

On the nature of shape coexistence and quantum phase transition phenomena

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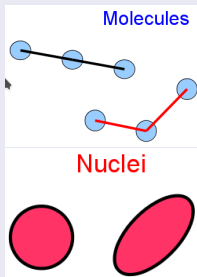
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What Shape Coexistence (SC) is?

It appears in quantum systems where eigenstates with very different density distribution coexist.

Therefore, the existence of a geometric interpretation is implicit.



Quadrupole shape invariants

$$q_{2,i} = \sqrt{5} \langle 0_i^+ | [\hat{Q} \times \hat{Q}]^{(0)} | 0_i^+ \rangle,$$

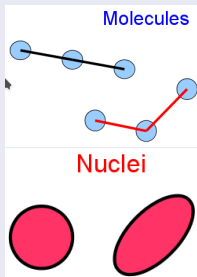
$$q_{3,i} = -\sqrt{\frac{35}{2}} \langle 0_i^+ | \hat{Q} \times \hat{Q} \times \hat{Q}^{(0)} | 0_i^+ \rangle,$$

$$q_2 = q^2, q_3 = q^3 \cos 3 \delta.$$

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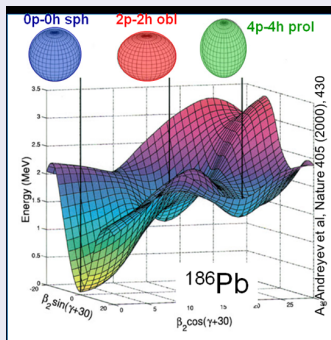
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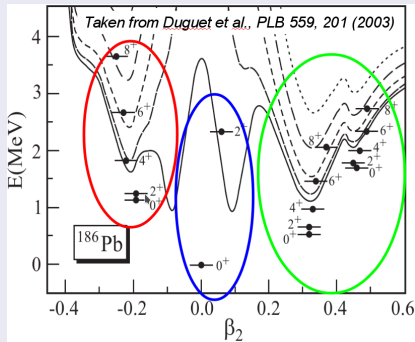
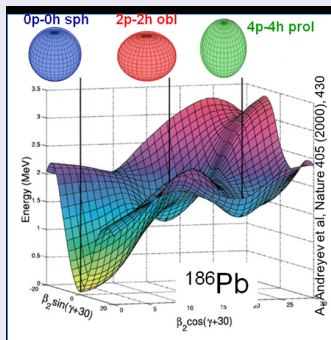
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Mean field: example of triple coexistence



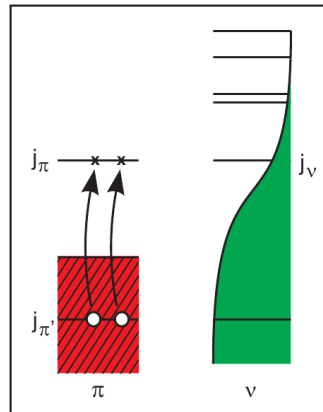
Mean field: example of triple coexistence



The angular momentum projected mean field plus the Generator Coordinate Method generates different bands with very different deformation.

Shell model. Where to be used

- For nuclei near to closed shells, either for neutrons or for protons, it can be energetically favorable to have excitations of 2p-2h, 4p-4h ... crossing the energy gap.
- The np-nh excitations have a lower excitation energy than expected due to the correlation energy: pairing and deformed correlations.
- Restricted to light and medium-heavy nuclei, at present.



"Sum" of configurations

$$\phi(J, M) = a(J, M)$$



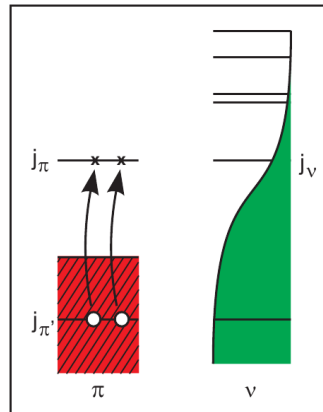
$$+ b(J, M)$$



In heavy nuclei the huge model space imposes some kind of truncation: symmetry dictated truncation.

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"Sum" of configurations

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A symmetry guided approximation: the IBM

Nucleons couple preferably in pairs with angular momentum either equal to 0 (S) or equal to 2 (D). Those pairs are then described by means of bosons: s and d.

$$s^\dagger, d_m^\dagger (m = 0, \pm 1, \pm 2)$$

$$s, d_m (m = 0, \pm 1, \pm 2)$$

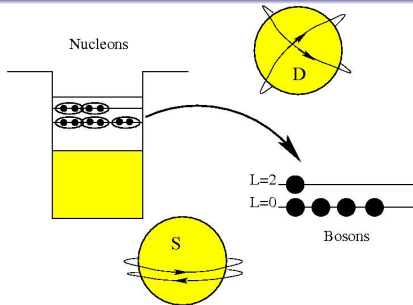
with

$$[\gamma_{lm}, \gamma_{l'm'}^\dagger] = \delta_{ll'} \delta_{mm'},$$

$$[\gamma_{lm}^\dagger, \gamma_{l'm'}^\dagger] = 0, [\gamma_{lm}, \gamma_{l'm'}] = 0$$

Simplified Hamiltonian

$$\hat{H}_{ECQF} = \varepsilon \hat{n}_d + \kappa \hat{Q} \cdot \hat{Q} + \kappa' \hat{L} \cdot \hat{L}$$

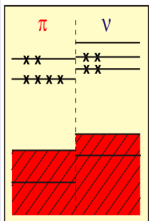


Model based on a $u(6)$ spectrum generator algebra. It is especially suited for medium and heavy-mass nuclei.

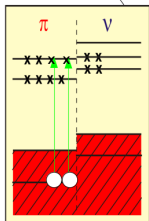
The number of bosons, N , corresponds the number of nucleons pairs, regardless its proton, neutron, particle or hole nature.

How IBM with configuration mixing works

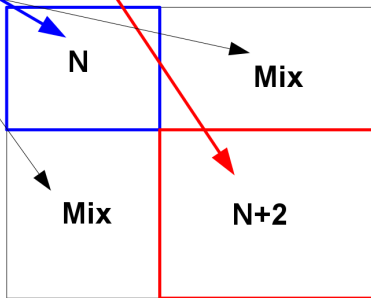
$$\hat{H} = \hat{P}_N^\dagger \hat{H}_{ECQF}^N \hat{P}_N + \hat{P}_{N+2}^\dagger (\hat{H}_{ECQF}^{N+2} + \Delta^{N+2}) \hat{P}_{N+2} + \hat{V}_{mix}^{N,N+2}$$



N



N+2

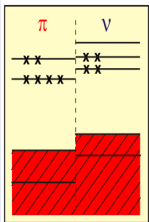


A different Hamiltonian, \hat{H}_{ECQF}^N and \hat{H}_{ECQF}^{N+2} , acts on the regular [N] and intruder [N+2] sectors, separately.

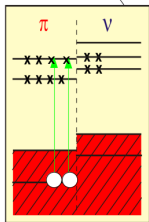
The offset Δ^{N+2} and the mixing interaction $\hat{V}_{mix}^{N,N+2}$ should be provided.

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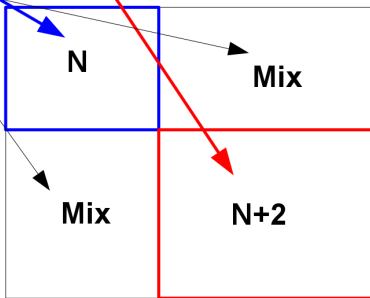
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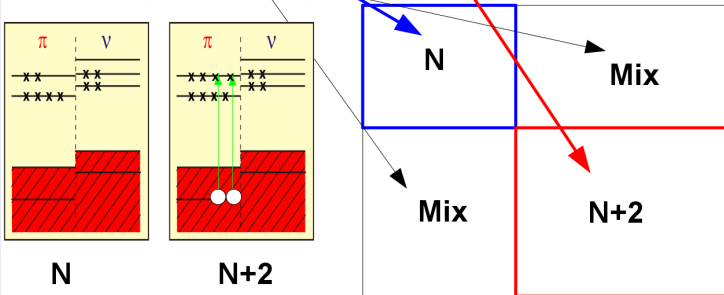


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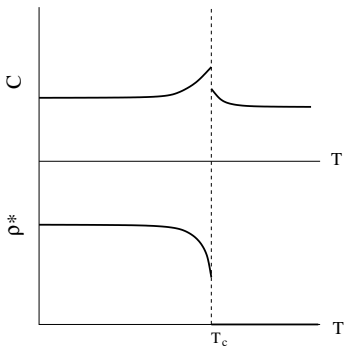
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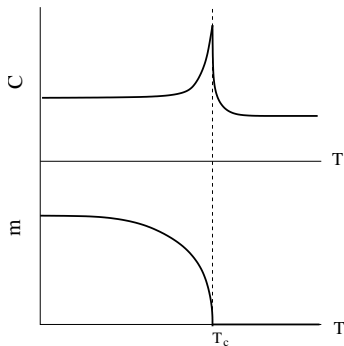
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Examples of Macroscopic Phase Transitions



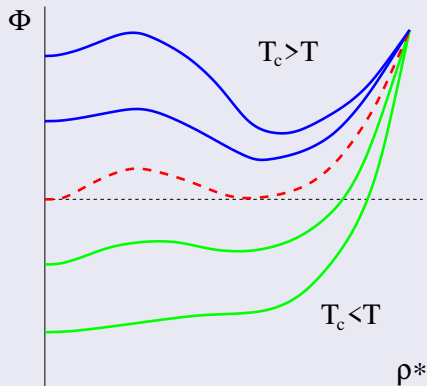
First order phase transition.
Liquid-gas



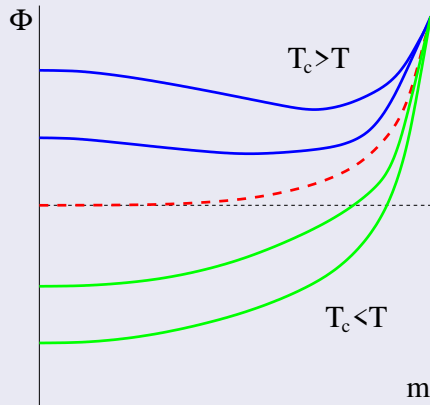
Second order phase transition.
Paramagnetic-ferromagnetic

Inside a Quantum Phase Transition

First order



Second order

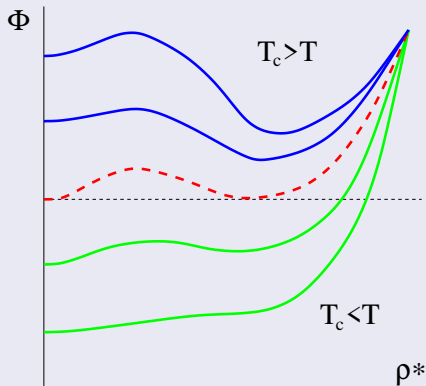


Φ in the Landau theory

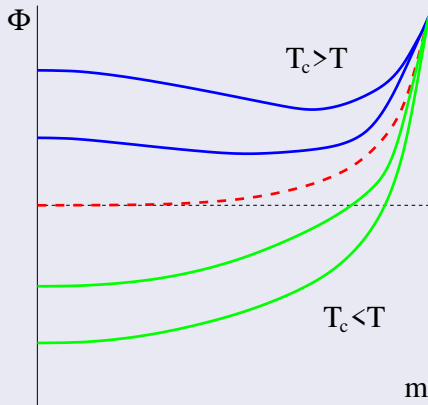
$$\Phi = A(T, \dots)\beta^4 + B(T, \dots)\beta^2 + C(T, \dots)\beta$$

Inside a Quantum Phase Transition

First order



Second order



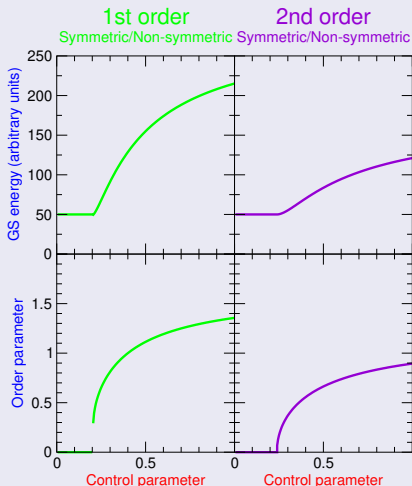
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What a Quantum Phase Transition (QPT) is?

A QPT appears when the ground state quantum system experiences a sudden change in its structure (order parameter) when a parameter that affects the Hamiltonian (control parameter) slightly changes around its critical value. This transitions are assumed to occurs at zero temperature.

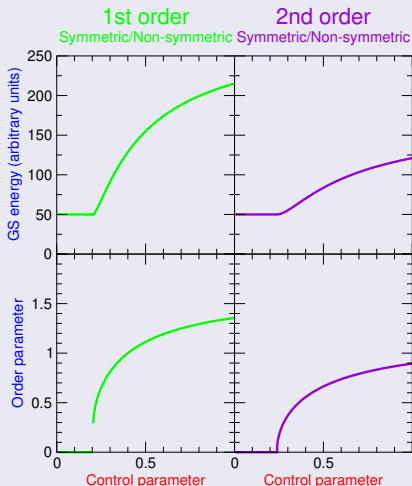
$$\hat{H} = (1 - \xi)\hat{H}_1 + \xi\hat{H}_2$$



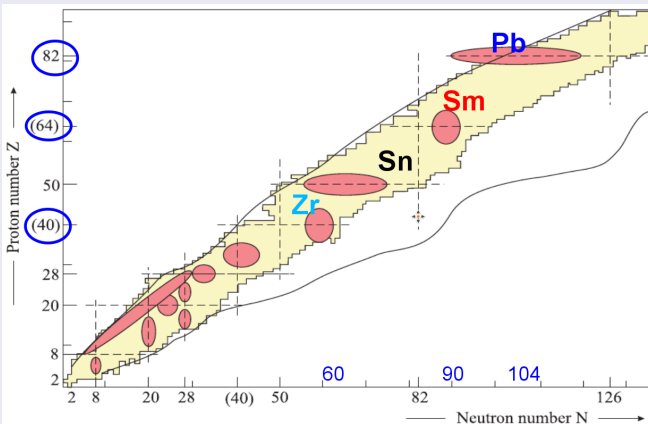
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Regions of interest



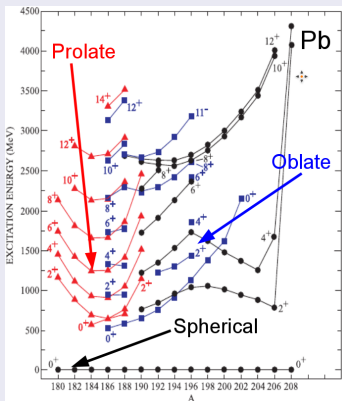
Pb and Sn regions are ideal regions to study the importance of Shape Coexistence (SC).

Sm region is the paradigm of Quantum Phase Transition (QPT) region.

Zr region seems to be the ideal region to study the interplay between SC and QPT.

Shape coexistence

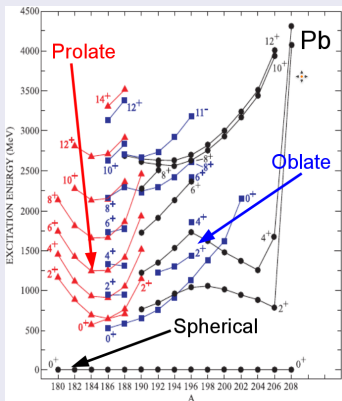
Pb isotopes



Three families of states are present.

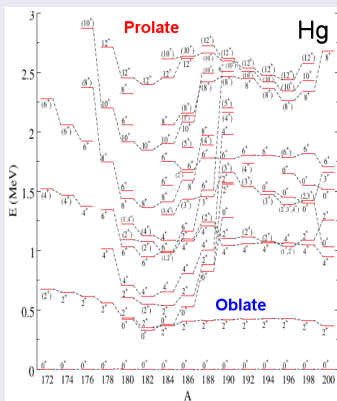
Shape coexistence

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Three families of states are present.

Hg isotopes

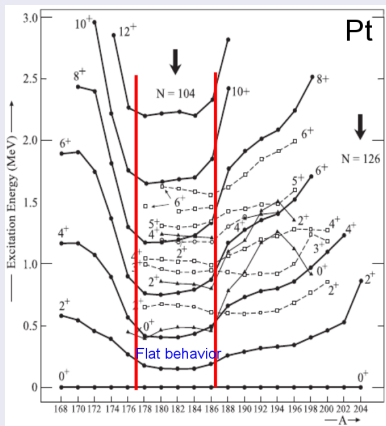


The presence of two families of states is self-evident.

Shape coexistence indicators

Lead region

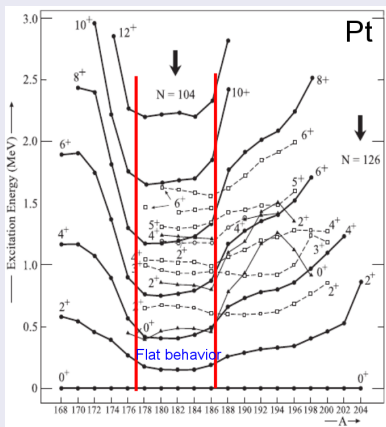
Pt isotopes



In this case only a *suspicious* flat area appears at midshell.

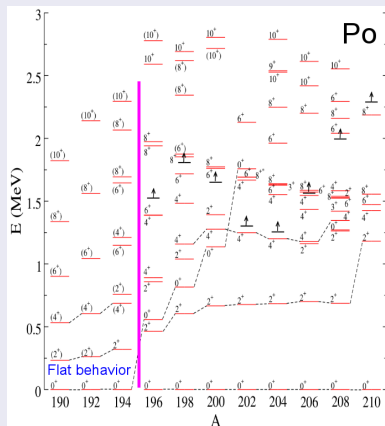
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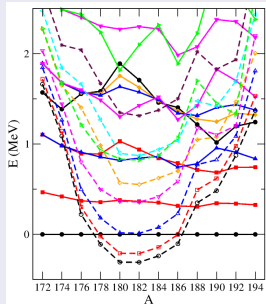
Po isotopes



Here, we hardly reach the midshell and no clear conclusions can be obtained.

Unperturbed energies

Pt isotopes

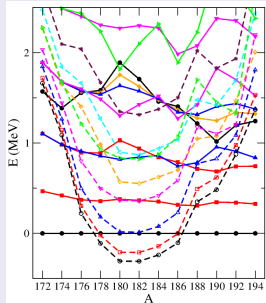


The parabolic energy systematics is clear and the intruder configuration becomes the ground state.

JEGR and K. Heyde, NPA **825**, 39 (2009).

Unperturbed energies

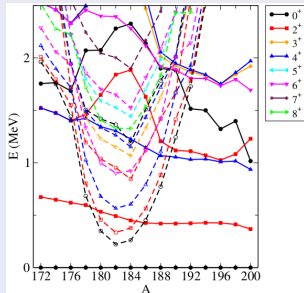
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Hg isotopes

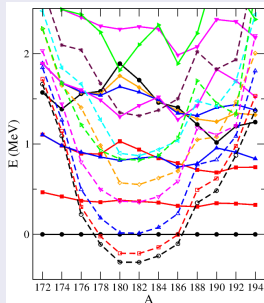


The parabolic energy systematics is obvious, but the ground state always presents a regular nature. JEGR and K. Heyde,

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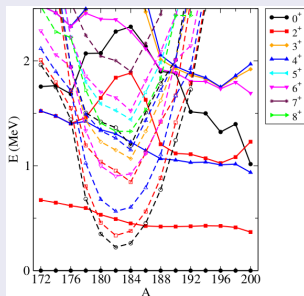
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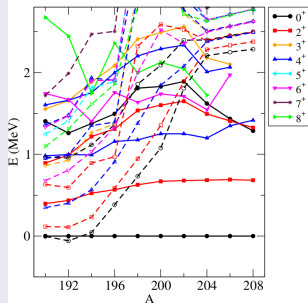
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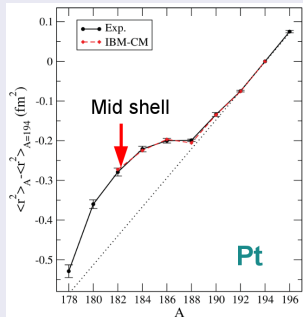
Po isotopes



Intruder and regular configurations are almost degenerated at midshell. JEGR and K. Heyde, PRC **92**, 034309 (2015).

Radii

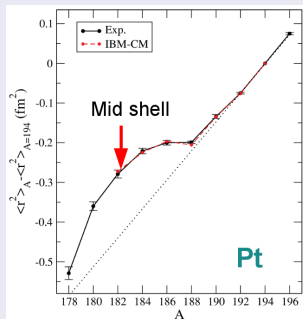
Pt isotopes



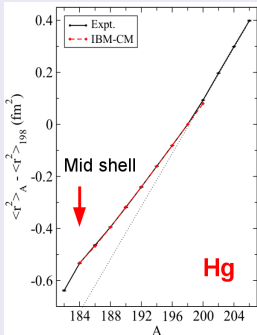
The three cases show a clear departure from the spherical trend.

Radii

Pt isotopes



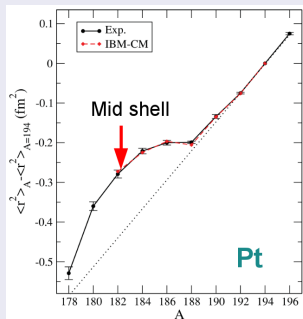
Hg isotopes



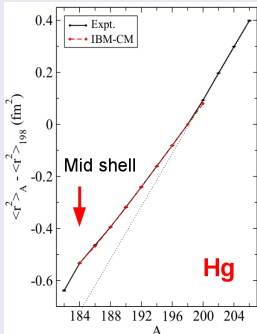
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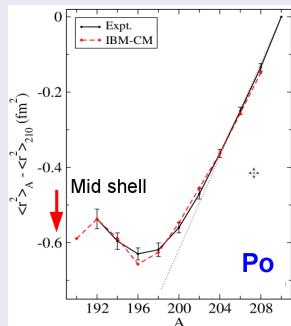
Pt isotopes



Hg isotopes

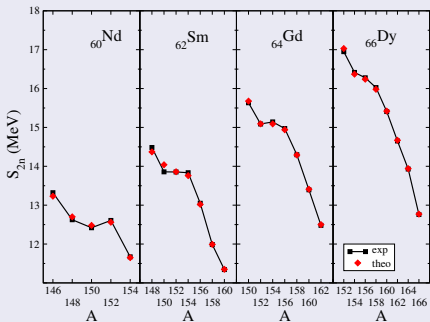


Po isotopes



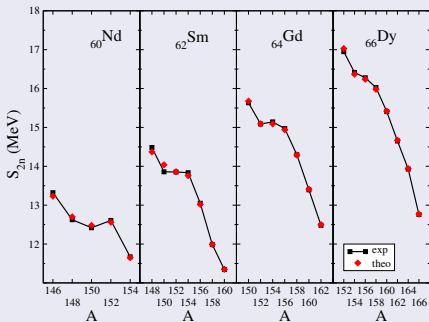
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Quantum Phase Transition indicators in the rare-earth region: Type I

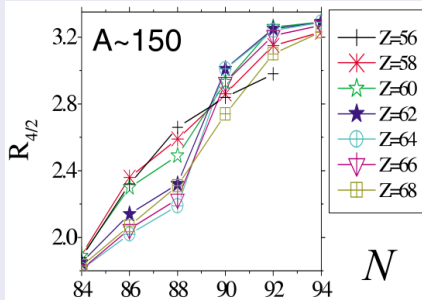
Two-neutron separation energy.
Why?

S_{2n} is connected with the first derivative of the binding energy. Its discontinuity is a hint for the onset a first order QPT.

Quantum Phase Transition indicators in the rare-earth region: Type I

Two-neutron separation energy.
Why?

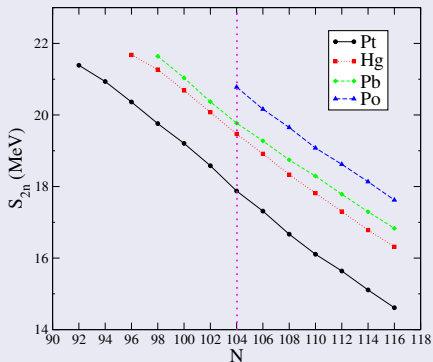
S_{2n} is connected with the first derivative of the binding energy. Its discontinuity is a hint for the onset a first order QPT.

 $E(4_1^+)/E(2_1^+)$ 

$E(4_1^+)/E(2_1^+)$ can be used as an order parameter and, therefore, it is a key observable to find where a QPT develops.

Hints for QPTs in lead region?

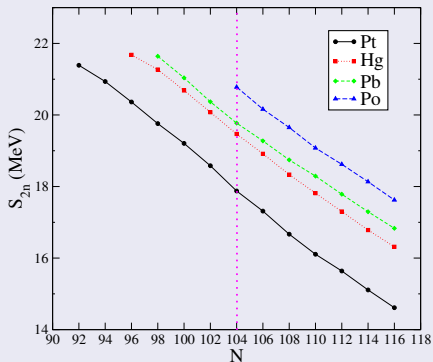
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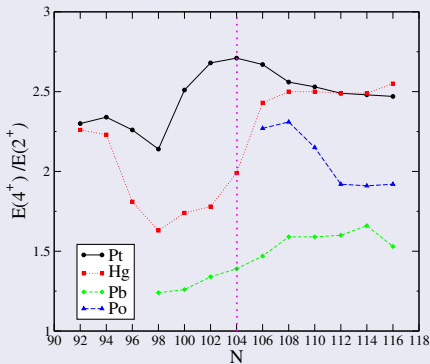
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Hints for QPTs in lead region?

Two-neutron separation energy



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 $E(4_1^+)/E(2_1^+)$ 

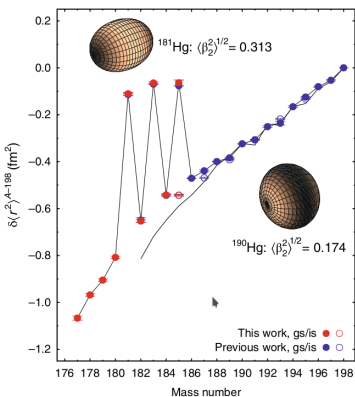
$E(4_1^+)/E(2_1^+)$ does not present neither the typical behaviour of an order parameter. Only Pt isotopes resemble the expected trend for an order parameter when approaching midshell from the left.

Something in common?

- Rapid change in the structure of certain states, including the ground-state.
- Lowering of certain 0^+ states.
- At the mean-field level several minima coexist.
- Onset of deformation: radii and isotopic shift.

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<https://doi.org/10.1038/s41567-018-0292-8>

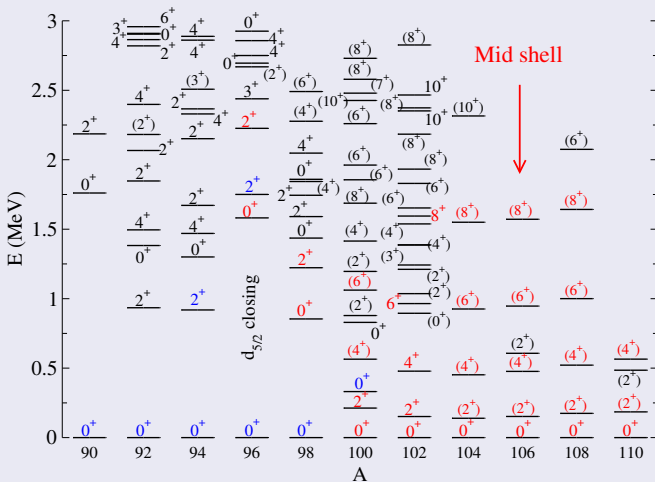
Characterization of the shape-staggering effect in mercury nuclei

B. A. Marsh^{1*}, T. Day Goodacre^{1,2,3*}, S. Sels^{3,3*}, Y. Tsunoda⁴, B. Andel⁵, A. N. Andreyev^{6,7},
 N. A. Althubiti², D. Atanasov⁸, A. E. Barzakh⁹, J. Billowes², K. Blaum¹⁰, T. E. Cocolios^{2,3}, J. G. Cubiss⁶,
 J. Dobaczewski⁶, G. J. Farooq-Smith^{2,3}, D. V. Fedorov⁹, V. N. Fedosseev¹, K. T. Flanagan⁷, L. P. Gaffney^{3,10},
 L. Ghys⁷, M. Huyse³, S. Kreim⁸, D. Lunney¹¹, K. M. Lynch¹, V. Manea⁸, Y. Martinez Palenzuela³, P. L. Molkanov⁹,
 T. Otsuka^{3,4,12,13,14}, A. Pastore⁵, M. Rosenbusch^{13,15}, R. E. Rossel¹, S. Rothe¹², L. Schweikhard¹⁵, M. D. Seliverstov⁹,
 P. Spagnoletti¹⁰, C. Van Bevern³, P. Van Duppen³, M. Veinhard¹, E. Verstraelen¹, A. Welker¹⁶, K. Wendt¹⁷,
 F. Wienholtz¹⁵, R. N. Wolf⁸, A. Zadornaya³ and K. Zuber¹⁶

“The shape staggering effect manifests characteristic features of a **quantum phase transition**: in a given nucleus, different phases ... By making small changes in the control parameter, which in this case is the neutron number, the system alternates between the two phases...”

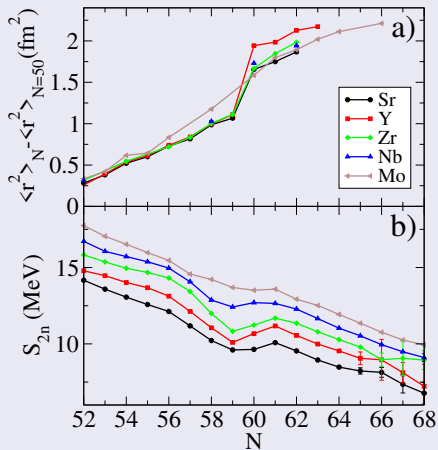
Experimental evidences

Energy systematics for even-even Zr nuclei



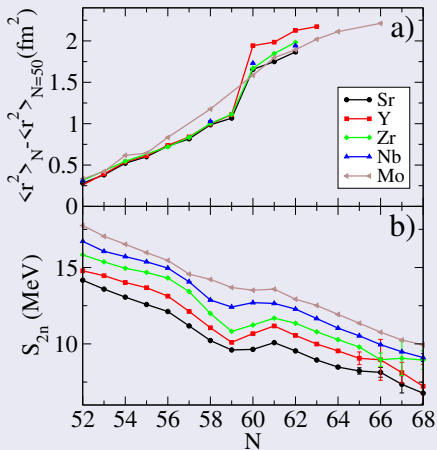
Blue labels for spherical states while red labels for deformed ones.

Radii and two-neutron separation energies



- Radii show a sudden increase at $N = 60$ for Sr, Y, Zr, and Nb being almost smoothed out for Mo.
- S_{2n} present a similar trend that the observed one in rare-earth region, although, once more, the *discontinuity* is smoothed out for Mo.

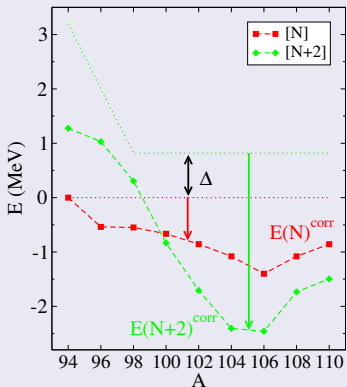
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Unperturbed energies

Correlation energies (Zr case)



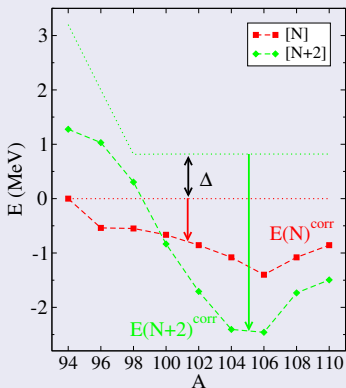
The intruder configuration becomes the ground state for

$A = 100$ and onwards. JEGR and K. Heyde, PRC **100**,

044315 (2019).

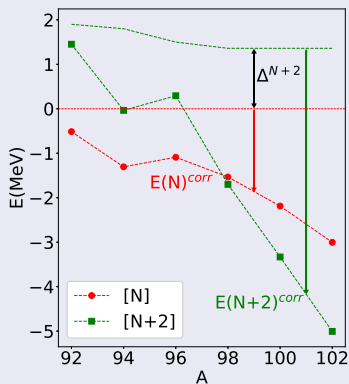
Unperturbed energies

Correlation energies (Zr case)



The intruder configuration becomes the ground state for $A = 100$ and onwards. JEGR and K. Heyde, PRC **100**, 044315 (2019).

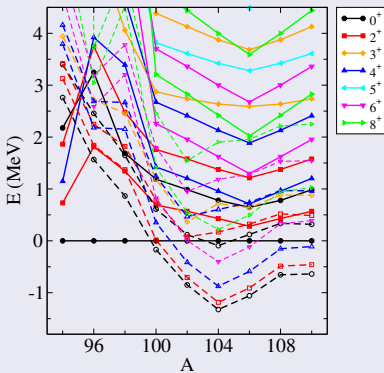
Correlation energies (Sr case)



The intruder configuration becomes the ground state for $A = 98$ and onwards. E. Maya-Barbecho and JEGR, PRC **105**, 034341 (2022).

Unperturbed energies

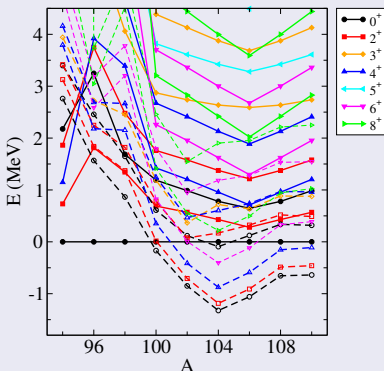
Unperturbed spectra (Zr case)



Intruder states present a *parabolic* behaviour while regular ones *flat*.

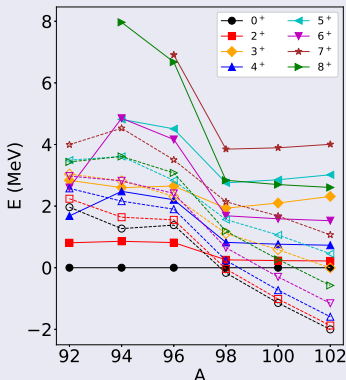
Unperturbed energies

Unperturbed spectra (Zr case)



Intruder states present a *parabolic* behaviour while regular ones *flat*.

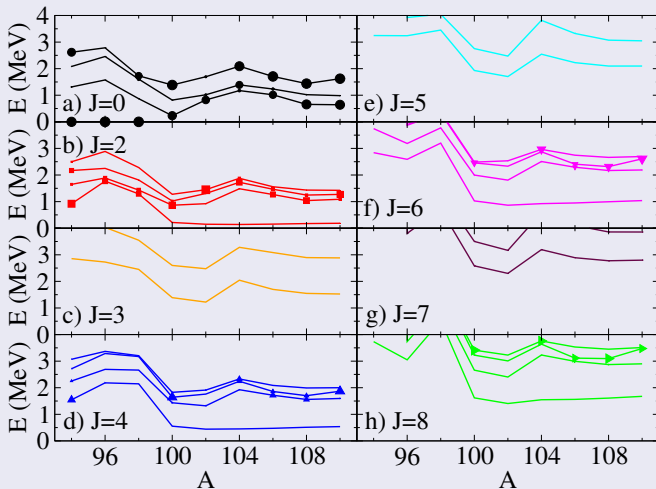
Unperturbed spectra (Sr case)



Intruder states present a *parabolic* behaviour while *flat* the regular ones.

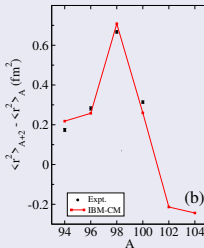
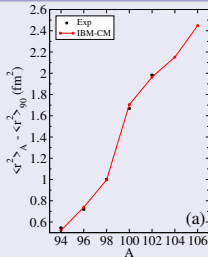
Wave function

Regular component and energy (Zr case)

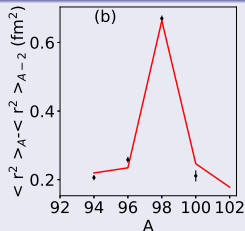
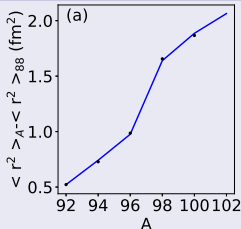


Radii and isotope shift

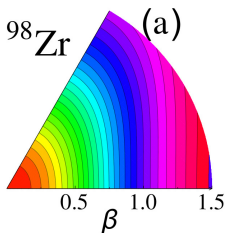
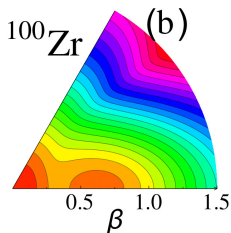
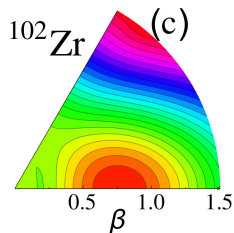
Zr isotopes



Sr isotopes

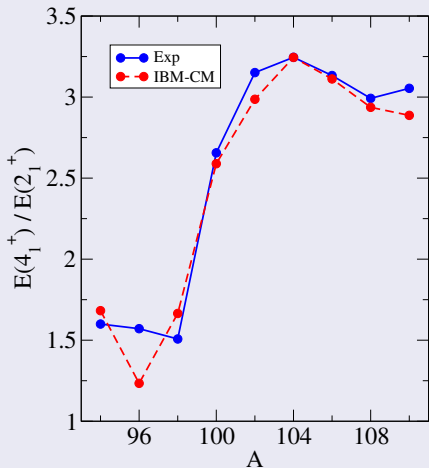


Mean-field energy surfaces

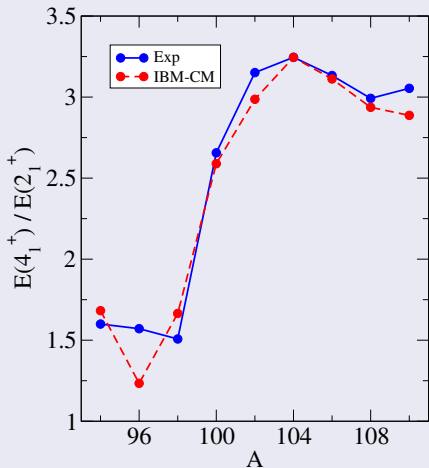
 ^{98}Zr  ^{100}Zr  ^{102}Zr 

Mean field energy surface shows up a rapid evolution from a spherical to a well deformed shape. ^{100}Zr shows the coexistence of two minima.

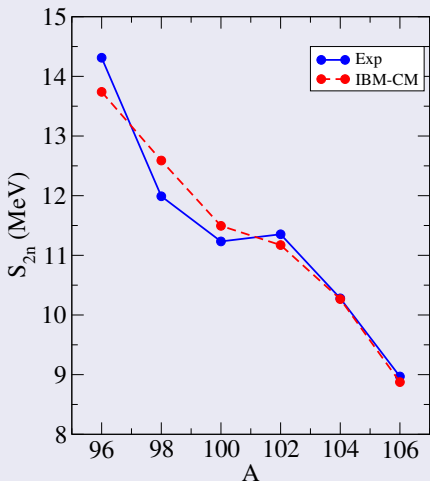
Hints pointing to a QPT: Type II

 $E(4_1^+)/E(2_1^+)$ (Zr case)

Hints pointing to a QPT: Type II

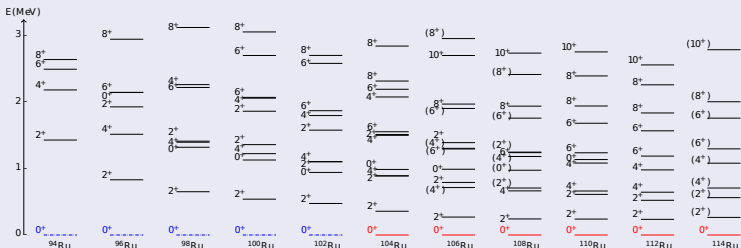
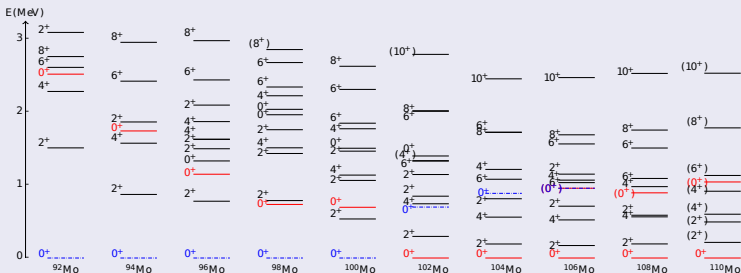
 $E(4_1^+)/E(2_1^+)$ (Zr case)

Two-neutron separation energy (Zr case)



Experimental evidences

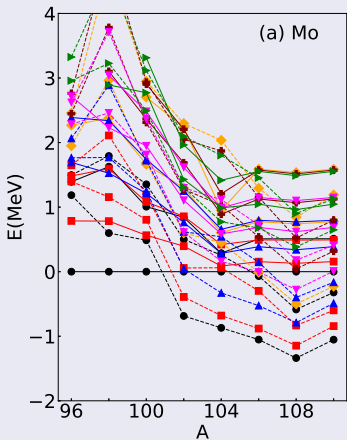
Energy systematics of Mo and Ru isotopes



Blue labels for spherical states while red labels for deformed ones.

Crossing or not

Mo

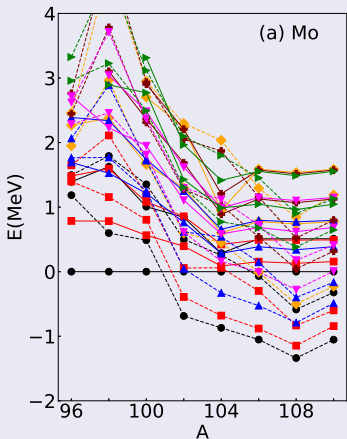


Intruder states present a *parabolic* behaviour while regular ones *flat*. E. Maya-Barbecho, S. Baid, J.M. Arias, and

JEGR, PRC **108**, 034316 (2023).

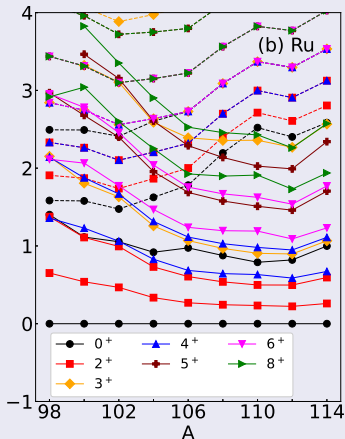
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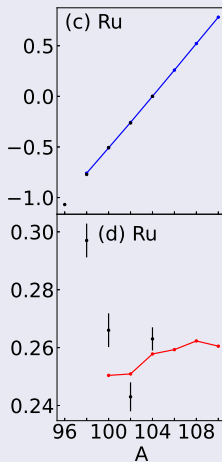
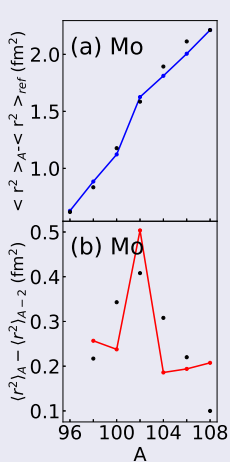
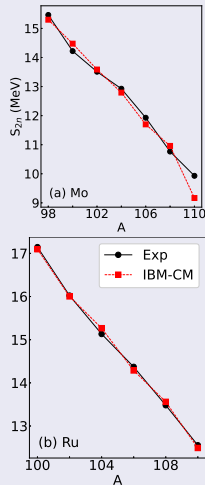
Ru



Intruder states present a *parabolic* behaviour while regular ones *flat*. E. Maya-Barbecho, S. Baid, J.M. Arias, and JEGR, PRC **108**, 034316 (2023)

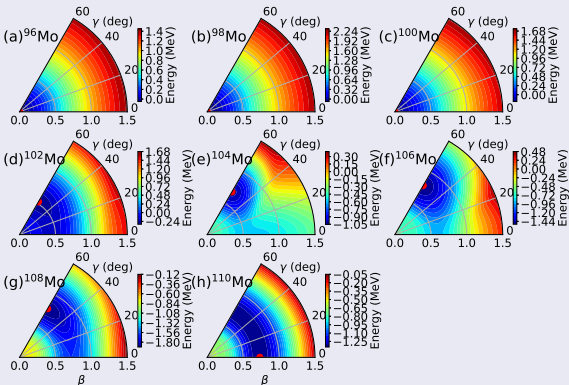
Radii and S_{2n}

Radii and isotope shift

 S_{2n} 

Mean-field energy surfaces

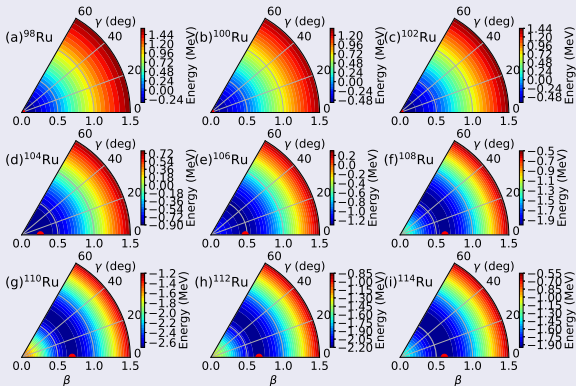
Mo isotopes



Mean field energy surface shows up a rapid evolution from a spherical to a well deformed shape. $^{102}\text{--}^{104}\text{Mo}$ shows the coexistence of two minima.

Mean-field energy surfaces

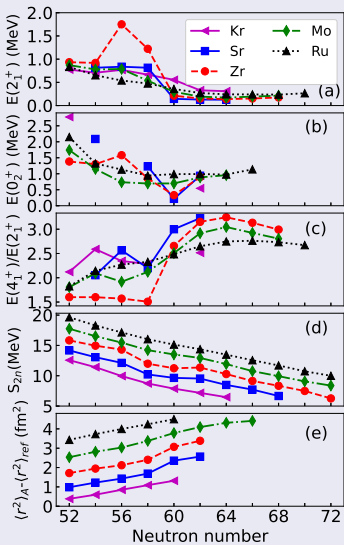
Ru isotopes



Mean field energy surface shows up a slow evolution from a spherical to a well deformed shape. ^{104}Ru shows a rather flat minimum.

Global view

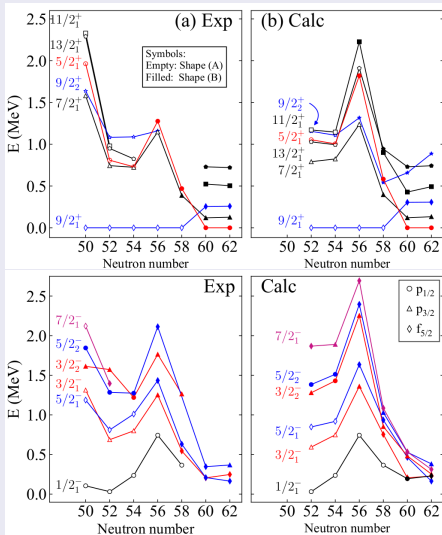
Relevant systematics



The different systematics points toward a QPT/Shape coexistence area in Sr, Zr, and Mo, being Kr and Ru at the border of this area.

Experimental evidences

Nb energy systematics and IBFM-CM results



N. Gavrielov, Phys. Rev. C 108, 014320 (2023); N. Gavrielov, A. Leviatan, and F. Iachello, Phys. Rev. C 106, L051304 (2022) introduced the IBFM with configuration mixing (IBFM-CM) in two seminal works.

$g_{9/2}$ for positive parity and $p_{1/2}$, $p_{3/2}$, and $f_{5/2}$ for negative parity.

The IBM-CM intrinsic state: the shape of odd-even nuclei

The extension of the IBM-CM intrinsic state

$$H_{CM}^{IBM} = \begin{pmatrix} E^B(N, \beta, \gamma) & \Omega^B(\beta) \\ \Omega^B(\beta) & E^B(N+2, \beta, \gamma) \end{pmatrix} \rightarrow \begin{array}{l} \text{Energy surface and} \\ \text{equilibrium value} \\ \text{of deformation parameters} \end{array}$$

A. Frank, O. Castaños, P. Van Isacker, and E. Padilla, AIP Conf. Proc. 638, 23 (2002); A. Frank, P. Van Isacker, and F. Iachello, PRC 73, 061302(R) (2006).

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The IBFM-CM formalism for multiple j 's (preliminary)

$$H_{CM}^{IBFM} = \begin{pmatrix} [E^{IBFM}(N, \beta, \gamma)] & [\Omega(\beta)] \\ [\Omega(\beta)] & [E^{IBFM}(N+2, \beta, \gamma)] \end{pmatrix}$$

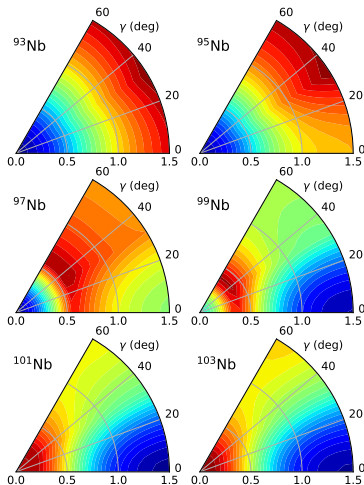
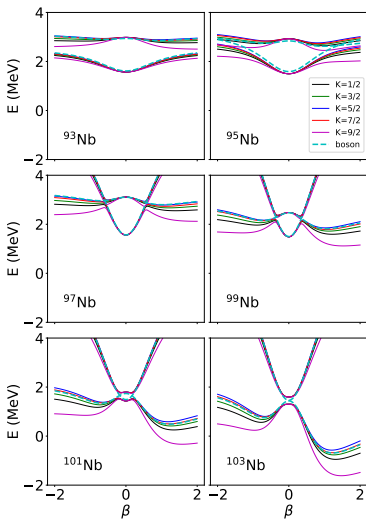
$$E^{IBFM}(N, \beta, \gamma)_{jmj'm'} = \delta_{jj'}\delta_{mm'}(E^B(N, \beta, \gamma) + \epsilon_j) + V^{BF}(N, \beta, \gamma)_{jmj'm'}$$

$$\Omega(\beta)_{jmj'm'} = \delta_{jj'}\delta_{mm'}\Omega^B(\beta)$$

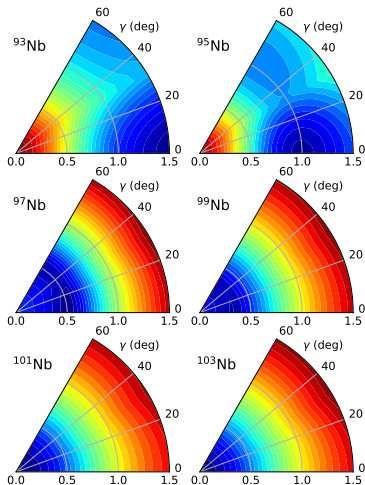
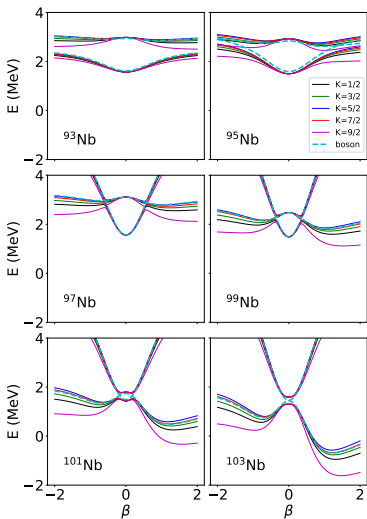
$\sum_i (2j_i + 1)$ energy surfaces and corresponding equilibrium parameters

(A. Leviatan, PLB 209, 415 (1988); C.E. Alonso, J.M. Arias, F. Iachello, and A. Vitturi, NPA 539, 59 (1992).)

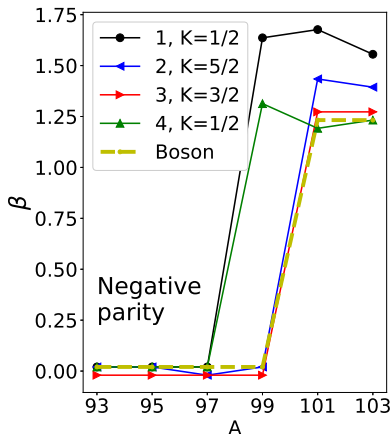
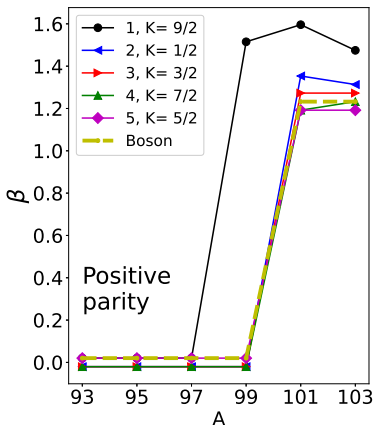
Nb positive parity: ground state energy surface (preliminary)



Nb positive parity: first intruder energy surface (preliminary)



Quantum Phase Transitions in Nb: the onset of deformation (preliminary)



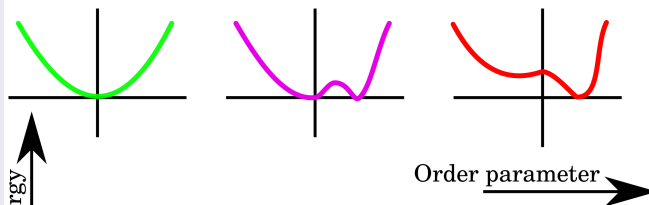
(Studied in the case of a single configuration in [D. Petrellis, A. Leviatan, F. Iachello, PLB 705, 379 \(2011\); Ann.](#)

[Phys. 326, 926 \(2011\).](#))

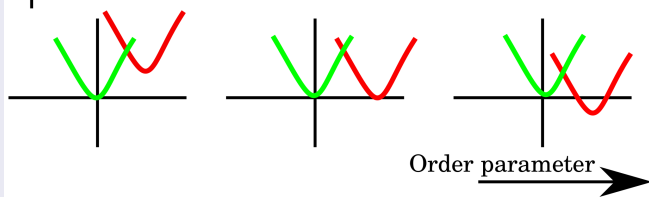
Schematic view

Two minima

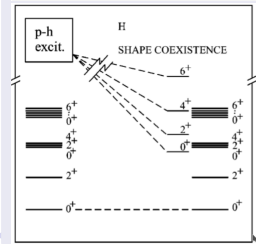
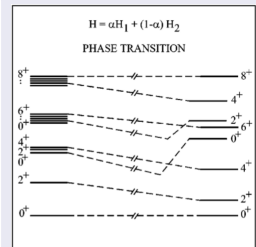
Phase transition



Shape coexistence



PRC 69, 054304 (2004)



Conclusions or rather open questions

- Lead region clearly shows up the onset of shape coexistence. Large mixing and relative energies hinder the onset of a Quantum Phase Transition.
- Rare-earth region is the most clear cut example of *critical region*, but without clear influence of shape coexistence, **although the SU3-proxy symmetry supports the presence of neutron particle-hole excitations (Bonatsos et al).**
- Are both descriptions compatible? **The answer is in Zr region: type I and type II QPT introduced by Gavrielov, Leviatan and Iachello**
- Can a Quantum Phase Transition be described in terms of the onset of intruder configurations?
- How things change in odd-even nuclei?
- Is shape coexistence always present *before* a Quantum Phase Transition sets in, or are they fully disconnected?

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Thanks for your attention

Also to my collaborators:

Kris Heyde (U. Gent),

Esperanza Maya-Barbacho and Pablo Martín-Higueras (U. Huelva),

Samira Baid and Pepe Arias (U. Sevilla)



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