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β- and γ-bands in the IBM

- Introduction
- Deformed nuclei
- Quantum phase transitions & IBM
- Intrinsic energies
- E2 matrix elements
- Summary and conclusions

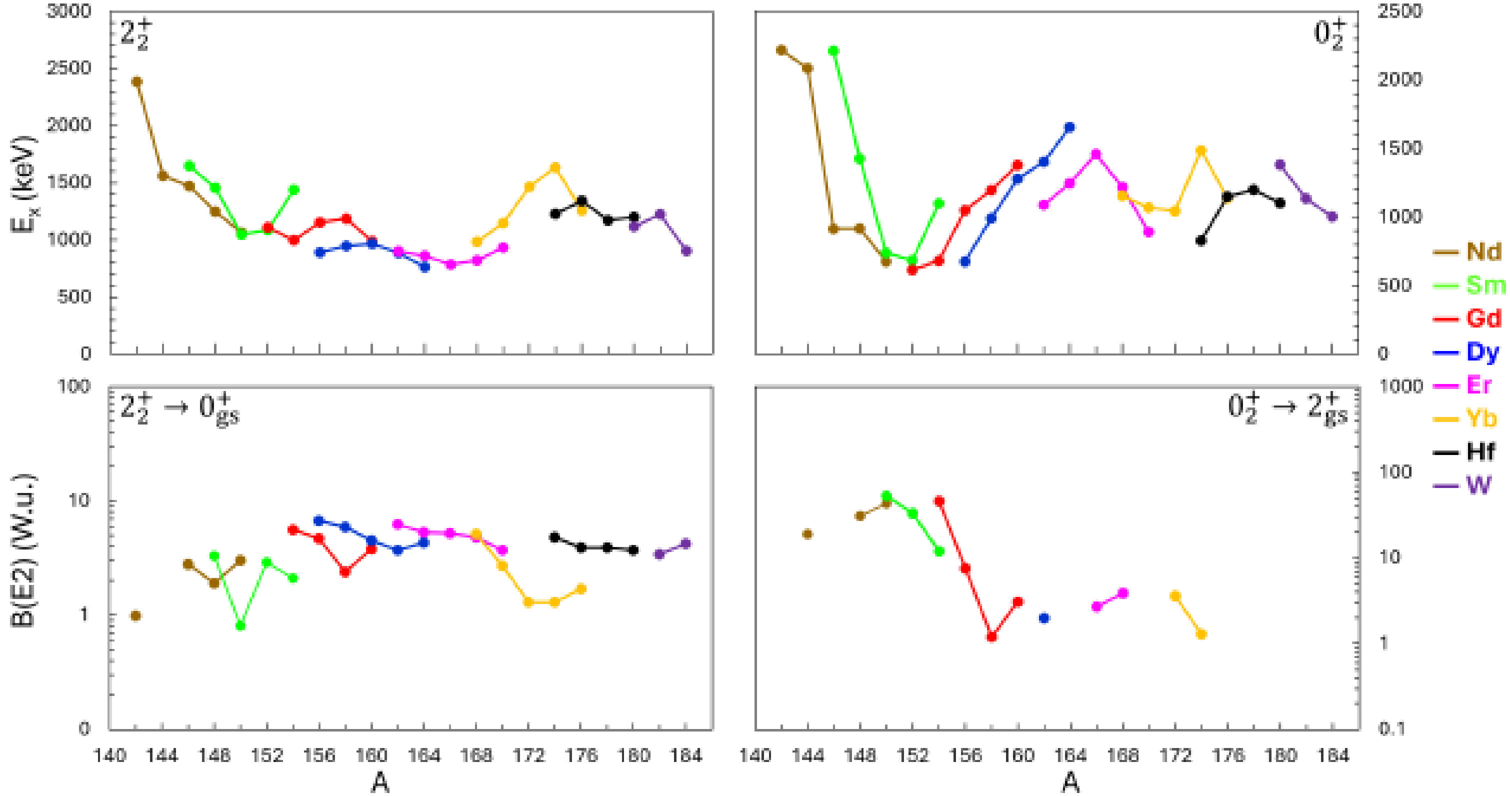

Figure 6: The systematics of the first excited $K^\pi = 2^+$ $K^\pi = 0^+$ bands in the Z=50-82 closed shell region for Nd to W nuclei and the associated B(E2) values connecting to the ground state for both types of bands.

Aprahamian et al, PPNP, in preparation

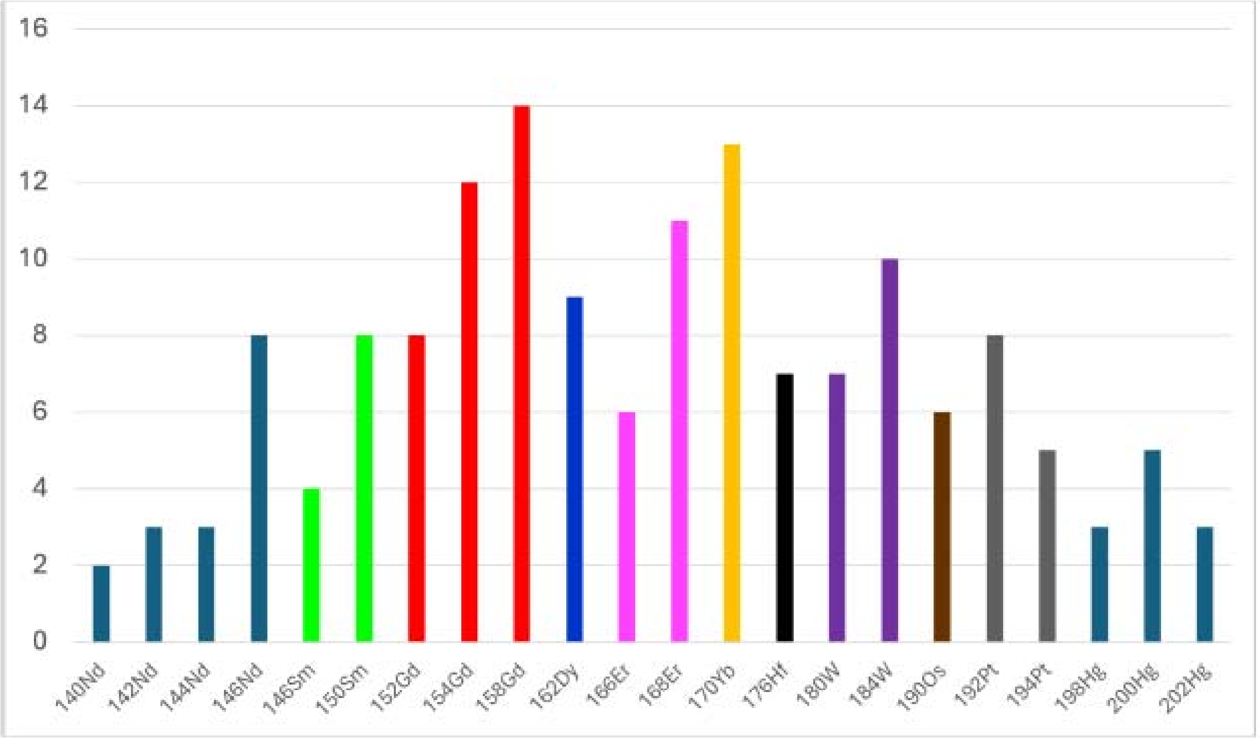

Figure 9: The number of 0^+ states observed with experiments at the Munich Q3D up to 3.0 MeV in excitation energy for Sm through Hg nuclei. We observe that fewer 0^+ states in the Sm and Hg than in the well-deformed nuclei such as the Gd, Er, Yb, and Hf. The Pt, and Os, nuclei are known to be γ-unstable. The Hg isotopes have well known coexisting excitations. In 2002, one of the first Q3D measurements observed 13 0^+ states in one nucleus below 3.1 MeV [35]. This work was followed by many others [36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49] to reveal that, in some cases, tens of 0^+ states exist in the low lying structure of deformed nuclei.

Aprahamian et al, PPNP, in preparation

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **27** (2001) R1–R22 www.iop.org/Journals/jg PII: S0954-3899(01)18337-4

TOPICAL REVIEW

Characterization of the β vibration and 0_2^+ states in deformed nuclei

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Abstract

A summary of the experimental properties of the first excited 0^+ states in deformed rare-earth nuclei is presented. By appealing to the original definition of a β vibration laid down in the Bohr–Mottelson picture, it is re-emphasized that most of the 0_2^+ states are not β vibrations. A consideration of all available data, especially that from transfer reactions, and of microscopic calculations of 0^+ states underscores the need to consider the role of pairing in the description, and labelling, of these states.

PHYSICAL REVIEW C VOLUME 26, NUMBER 6 DECEMBER 1982

Comments

Comments are short papers which comment on papers of other authors previously published in **Physical Review C**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract and keyword abstract.*

Properties of the intrinsic matrix elements of the interacting-boson-approximation $E2$ operator in the rotational limit

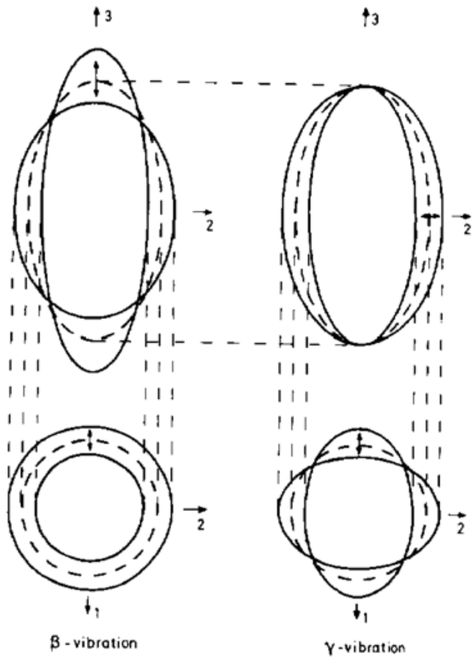
R. Bijker and A. E. L. Dieperink
Kernfysisch Versneller Instituut, Rijksuniversiteit Groningen, Nederland

(Received 14 July 1982)

It is shown that the dominance of $\beta\rightarrow\gamma$ and $\gamma\rightarrow g$ over $\beta\rightarrow g$ $E2$ transitions in the SU(3) limit of the interacting-boson-approximation model reported by Warner and Casten can be explained simply in terms of properties of the intrinsic $E2$ matrix elements.

Warner & Casten, PRC25, 2019 (1982)
Warner & Casten, PRC26, 2690 (1982)

β and γ vibrations



$$\langle g|T(E2)|g\rangle \sim N$$
$$\langle \beta|T(E2)|g\rangle \sim \sqrt{N}$$
$$\langle \gamma|T(E2)|g\rangle \sim \sqrt{N}$$
$$\langle \gamma|T(E2)|\beta\rangle \sim 1$$

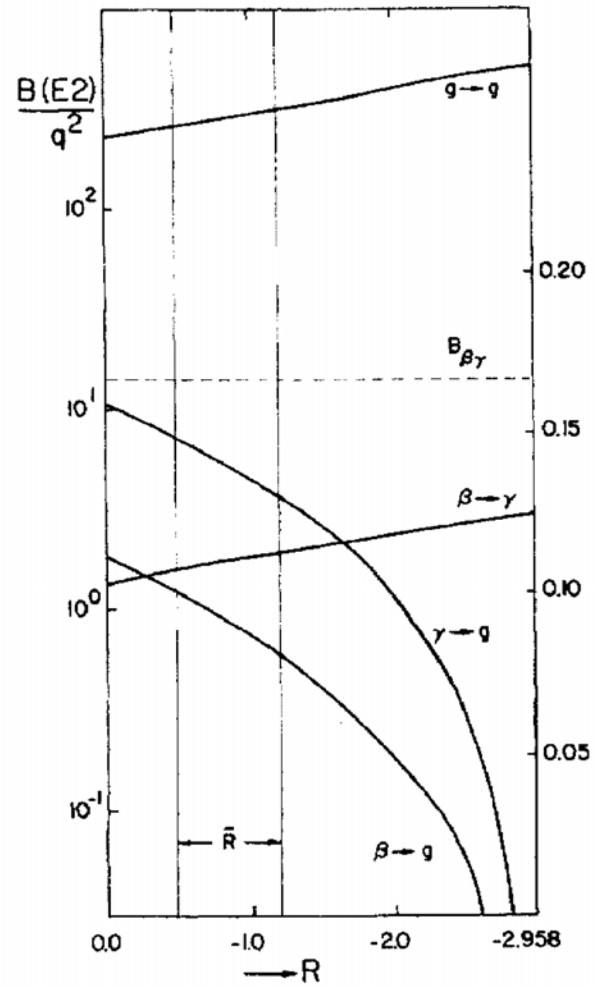
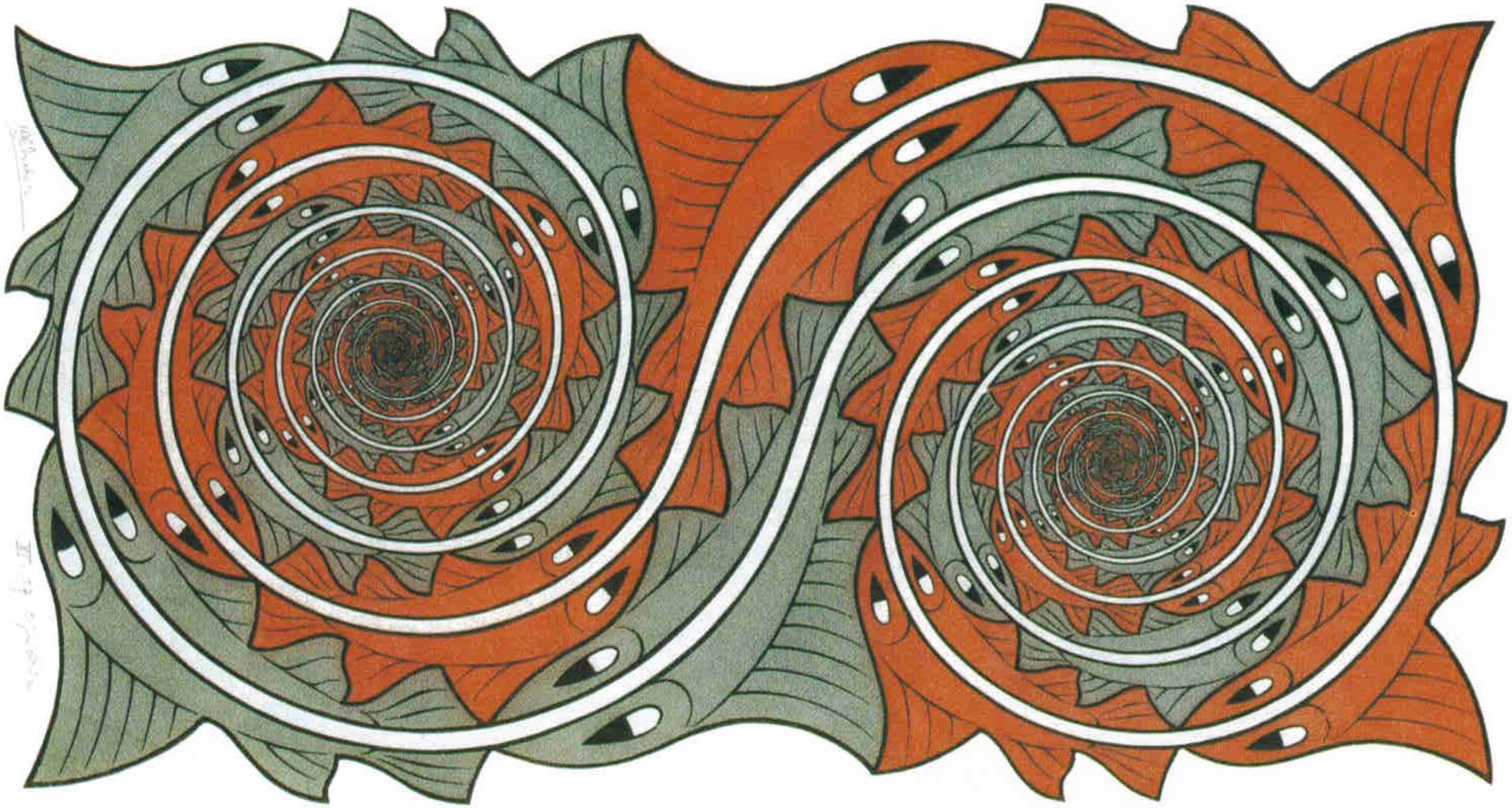


FIG. 1. Intrinsic $B(E2)$ values in the SU(3) limit (left scale) and the ratio $B_{\beta\gamma}$ (right scale) as a function of R for $N=16$. The range of R in the rare-earth nuclei, $-1.2 < \bar{R} < -0.5$, is indicated by vertical lines.



IBM Hamiltonian

$$H = (1-\xi)\hat{n}_d - \frac{\xi}{4(N-1)} Q(\chi)\cdot Q(\chi)$$

$$\hat{n}_d = \sqrt{5}\,(d^\dagger \times \tilde{d})^{(0)}_0$$

$$Q_\mu(\chi) = (s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s})^{(2)}_\mu + \chi\,(d^\dagger \times \tilde{d})^{(2)}_\mu$$

$$T_\mu(E2) = e_B\, Q_\mu(\chi)$$

Special Solutions: Dynamical Symmetries

$\xi = 0$	$U(5)$	spherical
$\xi = 1 \quad \chi = 0$	$SO(6)$	γ-unstable
$\xi = 1 \quad \chi = -\frac{1}{2}\sqrt{7}$	$SU(3)$	prolate deformed
$\xi = 1 \quad \chi = +\frac{1}{2}\sqrt{7}$	$SU(3)$	oblate deformed

Classical Limit

$$
\begin{aligned}
|N,\beta,\gamma\rangle &= \frac{1}{\sqrt{N!}}\left(b_c^\dagger(\beta,\gamma)\right)^N|0\rangle \\
b_c^\dagger(\beta,\gamma) &= \frac{1}{\sqrt{1+\beta^2}}\left(s^\dagger+\beta\cos\gamma\, d_0^\dagger+\frac{1}{\sqrt{2}}\beta\sin\gamma\,(d_2^\dagger+d_{-2}^\dagger)\right) \\
\frac{1}{N}E(\beta,\gamma) &= \frac{1}{N}\langle N,\beta,\gamma|H|N,\beta,\gamma\rangle \\
&= (1-\xi)\frac{\beta^2}{1+\beta^2}-\xi\frac{\frac{1}{7}\chi^2\beta^4-2\sqrt{\frac{2}{7}}\chi\beta^3\cos 3\gamma+2\beta^2}{2(1+\beta^2)^2}
\end{aligned}
$$

Equilibrium Shape

spherical	$U(5)$	$\xi=0$		$\beta_0=0$	
γ-unstable	$SO(6)$	$\xi=1$	$\chi=0$	$\beta_0=1$	
prolate deformed	$SU(3)$	$\xi=1$	$\chi=-\frac{1}{2}\sqrt{7}$	$\beta_0=\sqrt{2}$	$\gamma_0=0^\circ$
oblate deformed	$SU(3)$	$\xi=1$	$\chi=+\frac{1}{2}\sqrt{7}$	$\beta_0=\sqrt{2}$	$\gamma_0=180^\circ$

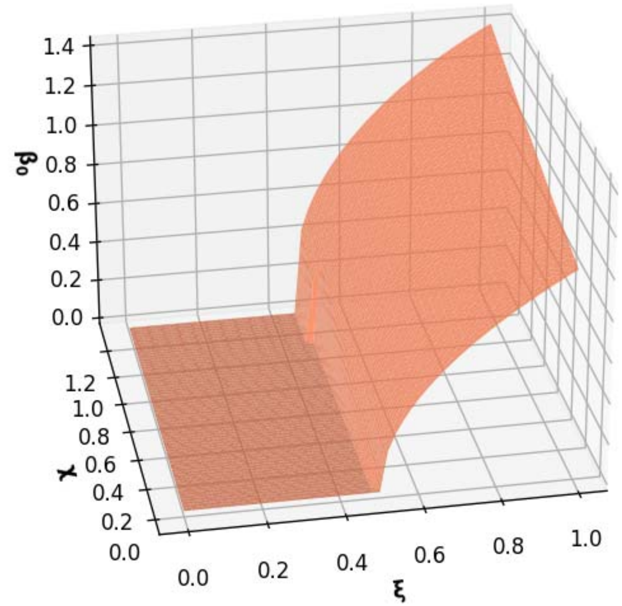
Quantum Phase Transitions

Hamiltonian: $H = (1 - \xi)\hat{n}_d - \frac{\xi}{4(N-1)} Q(\chi) \cdot Q(\chi)$ $\quad 0 \leq \xi \leq 1$

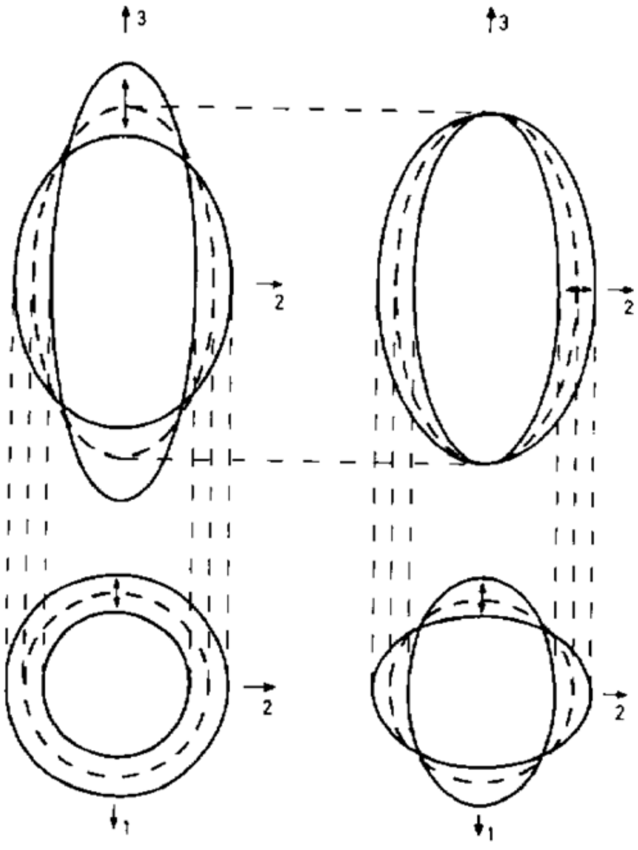
$\chi = 0$	$U(5) \leftrightarrow SO(6)$	$\xi_c = \frac{1}{2}$
$\chi = \mp\frac{1}{2}\sqrt{7}$	$U(5) \leftrightarrow SU(3)$	$\xi_c = \frac{8}{17}$
$\chi \neq 0, \mp\frac{1}{2}\sqrt{7}$	spherical $\leftrightarrow$ deformed	ξ_c

First order phase transition

$\xi < \xi_c$: spherical
$\xi > \xi_c$: deformed



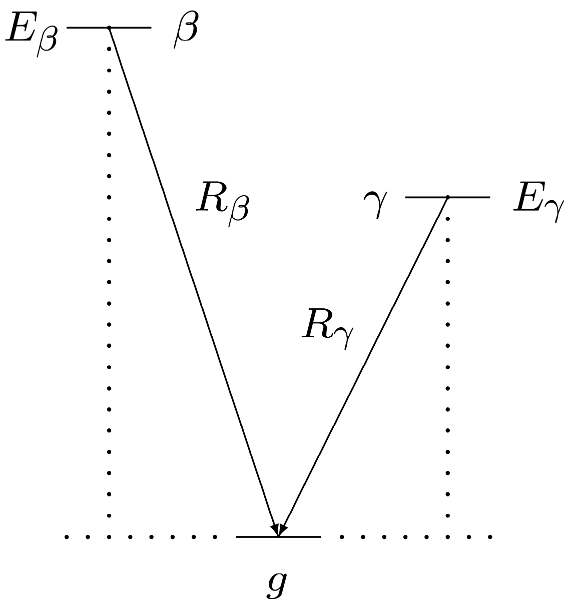
β- and γ-vibrations



$$
\begin{aligned}
|g\rangle &= \frac{1}{\sqrt{N!}} \left(b_c^\dagger\right)^N |0\rangle \\
|\beta\rangle &= \frac{1}{\sqrt{N}} b_\beta^\dagger b_c \, |g\rangle \\
|\gamma\rangle &= \frac{1}{\sqrt{N}} b_\gamma^\dagger b_c \, |g\rangle
\end{aligned}
$$

$$
\begin{aligned}
b_c^\dagger &= \frac{1}{\sqrt{1+\beta_0^2}}(s^\dagger + \beta_0 d_0^\dagger) \\
b_\beta^\dagger &= \frac{1}{\sqrt{1+\beta_0^2}}(-\beta_0 s^\dagger + d_0^\dagger) \\
b_\gamma^\dagger &= \frac{1}{\sqrt{2}}(d_2^\dagger + d_{-2}^\dagger)
\end{aligned}
$$

Intrinsic Energies & ME

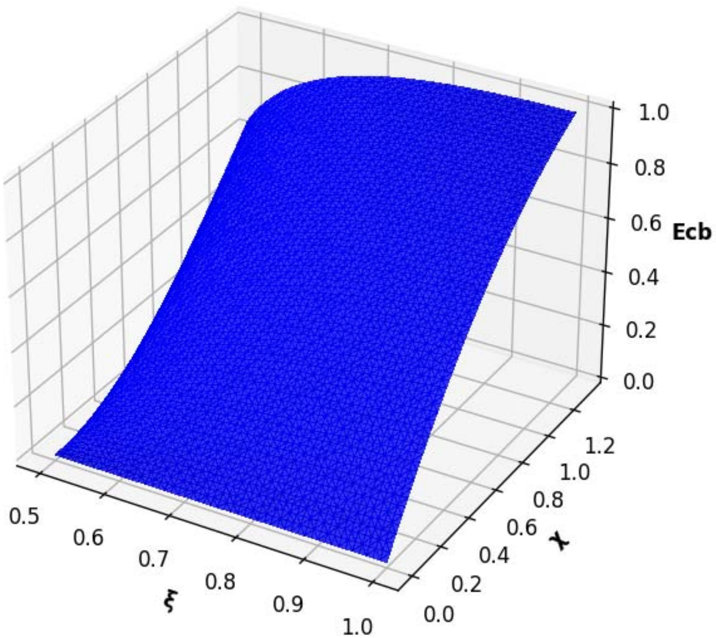


$$
\begin{aligned}
E_\beta &= \frac{\xi}{2(1+\beta_0^2)}\left[4+\sqrt{\frac{2}{7}}|\chi|\beta_0(\beta_0^2+3)\right] \\
E_\gamma &= \frac{9\xi}{2}\sqrt{\frac{2}{7}}|\chi|\frac{\beta_0}{1+\beta_0^2} \\
R_\beta &= \langle\beta|T_0(E2)|g\rangle \\
&= \frac{e_B\sqrt{N}}{1+\beta_0^2}\left(1+\sqrt{\frac{2}{7}}|\chi|\beta_0-\beta_0^2\right) \\
R_\gamma &= \langle\gamma|T_2(E2)+T_{-2}(E2)|g\rangle \\
&= \frac{e_B\sqrt{2N}}{\sqrt{1+\beta_0^2}}\left(1-\sqrt{\frac{2}{7}}|\chi|\beta_0\right)
\end{aligned}
$$

Bijker & Dieperink, PRC 26, 2688 (1982)
Leviatan, AP 179, 201 (1987)

Intrinsic Energies

$$\frac{E_\gamma}{E_\beta} = \frac{9\sqrt{\frac{2}{7}}\chi\beta_0}{4+\sqrt{\frac{2}{7}}\chi\beta_0(\beta_0^2+3)}$$



$\xi > \xi_c$: deformed region

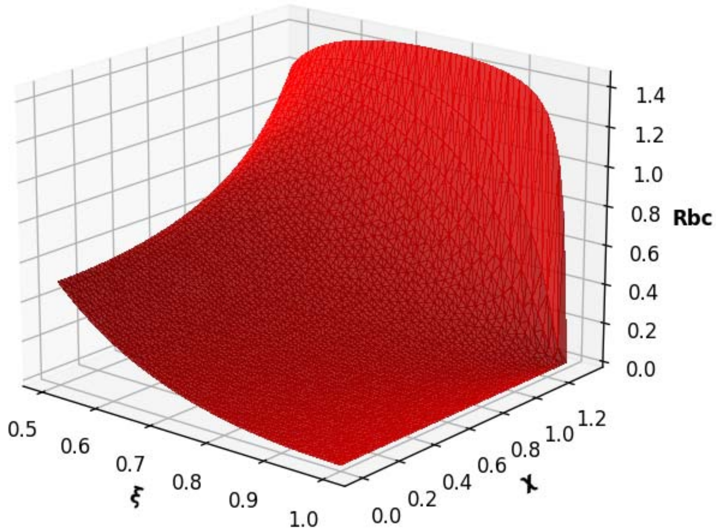
Ratio: $E_\gamma/E_\beta \leq 1$

SU(3) limit

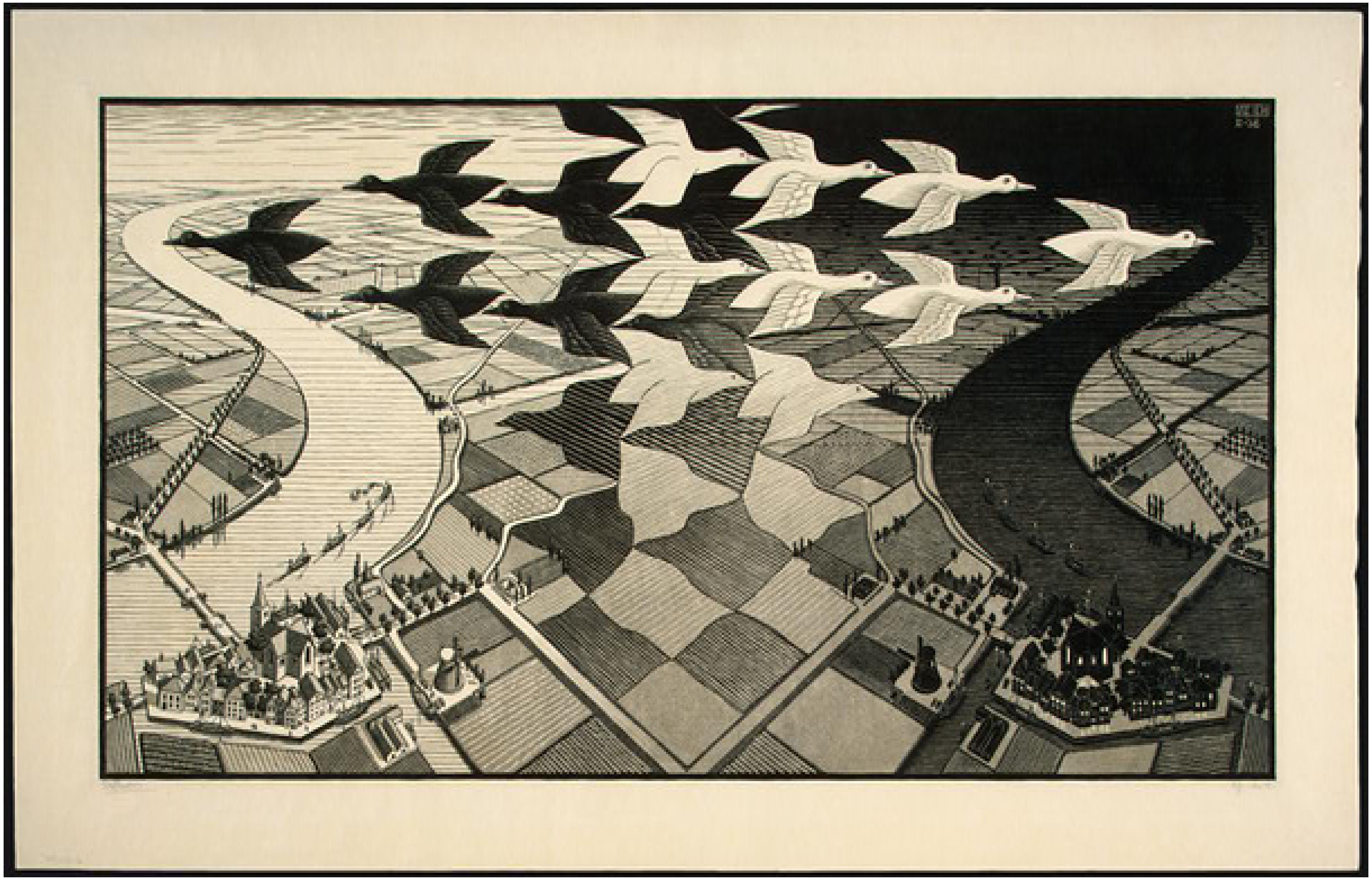
$$\begin{aligned} \xi &= 1 \\ \chi &= \mp\frac{1}{2}\sqrt{7} \\ \beta_0 &= \sqrt{2} \\ E_\gamma/E_\beta &= 1 \end{aligned}$$

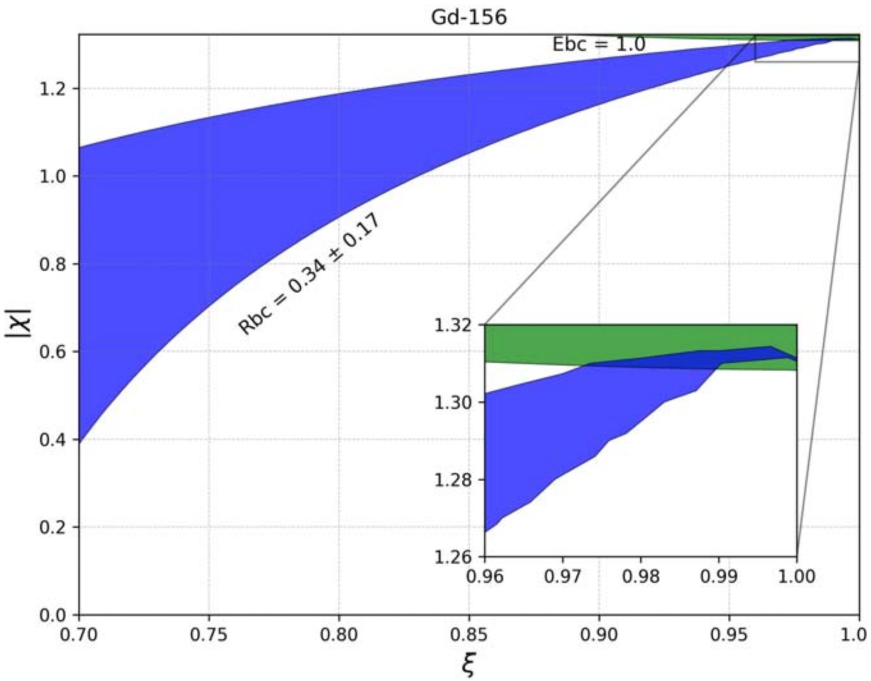
E2 Transitions

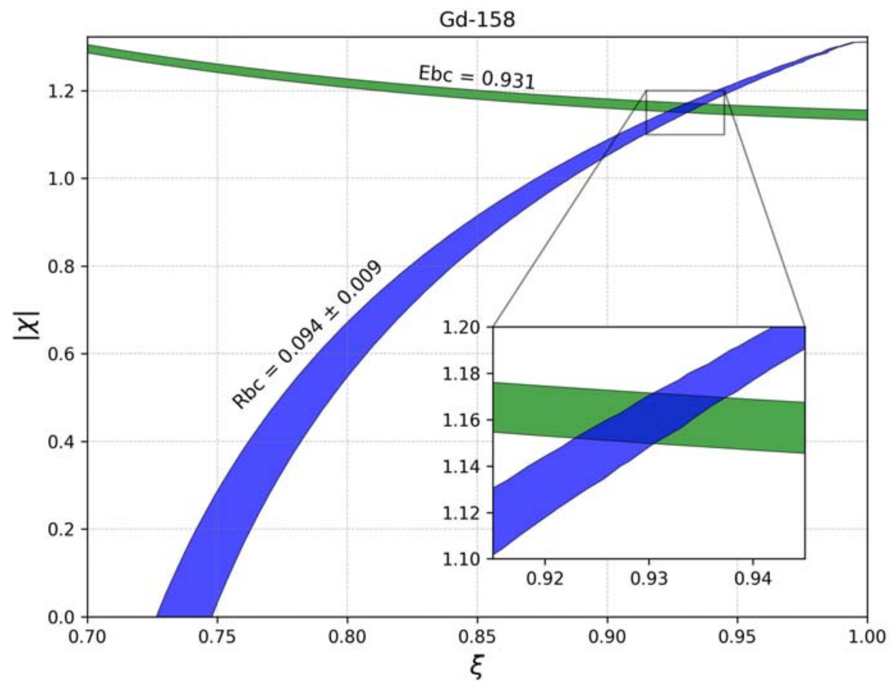
$$R_{\beta\gamma} = \frac{B(E2;0_1^+ \rightarrow 2_\beta^+)}{B(E2;0_1^+ \rightarrow 2_\gamma^+)} = \frac{1}{2(1+\beta_0^2)} \left[\frac{1+\sqrt{\frac{2}{7}}|\chi|\beta_0 - \beta_0^2}{1-\sqrt{\frac{2}{7}}|\chi|\beta_0} \right]^2$$



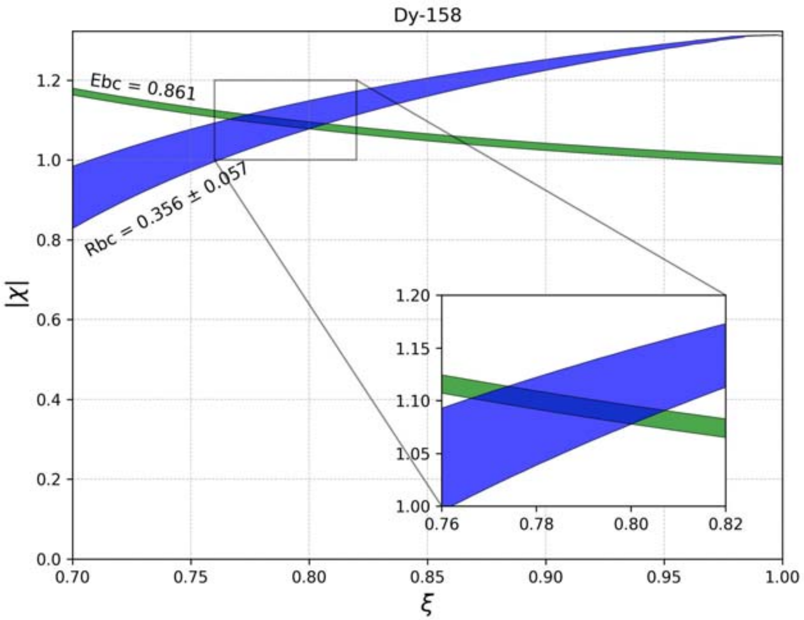
ξ	χ	$R_{\beta\gamma}$
	0	$\frac{(1-\xi)^2}{\xi}$
1	$\neq \mp\frac{1}{2}\sqrt{7}$	0
1	$\mp\frac{1}{2}\sqrt{7}$	$\frac{3}{2}$
< 1	$\mp\frac{1}{2}\sqrt{7}$	$\frac{(\sqrt{2}\beta_0+1)^2}{2(1+\beta_0^2)}$

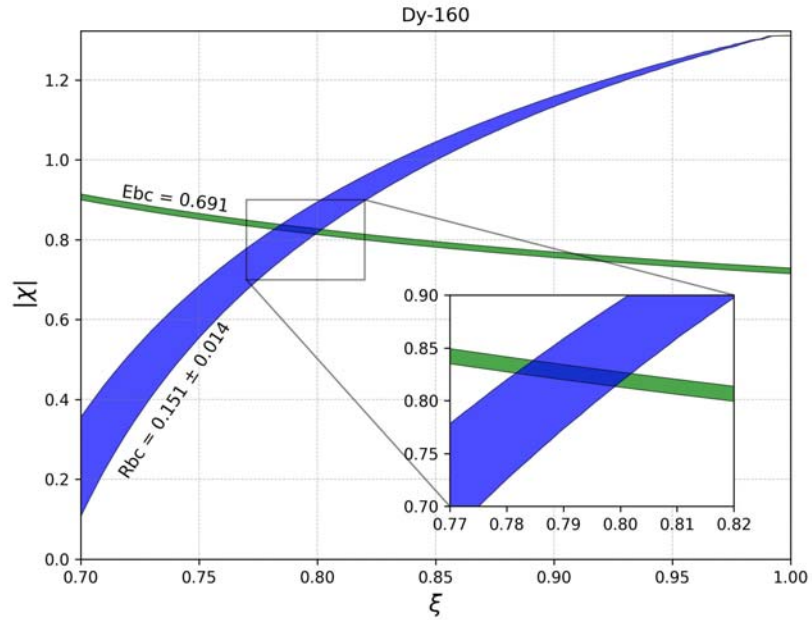


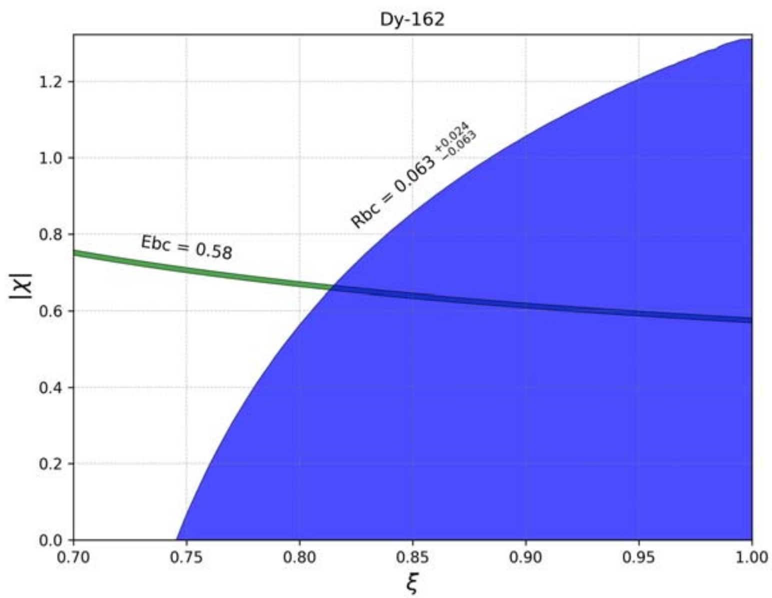




$^{156,158}\mathrm{Gd} \quad Z = 64$



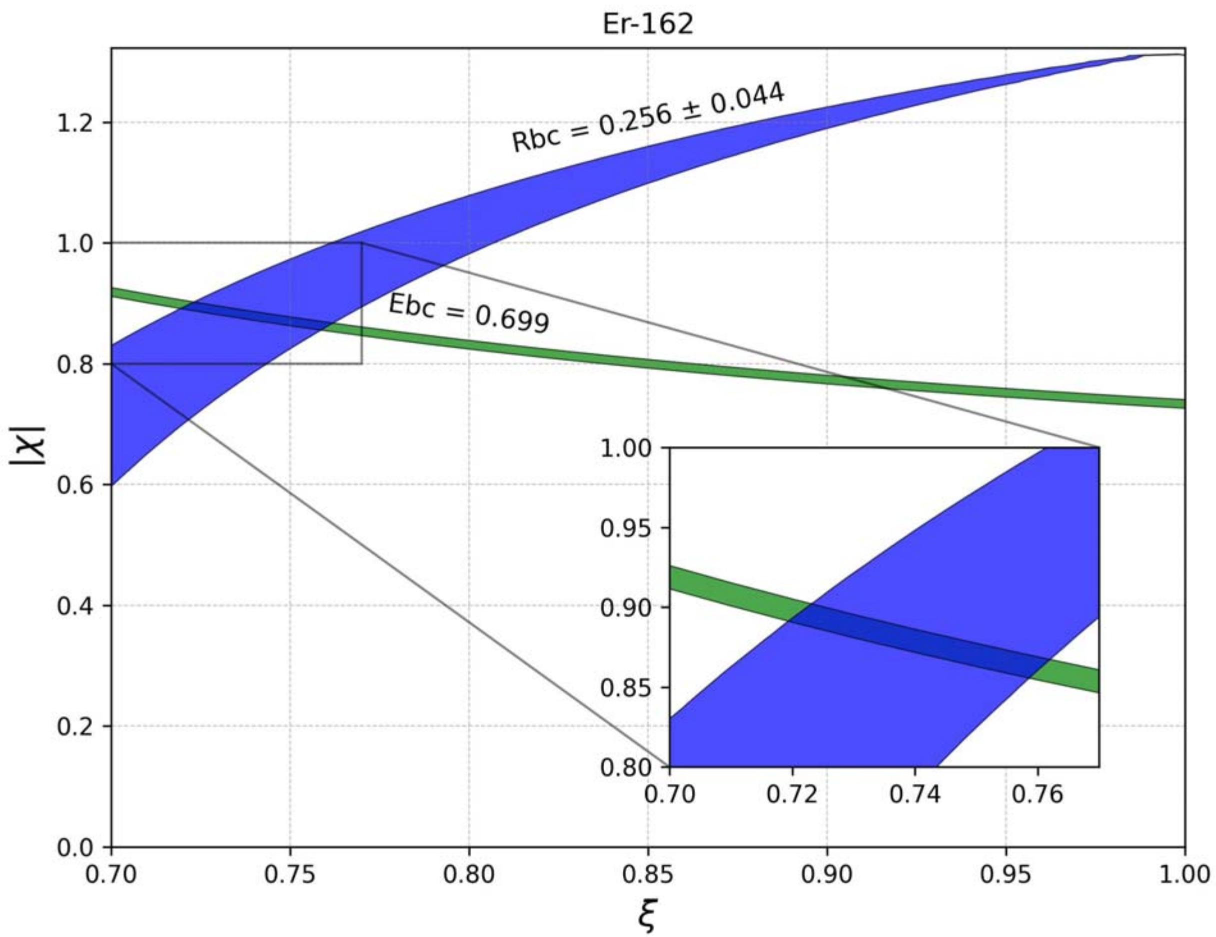


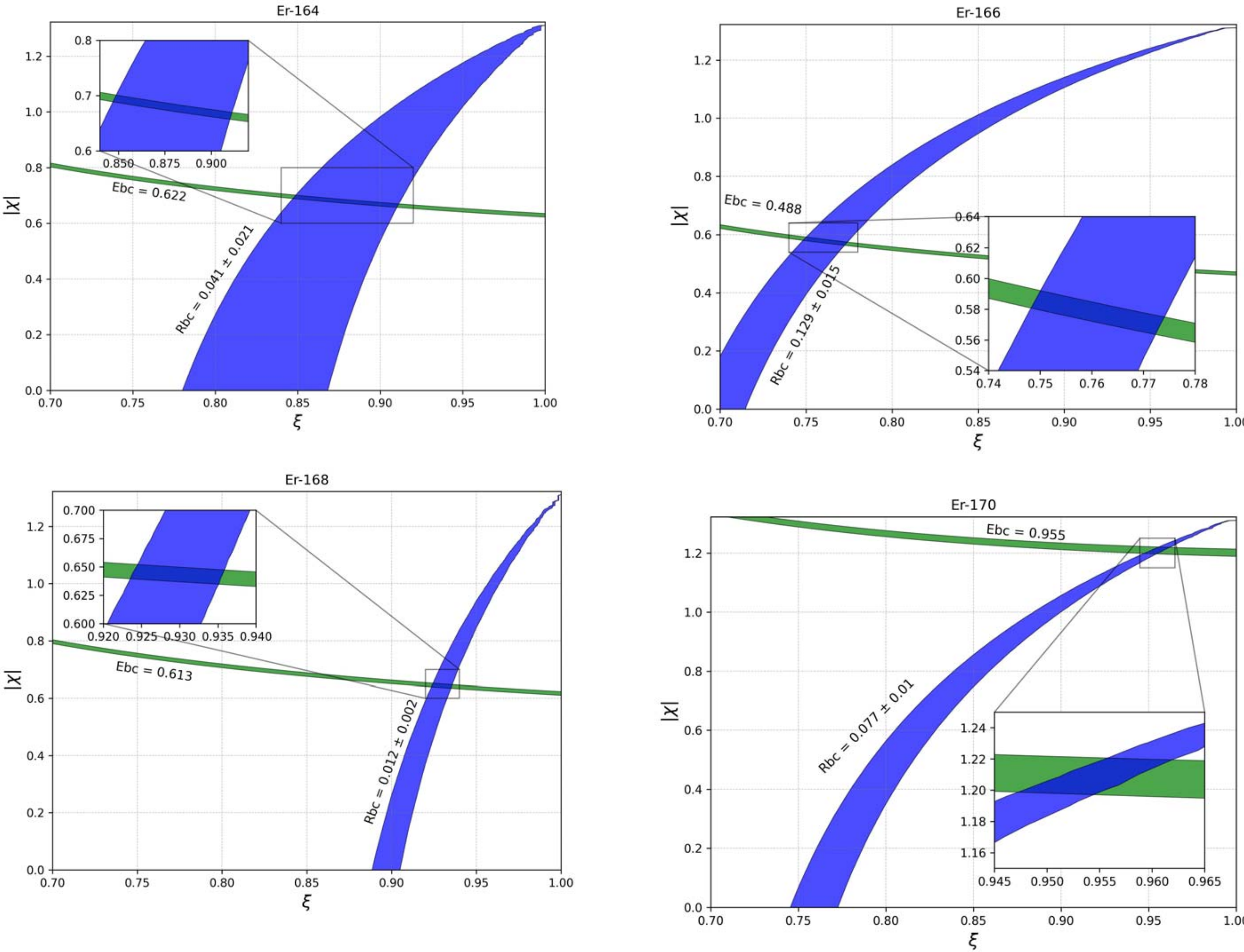


$^{158,160,162}\mathrm{Dy} \quad Z = 66$

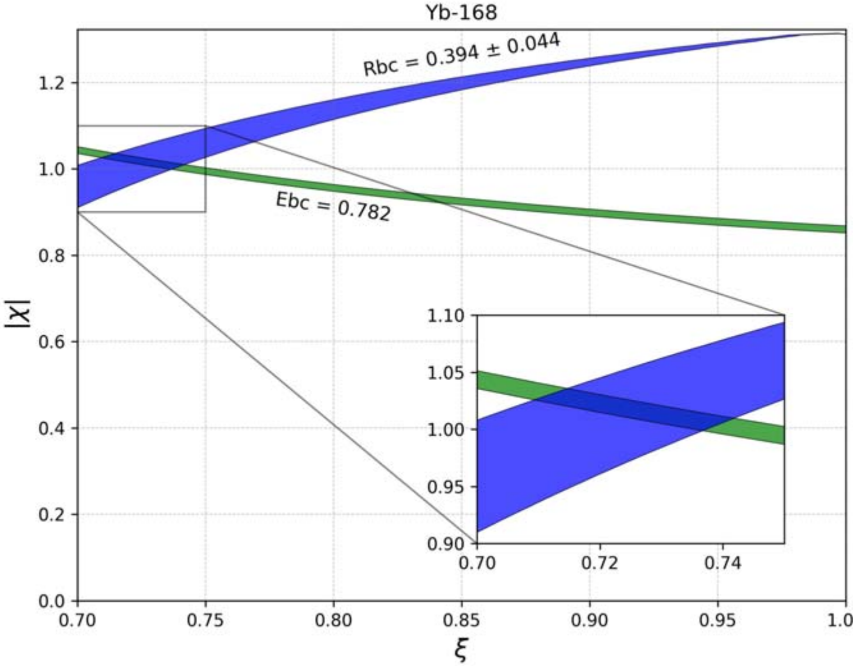
Very small range
in the ξ-χ plane !!

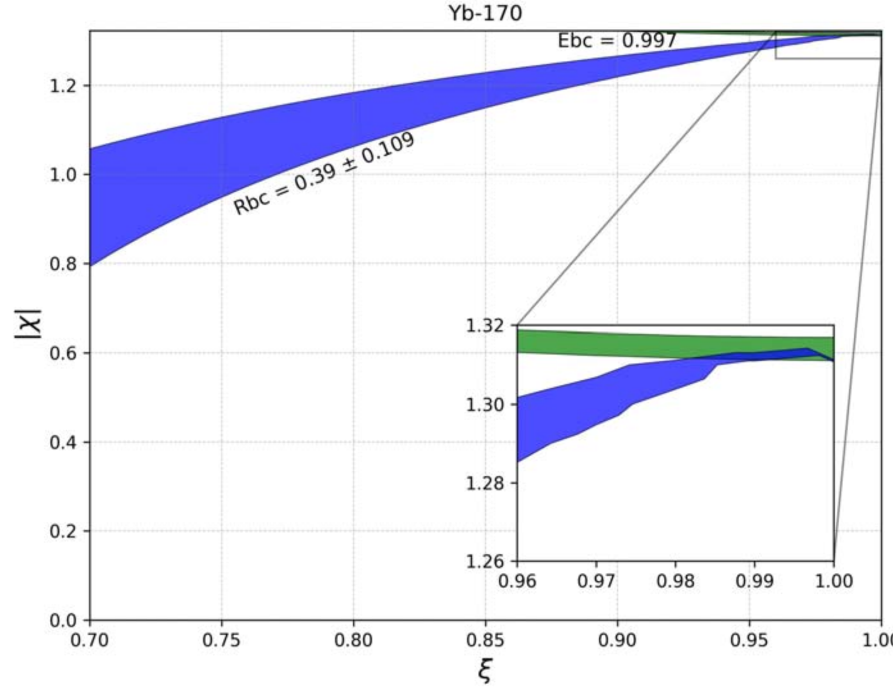
^{162,164,166,168,170}Er $\quad Z = 68$



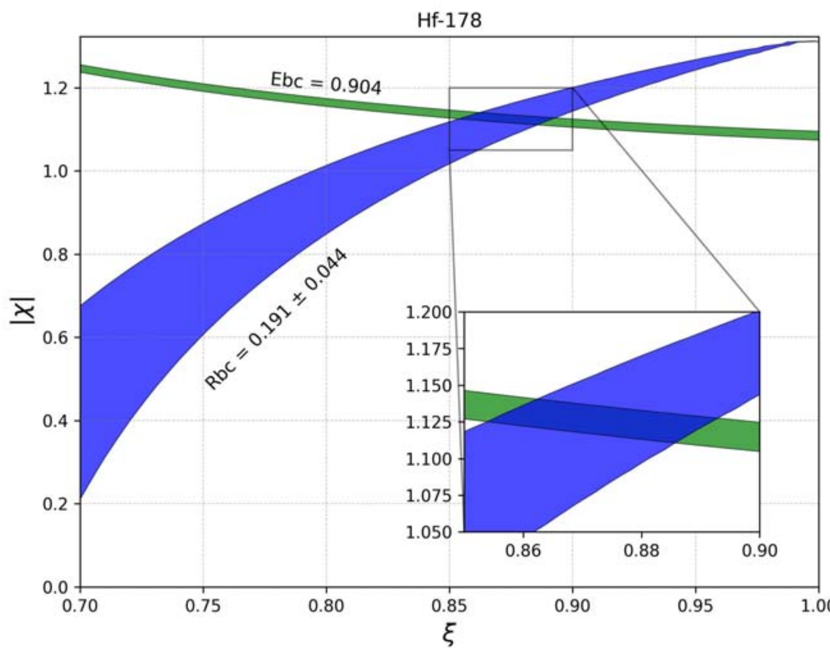


Er-164
Ebc = 0.622
Rbc = 0.041 ± 0.021
|χ|
ξ
Er-166
Ebc = 0.488
Rbc = 0.129 ± 0.015
|χ|
ξ
Er-168
Ebc = 0.613
Rbc = 0.012 ± 0.002
|χ|
ξ
Er-170
Ebc = 0.955
Rbc = 0.077 ± 0.01
|χ|
ξ

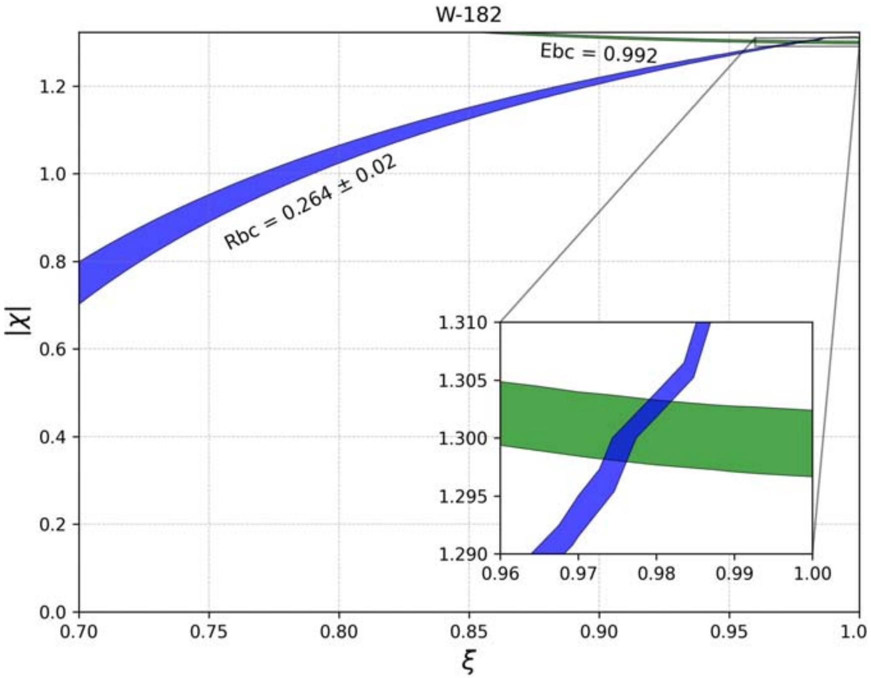


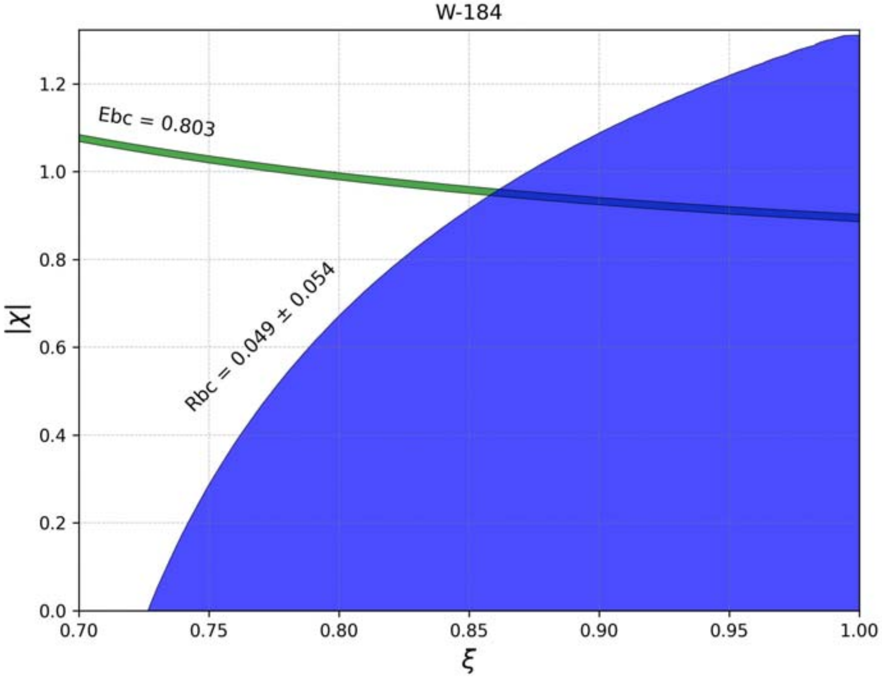


^{168,170}Yb $\quad Z = 70$



^{178}Hf $\quad Z = 72$



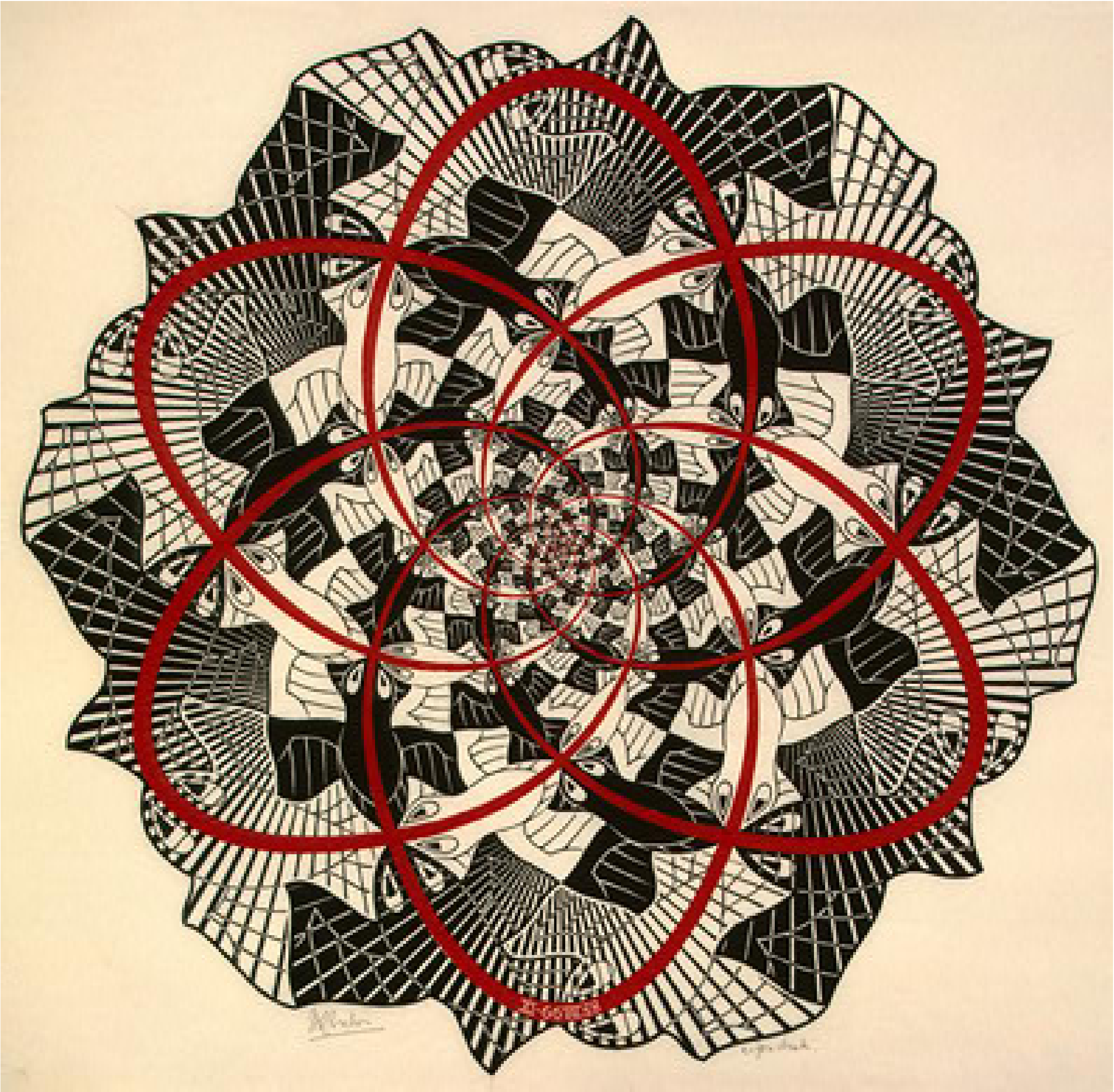


$^{182,184}\mathrm{W} \quad Z = 74$

Case Studies

		Intrinsic IBM		Experiment		
ξ	χ	$E\gamma/E_\beta$	$R_{\beta\gamma}$	$E\gamma/E_\beta$	$R_{\beta\gamma}$	
0.97	-1.30	0.99	0.34	1.02	0.34 ± 0.17	^{156}Gd
0.84	-0.64	0.58	0.063	0.58	$0.063^{+0.024}_{-0.063}$	^{162}Dy
0.83	-0.70	0.62	0.078	0.61	$0.079^{+0.030}_{-0.079}$	^{168}Er

Very small range in the ξ-χ plane !!



Summary and Conclusions

- One-phonon β and γ vibrations
- Intrinsic energies: $E_\gamma/E_\beta \leq 1$
- E2 transitions: dominance of $\gamma \rightarrow g$ over $\beta \rightarrow g$
- Case studies: Gd, Dy, Er, Yb, Hf, W (Z=64-72)
- Finite N effects
- Double-phonon excitations: $\beta\beta$, $\beta\gamma$ and $\gamma\gamma$

The Nature of 0^+ Excitations in Nuclei

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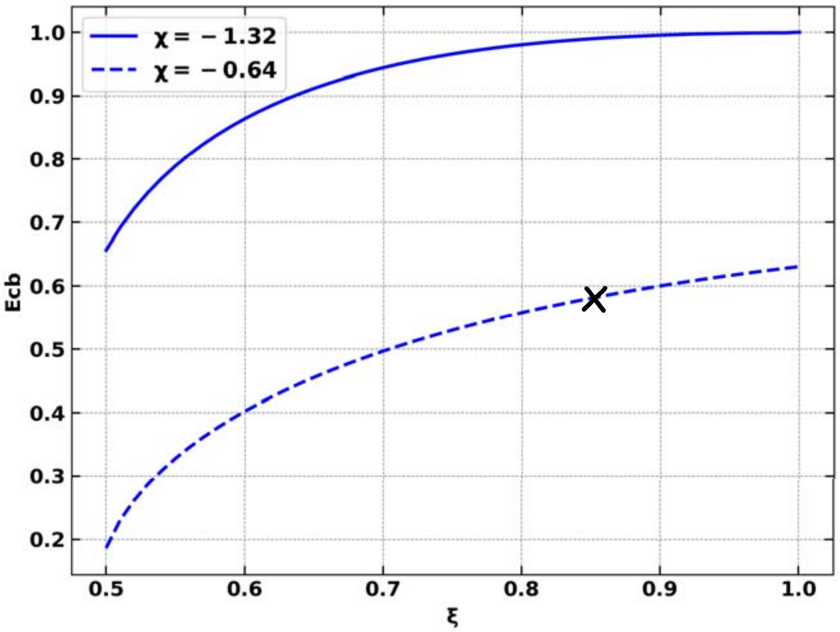
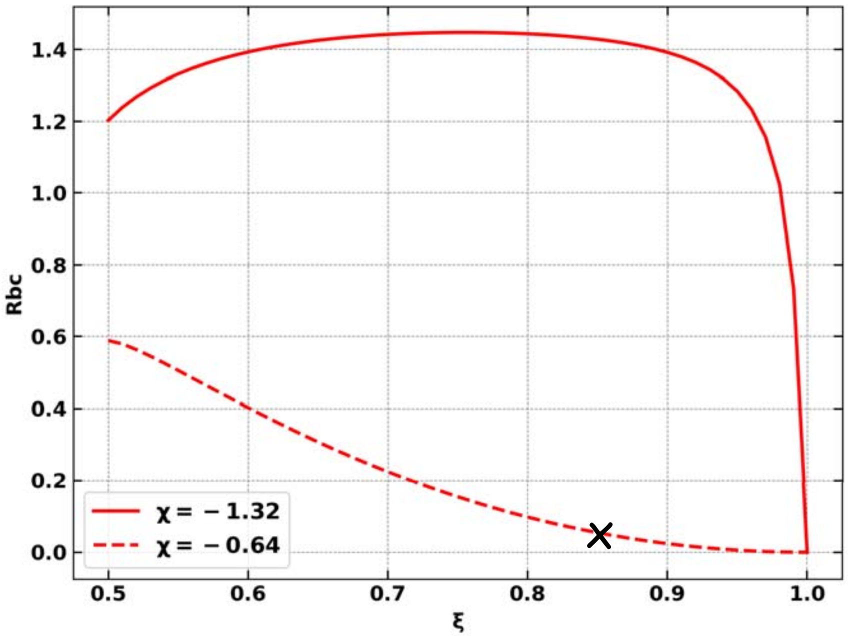
Review, to be published in Prog Part Nucl Phys

Bijker & Mas, EPJ Web of Conferences (2024), in press
Mas & Bijker, in preparation

^{162}Dy





		Intrinsic IBM		Experiment		
ξ	χ	$E\gamma/E_\beta$	$R_{\beta\gamma}$	$E\gamma/E_\beta$	$R_{\beta\gamma}$	
0.84	-0.64	0.58	0.063	0.58	$0.063^{+0.024}_{-0.063}$	^{162}Dy