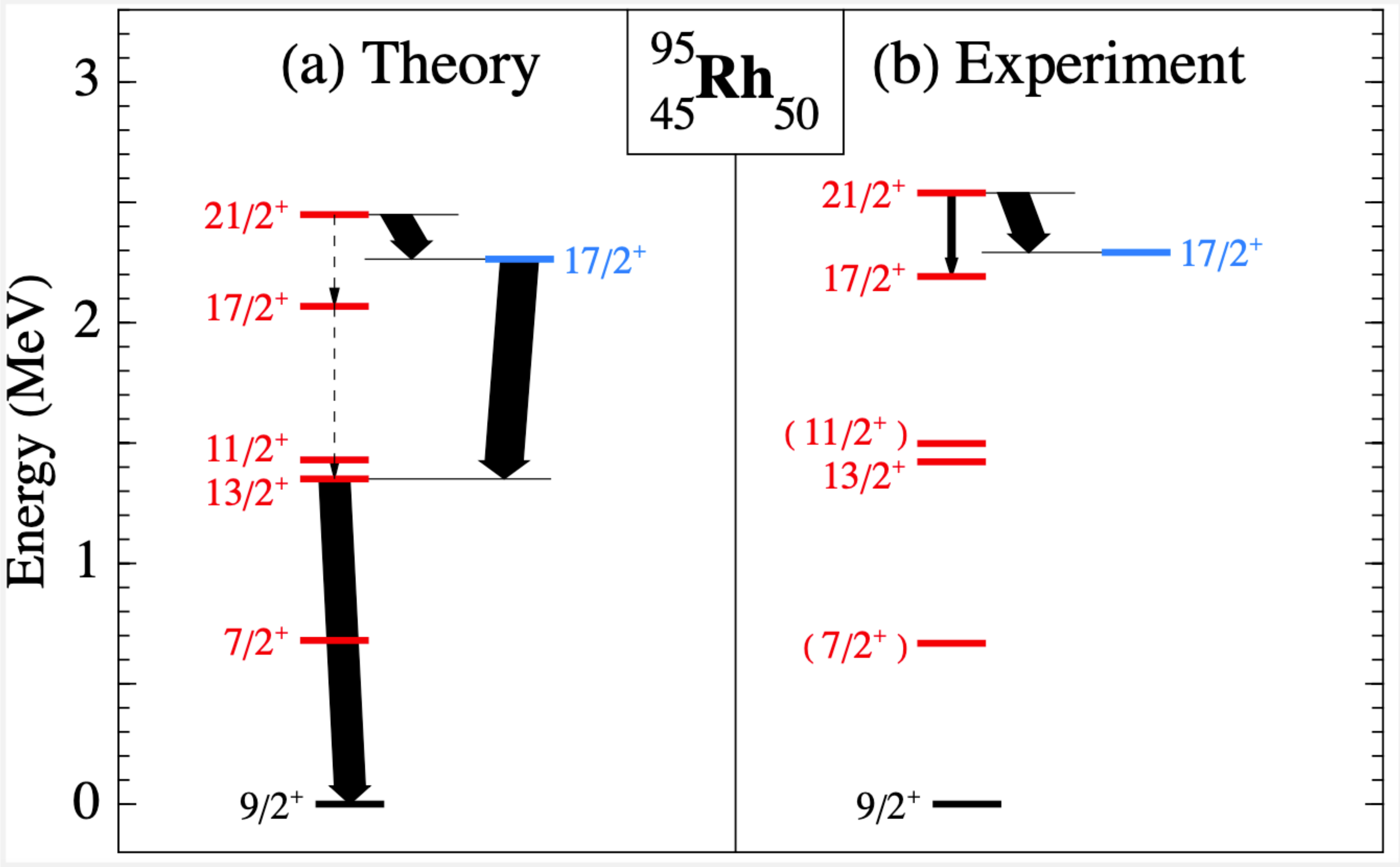
$$\left\langle j^{(2j+1)/2} \upsilon J \left\| \hat{T}(E2) \right\| j^{(2j+1)/2} \upsilon J' \right\rangle$$

$$= -\left\langle j^{(2j+1)/2} \upsilon J \left\| \hat{T}(E2) \right\| j^{(2j+1)/2} \upsilon J' \right\rangle = 0$$

∴ E2 transitions between states with the same seniority are forbidden in mid-shell nuclei.

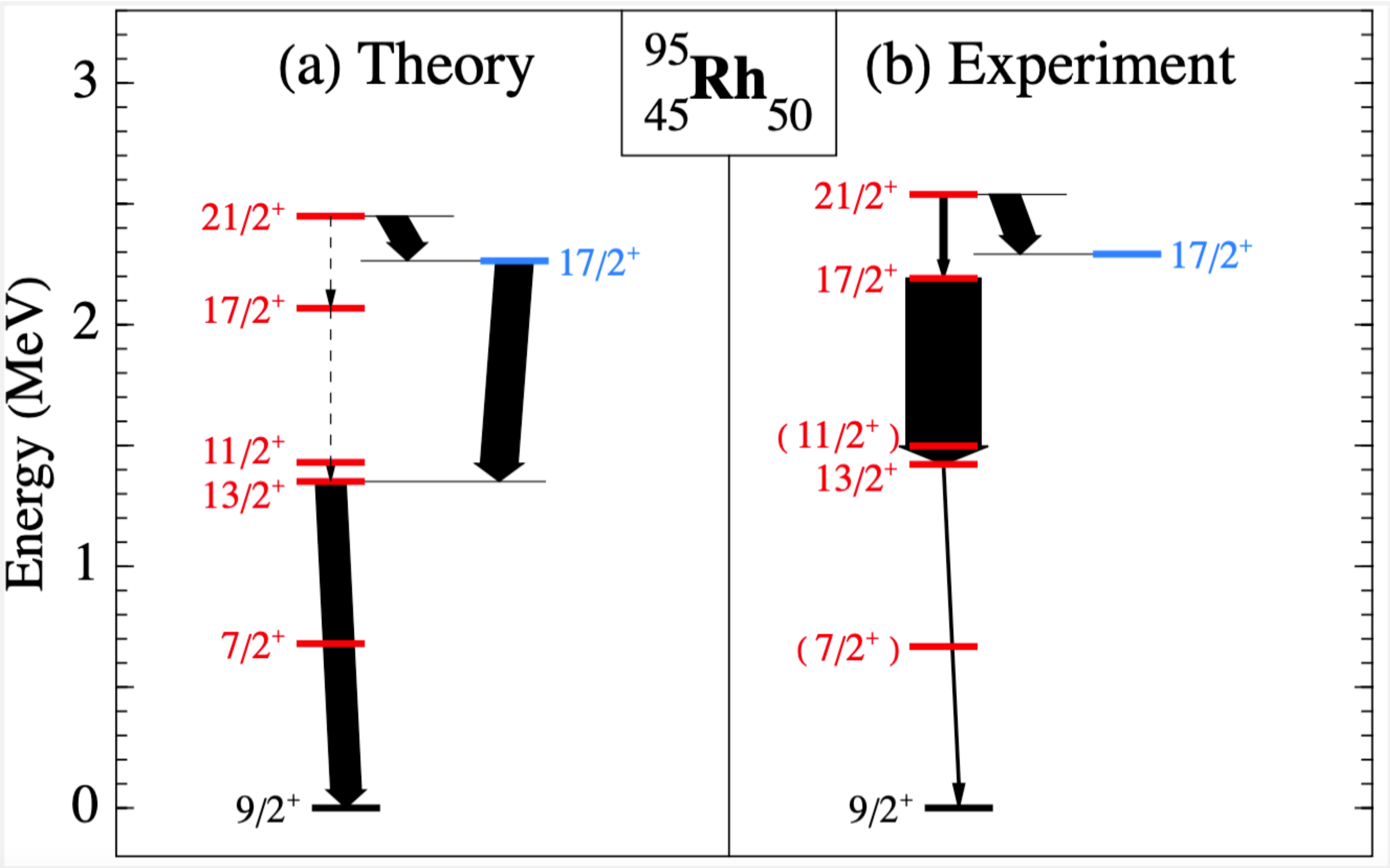
R.D. Lawson, *Theory of the Nuclear Shell Model*

E2 decay in ^{95}Rh



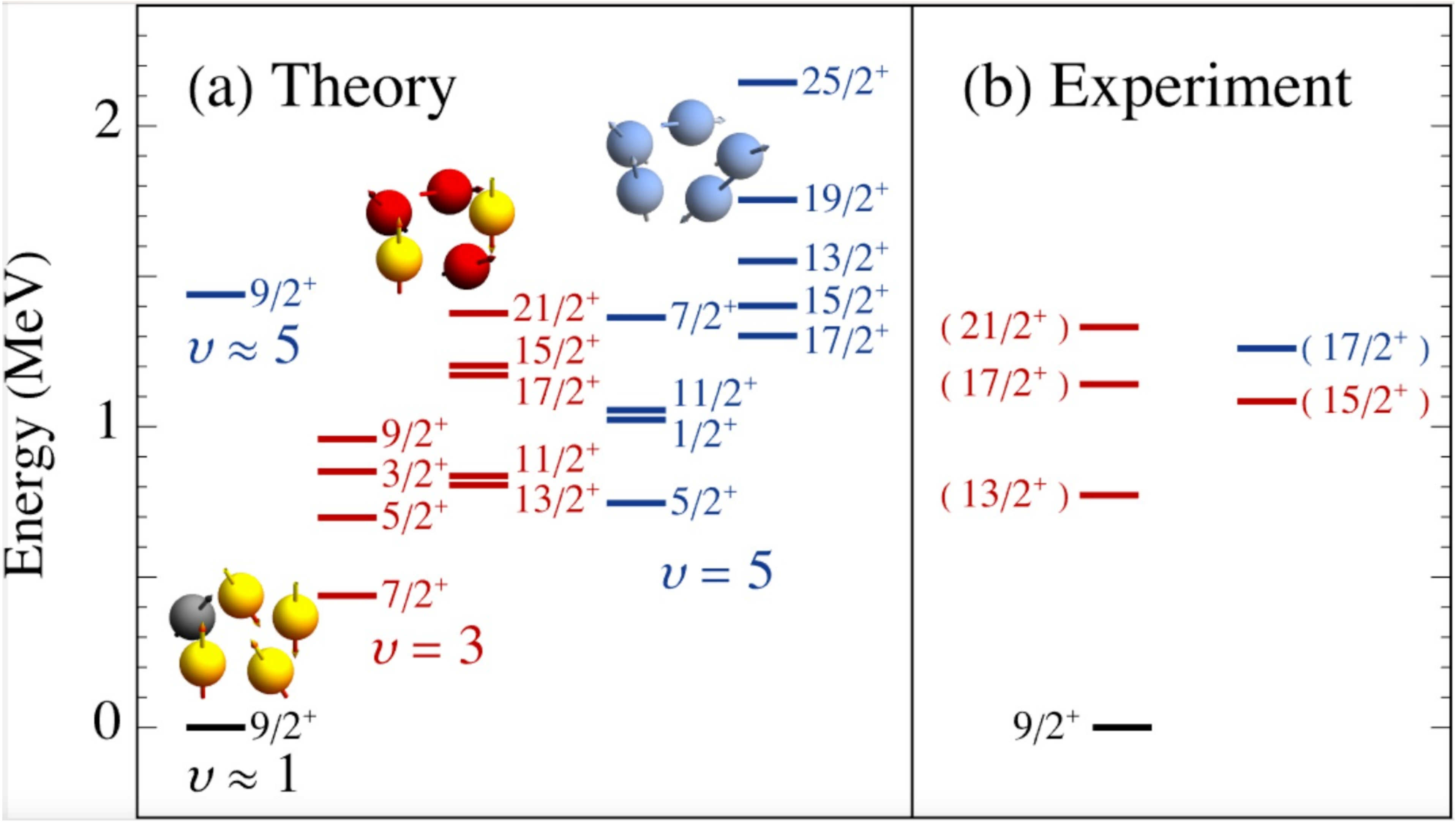
A. Amusa & R.D. Lawson, *High spin states in ^{94}Ru and ^{95}Rh*, Z. Phys. A **307** (1982) 333

E2 decay in ^{95}Rh



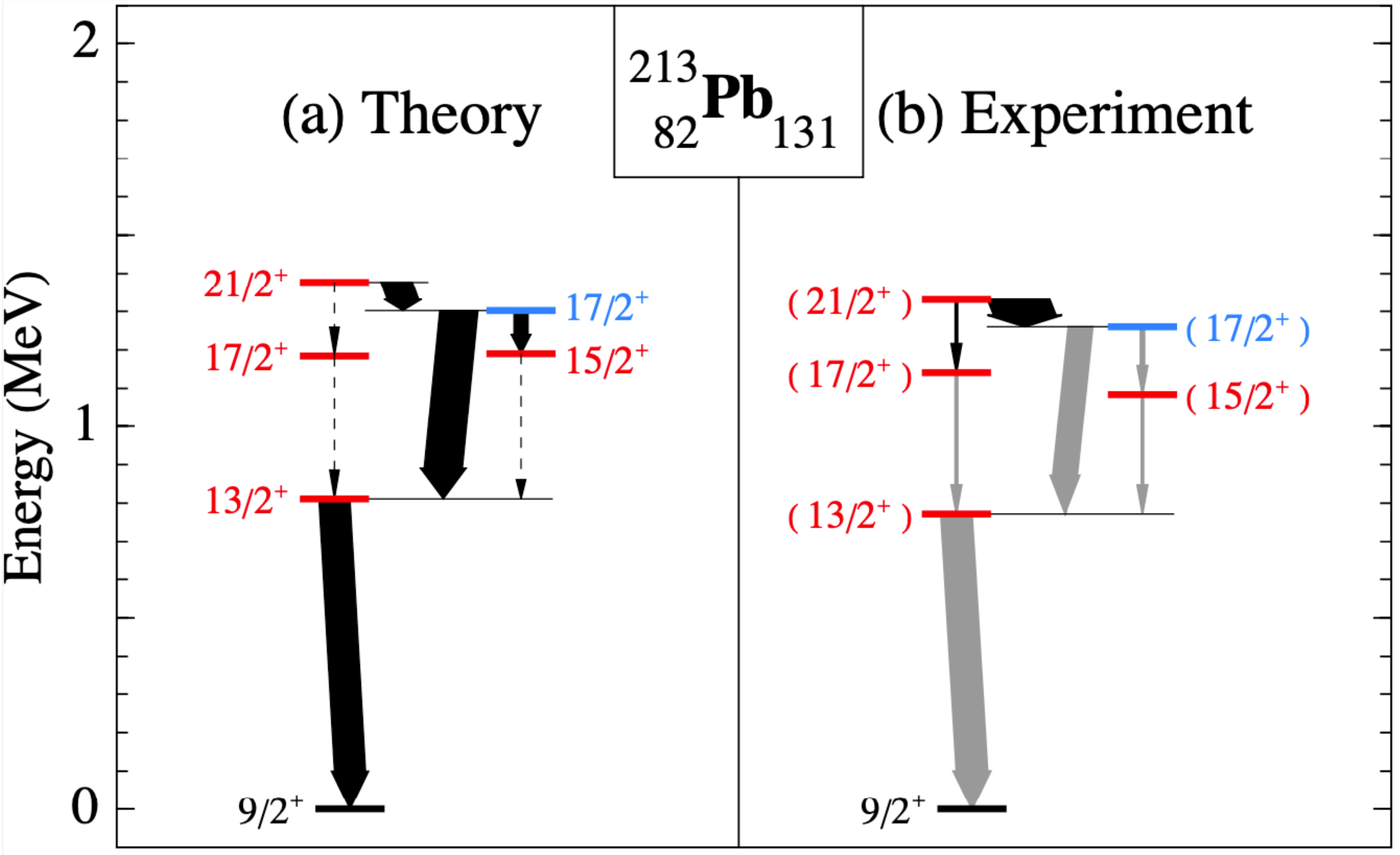
B. Das *et al.*, *Broken seniority symmetry in the semimagic nucleus ^{95}Rh*, unpublished

Spectrum of ^{213}Pb



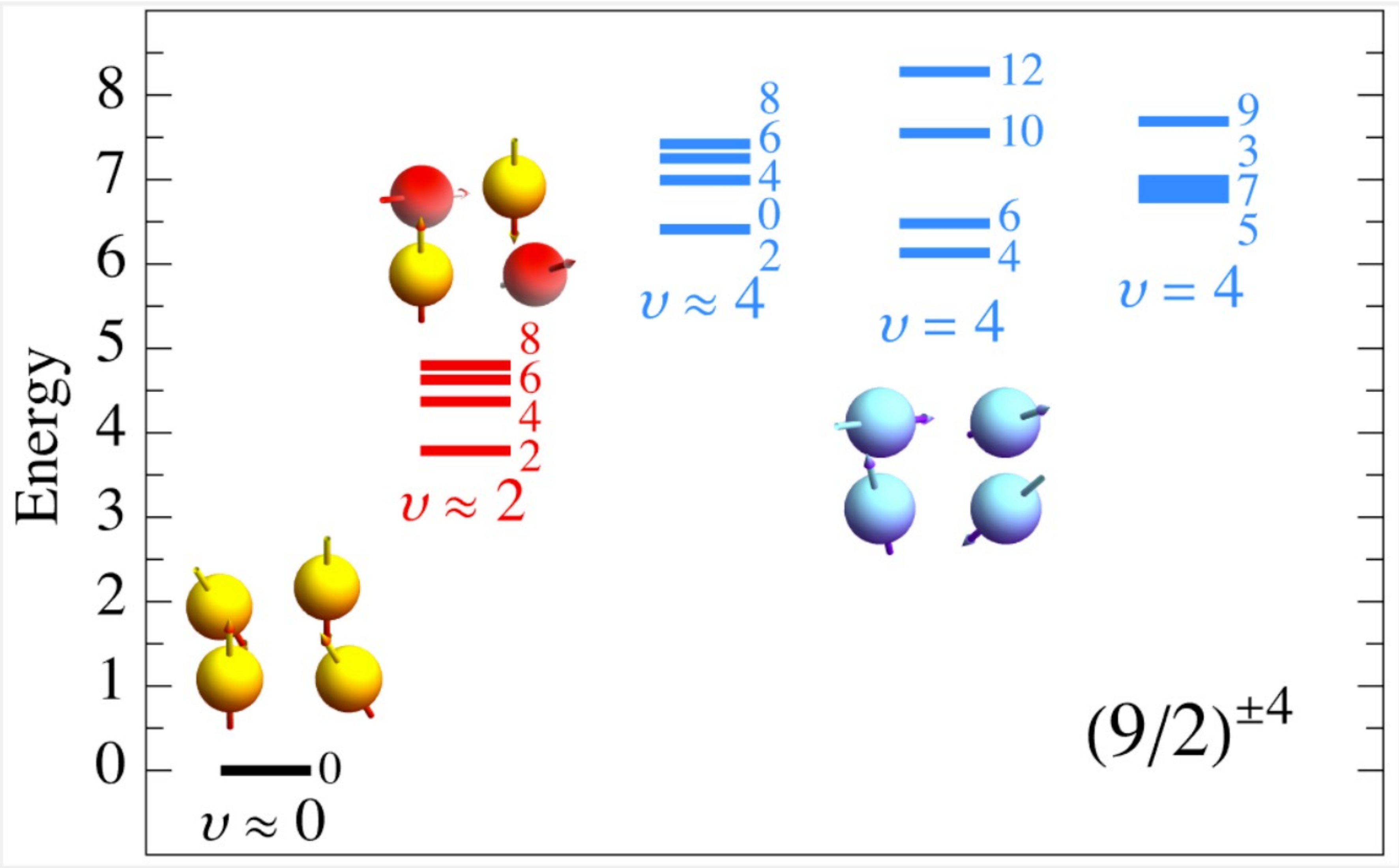
J.J. Valiente-Dobón *et al.*, *Manifestation of the Berry phase in the atomic nucleus ^{213}Pb*, Phys. Lett. B **816** (2021) 136183

E2 transitions in ^{213}Pb

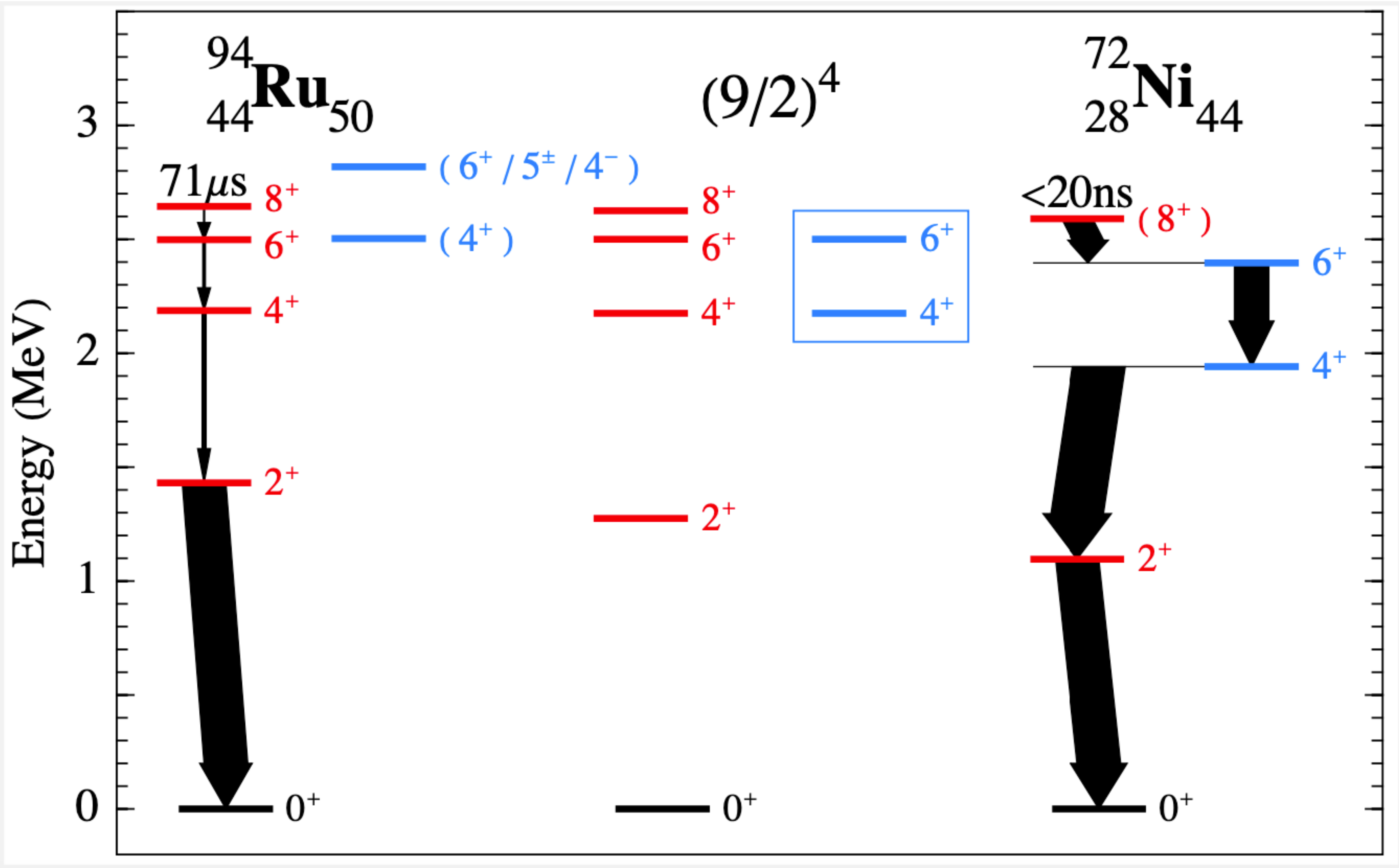


J.J. Valiente-Dobón *et al.*, *Manifestation of the Berry phase in the atomic nucleus ^{213}Pb*, Phys. Lett. B **816** (2021) 136183

Four/six nucleons in a j=9/2 shell

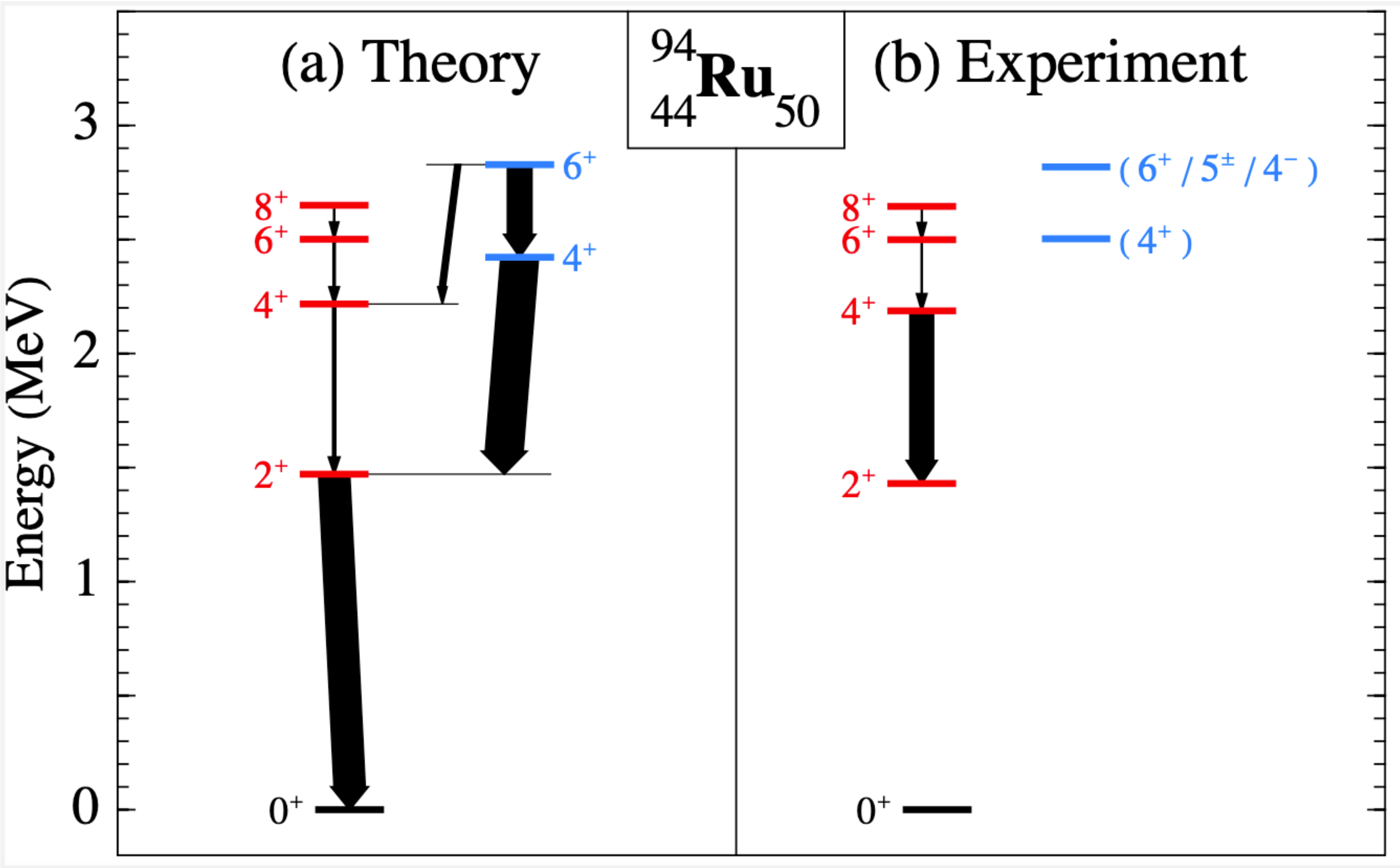


E2 decay in ^{94}Ru versus ^{72}Ni



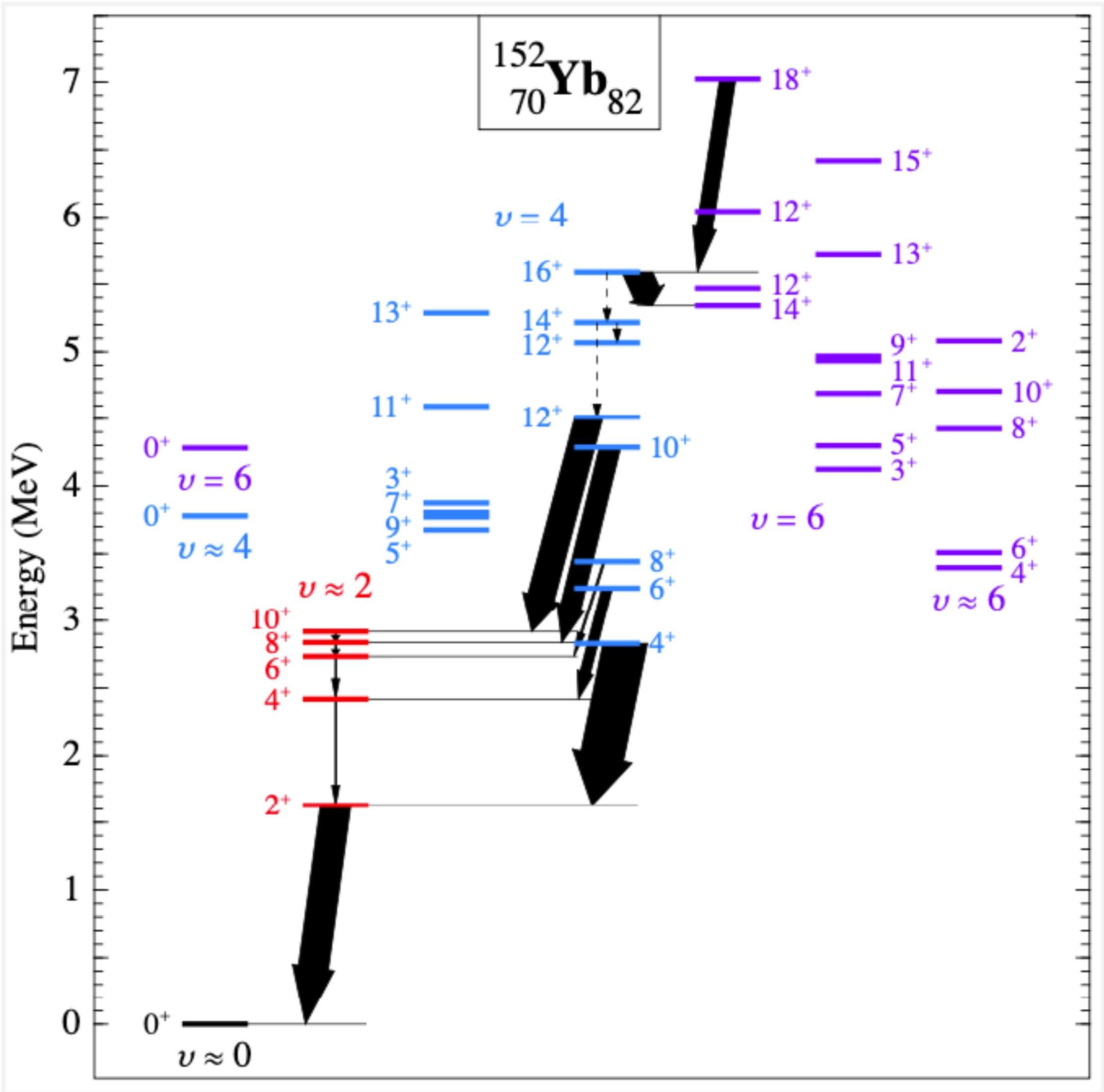
P. Van Isacker & S. Heinze, *Partial conservation of seniority and nuclear isomerism*, Phys. Rev. Lett. **100** (2008) 052501

E2 decay in ^{94}Ru



B. Das *et al.*, *Nature of seniority symmetry breaking in the semimagic nucleus ^{94}Ru*, Phys. Rev. C **105** (2022) L031304

Predicted isomers in ^{152}Yb



P. Van Isacker, *Seniority isomers and particle-hole conjugation*, Eur. Phys. J. Spec. Top. **108** (2023) 1

Conclusions

Symmetry considerations are useful to obtain insight in the structure of nuclear models.

Seniority is a relevant quantum number in semi-magic nuclei.

Seniority conservation in mid-shell nuclei is the consequence of a geometric phase associated with particle-hole conjugation.

The search for seniority isomers continues to inspire present-day experiments.

On exactitude in science

Del Rigor en la Ciencia

Jorge Luis Borges

En aquel Imperio, el Arte de la Cartografía logró tal Perfección que el mapa de una sola Provincia ocupaba toda una Ciudad, y el mapa del Imperio, toda una Provincia. Con el tiempo, estos Mapas Desmesurados no satisficieron y los Colegios de Cartógrafos levantaron un Mapa del Imperio, que tenía el tamaño del Imperio y coincidía puntualmente con él.

Menos Adictas al Estudio de la Cartografía, las Generaciones Siguientes entendieron que ese dilatado Mapa era Inútil y no sin Impiedad lo entregaron a las Inclemencias del Sol y los Inviernos. En los desiertos del Oeste perduran despedazadas Ruinas del Mapa, habitadas por Animales y por Mendigos; en todo el País no hay otra reliquia de las Disciplinas Geográficas.

Suárez Miranda, *Viajes de Varones Prudentes*, Libro Cuarto, Cap. XLV, Lérida, 1658.

On exactitude in science

Lewis Carroll's "*Sylvie and Bruno concluded*":

What is the largest map that would be useful?

> *"We very soon got the six yards to the mile. Then we tried a hundred yards to the mile. And then came the grandest idea of all! We actually made a map of the country, on the scale of a mile to the mile!"*
>
> *"Have you used it much?"*
>
> *"It has never been spread out, yet. The farmers objected: they said it would cover the whole country, and shut out the sunlight! So now we use the country itself, as its own map, and I assure you it does nearly as well."*