Quark and proton anomalous magnetic moments in confining models

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Outline

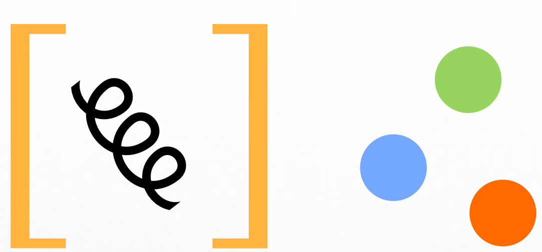
- Motivation: ***confinement***
- ***Confining models*** *as an alternative approach to IR QCD*
- Status of Refined Gribov-Zwanziger framework
- Quark and proton anomalous magnetic moment: consistent results? model contraints?

Motivation: the confinement problem

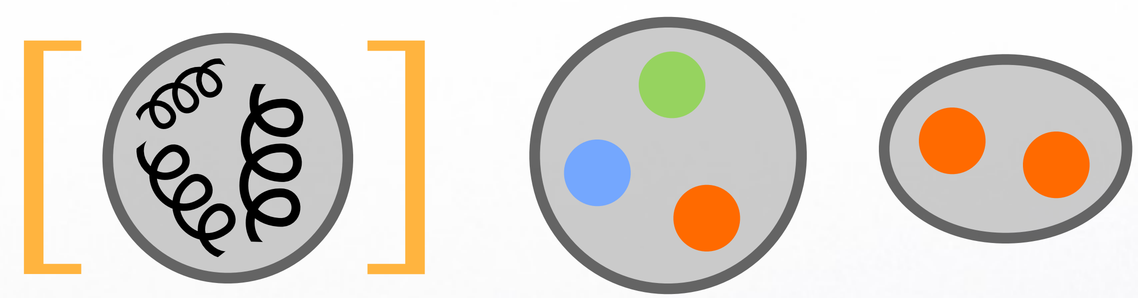
Fundamental degrees of freedom are not part of the spectrum $\xrightarrow{?}$ **Physical spectrum of bound states** dynamically generated at low energies.

[Yang-Mills gauge theories] and QCD:

$$\mathcal{L}^{\rm E}_{\rm YM} = \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}\right] + \overline{\Psi}_f[\gamma_\mu D_\mu + m_f]\Psi_f$$



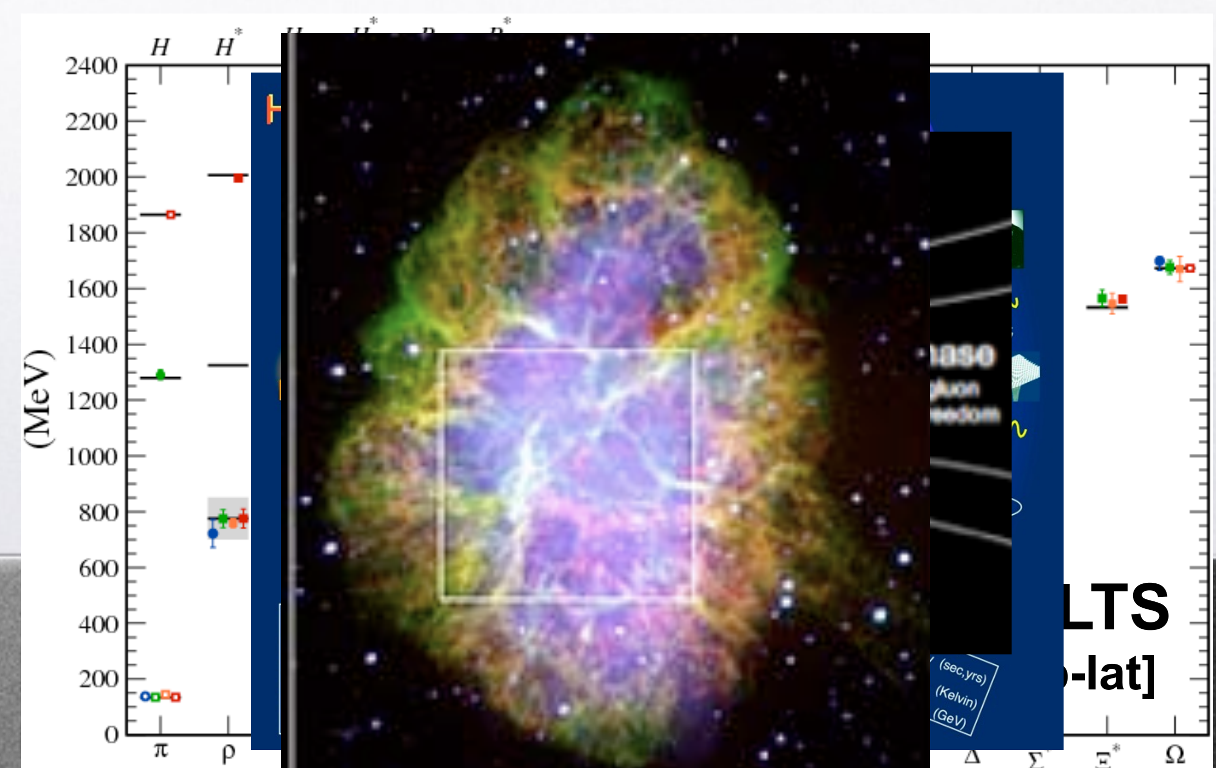
$\xrightarrow{?}$



[Gluons] and quarks **[Glueballs,] baryons and mesons**

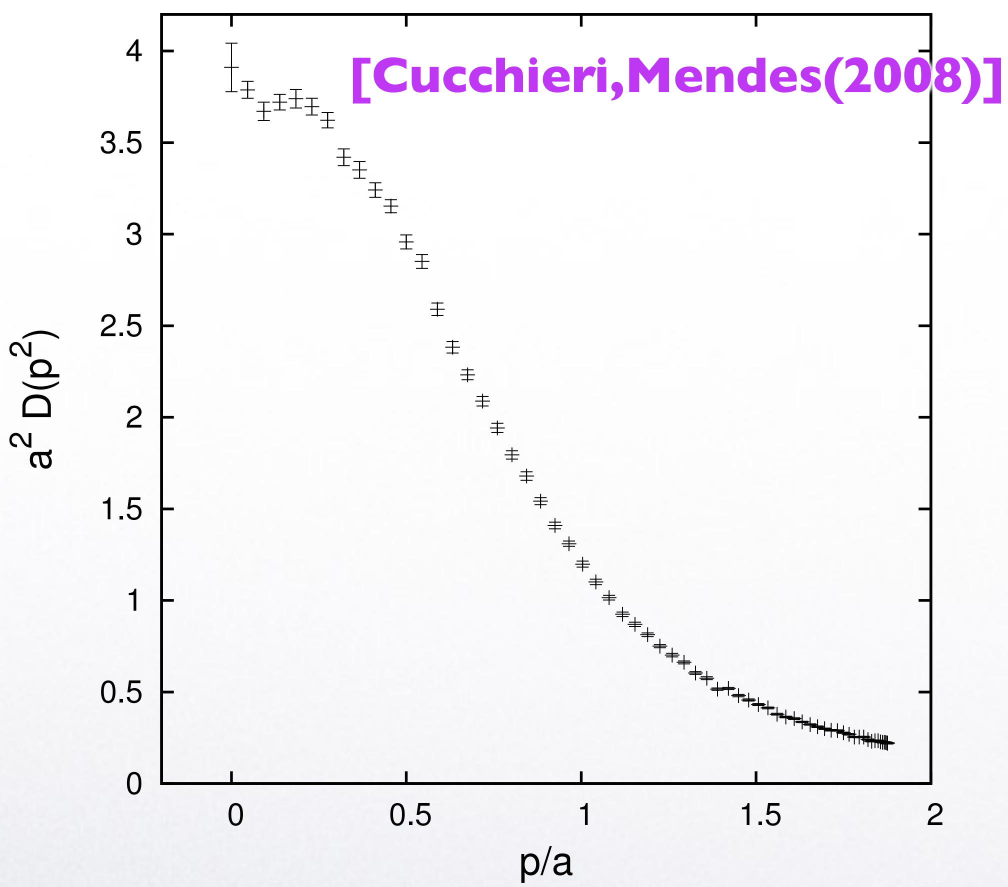
What is the mechanism??

What happens to quarks and gluons in the IR??

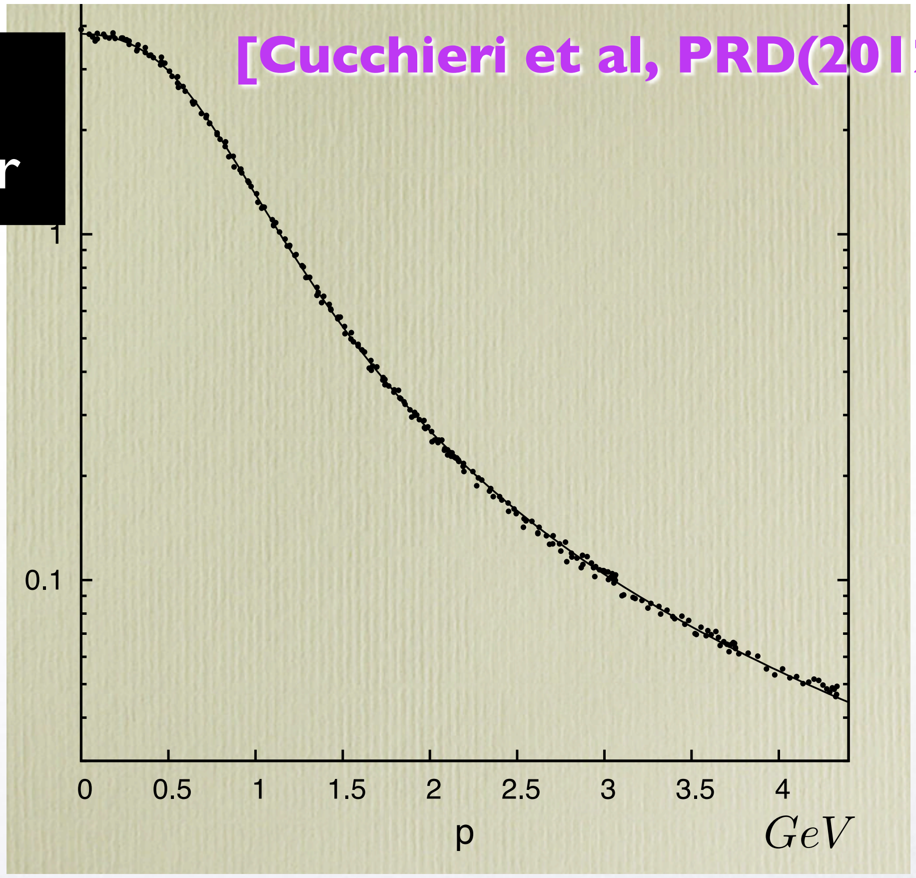


Motivation: gluon propagator in the infrared

- **Finite infrared gluon propagator in Landau gauge:**
  - early predictions in Dyson-Schwinger studies [Aguilar, Natale (2004); Frasca (2007)]
  - High-precision lattice YM results for large systems [Cucchieri, Mendes (2008)]



Gluon Propagator



Also confirmed by other lattice groups: [Bogolubsky et al (2009); Oliveira & Silva (2009)]

- FRG: Cyrol, Fister, Mitter, Pawlowski, Strodthoff (PRD 2016)
- Curci-Ferrari (massive) models: Pelaez, Reinosa, Serreau, Tissier, Wschebor (2015,2016)
- Gluon condensate from lattice QCD: Boucaud, Pene, Rodriguez-Quintero et al (2001)

[see talk by M. Pelaez]

[Gribov (1978)]

The Gribov problem:

- In the Landau gauge, for instance, the theory assumes the form

$$\int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c\mathcal{D}b\, e^{-S_{YM}+S_{gf}}$$

$$S_{gf} = b^a\partial_\mu A^a_\mu - \bar{c}^a\mathcal{M}^{ab}c^b \,, \qquad \mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + gf^{abc}A^c_\mu\right)$$

- Gribov copies $\rightarrow$ zero eigenvalues of the Faddeev-Popov operator $\mathcal{M}^{ab}$.
- Copies cannot be reached by small fluctuations around $A = 0$ (perturbative vacuum) $\rightarrow$ pertubation theory works.
- Once large enough gauge field amplitudes have to be considered (non-perturbative domain) the copies will show up enforcing the effective breakdown of the Faddeev-Popov procedure.

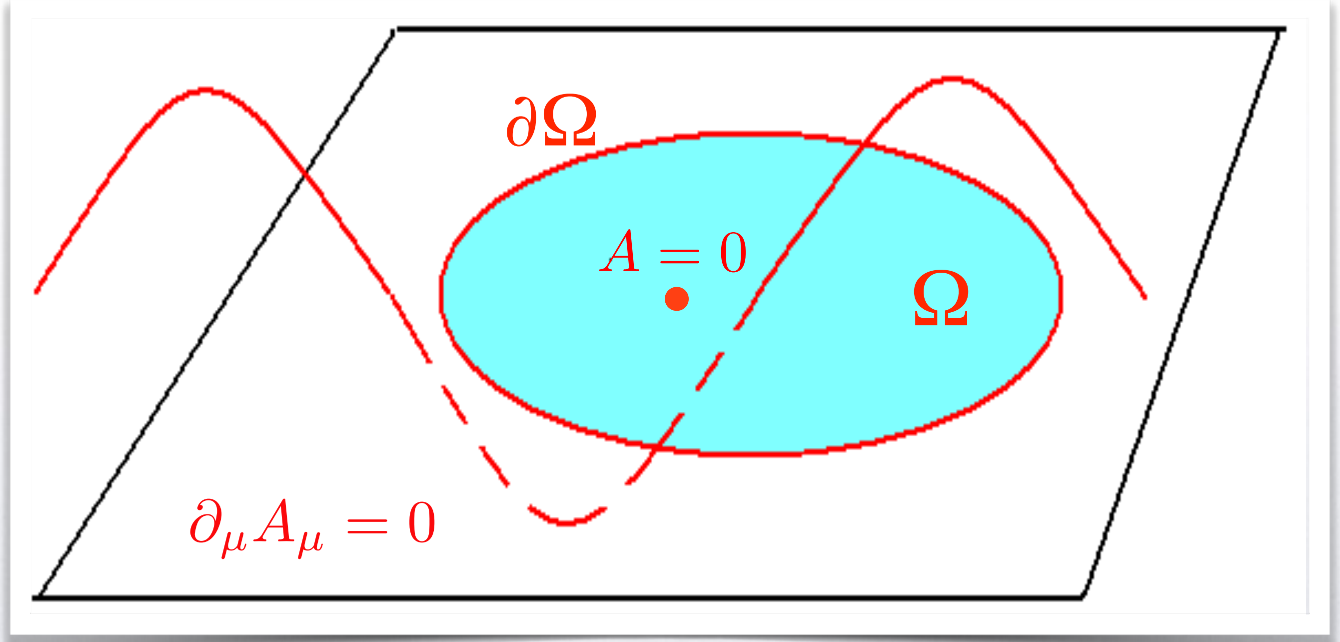
- Gribov proposed a way to eliminate (infinitesimal) Gribov copies from the integration measure over gauge fields: **the restriction to the (first) Gribov region Ω**

$$\int [DA]\delta(\partial A)\det(\mathcal{M})e^{-S_{\rm YM}} \longrightarrow \int_\Omega [DA]\delta(\partial A)\det(\mathcal{M})e^{-S_{\rm YM}} \qquad S_{\rm YM} = \frac{1}{4}\int_x F^2$$

with $\Omega = \left\{A^a_\mu \ ; \ \partial A^a = 0, \mathcal{M}^{ab} > 0\right\}$

$$\boxed{\mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + f^{abc}A^c_\mu\right) = -\partial_\mu D^a_\mu}$$

(Faddeev-Popov operator)



The Gribov-Zwanziger action

- The **restriction** can be implemented as a **gap equation** for the vacuum energy obtained as:

[Zwanziger (1989,...)]

$$Z = e^{-V\mathcal{E}(\gamma)} = \int \mathcal{D}A\, \delta(\partial A)\ \det \mathcal{M}\ e^{-\left(S_{YM} \underbrace{+\gamma^4 H(A) - \gamma^4 V D(N^2-1)}_{=:\,+\gamma^4 \mathcal{H}}\right)}$$

Gap equation: $$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = V D(N^2-1)$$

$$H(A) = \int_p \int_q A^a_\mu(-p) \left(\mathcal{M}^{ab}\right)^{-1} A^b_\mu(q)$$

- Using auxiliary fields, this can be cast in a *local* form: $Z = \int [\mathcal{D}\Phi]\ \delta(\partial A)\ \det \mathcal{M}\ e^{-S_{GZ}}$

The Refined Gribov-Zwanziger action

- The GZ theory is unstable against the formation of certain dimension 2 condensates, giving rise to a refinement of the effective IR action:

[Dudal et al (2008)]

$$S_{\mathrm{YM}} \xrightarrow[\text{Gribov restriction(UV}\to\text{IR)}]{} S_{\mathrm{GZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H}$$

Dynamical generation of dim.2 condensates

$$\boxed{S_{\mathrm{RGZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H} + \frac{m^2}{2} AA - M^2 \left(\overline{\varphi}\varphi - \overline{\omega}\omega\right)}$$

Gap equation for the Gribov param.:

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = VD(N^2-1)$$

The parameters *M* and *m* are obtained via minimization of an effective potential for:

$$\langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle \neq 0 \qquad\qquad \langle A^2 \rangle \neq 0$$

- Non-perturbative effects included: $(\gamma, M, m) \propto \mathrm{e}^{-\frac{1}{g^2}}$

A checklist for RGZ

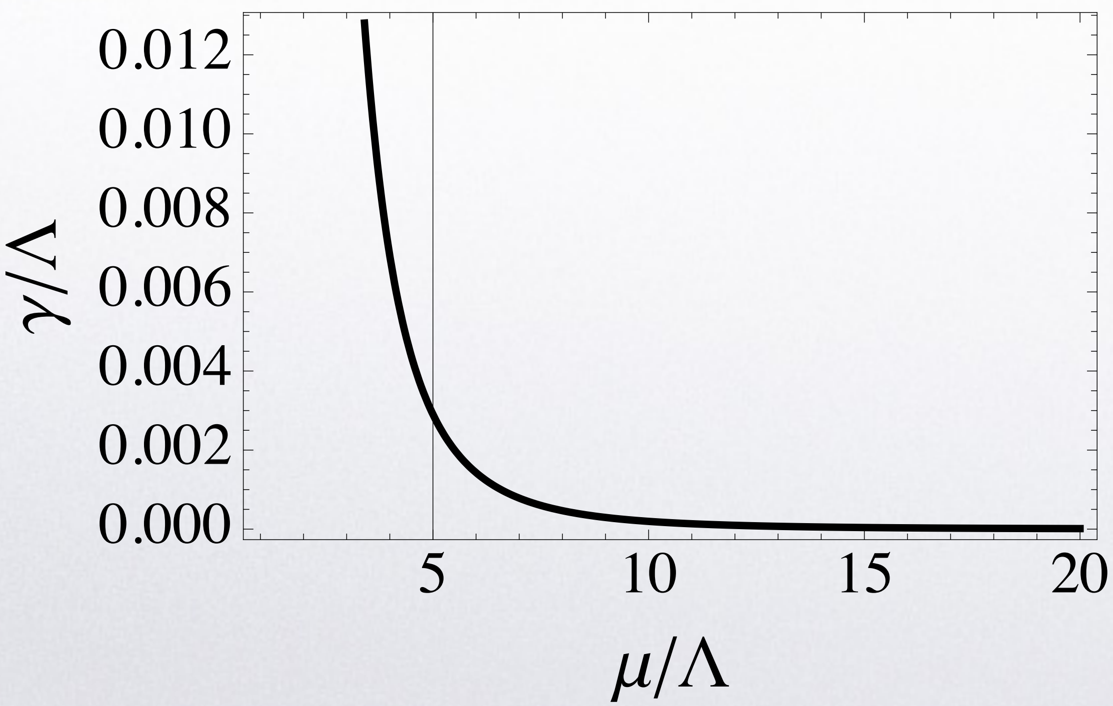
- ✓ *(can be cast in a) local and renormalizable action*
- ✓ *reduces to QCD (pure gauge) at high energies?*

- The one-loop solution of the gap equation in the GZ theory gives:

$$2Ng^2\gamma^4 = \tilde{\gamma}^4 = \mu^4 \mathrm{e}^{\frac{5}{3} - \frac{128\pi^2}{3Ng^2}}$$

- Using the definition of the MSbar YM scale Λ (RG-invariant scale):

$$\frac{\tilde{\gamma}^4}{\Lambda} = \mathrm{e}^{5/12}\left[\frac{\Lambda}{\mu}\right]^{\frac{ab_0\pi}{2N}} \qquad \frac{ab_0\pi}{2N} \sim 3.9$$



$$\Lambda = 300\mathrm{MeV}$$

$$\tilde{\gamma}(\mu = 1\,\mathrm{GeV}) \sim 4\,\mathrm{MeV}$$
$$\tilde{\gamma}(\mu = 5\,\mathrm{GeV}) \sim 0.008\,\mathrm{MeV}$$

A checklist for RGZ

✓ ***(can be cast in a) local and renormalizable action***

✓ ***reduces to QCD (pure gauge) at high energies***

✓ ***consistent with gluon 'confinement'? Confining propagator (no physical propagation; violation of reflection positivity)***

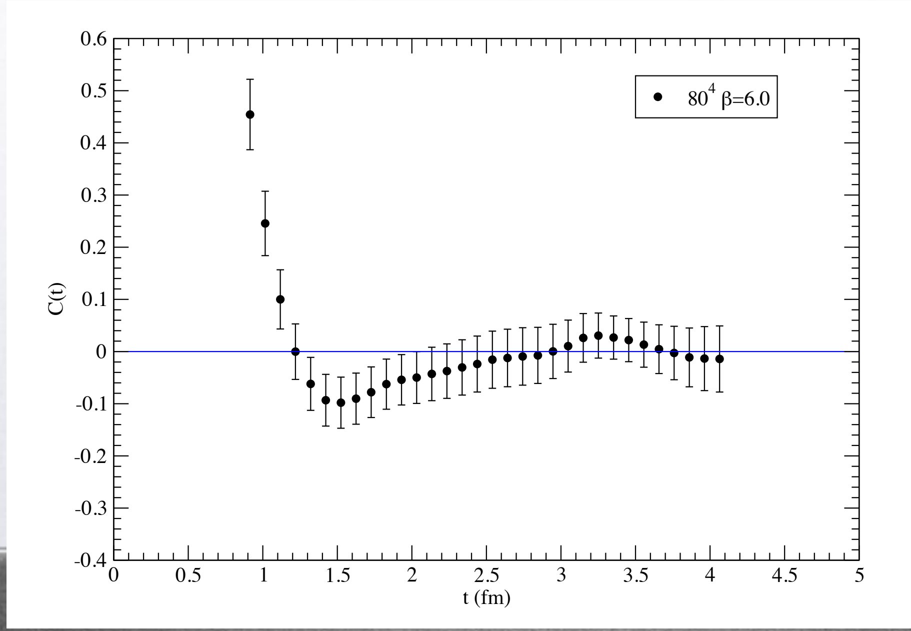
Schwinger function (computed directly from the gluon propagator):

$$C(t) = \int_{-\infty}^{\infty} \frac{\mathrm{d}p}{2\pi} D(p^2) \exp(-ipt).$$

Strictly positive if the gluon spectral function is physical:

$$C(t) = \int_0^{\infty} \mathrm{d}\omega \rho(\omega^2) e^{-\omega t}, \qquad D(p^2) = \int_0^{\infty} \mathrm{d}\mu \frac{\rho(\mu)}{\mu + p^2}.$$

Positivity violation: for the RGZ gluon and in lattice data



SU(3) latt.: [Silva et al (2014)]

A checklist for RGZ

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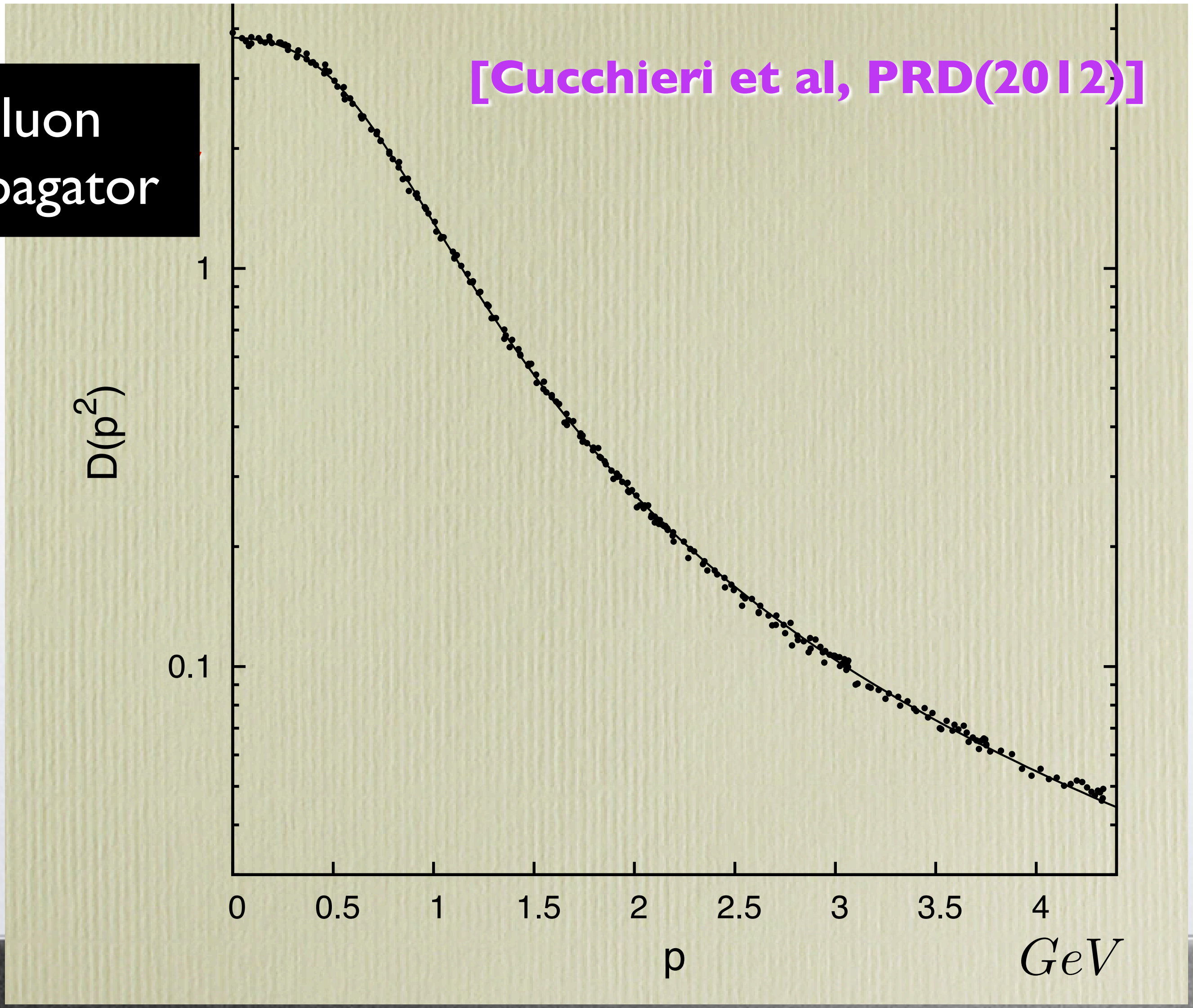
✓ *consistent with **gluon 'confinement':** confining propagator (no physical propagation; violation of reflection positivity)*

✓ *consistent with lattice IR results ?*

Gluon Propagator



$$\langle A^a_\mu A^b_\nu \rangle_p = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(p^2)$$

$$D_{\rm fit}(p^2) = C\,\frac{p^2+s}{p^4+u^2\,p^2+t^2}$$

$$C = 0.56(0.01)\,,\; u = 0.53(0.04)\;\mathrm{GeV}\,,$$

$$t = 0.62(0.01)\;\mathrm{GeV}^2\,,\; u = 2.6(0.2)\;\mathrm{GeV}^2$$

poles: $m^2_\pm \;=\; (0.352 \pm 0.522i)\mathrm{GeV}^2$

$$D_{\rm RGZ}(p^2) = \frac{p^2+M^2}{p^4+(M^2+m^2)p^2+2g^2N\gamma^4}$$

NB.: Complex conjugated poles!

A checklist for RGZ

✓ *(can be cast in a) local and renormalizable action*

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✓ *consistent with gluon 'confinement': confining propagator (no physical propagation; violation of reflection positivity)*

✓ *consistent with lattice IR results (propagators, ghost-gluon vertex)*

[See talk by B. Mintz]

A checklist for RGZ

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✓ *physical spectrum of bound states ??*

Glueball masses are obtained by computing two-point correlation functions of composite operators with the appropriate quantum numbers and casting them in the form of a Källén-Lehmann spectral representation. ***A lot of caveats of course!***

J^{PC}	confining gluon propagator
0^{++}	2.27
2^{++}	2.34
0^{-+}	2.51
2^{-+}	2.64

[Dudal,Guimaraes,Sorella, PRL(2011), PLB(2014)]

-Lattice: (1) Y. Chen *et al.* PRD **73**, 014516 (2006)
-Flux tube model: M. Iwasaki *et al.* PRD **68**, 074007 (2003).
-Hamiltonian QCD: A. P. Szczepaniak and E. S. Swanson, PLB **577**, 61 (2003).
-AdS/CFT: K. Ghoroku, K. Kubo, T. Taminato and F. Toyoda, arXiv:1111.7032.
-AdS/CFT: H. Boschi-Filho, N. R. F. Braga JHEP 0305, 009 (2003)

J^{PC}	Lattice	Flux tube model	Hamiltonian QCD	ADS/CFT
0^{++}	1.71	1.68	1.98	1.21
2^{++}	2.39	2.69	2.42	2.18
0^{-+}	2.56	2.57	2.22	3.05
2^{-+}	3.04	–	–	–

RGZ: Correct hierarchy of masses

A checklist for RGZ

✓ *(can be cast in a) local and renormalizable action*

✓ *reduces to QCD (pure gauge) at high energies*

✓ *consistent with gluon 'confinement': confining propagator (no physical propagation; violation of reflection positivity)*

✓ *consistent with lattice IR results (propagators, ghost-gluon vertex)*

✓ *physical spectrum of bound states? Glueballs w/ masses compatible w/ lattice*

✓ *other applications…* [Canfora *et al*, Sobreiro *et al*, …]

✓ *Exact modified BRST invariance => gauge-parameter independence*

BRST-invariant (R)GZ framework in a nutshell

- A gauge-invariant gluon field: [Dell'Antonio & Zwanziger ('89), van Ball ('92), Lavelle & McMullan ('96)]

$$f_A[u] \equiv \min_{\{u\}} \mathrm{Tr} \int d^d x\, A^u_\mu A^u_\mu \qquad A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right)\phi_\nu$$

$$A^u_\mu = u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u \qquad \phi_\nu = A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right] + \mathcal{O}(A^3)\,.$$

- Localization is possible through the introduction of a Stueckelberg field ξ^a:

$$A^h_\mu = (A^h)^a_\mu T^a = h^\dagger A^a_\mu T^a h + \frac{i}{g} h^\dagger \partial_\mu h\,, \qquad h = e^{ig\,\xi^a T^a}$$

- The BRST-invariant Gribov region and condensates: **[Capri *et al* (2016,2017)]**

$$\Omega = \{A^a_\mu;\ \partial_\mu A^a_\mu = i\alpha b^a, \qquad \mathcal{M}^{ab}(A^h) = -\partial_\mu D^{ab}_\mu(A^h) > 0\}$$ (ex. in Linear Cov. Gauges)

$$\langle A^a_\mu(p) A^b_\nu(-p)\rangle = \frac{p^2 + M^2}{p^4 + (M^2+m^2)p^2 + M^2 m^2 + \lambda^4}\mathcal{P}_{\mu\nu}(p)\delta^{ab} + \frac{\alpha_g}{p^2} L_{\mu\nu}\delta^{ab}$$

$\langle \bar\varphi^{ab}_\mu \varphi^{ab}_\mu \rangle$

$\langle A^{h,a}_\mu A^{h,a}_\mu \rangle$

$$\begin{aligned}
&sA^a_\mu = -D^{ab}_\mu c^b\,,\\
&sc^a = \frac{g}{2} f^{abc} c^b c^c\,, \qquad s\bar c^a = ib^a\\
&sb^a = 0\,,\\
&s\varphi^{ab}_\mu = 0\,, \qquad s\omega^{ab}_\mu = 0\,,\\
&s\bar\omega^{ab}_\mu = 0\,, \qquad s\bar\varphi^{ab}_\mu = 0\,,\\
&s\varepsilon^a = 0\,, \qquad s(A^h)^a_\mu = 0\,,\\
&sh^{ij} = -igc^a (T^a)^{ik} h^{kj}\,.
\end{aligned}$$

A checklist for RGZ

✓ *(can be cast in a) local and renormalizable action*

✓ *reduces to QCD (pure gauge) at high energies*

✓ *consistent with gluon 'confinement': confining propagator (no physical propagation; violation of reflection positivity)*

✓ *consistent with lattice IR results (propagators, ghost-gluon vertex)*

✓ *physical spectrum of bound states? Glueballs w/ masses compatible w/ lattice*

✓ *other applications…* **[Canfora et *al*]**

✓ *Exact BRST invariance*

[Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]

✗ *no quantitative prediction without fitting lattice data for propagators*

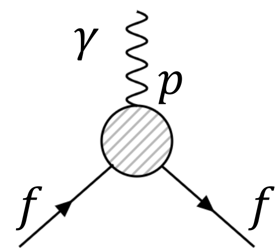
✗ *no general definition of physical operators, unitarity*

✗ *quark confinement properties: linear potential, etc…*

✗ *Minkowski space*

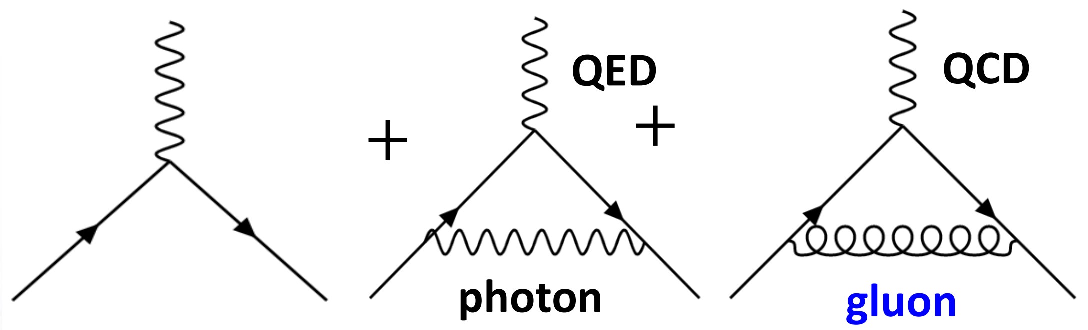
✗ *results for other observables? q-qbar-photon vertex and the AMM*

The quark-photon vertex



$$= e\, Q_f\, \overline{\boldsymbol{U}}(q_2) \left[\gamma_\mu\, \boldsymbol{F_1}(p^2) + \frac{p_\nu \sigma_{\mu\nu}}{2m}\, \boldsymbol{F_2}(p^2)\right] \boldsymbol{U}(q_1) \Rightarrow g = 2\,[\cancelto{1}{F_1(0)} + F_2(0)]$$

- At one-loop:



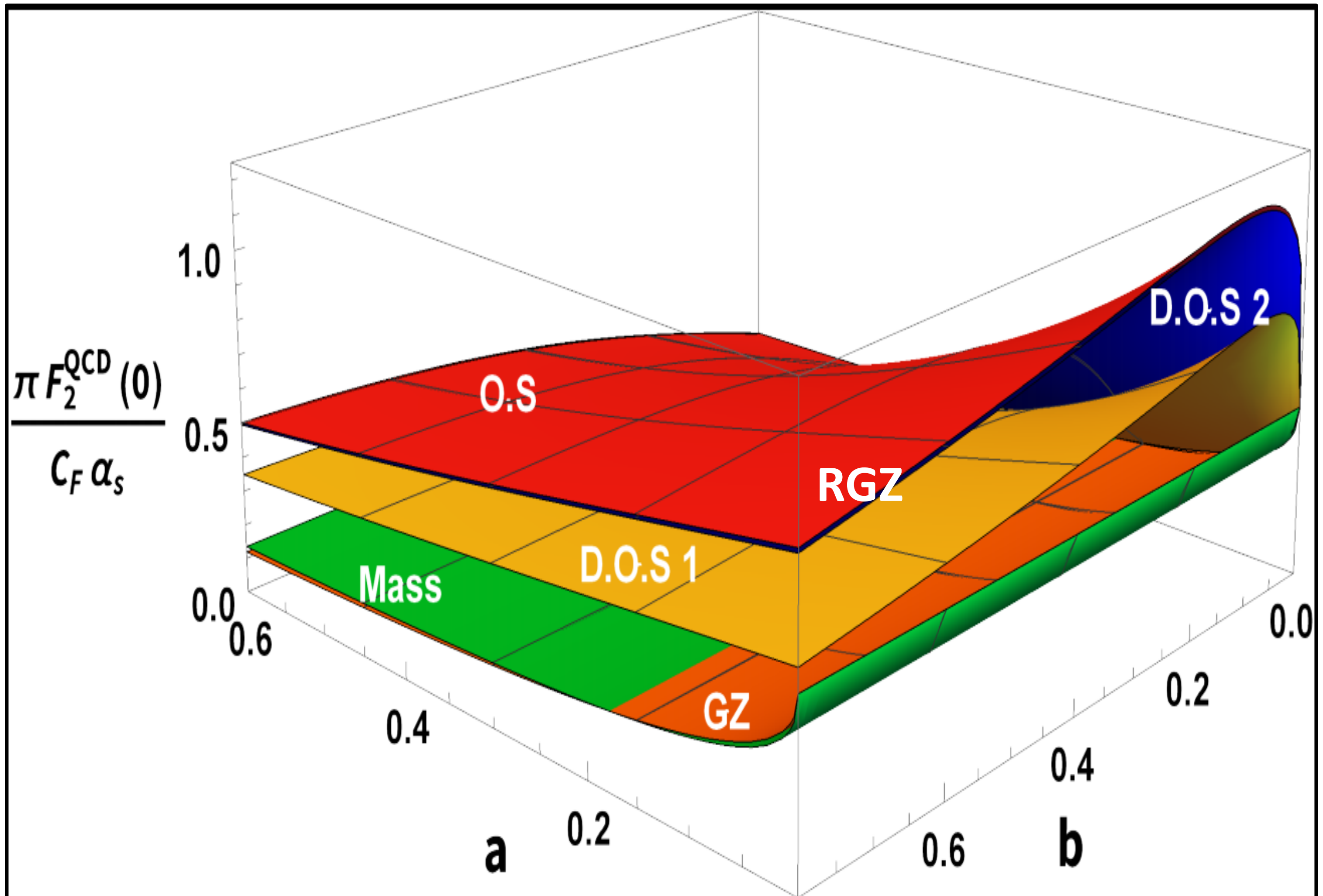
…yielding a quark anomalous magnetic moment:

$$\mu_q = Q_q \left(\frac{e}{2m_q}\right)\left(1 + Q_q^2 \left(\frac{\alpha}{2\pi}\right) + C_F \left(\frac{\alpha_s}{\pi}\right) \overline{F}_2(0)\right)$$

Gluon Confining models	Massive	GZ	RGZ
	$\frac{1}{l^2 + m_g^2}$	$\frac{l^2}{l^4 + \lambda^4}$	$\frac{l^2 + M_1^2}{l^4 + l^2 M_2^2 + M_3^4}$

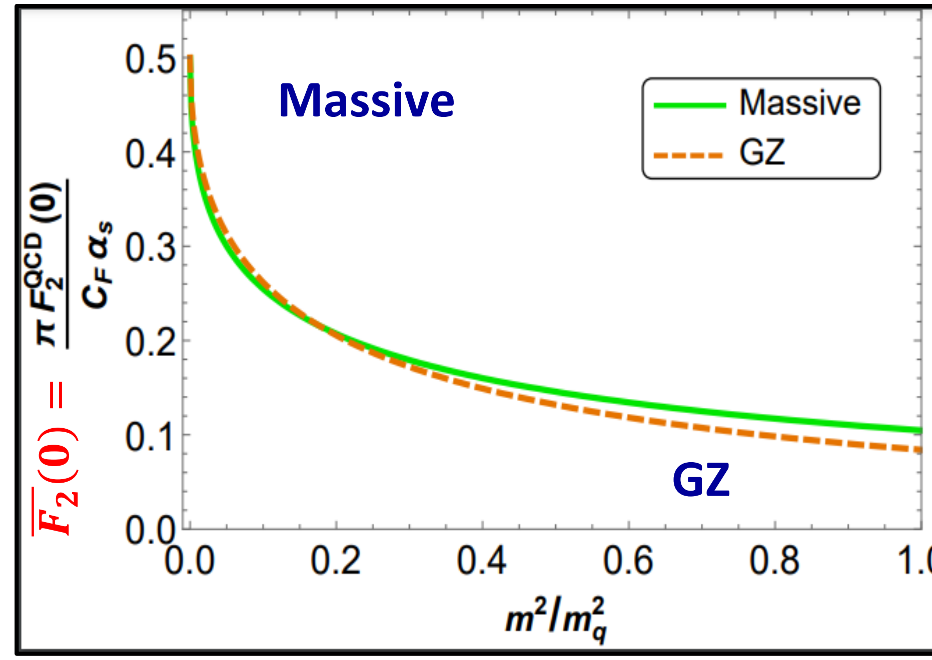
[Mena & LFP, to appear]

$\overline{F_2}(0)$ RESULTS



$m_q = 363$ MeV $M_p \to M_p^{exp} \approx 939$ MeV

*a and b are ratios of the gluon mass parameters and the quark mass.



RGZ:

DATA	M [GeV]	m [GeV]	λ [GeV]	$\overline{F_2}(0)$
O.S	2.1150	1.9414 i	2.0381	0.2456
D.O.S 2	2.0389	1.8881 i	1.9730	0.2658

Massive parameters fit from lattice data:
Oliveira & Silva, PRD (2012)
Dudal, Oliveira & Silva, Ann. Phys. (2018)

Estimating the proton AMM from confining models…

We adopt the simplest Constituent Quark Model to estimate the effect on the proton AMM:

$$\boxed{\mu_p^{CQM} = \left[\frac{4}{3}\mu_u - \frac{1}{3}\mu_d\right]} \qquad \text{with} \qquad \mu_q = Q_q\left(\frac{e}{2m_q}\right)\left(1 + Q_q^2\left(\frac{\alpha}{2\pi}\right) + C_F\left(\frac{\alpha_s}{\pi}\right)\overline{F}_2(0)\right)$$

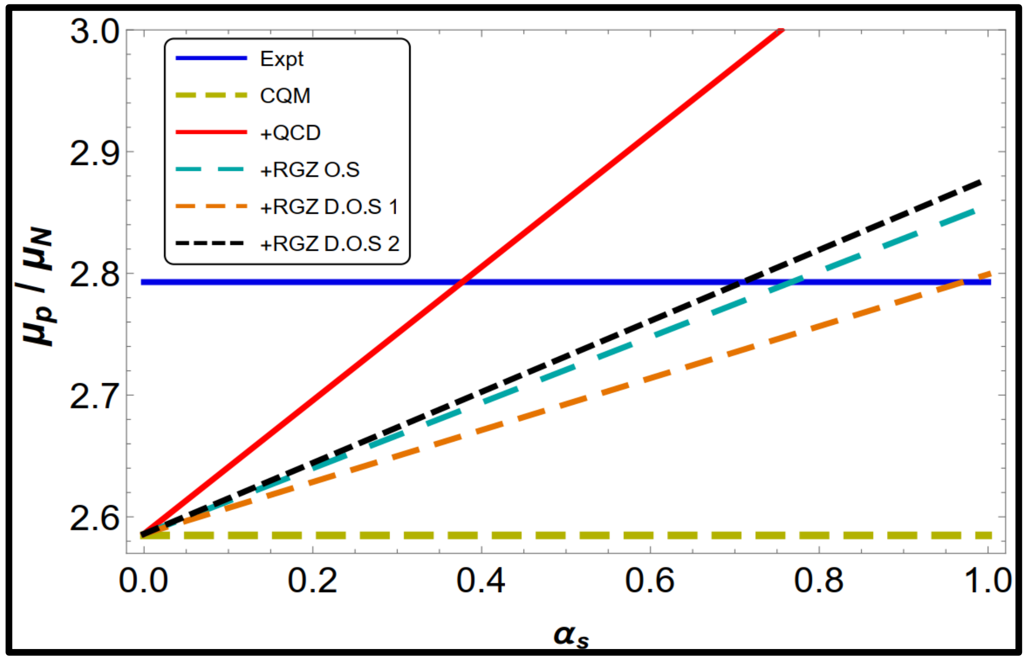
- CQM parameters:
  constituent quark mass fixed to proton mass

  $$m_q = 363 \text{ MeV} \;\; M_p \to M_p^{exp} \approx 939 \text{ MeV}$$

- Confining model parameters:
  dynamically generated gluon mass(es)
  +
  strong coupling in the deep IR

m_g	0 MeV	140 MeV	185.64 MeV	600 MeV
$\alpha_s \parallel \lambda_{CF} = 3\alpha_s/4$	0.38 ‖ 0.091	0.83 ‖ 0.198	1.00 ‖ 0.239	3.24 ‖ 0.773

RGZ model



Estimating the proton AMM from confining models…

1. Confining models — even with complex conjugated poles — yield reasonable results;

2. Dynamically generated gluon masses can be accomodated
   if the strong coupling is large enough in the IR (or changing other CQM parameters…)

3. Still hard to constrain models, but lattice data may help.

- CQM parameters:
  constituent quark mass fixed to proton mass

  $$m_q = 363 \text{ MeV} \;\; M_p \to M_p^{exp} \approx 939 \text{ MeV}$$

- Confining model parameters:
  dynamically generated gluon mass(es)
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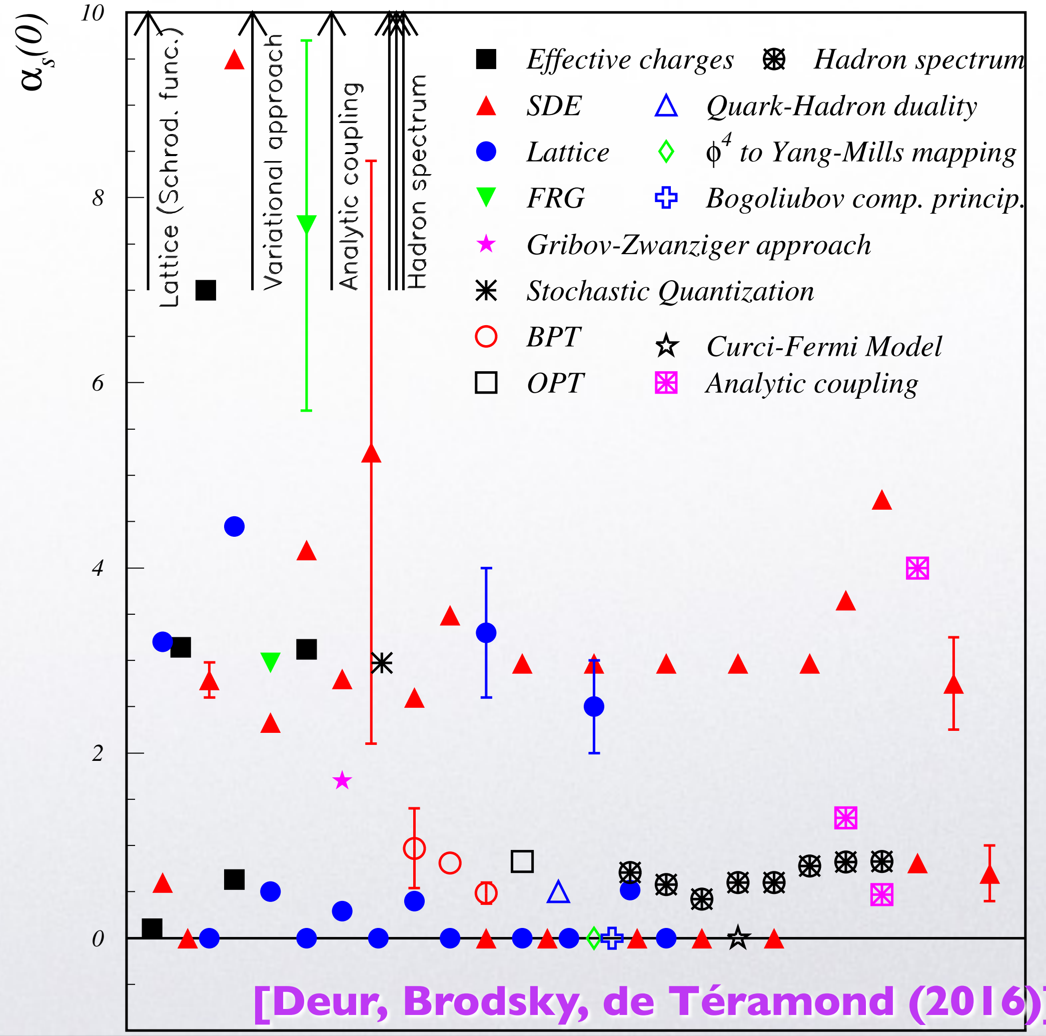


[Deur, Brodsky, de Téramond (2016)]

Final comments

- ***Dynamical gluon mass generation should occur in IR YM theories.***
- The ***Gribov problem*** is present and should profoundly affect the IR regime of gauge-fixed non-Abelian gauge theories.
- The ***RGZ framework*** represents a consistent scenario to study the non-perturbative IR physics and has provided ***interesting results for correlation functions in the gluon sector fitting lattice propagators***.
- The q-qbar-photon may be calculated on the lattice and offers a window to observables like the anomalous magnetic moment (possibility of parameter and/or model constraining)
- Extend calculations to the quark sector and to observables, in order to further test RGZ predictions.

Thank you for your attention!

BACKUP SLIDES

- To explicitly calculate the values of the condensates in RGZ, one should construct an effective potential for the composite operators:

$$\Sigma[\cdots,\tau,Q] = S + \tau \underbrace{\boxed{\frac{1}{2}A_\mu^{h,a}A_\mu^{h,a}}}_{O_A} + Q \underbrace{\boxed{\bar{\varphi}_\mu^{ac}\varphi_\mu^{ac}}}_{O_\varphi} \xrightarrow{\text{Legendre transf.}} \Gamma[O_A, O_\varphi] \xrightarrow{\text{Minimize}} \langle O_I \rangle$$

- For composite operators (mass dimension 2 or higher) a lot of complications appear!

- In the non-BRST-invariant formulation of RGZ, there could be many more condensates and the full effective potential calculation was never achieved.

[cf. Dudal, Sorella & Vandersickel (2011)]

The LCO effective potential of GZ theory

- For composite operators (dim 2 or higher), this is not so straightforward… **[Verschelde ('95)]**

$$\boxed{\Sigma = S + S_{A^2} + S_{\varphi\bar{\varphi}} + S_{\mathrm{vac}}} \quad \begin{cases} S_{A^2} = \int d^dx Z_A (Z_{\tau\tau}\tau + Z_{\tau Q} Q)\frac{1}{2} A^{h,a}_\mu A^{h,a}_\mu \,, \\ S_{\bar{\varphi}\varphi} = \int d^dx Z_{QQ} Z_\varphi Q \bar{\varphi}^{ac}_\mu \varphi^{ac}_\mu \,, \\ S_{\mathrm{vac}} = -\int d^dx \left(\frac{Z_\zeta \zeta}{2}\tau^2 + Z_\alpha \alpha Q^2 + Z_\chi \chi Q \tau \right) \end{cases}$$

- Nonlinear terms in the currents necessary to cancel divergences + Mixing

- The usual Legendre transform does not work, but one can use Hubbard-Stratonovich transformations to eliminate these nonlinear terms in the currents and construct an effective potential that can be properly minimized.

- Finite parts of the LCO parameters ζ, α, ξ have to be computed separately, requiring that the effective potential obeys the usual RG equation.

$\boxed{\text{needed (n+1) loops for n-loop results}}$ **[cf. Dudal, Sorella & Vandersickel (2011)]**

- **In this talk:** [Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]
    - BRST-invariance allows us to work with Landau gauge and "only" two condensates (still 4-dim. parameter space, with the Gribov parameter and renormalization scale)
    - More convenient Hubbard-Stratonovich transformation that eliminates the necessity of n+1-loop calculations. Similar technique first used in [Lemes, Sarandy, Sorella (2003)]

$$1 = \int [\mathcal{D}\sigma_1]\; e^{-\frac{1}{2Z_\zeta}\int d^dx\left(\sigma_1 + \frac{\bar{a}}{2}A^2 + \bar{b}Q + \bar{c}\tau\right)^2},$$

$$1 = \int [\mathcal{D}\sigma_2]\; e^{+\frac{1}{2Z_\alpha}\int d^dx\left(\sigma_2 + \bar{d}\bar{\varphi}\varphi + \bar{e}Q + \frac{\bar{f}}{2}A^2\right)^2},$$

(auxiliary fields σ_1, σ_2 play the role of the composite fields)

$$\begin{aligned}
\bar{a} &= \frac{Z_A Z_{\tau\tau}}{\sqrt{\zeta}}\mu^{\epsilon/2} \\
\bar{b} &= -\frac{Z_\chi \chi}{\sqrt{\zeta}}\mu^{-\epsilon/2}, \\
\bar{c} &= -Z_\zeta\sqrt{\zeta}\mu^{-\epsilon/2}, \\
\bar{d} &= \frac{Z_\varphi Z_{QQ}}{\sqrt{-2\alpha + \frac{Z_\chi^2\chi^2}{Z_\alpha Z_\zeta \zeta}}}\mu^{\epsilon/2}, \\
\bar{e} &= Z_\alpha\sqrt{-2\alpha + \frac{Z_\chi^2\chi^2}{Z_\alpha Z_\zeta\zeta}}\;\mu^{-\epsilon/2}, \\
\bar{f} &= -\frac{\frac{Z_A Z_{\tau\tau} Z_\chi \chi}{Z_\zeta \zeta} + Z_A Z_{\tau Q}}{\sqrt{-2\alpha + \frac{Z_\chi^2\chi^2}{Z_\alpha Z_\zeta \zeta}}}\mu^{\epsilon/2}.
\end{aligned}$$

coefficients chosen to eliminate nonlinear terms in the currents

- After introducing the HS identities, current terms are now linear:

$$e^{-\Gamma(Q,\tau)} = \int [\mathcal{D}\Phi][\mathcal{D}\sigma_1 \mathcal{D}\sigma_2'] \exp\left[-S_{GZ} - \int d^dx \left(\frac{\sigma_1^2}{2Z_\zeta}\left(1-\frac{\bar{b}^2}{\bar{e}^2}\frac{Z_\alpha}{Z_\zeta}\right) - \frac{\sigma_2'^2}{2Z_\alpha} - \frac{\bar{b}}{\bar{e}}\frac{\sigma_1\sigma_2'}{Z_\zeta} \right.\right.$$
$$+\left(\frac{1}{2Z_\zeta}\left(\bar{a}-\frac{\bar{f}\bar{b}}{\bar{e}}\right)\sigma_1 - \frac{\bar{f}}{2Z_\alpha}\sigma_2'\right)A^2 - \left(\frac{\bar{b}\bar{d}}{\bar{e}}\frac{1}{Z_\zeta}\sigma_1 + \frac{\bar{d}}{Z_\alpha}\sigma_2'\right)\overline{\varphi}\varphi$$
$$\left.\left. + \frac{\bar{a}^2}{8Z_\zeta}(A^2)^2 - \frac{1}{2Z_\alpha}\left(\frac{\bar{f}}{2}A^2 + \bar{d}\overline{\varphi}\varphi\right)^2 + \underline{\frac{\bar{c}}{Z_\zeta}\sigma_1\tau - \frac{\bar{e}}{Z_\alpha}\sigma_2' Q}\right)\right]$$

- The condensates are directly related to the sigma fields:

$$\langle A_\mu^{h,a} A_\mu^{h,a}\rangle \longleftrightarrow m^2 = \sqrt{\frac{13Ng^2}{9(N^2-1)}}\,\langle\sigma_1\rangle$$

$$\langle \bar{\varphi}_\mu^{ab}\varphi_\mu^{ab}\rangle \longleftrightarrow M^2 = \sqrt{\frac{35Ng^2}{48(N^2-1)^2}}\,\langle\sigma_2'\rangle$$

- The one-loop effective potential will only involve the quadratic terms in the fluctuations around the condensates. A standard calculation (Tr log of quadratic operators) gives the final analytic result (MSbar scheme):

$$
\begin{aligned}
\Gamma(m^2, M^2, \lambda^4) = &-\frac{2(N^2-1)}{Ng^2}\lambda^4\left(1-\frac{3}{8}\frac{Ng^2}{16\pi^2}\right)+\frac{9(N^2-1)}{13Ng^2}\frac{m^4}{2}-\frac{48(N^2-1)^2}{35Ng^2}\frac{M^4}{2}+\frac{(N^2-1)^2}{8\pi^2}M^4\left(-1+\ln\frac{M^2}{\bar{\mu}^2}\right)\\
&+\frac{3(N^2-1)}{64\pi^2}\left(-\frac{5}{6}(m^4-2\lambda^4)+\frac{m^4+M^4-2\lambda^4}{2}\ln\frac{m^2M^2+\lambda^4}{\bar{\mu}^4}\right.\\
&\left.-(m^2+M^2)\sqrt{4\lambda^4-(m^2-M^2)^2}\arctan\frac{\sqrt{4\lambda^4-(m^2-M^2)^2}}{m^2+M^2}-M^4\ln\frac{M^2}{\bar{\mu}^2}\right).
\end{aligned}
$$

<u>To determine the condensates, one needs to:</u>

1. compute the Gribov parameter lambda through the gap equation: $\frac{\partial\Gamma}{\partial\lambda^4}=0$
2. minimize the effective potential as a function of the condensates

Numerics: minimizing the one-loop effective potential of GZ

- The coupling constant and the renormalization scale (related by the RGE) must also be chosen in order to guarantee:

  (i) a valid perturbative approx. (above the nonperturbative background);
  (ii) valid solutions of the multi-dimensional extremization problem… **NOT EASY…**

- We were only able to find solutions meeting these criteria by considering a generic renormalization scheme, changing the first term in the effective potential to:

$$-\frac{2(N^2-1)}{Ng^2}\lambda^4\left(1-\left(\frac{3}{8}-b_0\right)\frac{Ng^2}{16\pi^2}\right)$$

- Now we have acceptable solutions, but the parameter b0 is NOT self-consistently determined. Applying the Principle of Minimal Sensitivity also does not work…
- A robust result is the instability of the zero-condensate case, meaning that GZ (scaling solution) is not even an acceptable phase of the theory [d=4].

[Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]

Numerics: minimizing the one-loop effective potential of GZ

[Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]

- We were able to show that for both SU(3) and SU(2) the renormalization scheme (i.e. b0) can be chosen to give proper minima of the effective potential describing the available lattice data:

SU(3)

$$\frac{g^2N}{16\pi^2} = 0.40\,, \qquad \bar{\mu} = 1.41\,\Lambda_{\overline{MS}} = 0.31\text{ GeV}\,,$$
$$\Gamma = -24\,\Lambda^4_{\overline{MS}} = -0.059\text{ GeV}^4\,, \qquad \lambda^4 = 28\,\Lambda^4_{\overline{MS}} = 0.071\text{ GeV}^4\,,$$
$$m^2 = 2.6\,\Lambda^2_{\overline{MS}} = 0.13\text{ GeV}^2\,, \qquad M^2 = 7.8\,\Lambda^2_{\overline{MS}} = 0.39\text{ GeV}^2\,.$$

SU(3) Lattice data: [Cucchieri,Dudal,Mendes,Vanderscikel (2012)]

SU(2)

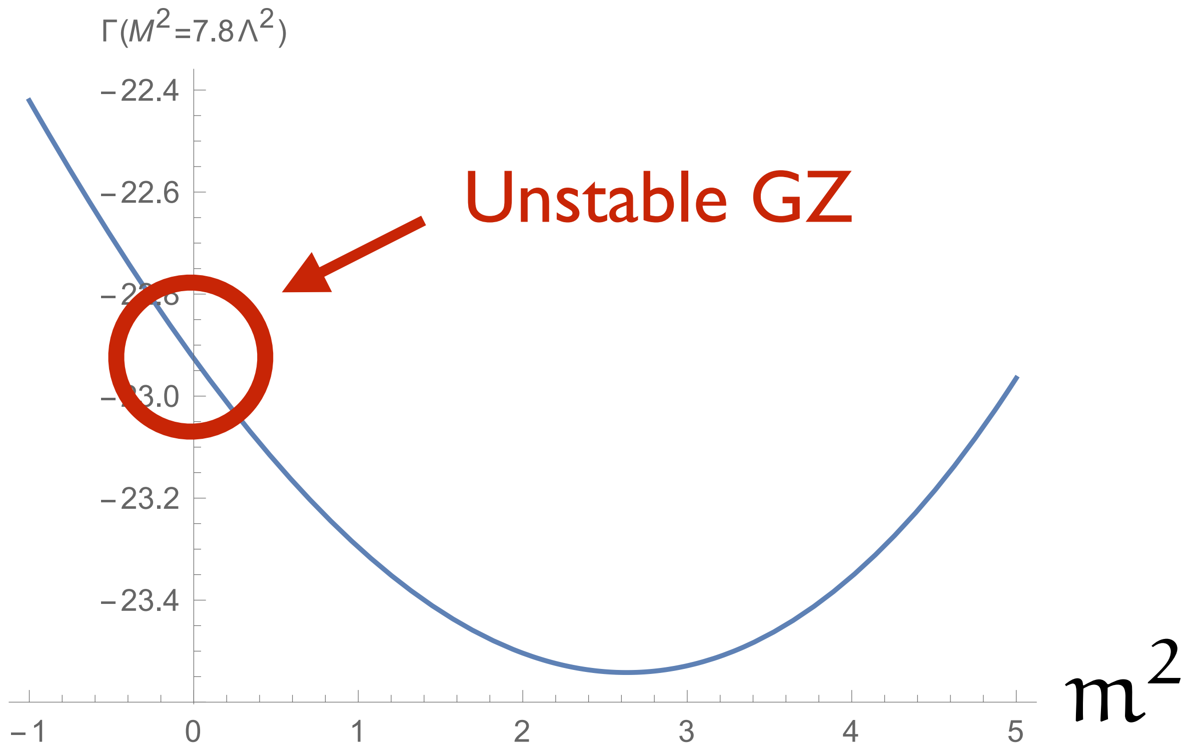
$$\frac{g^2N}{16\pi^2} = 1.24\,, \qquad \bar{\mu} = 1.12\,\Lambda_{\overline{MS}} = 0.37\text{ GeV}\,,$$
$$\Gamma = -0.38\,\Lambda^4_{\overline{MS}} = -0.0046\text{ GeV}^4\,, \qquad \lambda^4 = 9.1\,\Lambda^4_{\overline{MS}} = 0.109\text{ GeV}^4$$
$$m^2 = 2.3\,\Lambda^2_{\overline{MS}} = 0.25\text{ GeV}^2\,, \qquad M^2 = 2.9\,\Lambda^2_{\overline{MS}} = 0.32\text{ GeV}^2\,.$$

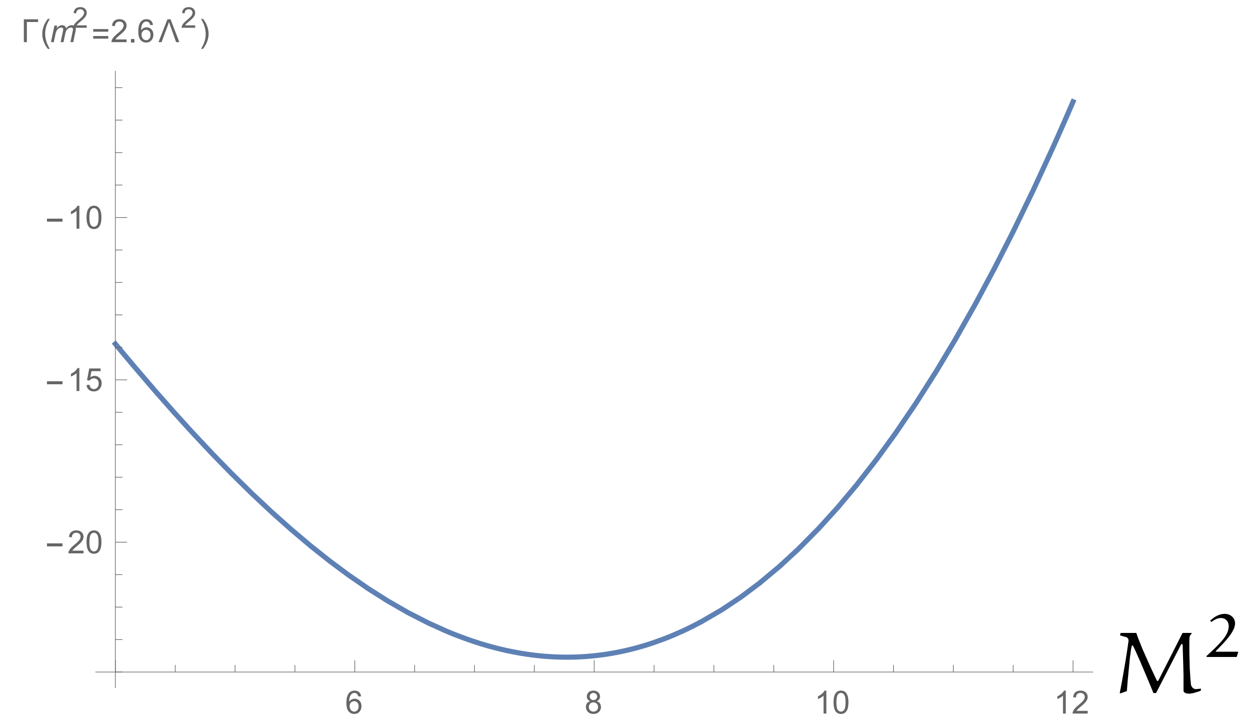
SU(2) Lattice data: [Dudal,Oliveira,Silva (2018)]

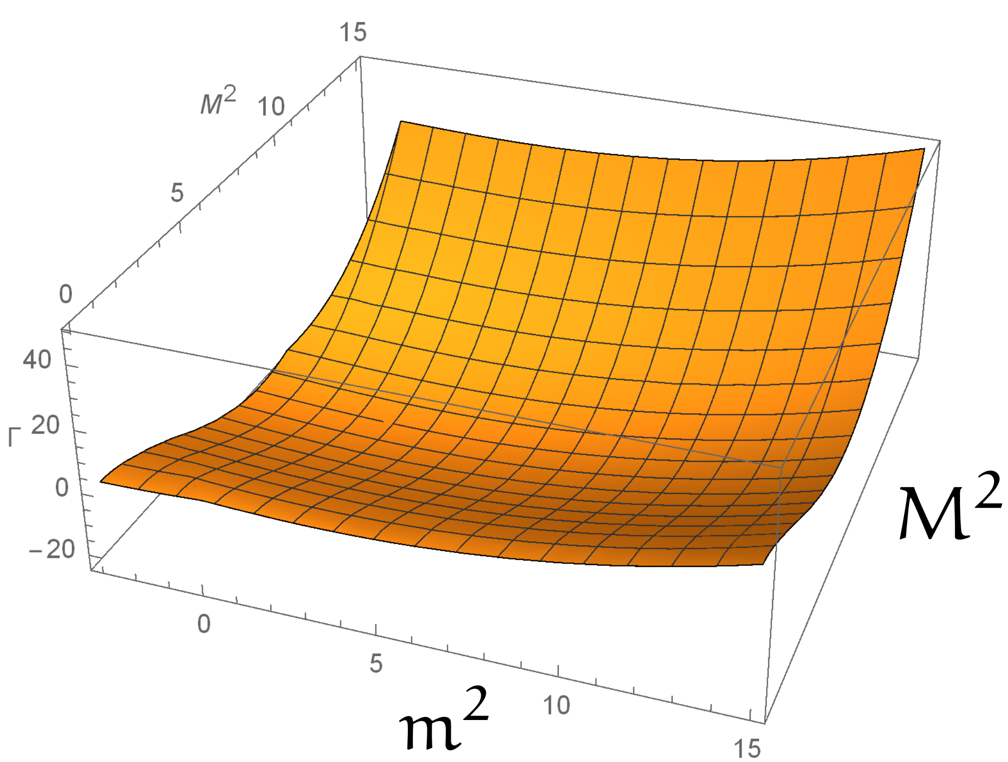
Unstable GZ and (stable) RGZ minimum

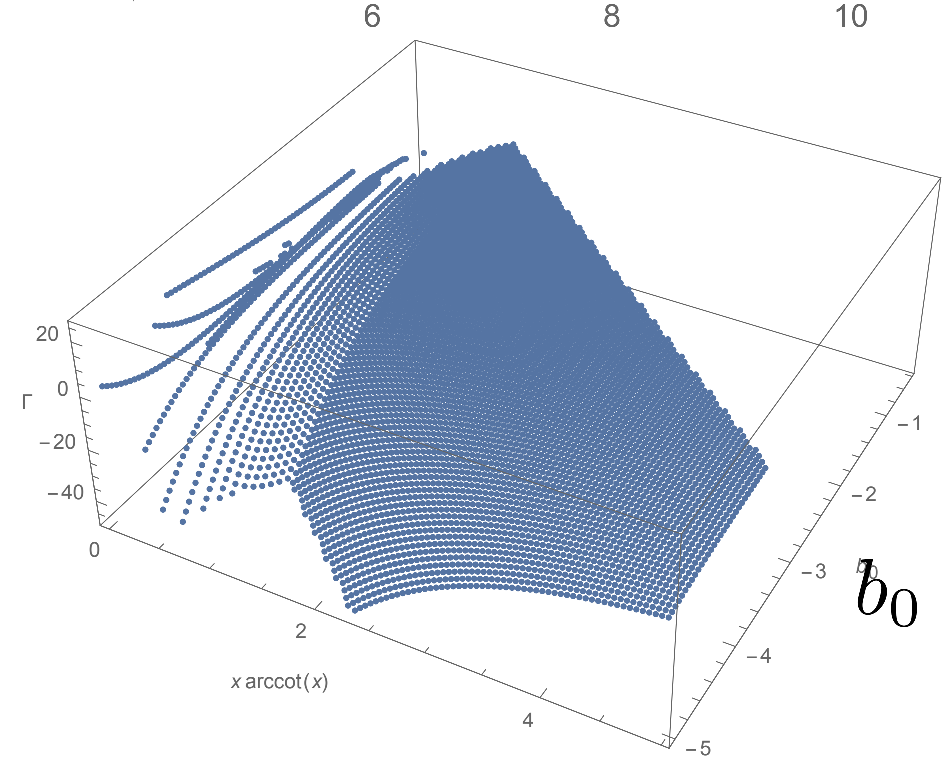
Effective Potential Plots [SU(3)]:

[Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]









LCO parameter

J^2 is always needed in the counterterm, and the starting action needs to display a term[3] ζJ^2. The novel parameter ζ, called the LCO parameter, is needed to absorb the divergences in J^2, i.e. $\delta\zeta J^2$. With the inclusion of the term ζJ^2, the functional $W(J)$ obeys the following homogeneous RGE

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g^2)\frac{\partial}{\partial g^2}-\gamma_J(g^2)\int d^4x J\frac{\delta}{\delta J}+\eta(g^2,\zeta)\frac{\partial}{\partial\zeta}\right)W(J)=0\,, \tag{21}$$

with $\eta(g^2,\zeta)$ the running of ζ,

$$\mu\frac{\partial}{\partial\mu}\zeta=\eta(g^2,\zeta)\,. \tag{22}$$

Notice that it is necessary to include the running of ζ at this point.

Now the question is, how can we determine this seemingly arbitrary parameter ζ? This is possible by employing the renormalization group equations. We can write

$$\zeta_0 J_0^2=\mu^{-\varepsilon}(\zeta J^2+\delta\zeta J^2)\,, \tag{23}$$

whereby the second term of the r.h.s. represents the counterterm. As the l.h.s. is independent from μ, we can derive both sides w.r.t. μ to find:

$$-\varepsilon(\zeta+\delta\zeta)+\left(\mu\frac{\partial}{\partial\mu}\zeta+\mu\frac{\partial}{\partial\mu}(\delta\zeta)\right)-2\gamma_J(g^2)(\zeta+\delta\zeta)=0\,, \tag{24}$$

whereby $\gamma_J(g^2)$ is the anomalous dimension of J. As we can consider ζ to be a function of g^2, and by evoking the β function,

$$\beta(g^2)\frac{\partial}{\partial g^2}\zeta(g^2) = 2\gamma_J(g^2)\zeta + f(g^2)\,. \tag{26}$$

with $f(g^2) = \varepsilon\delta\zeta - \beta(g^2)\frac{\partial}{\partial g^2}(\delta\zeta) + 2\gamma_G(g^2)\delta\zeta$. The general solution of this differential equation reads

$$\zeta(g^2) = \zeta_p(g^2) + \alpha\exp\left(2\int_1^{g^2}\frac{\gamma_J(z)}{\beta(z)}\mathrm{d}z\right)\,, \tag{27}$$

with $\zeta_p(g^2)$ a particular solution of (26). A possible particular solution is given by

$$\zeta_p(g^2) = \frac{c_0}{g^2} + c_1\hbar + c_2 g^2\hbar^2 + \ldots\,. \tag{28}$$

whereby we have temporarily introduced the dependence on $\hbar$. Notice therefore that the n-loop result for $\zeta(p^2)$ will require the $(n+1)$ loop results of $\beta(g^2)$, $\gamma_J(g^2)$ and $f(g^2)$. As we would like ζ to be multiplicatively renormalizable, we set $\alpha = 0$. In this case we have that

$$\zeta(g^2) + \delta\zeta(g^2) = \zeta_0 = Z_\zeta\zeta(g^2)\,, \tag{29}$$

and we have removed the independent parameter α. Also, now that ζ is a function of g^2, the RGE (21) becomes

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g^2)\frac{\partial}{\partial g^2} - \gamma_J(g^2)\int \mathrm{d}^4x J\frac{\delta}{\delta J}\right)W(J) = 0\,, \tag{30}$$

as deriving w.r.t. ζ is now incorporated in deriving w.r.t. g^2.

After determining the LCO parameter ζ, the next step is to calculate the effective action by doing a Legendre transformation. However, it shall be easier to perform a Hubbard-Stratonovich transformation on $W(J)$, whereby we introduce an auxiliary field σ describing the composite operator O. In this way, we can get rid of the quadratic term in J^2 and a clear relation with the effective action emerges, as it will

Quantizing Yang-Mills theory

$$S_{YM} = \frac{1}{4}\int dx\, F^a_{\mu\nu} F^a_{\mu\nu},$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- The quantum theory may be formulated in a path-integral approach:

$$\int \mathcal{D}A\, e^{-S_{YM}}$$

- Gauge reduncancy must be properly fixed to work with these dofs:

$$A_\mu \to U A_\mu U^\dagger - U \partial_\mu U^\dagger$$

- We only know how to do it perturbatively!

Faddeev-Popov procedure:

- The procedure amounts to disentangle the gauge redundancy from the integral measure

$$\int \mathcal{D}A\, e^{-S_{YM}} \rightarrow \int \mathcal{D}\Omega \int \mathcal{D}A\, \delta[G(A)]\, \det \mathcal{M} e^{-S_{YM}}$$

supposing we can write

$$\int \mathcal{D}\Omega\, \delta[G(A)]\, \det \mathcal{M} = \mathbb{1}$$

with the Faddeev-Popov operator

$$\mathcal{M}^{ab}(A) = \left.\frac{\delta G^a[A^{(g)}]}{\delta \Omega^b}\right|_{\Omega=0}$$

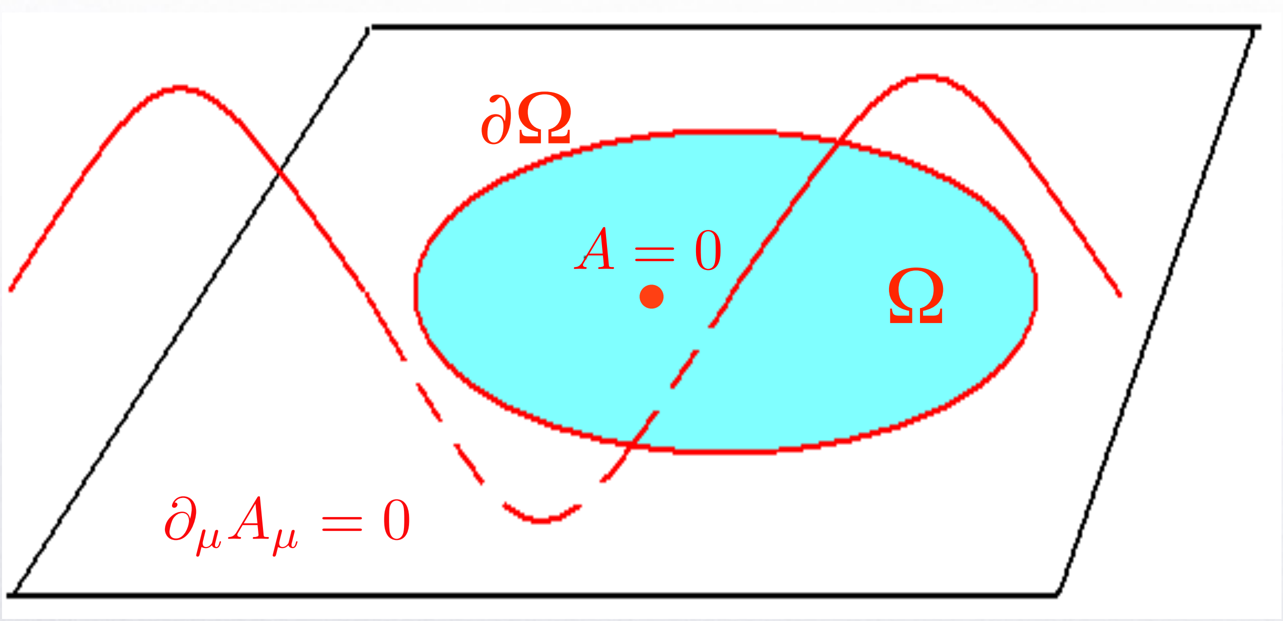
- Gribov proposed a way to eliminate (infinitesimal) Gribov copies from the integration measure over gauge fields: **the restriction to the (first) Gribov region Ω**

$$\int [DA]\delta(\partial A)\,\det(\mathcal{M})\mathrm{e}^{-S_{\rm YM}} \longrightarrow \int_\Omega [DA]\delta(\partial A)\,\det(\mathcal{M})\mathrm{e}^{-S_{\rm YM}} \qquad S_{\rm YM}=\frac{1}{4}\int_x F^2$$

with $\Omega = \left\{ A_\mu^a \;;\; \partial A^a = 0, \mathcal{M}^{ab} > 0 \right\}$

$$\boxed{\mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + f^{abc}A_\mu^c\right) = -\partial_\mu D_\mu^a}$$

(Faddeev-Popov operator)



- The FP operator is related to the ghost 2-point funtion:

$$\mathcal{G}^{ab}(k;A) = \langle k|c^a\bar{c}^b|k\rangle = \langle k|\left(\mathcal{M}^{ab}\right)^{-1}|k\rangle$$

positivity of $\mathcal{M}^{ab}$ $\longleftrightarrow$ ***No-pole condition*** for the ghost prop.

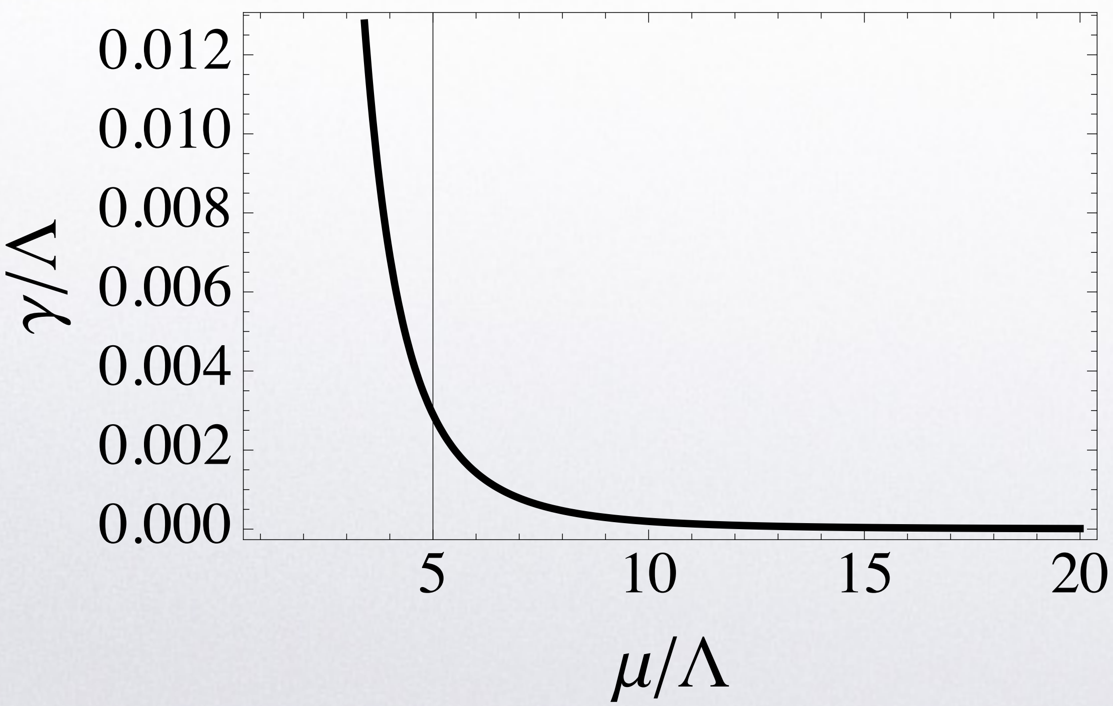
[Gribov (1978)]

- The one-loop solution of the gap equation in the GZ theory gives:

$$2Ng^2\gamma^4 = \tilde{\gamma}^4 = \mu^4 \mathrm{e}^{\frac{5}{3} - \frac{128\pi^2}{3Ng^2}}$$

- Using the definition of the MSbar YM scale Λ (RG-invariant scale):

$$\frac{\tilde{\gamma}^4}{\Lambda} = \mathrm{e}^{5/12}\left[\frac{\Lambda}{\mu}\right]^{\frac{ab_0\pi}{2N}} \qquad \frac{ab_0\pi}{2N} \sim 3.9$$



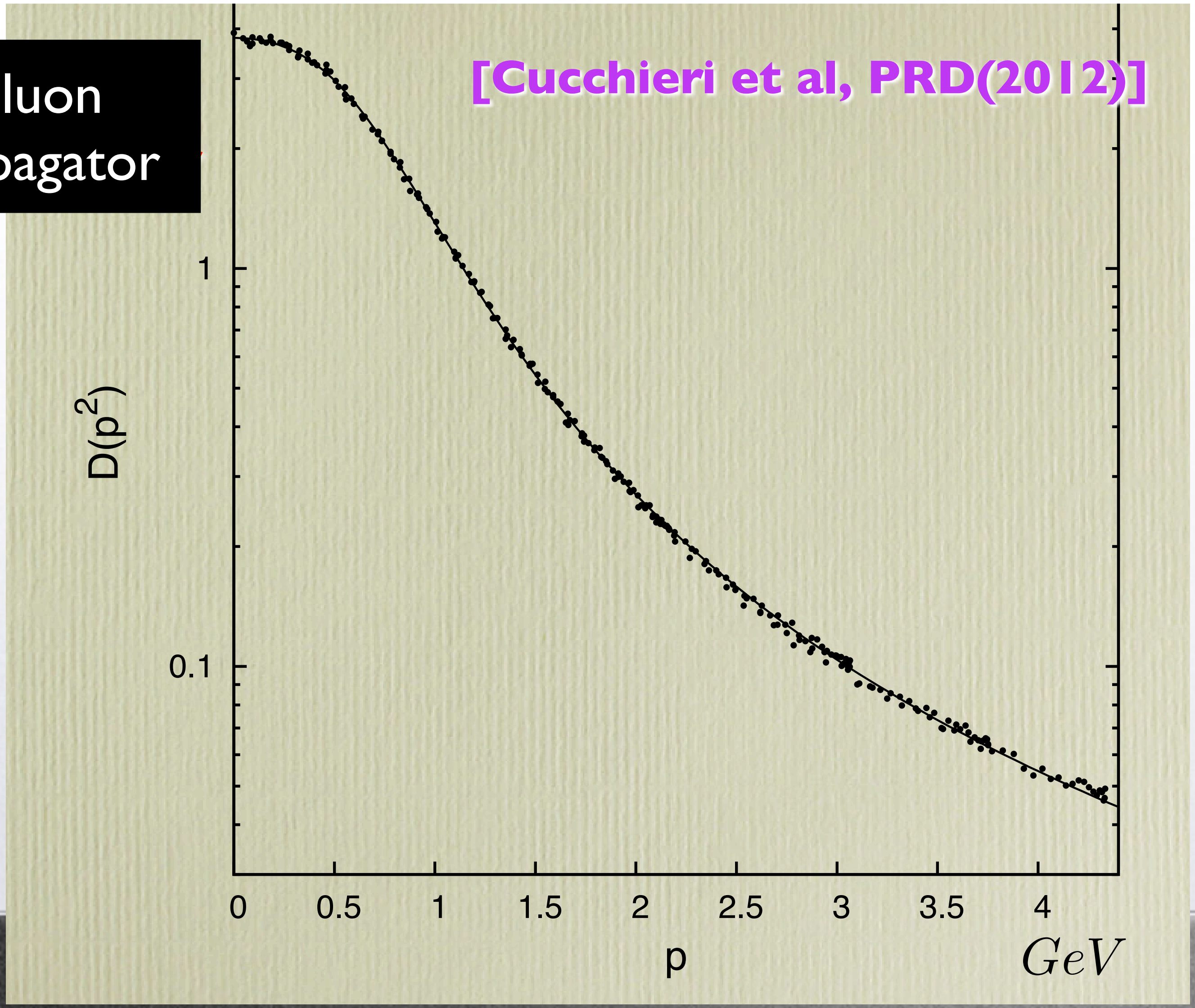
$$\Lambda = 300\mathrm{MeV}$$

$$\tilde{\gamma}(\mu = 1\,\mathrm{GeV}) \sim 4\,\mathrm{MeV}$$
$$\tilde{\gamma}(\mu = 5\,\mathrm{GeV}) \sim 0.008\,\mathrm{MeV}$$

A checklist for RGZ

- ✓ *(can be cast in a) local and renormalizable action*
- ✓ *reduces to QCD at high energies*
- ✓ *gluon confinement: confining propagator (no physical propagation; violation of reflection positivity)*
- ✓ *consistent with lattice IR results ?*

Gluon Propagator



$$\langle A^a_\mu A^b_\nu \rangle_p = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(p^2)$$

$$D_{\rm fit}(p^2) = C\,\frac{p^2+s}{p^4+u^2\,p^2+t^2}$$

$$C = 0.56(0.01)\,,\ u = 0.53(0.04)\,\mathrm{GeV}\,,$$

$$t = 0.62(0.01)\,\mathrm{GeV}^2\,,\ u = 2.6(0.2)\,\mathrm{GeV}^2$$

poles: $m^2_\pm \;=\; (0.352 \pm 0.522i)\mathrm{GeV}^2$

$$D_{\rm RGZ}(p^2) = \frac{p^2+M^2}{p^4+(M^2+m^2)p^2+2g^2N\gamma^4}$$

✓ *consistent with lattice IR results*

A checklist for RGZ

✓ *(can be cast in a) local and renormalizable action*

✓ *reduces to QCD at high energies*

✓ *gluon confinement: confining propagator (no physical propagation; violation of reflection positivity)*

✓ *consistent with lattice IR results*

✓ *physical spectrum of bound states ??*

Glueball masses are obtained by computing two-point correlation functions of composite operators with the appropriate quantum numbers and casting them in the form of a Källén-Lehmann spectral representation.

J^{PC}	confining gluon propagator
0^{++}	2.27
2^{++}	2.34
0^{-+}	2.51
2^{-+}	2.64

[Dudal,Guimaraes,Sorella, PRL(2011), PLB(2014)]

-Lattice: (1) Y. Chen *et al.* PRD **73**, 014516 (2006)
-Flux tube model: M. Iwasaki *et al.* PRD **68**, 074007 (2003).
-Hamiltonian QCD: A. P. Szczepaniak and E. S. Swanson, PLB **577**, 61 (2003).
-AdS/CFT: K. Ghoroku, K. Kubo, T. Taminato and F. Toyoda, arXiv:1111.7032.
-AdS/CFT: H. Boschi-Filho, N. R. F. Braga JHEP 0305, 009 (2003)

J^{PC}	Lattice	Flux tube model	Hamiltonian QCD	ADS/CFT
0^{++}	1.71	1.68	1.98	1.21
2^{++}	2.39	2.69	2.42	2.18
0^{-+}	2.56	2.57	2.22	3.05
2^{-+}	3.04	–	–	–

RGZ: Correct hierarchy of masses

- The Faddeev-Popov action in the Landau gauge

$$S_{\rm YM}=\int_x\left(\frac{1}{4}F^2+b^a\partial A^a+\bar{c}^a(\partial_\mu D_\mu)^{ab}c^b\right)$$

is invariant under BRST transformations:

$$sA^a_\mu=-D^{ab}_\mu c^b \qquad sc^a=\frac{g}{2}f^{abc}c^bc^c \qquad s\bar{c}^a=b^a \qquad sb^a=0$$

- ***The restriction to the Gribov region breaks this BRST symmetry:*** the horizon term is not invariant

$$S_{\rm GZ}\;=\;S_{\rm YM}\;+\;\gamma^4\mathcal{H}\longrightarrow \quad sS_{\rm GZ}\;=\;\gamma^2\Delta$$

$$\Delta=\int_x\left[-gf^{abc}(D^{am}_\mu c^m)(\varphi^{bc}_\mu+\bar{\varphi}^{bc}_\mu)+gf^{abc}A^a_\mu\omega^{bc}_\mu\right]$$

- The breaking terms have dimension less than D=4, so that they correspond to ***SOFT*** terms.

- The softness is crucial for keeping the UV intact. ***Perturbative results recovered at high energies!***
- Can be understood as a ***non-perturbative BRST symmetry, that controls gauge-par. dependence and the extension to other gauges*** **[Capri et al, (2015,20**

- The restriction to the Gribov region is a constraint on a quantum operator: $\mathcal{M}^{ab} \geq 0$

➠ **should naturally affect all correlation functions built with it...**

$$\boxed{\int [D\mu]\mathrm{e}^{-S} \overset{\mathrm{IR}}{\mapsto} \int [D\mu]_\Omega \mathrm{e}^{-S}} \quad \left\{ \begin{array}{l} \langle \tilde{R}\tilde{R} \rangle \sim 1/k^4 \\ \langle \mathcal{M}^{-1}\mathcal{M}^{-1} \rangle \sim 1/k^4 \\ \quad \ldots \end{array} \right.$$

$$\mathcal{R} \approx \int A\mathcal{M}^{-1}$$

... including the ones with <u>matter fields</u>: $F^I = (\psi^i, \phi^a)$

$$\mathcal{R}_F = g \int_z (\mathcal{M}^{-1})^{ab}(x,z)(T^b)^{IJ} F^J(z) \qquad \langle \tilde{\mathcal{R}}_F \tilde{\mathcal{R}}_F \rangle$$

Can in principle be checked on the lattice!

How to construct an IR effective ***gauge-matter*** action that can account for these correlations?

Violation of positivity

Osterwalder-Schrader axiom of reflection positivity

$$\int d^4x d^4y f^\dagger(-x_0,\vec{x})\Delta(x-y)f(y_0,\vec{y}) \geq 0$$

where f is a test function with support for positive times. Making a Fourier transformation, one gets the condition

$$\int dt dt' d^4y f^\dagger(t',\vec{p})\Delta((t'-t),\vec{p})f(t,\vec{p}) \geq 0 \;\; \forall\vec{p}$$

If Δ(t'-t, p) is negative within any domain, it is easy to find a test function which will pinpoint this negative region, thus violating the O-S positivity. In practice, one looks at p=0

$$\Delta(t) = \int dp\, e^{itp}\tilde{\Delta}(p^2 = p_4^2)$$

Confined quarks: violation of positivity

For the quark propagator, we have two form factors:

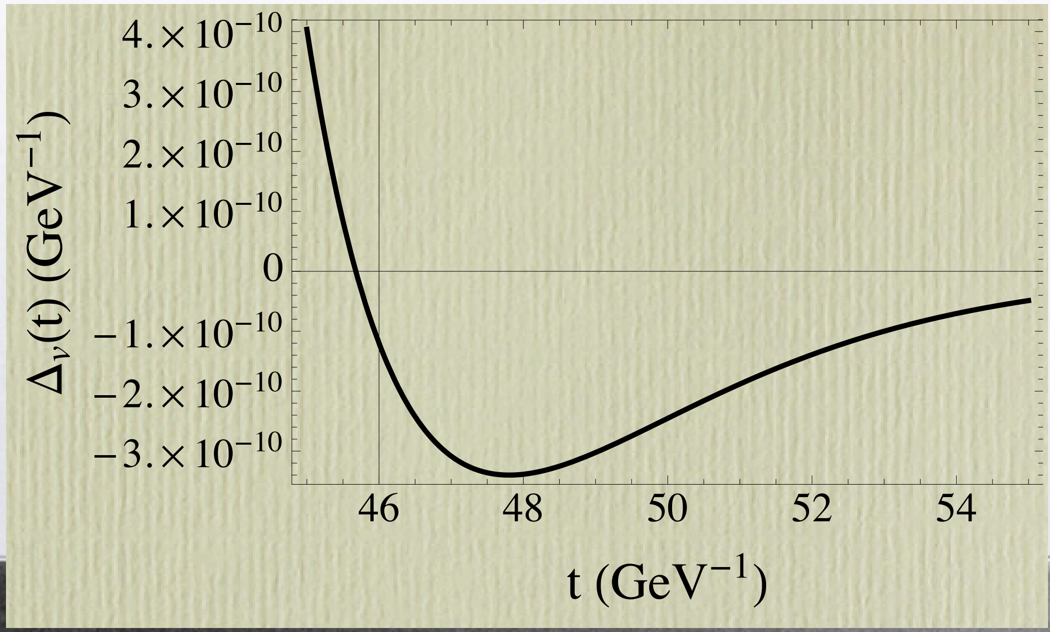
$$S(p) = -i\gamma_\mu p_\mu \, \sigma_v(p^2) + \sigma_s(p^2)$$

We need to study two functions:

$$\Delta_v(t) = \int dp \, e^{ipt} \sigma_v(p^2) \qquad \Delta_v(t) = \int dp \, e^{ipt} \sigma_s(p^2)$$

In the fermionic case the O-S positivity implies that

$$\Delta_v(t) \geq 0 \qquad -\partial_t \Delta_v(t) \geq \Delta_s(t)$$

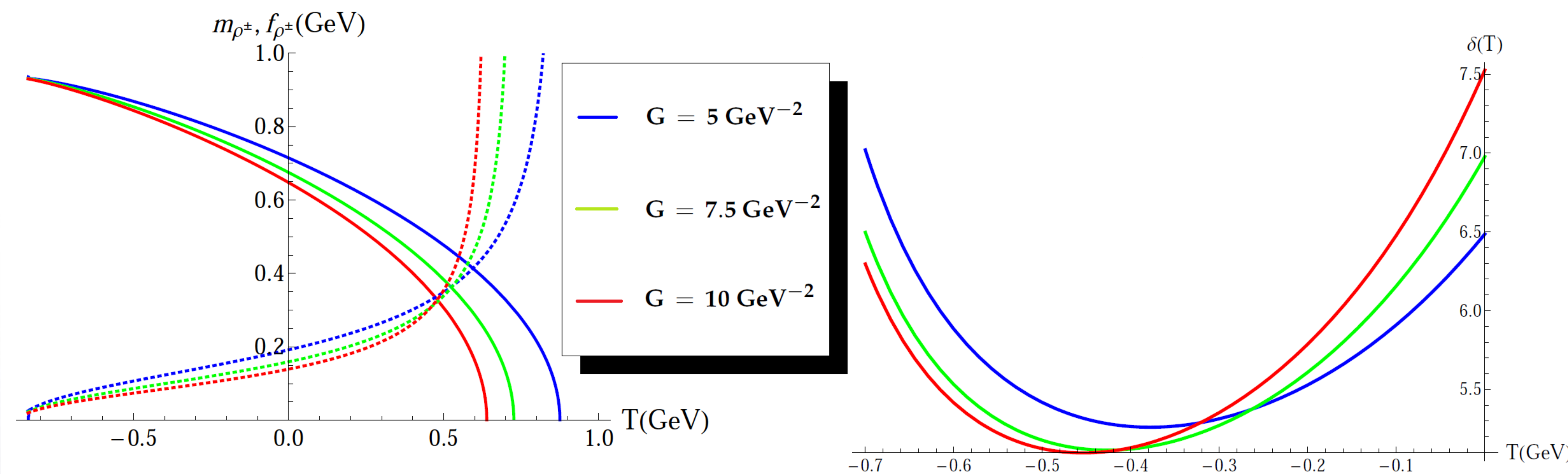


[Dudal,Guimaraes,LFP,Sorella, Annals Phys. 365 (2016

From confined quarks to a meson...

[Dudal,Guimaraes,LFP,Sorella, Annals Phys. 365 (201

- Estimate of the rho meson mass and decay constant:



We obtain for the masses
$m_{\rho^\pm}^{G=5,\ 7.5,\ 10} \approx 0.84, 0.83, 0.83$ GeV ($m_{\rho^\pm}^{exp} = 775.49 \pm 0.34$ MeV)
and decay constants
$f_{\rho^\pm}^{G=5,\ 7.5,\ 10} \approx 0.13, 0.10, 0.09$ GeV. ($f_{\rho^\pm}^{exp} \approx 0.208$ GeV,
$f_{\rho^\pm}^{latt} \approx 0.25$ GeV)

[Guimaraes,Mintz,LFP, PRD92 (2015) 085029]

After the determinant in spinor space,

$$\log Z(T,\mu) = 2N_c N_f \beta V \sum_{n,\vec{p}} \log\left[\beta^2\left(\omega_n^2 + \vec{p}^2 + M_{n,\vec{p}}^2(0)\right)\right]$$

$$M_{n,\vec{p}}(\mu) = \frac{M_3}{-(i\omega_n^2 + \mu)^2 + \vec{p}^2 + m^2} + m_0.$$

- The argument of the log is

$$\omega_n^2 + \vec{p}^2 + M_{n,\vec{p}}^2(0) = \omega_n^2 + \vec{p}^2 + \left[\frac{M_3}{\omega_n^2 + \vec{p}^2 + m^2} + m_0\right]^2 \equiv \frac{P_3(\omega_n^2)}{(\omega_n^2 + \vec{p}^2 + m^2)^2}$$

- $P_3(\omega_n^2)$ is a polynomial of 3rd degree.

The result is the sum of five terms of the standard free-field form:

$$\log Z(T,\mu) = 2N_c N_f \beta V \sum_n \int \frac{d^3p}{(2\pi)^3}\left\{\sum_{i=1}^{3} \log\left[\beta^2(\omega_n^2 + \phi_i^2)\right] - 2\log\left[\beta^2(\omega_n^2 + \vec{p}^2 + m^2)\right]\right\}$$

where $-\phi_i^2$ are the roots of the polynomial $P(\omega_n^2)$. (Explicitly calculable.)

with the Matsubara sum yielding: $\left(T\sum_n \log[\beta^2(\omega_n^2 + E^2)]\right)_{\text{th}} = 2T\log[1 + \exp(-\beta E)]$

Confining toy model

- ***'Free' confining theory:*** reproduces a propagator of the RGZ type at tree level

The simplest model leading to this type of propagator is of the form

$$S = \int d^D x \frac{1}{2}\psi \left(-\partial^2 + M^2 + 2\frac{\theta^4}{-\partial^2 + m^2} \right) \psi \tag{2.3}$$

from which one easily checks that the scalar field ψ has a free propagator $\langle \psi(k)\psi(-k) \rangle = D(k)$. This model can be rewritten in a local form through the introduction of auxiliary fields

$$S = \int d^D x \left(\frac{1}{2}\psi(-\partial^2 + M^2)\psi - \bar{\phi}(-\partial^2 + m^2)\phi + \theta^2\psi(\phi + \bar{\phi}) + \bar{\omega}(-\partial^2 + m^2)\omega \right) \tag{2.4}$$

where $(\phi, \bar{\phi})$ is a pair of bosonic complex conjugated fields and $(\omega, \bar{\omega})$ is a pair of anticommuting fields.

Diagonalization of this quadratic action gives (omitting decoupled contribs.):

$$S = \int d^D x \left(\frac{1}{2}\lambda(-\partial^2 + m^2 + i\sqrt{2}\theta^2)\lambda + \frac{1}{2}\eta(-\partial^2 + m^2 - i\sqrt{2}\theta^2)\eta \right)$$

Why soft BRST breaking in quark sector?

➠ **The RGZ scenario is characterized by a *soft BRST breaking* generated by the restriction of the gauge path integral to the Gribov region.**

- BRST symmetry: an extension of the gauge symmetry which remains intact even for gauge fixed actions.
- Defined by a nilpotent operator s (i.e. $s^2 = 0$).
- Path integrals in Yang-Mills theory are dictated by a BRST-invariant action:

$$\mathcal{L}_{\rm YM} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \underbrace{ib^a\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D^{ab}_\mu c^b}_{\text{gauge fixing (Landau)}} = \underbrace{\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}}_{\substack{\text{gauge inv.}\\ \Rightarrow \text{BRST inv.}}} + \underbrace{s\Delta_{GF}}_{\text{BRST exact}}$$

- **The {Refined}Gribov-Zwanziger action breaks BRST softly through the 'horizon function'**

$$\boxed{\mathcal{L}_{\{\rm R\}GZ} = \mathcal{L}_{\rm YM} + H + \left\{\frac{m}{2}A^a_\mu A^a_\mu - M\overline{\varphi}^{ab}_\mu\varphi^{ab}_\mu\right\}}$$

How to extend this description to the quark sector?

Construct a soft BRST breaking model for the quark sector!

Confining propagator: light vs strange quarks

$$\langle \psi_f \bar{\psi}_f \rangle_k = \frac{i\gamma_\mu k_\mu + \mathcal{A}_f(k^2)}{k^2 + \mathcal{A}_f^2(k^2)}$$

$$\boxed{\mathcal{A}_f(k^2) = \frac{M_f^3}{k^2 + m_f^2} + \mu_f}$$

$\mu_f = 82.2\,\mathrm{MeV}$ **(strange)**

$\mu_f = 11.5\,\mathrm{MeV}$ **(up, down; degenerate)**

Lattice fits $\longrightarrow$

f	μ_f	m_f	M_f
u,d	11.5 MeV	870 MeV	662 MeV
s	82.2 MeV	1450 MeV	971 MeV

[Furui et al PRD73,074503(2006)]

- Decomposing the denominators we find **unphysical contributions**:

up, down:

$$\frac{1}{(k^2 + m_{u,d}^2)^2 \left[k^2 + \mathcal{A}_{u,d}^2\right]} = \frac{R}{k^2 + \omega} + \frac{R_r + i\alpha}{k^2 + \omega_r + i\theta} + \frac{R_r - i\alpha}{k^2 + \omega_r - i\theta}$$

strange:

$$\frac{1}{(k^2 + m_s^2)^2 \left[k^2 + \mathcal{A}_s^2\right]} = \frac{R_1}{k^2 + \omega_1} + \frac{-R_2}{k^2 + \omega_2} + \frac{R_3}{k^2 + \omega_3}$$

This model: ✓ *quark confinement*!

Meson masses and soft BRST breaking

$$\mathcal{L}_{\mathrm{IRQCD}} = \mathcal{L}_{\mathrm{RGZ}} - \overline{\psi}\left[\gamma_\mu D_\mu - \mu\right]\psi - 2M_1^2 M_2\,\bar{\psi}\left(\frac{1}{\partial^2 - m^2}\right)\psi$$

This model with soft BRST breaking in the quark sector:

- ✓ ***(can be cast in a) local and renormalizable action***
- ✓ ***reduces to QCD at high energies***
- ✓ ***quark confinement: confining propagator (no physical propagation; violation of reflection positivity: to be checked!) consistent with lattice IR results.***
- ✓ ***Meson spectrum?***

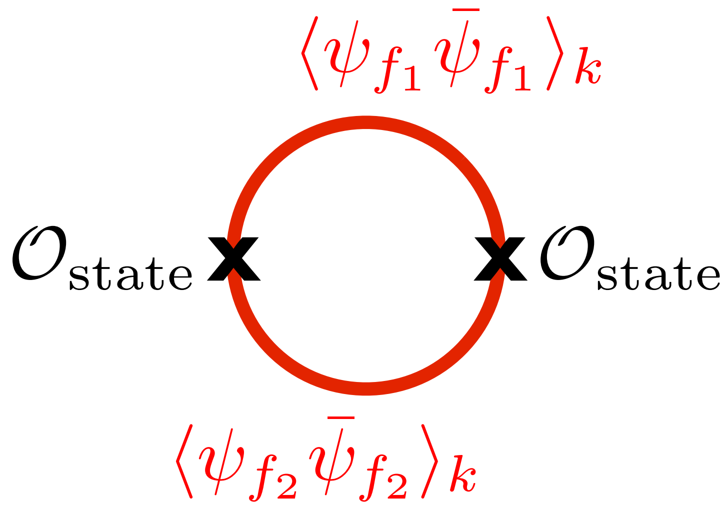
General Strategy: [cf. also Marcelo S. Guimarães' talk on Monday]

1. Write down ***pure composite operators*** $\mathcal{O}_{\rm state}$ which contain information of (and only of) the meson state of interest: with the same quantum numbers and being conserved;

2. Compute the associated **2-point function**:

$$\langle \mathcal{O}^\dagger_{\rm state}\mathcal{O}_{\rm state}\rangle_k = \int d^4x \; \mathrm{e}^{ikx} \; \langle \mathcal{O}^\dagger_{\rm state}(x)\mathcal{O}_{\rm state}(0)\rangle$$

- using ***confining quark propagators with parameters from lattice,***
- ***1-loop approximation*** in our model;



3. Obtain the **Källén-Lehmann spectral representation** of $\langle \mathcal{O}^\dagger_{\rm state}\mathcal{O}_{\rm state}\rangle_k$:

$$\langle \mathcal{O}^\dagger_{\rm state}\mathcal{O}_{\rm state}\rangle_k = \int_0^\infty d\tau \, \frac{\rho_{\rm state}(\tau)}{\tau + k^2} + \text{rest}$$

using ***Cutkosky rules*** (and their generalization for complex masses);

4. Estimate the **mass of the (physical) propagating mode** from the spectral function:

➠ using SVZ-like Sum Rule, Padé approximants, etc.

Mesonic *pure* composite operators

✓ Write down ***pure composite operators*** $\mathcal{O}_{\text{state}}$ which contain information of (and only of) the meson state of interest: with the **same quantum numbers** and being **conserved**;

pions: ***large explicit chiral symm. breaking ⇒ NO pseudo-Goldstone bosons***

ρ meson: $(\bar{u}d, \bar{d}u, \bar{u}u, \bar{d}d \approx \bar{q}q)$, **J^PC=1^--**

$$\mathcal{O}_\rho = \bar{q}\gamma_\mu q$$

a₁ meson: $(\bar{u}d, \bar{d}u, \bar{u}u, \bar{d}d \approx \bar{q}q)$, **J^PC=1^++**

$$\mathcal{O}_{a_1} = (\partial^2 \delta_{\mu\nu} - \partial_\mu \partial_\nu)\bar{q}\gamma_\mu\gamma_5 q$$

φ meson: $(\bar{s}s)$, **J^PC=1^--**

$$\mathcal{O}_\phi = \bar{s}\gamma_\mu s$$

Meson masses from a confining quark prop.

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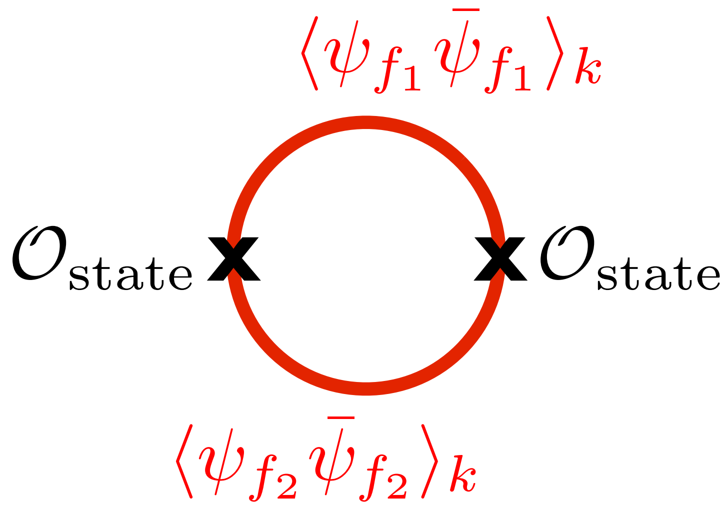

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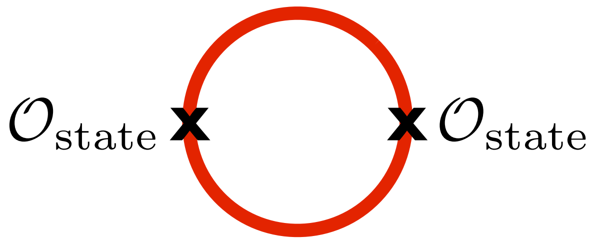
✓ Compute the associated **2-point function**:

confining quark prop.



confining quark prop.

$$\langle \psi_f \bar{\psi}_f \rangle_k = \frac{i\gamma_\mu k_\mu + \mathcal{A}_f(k^2)}{k^2 + \mathcal{A}_f^2(k^2)} \qquad \mathcal{A}_f(k^2) = \frac{M_f^3}{k^2 + m_f^2} + \mu_f$$

$$\langle [\mathcal{O}_\rho]^\dagger_\mu [\mathcal{O}_\rho]_\mu \rangle_k = \int \frac{d^4q}{(2\pi)^4} \frac{\mathrm{Tr}\Big\{[i\gamma_\delta(k_\delta - q_\delta) + \mathcal{A}\big((k-q)^2\big)]\gamma_\mu [i\gamma_\sigma q_\sigma + \mathcal{A}(q^2)]\gamma_\mu\Big\}}{[(k-q)^2 + \mathcal{A}^2\big((k-q)^2\big)][q^2 + \mathcal{A}^2(q^2)]}$$

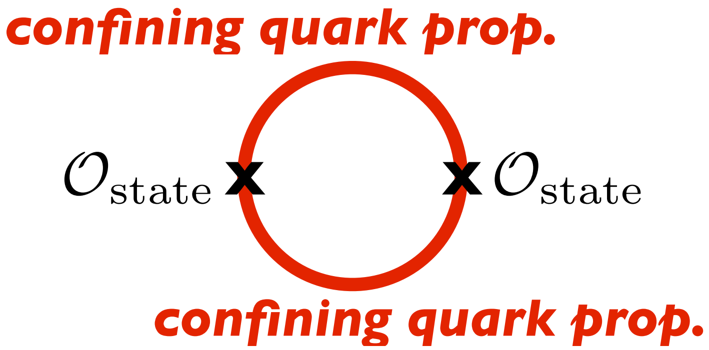
$$= \int \frac{d^4q}{(2\pi)^4} f_\rho(k,q) \frac{1}{\Big[(k-q)^2\big[(k-q)^2+m^2\big]^2 + \big\{M^3 + \mu\big[(k-q)^2+m^2\big]\big\}^2\Big]} \frac{1}{\Big[q^2\big[q^2+m^2\big]^2 + \big\{M^3 + \mu\big[q^2+m^2\big]\big\}^2\Big]}$$

Obs.: **different states** ⇒ different gamma matrix structure and/or parameters (light or strange sector); extra projectors for **a$_1$**.

Obs2.: **confining propagators:** to be decomposed in 3 poles each, for using ***cut rules***.

2-pt function of composite operators

✓ Compute the associated **2-point function**:



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$$= \int \frac{d^4q}{(2\pi)^4} f_\rho(k,q) \left\{ \frac{R}{(k-q)^2 + \omega} + \frac{R_r + i\alpha}{(k-q)^2 + \omega_r + i\theta} + \frac{R_r - i\alpha}{(k-q)^2 + \omega_r - i\theta} \right\}$$
$$\left\{ \frac{R}{q^2 + \omega} + \frac{R_r + i\alpha}{q^2 + \omega_r + i\theta} + \frac{R_r - i\alpha}{q^2 + \omega_r - i\theta} \right\}$$

Obs.: **different states** ⇒ different gamma matrix structure and/or parameters (light or strange sector); extra projectors for **a_1**.

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Meson masses from a confining quark prop.

General Strategy: [cf. also Marcelo S. Guimarães' talk on Monday]

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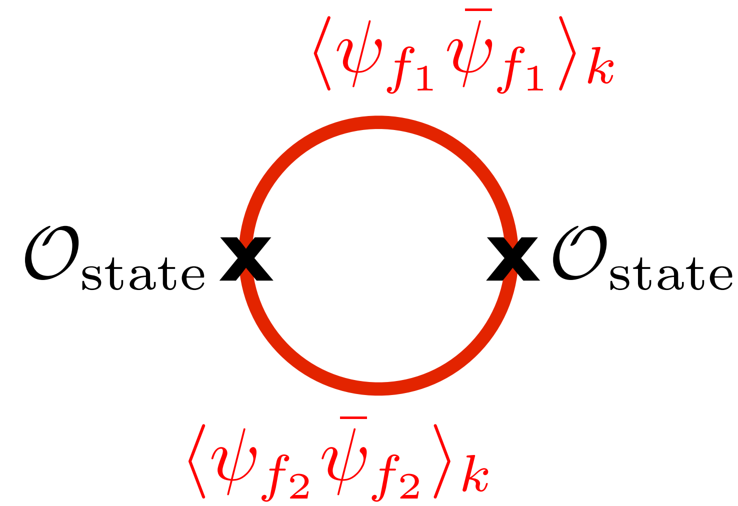

- using ***confining quark propagators with parameters from lattice,***
- ***1-loop approximation*** in our model;

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using ***Cutkosky rules*** (and their generalization for complex masses);

4. Estimate the **mass of the (physical) propagating mode** from the spectral function:

➠ using SVZ-like Sum Rule, Padé approximants, etc.

Cut rules and *physical* spectral function

✓ Obtain the **Källén-Lehmann spectral representation** of $\langle \mathcal{O}^\dagger_{\text{state}} \mathcal{O}_{\text{state}} \rangle_k$,

$$\langle \mathcal{O}^\dagger_{\text{st}} \mathcal{O}_{\text{st}} \rangle_k = \int_0^\infty d\tau \, \frac{\rho_{\text{state}}(\tau)}{\tau + k^2} + \text{rest}$$

with $\rho_{\text{state}}(\tau) = \left[\frac{1}{\pi} \text{Im} \, \langle \mathcal{O}^\dagger_{\text{st}} \mathcal{O}_{\text{st}} \rangle_{k=(E,0)} \right] \Big|_{\text{physical}}$

using ***Cutkosky rules*** ***(and their generalization for complex masses)*:**

[Dudal & Guimarães PRD83,045013(2011)]

$$\mathcal{F}(m_1^2, m_2^2) = \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(k-q)^2 - m_1^2} \frac{1}{q^2 - m_2^2} f(k,q) \quad \Rightarrow \quad \text{Im}\, \mathcal{F}$$

- Our composite correlator has nine contributions of this form: $\langle [\mathcal{O}_\rho]^\dagger_\mu [\mathcal{O}_\rho]_\mu \rangle_k = \sum_{i,j} \mathcal{F}(\omega_i, \omega_j)$

$$\langle [\mathcal{O}_\rho]^\dagger_\mu [\mathcal{O}_\rho]_\mu \rangle_k = \frac{1}{\pi} \int d\tau \sum_{i,j} \frac{\text{Im}\mathcal{F}(\omega_i, \omega_j)}{k^2 + \tau} \stackrel{!}{=} \int d\tau \frac{\rho_\rho(\tau)}{\tau + k^2} + \text{rest}$$

Criterium: only terms with ***good analytical behavior (i.e. positive)*** contribute to the physical spectral function.

Meson masses from a confining quark prop.

General Strategy: [cf. also Marcelo S. Guimarães' talk on Monday]

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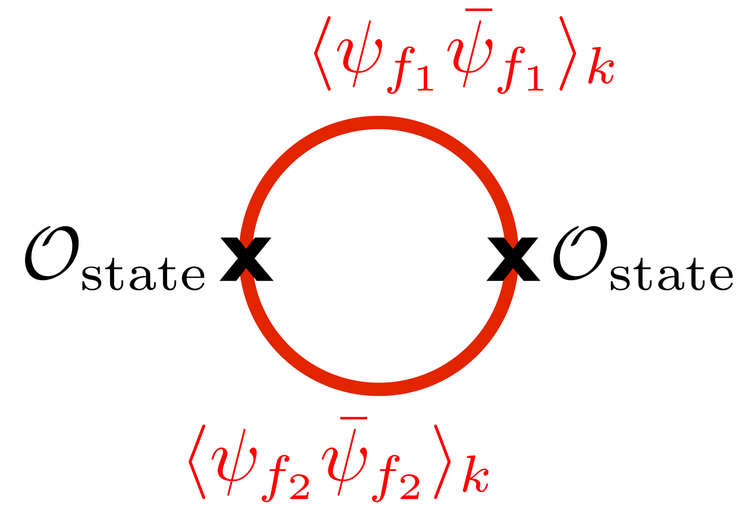

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➠ using SVZ-like Sum Rule, Padé approximants, etc.

Meson spectrum: **preliminary** results

State	Current quark mass μ_f^{phys}	μ_f on lattice	Physical mass	Estimates Confining prop.	Ratio
ρ	$1.7 < \mu_u < 3.1$ $4.1 < \mu_d < 5.7$	$\mu_u = \mu_d = 11.5$	770	1030	1.34
a_1			1260	2030	1.62
ϕ	$80 < \mu_s < 130$	$\mu_s = 82.2$	1020	1222	1.20

- Significantly larger deviations in the **light sector** due to ***large up and down masses on the lattice***:

$$\frac{m_\pi^{\mathrm{latt}}}{m_\pi^{\mathrm{phys}}} \approx \sqrt{\frac{\mu_u^{\mathrm{latt}} + \mu_d^{\mathrm{latt}}}{\mu_u^{\mathrm{phys}} + \mu_d^{\mathrm{phys}}}} \approx 1.60$$

- Fixing parameters to reproduce the rho meson mass, we get the a_1 mass ratio < 1.20.
- Extrapolation of lattice results to physical masses.

Conclusions and outlook

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm YM} + \overline{\psi}\gamma_\mu D_\mu \psi \longrightarrow$$

in the infrared $\longrightarrow$ **soft BRST breaking** triggered by Gribov horizon

$$\boxed{\mathcal{L}_{\rm IRQCD} = \mathcal{L}_{\rm RGZ} + \overline{\psi}\gamma_\mu D_\mu \psi + s\mathcal{L}_m(\xi, \lambda, \cdots) + \mathcal{L}_M(\xi, \lambda, \psi)}$$

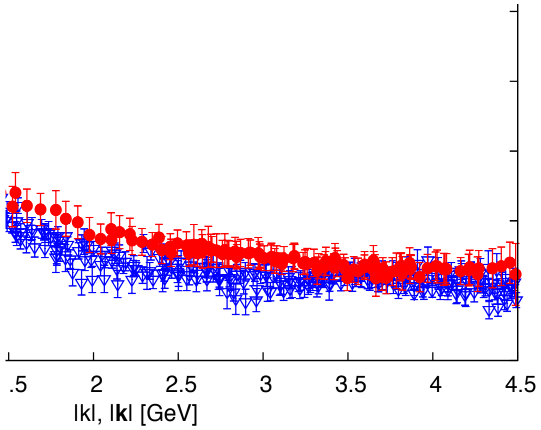
where $\mathcal{L}_M(\xi, \lambda, \psi) = M_1^2 \left(\bar{\xi}\psi + \bar{\psi}\xi\right) - M_2 \left(\bar{\lambda}\psi + \bar{\psi}\lambda\right).$

- ✓ *local and renormalizable action*
- ✓ *reduces to full QCD at high energies*
- ✓ *gluon and quark confinement: confining propagators (no physical propagation; violation of reflection positivity) consistent with lattice IR results.*
- ✓ *glueballs spectrum: good agreement with lattice*
- ✓ *meson spectrum: sensible results (Preliminary)*

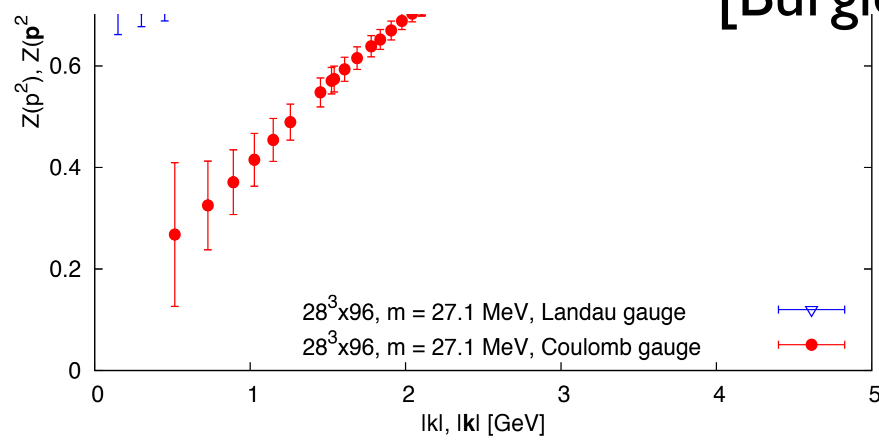
- ✗ *no dynamical description (work in progress on SSB model)*
- ✗ *pion and kaon masses*
- ✗ *no general definition of physical operators*
- ✗ *predictions depend on lattice data for propagators*

★ *connection between confinement and chiral symmetry?*

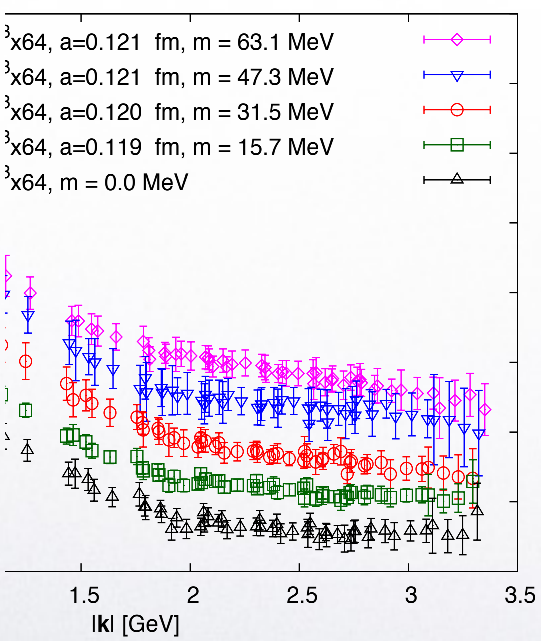
[Burgio et al, arXiv:1204.0716[hep-lat]]



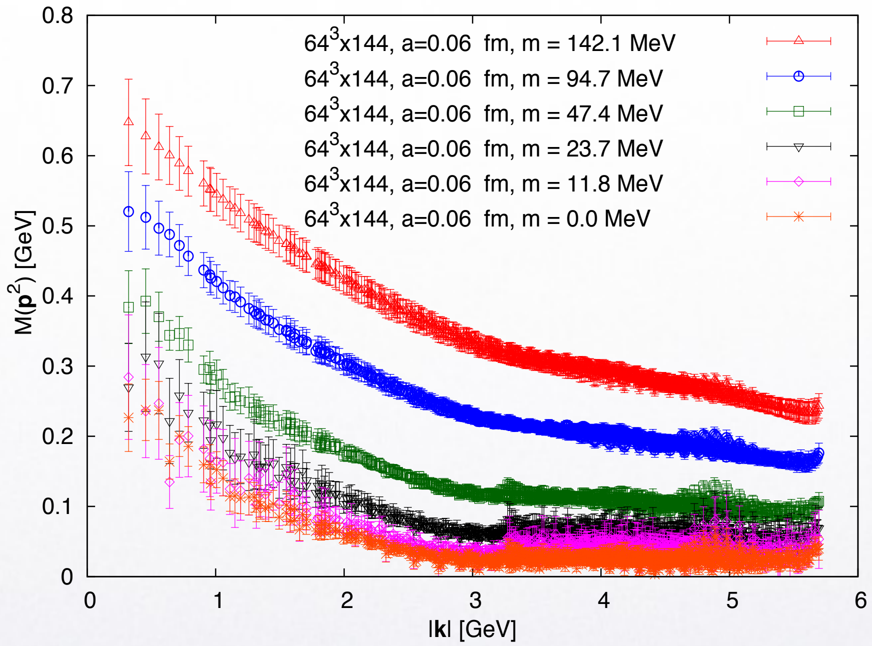
ı. vs. $M_{Lan.}$ from (f)



(b) $Z_{Coul.}$ vs. $Z_{Lan.}$ from (f)



M from (a)–(d)



(d) M from (i)

. 6. Landau vs. Coulomb gauge (top); chiral behaviour of M (bottom)

$$\ldots{}_{\lambda}F^{a}_{\rho\sigma}) - \frac{1}{6} P_{k\mu}\varepsilon_{\alpha\beta\rho\sigma}F^{a}_{\alpha\beta}F^{a}_{\rho\sigma}$$

ator $< O^{PC}(k)\ O^{PC}(-k) >$ and cast it i
sentation

$$\ldots))\rangle = \frac{1}{\pi}\int_{\tau_0}^{\infty} d\tau\, \frac{\rho(\tau)}{k^2+\tau}$$