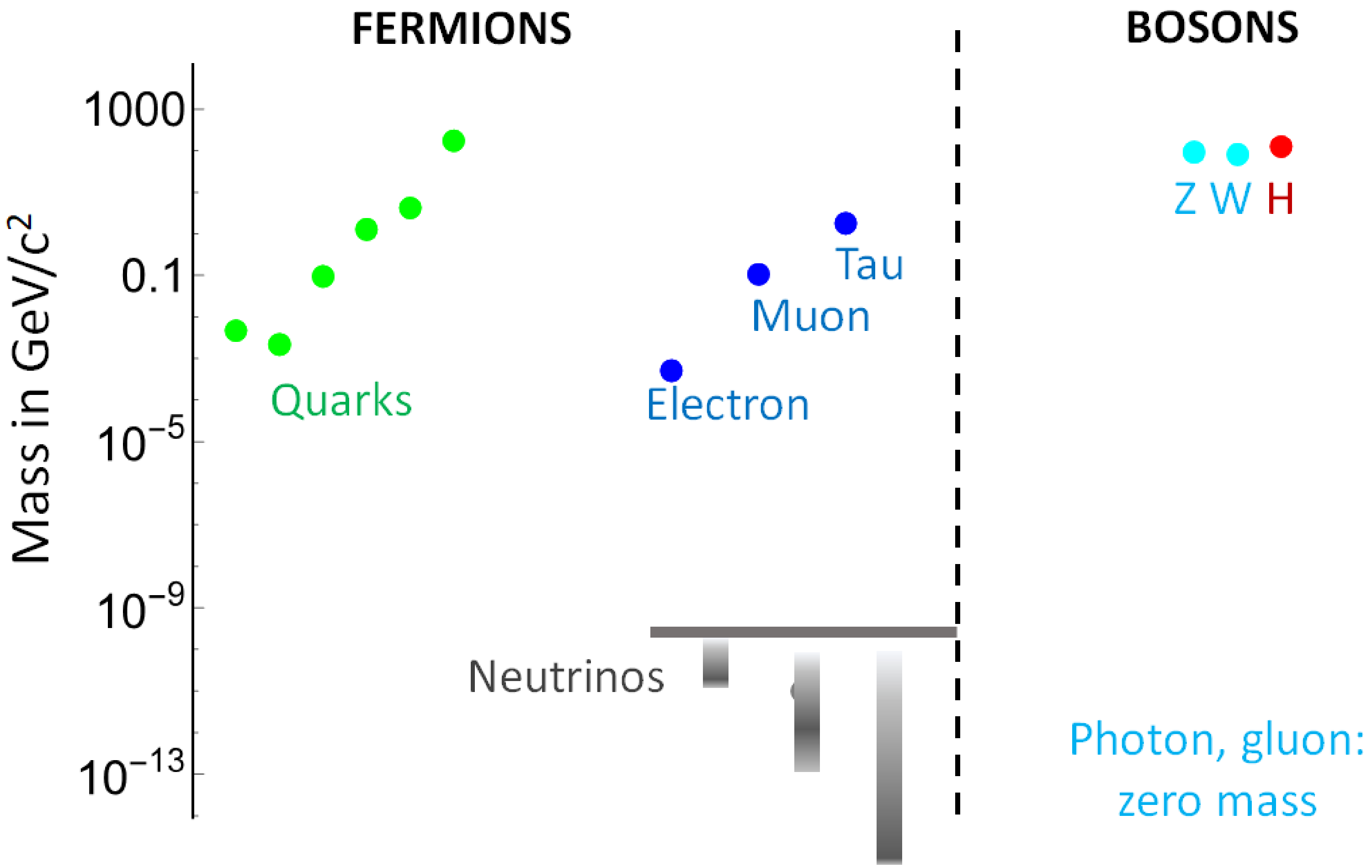
Nuclear physics input to charged lepton flavor violation (l->l' conversion in nuclei)

XIV LASNPA
Facultad de Ciencias, UNAM, June 17-21, 2024

Pablo Roig (Cinvestav, Mexico City)

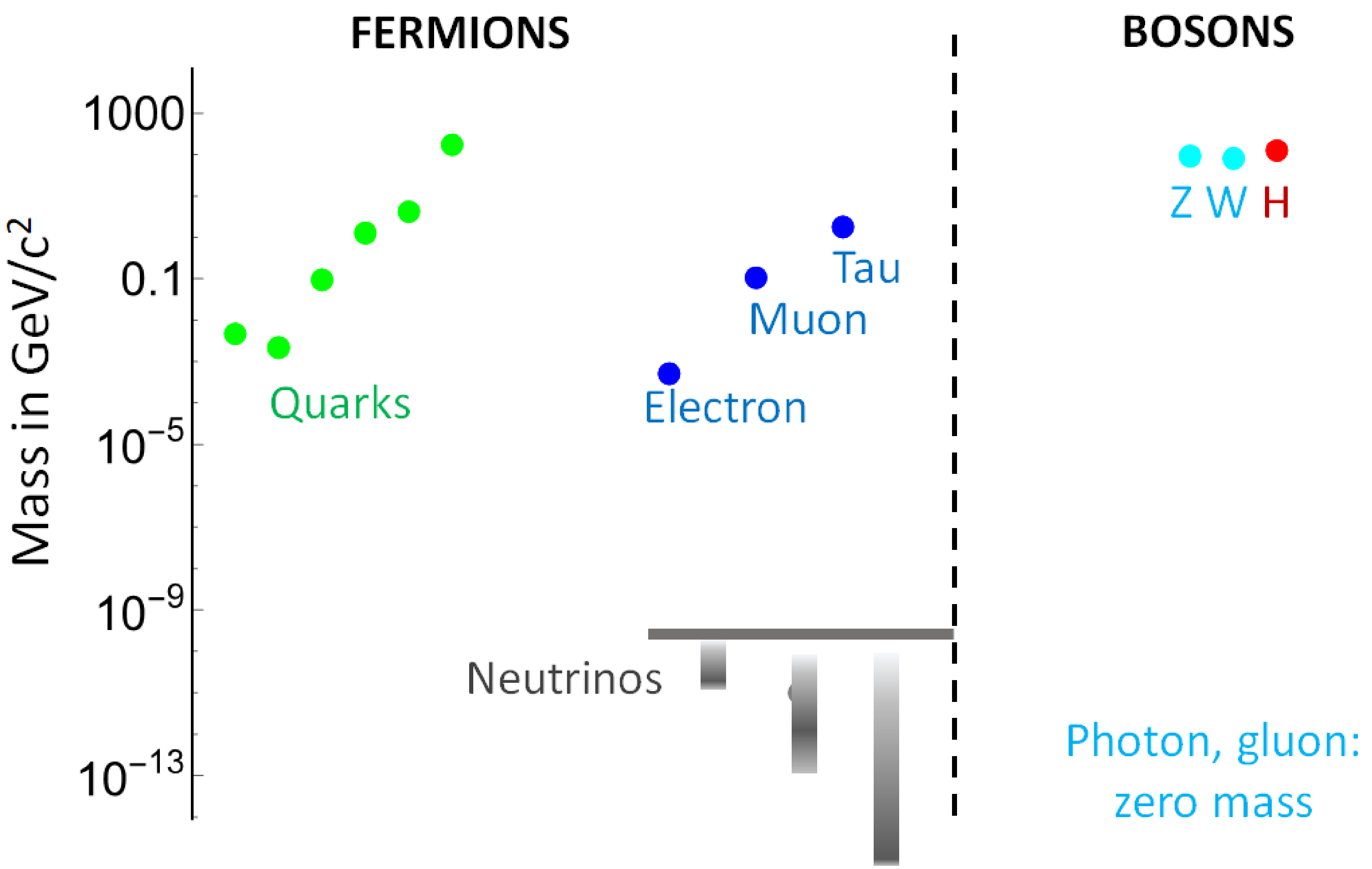
MOTIVATION

Small neutrino masses



MOTIVATION

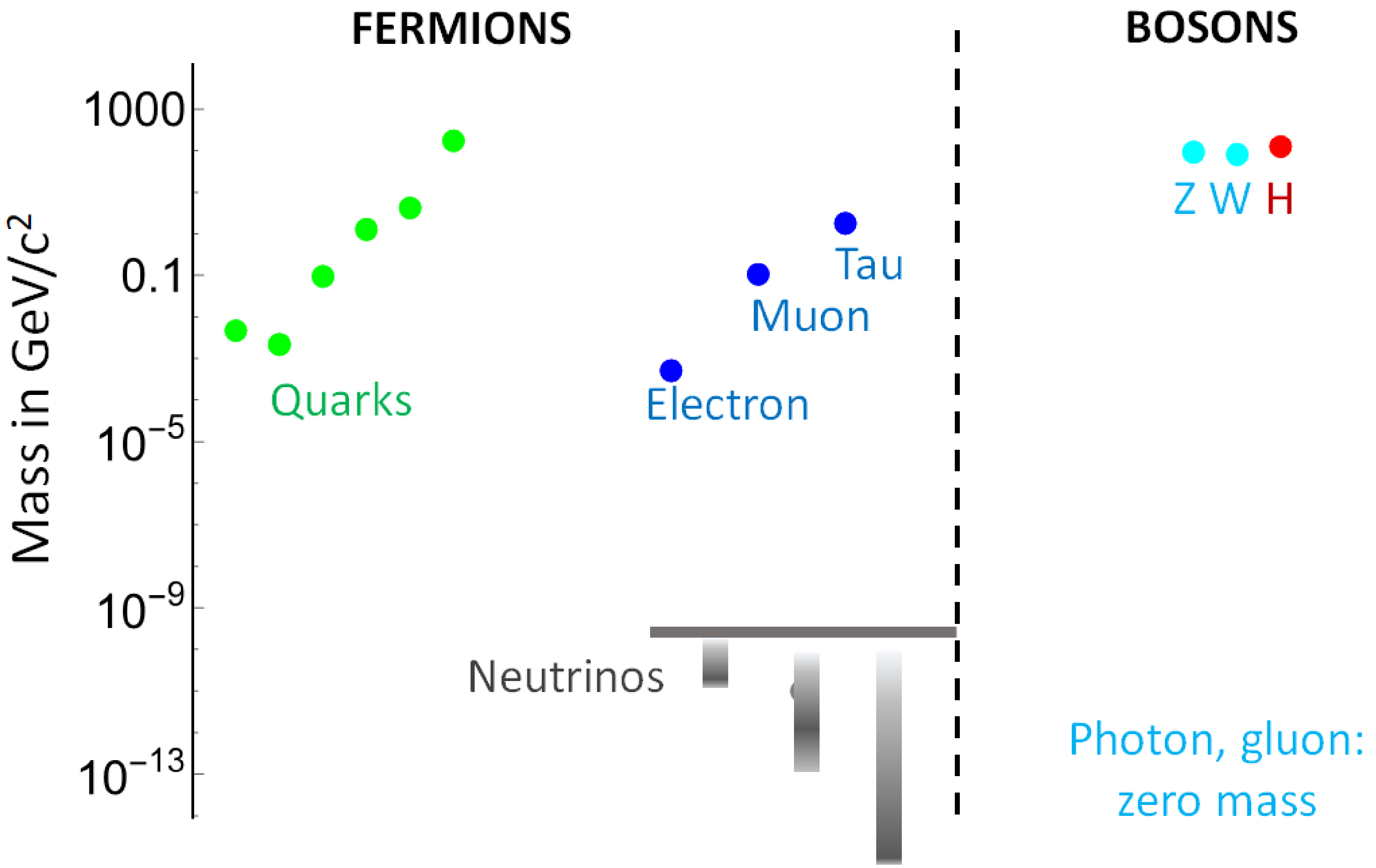
Small neutrino masses



The same (Higgs) mechanisms that endows mass to all known elementary particles, could also be responsible for the tiny neutrino masses, **BUT...**

MOTIVATION

Small neutrino masses



The same (Higgs) mechanisms that endows mass to all known elementary particles, could also be responsible for the tiny neutrino masses, **BUT**
once ν_R is introduced, a Majorana ν mass term is allowed as well, which –unless the UV theory forbids LNV (?)- makes neutrinos their own antiparticles & connects them to GUT scales (Weinberg operator)

MOTIVATION

Small neutrino masses

See-saw mechanism

$m_L \sim m_D^2/m_M$

$m_H \sim m_M$

$m_L \sim 50\text{meV} \Rightarrow m_M \sim 10^{[10,15]}\text{GeV}$

H ν_R decays produce leptogenesis at GUT scales

The same (Higgs) mechanisms that endows mass to all known elementary particles, could also be responsible for the tiny neutrino masses, **BUT**
once ν_R is introduced, a Majorana ν mass term is allowed as well, which –unless the UV theory forbids LNV (?)- makes neutrinos their own antiparticles & connects them to GUT scales (Weinberg operator)

MOTIVATION

Small neutrino masses

See-saw mechanism

$m_L \sim m_D^2/m_M$

$m_H \sim m_M$

$m_L \sim 50 meV \Rightarrow m_M \sim 10^{[10,15]} GeV$

H ν_R decays produce leptogenesis at GUT scales

The same (Higgs) mechanisms that endows mass to all known elementary particles, could also be responsible for the tiny neutrino masses, **BUT**

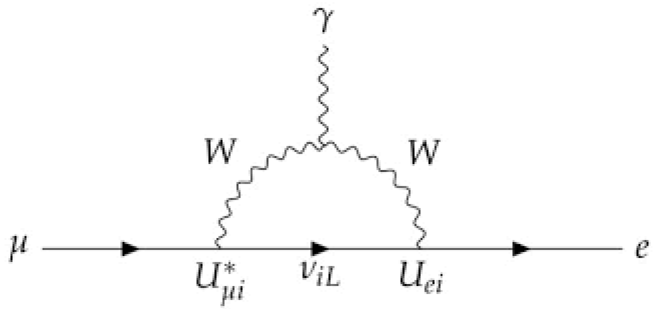
once ν_R is introduced, a Majorana ν mass term is allowed as well, which –unless the UV theory forbids LNV (?)- makes neutrinos their own antiparticles & connects them to GUT scales (Weinberg operator)

Theoretically beautiful but phenomenologically sad

MOTIVATION

Small neutrino masses

Theoretically beautiful
but phenomenologically sad



$BR(\mu\text{->}e\,\gamma) \sim (\alpha/\pi)\,\Sigma_i\,[U_{ei}{}^*\,U_{\mu i}\,(m_{\nu i}/M_W)^2]^2 \sim 10^{-50}$

(Cheng-Li '77; Petcov '77; Marciano-Sanda'77; Shrock-Lee '77)

See-saw mechanism

$m_L \sim m_D{}^2/m_M$

$m_H \sim m_M$

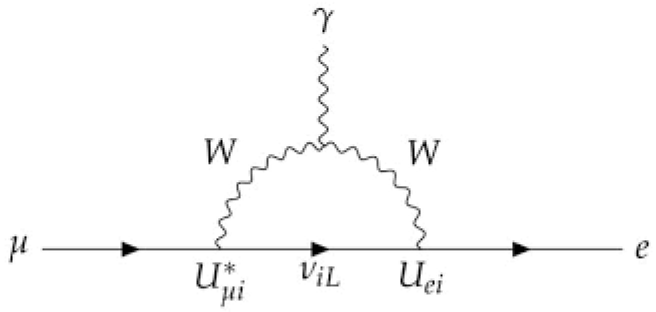
$m_L \sim 50\text{meV} \Rightarrow m_M \sim 10^{[10,15]}\text{GeV}$

H ν_R decays produce leptogenesis at GUT scales

MOTIVATION

Small neutrino masses

Theoretically beautiful but phenomenologically sad



$BR(\mu\text{->}e\,\gamma)\sim(\alpha/\pi)\,\Sigma_i\,[U_{ei}^*\,U_{\mu i}\,(m_{\nu i}/M_W)^2]^2\sim 10^{-50}$

(Cheng-Li '77; Petcov '77; Marciano-Sanda'77; Shrock-Lee '77)
With similar suppression in H/Z -> l l', L -> 3l, l -> l' in nuclei, etc. (Illana-Riemann'01,Arganda et al. '05, Hernández-Tomé—López-Castro—Roig'20, Blackstone-Fael-Passemar'20)

See-saw mechanism

$m_L \sim m_D^2/m_M$

$m_H \sim m_M$

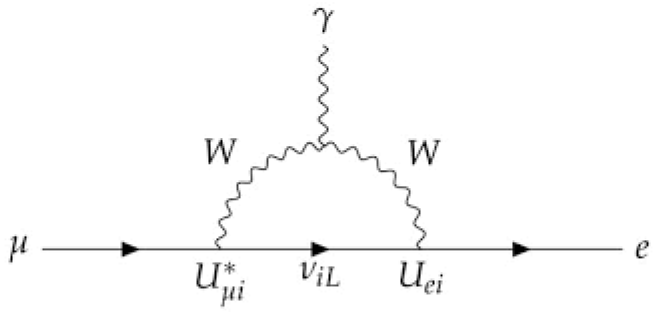
$m_L \sim 50\text{meV} \Rightarrow m_M \sim 10^{[10,15]}\text{GeV}$

H ν_R decays produce leptogenesis at GUT scales

MOTIVATION

Small neutrino masses

Theoretically beautiful
but phenomenologically sad



$BR(\mu\text{->}e\,\gamma)\sim(\alpha/\pi)\,\Sigma_i\,[U_{ei}{}^*\,U_{\mu i}\,(m_{\nu i}/M_W)^2]^2\sim10^{-50}$

(Cheng-Li '77; Petcov '77; Marciano-Sanda'77; Shrock-Lee '77)
With similar suppression in H/Z -> l l', L -> 3l, l -> l' in nuclei, etc. (Illana-Riemann'01,Arganda et al. '05, Hernández-Tomé—López-Castro—Roig'20, Blackstone-Fael-Passemar'20)

See-saw mechanism

$m_L\sim m_D{}^2/m_M$

$m_H\sim m_M$

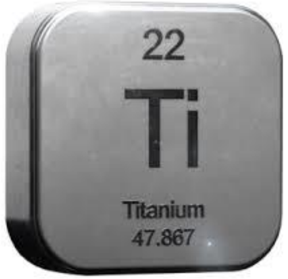
$m_L\sim 50\,meV \Rightarrow m_M\sim10^{[10,15]}GeV$

H ν_R decays produce leptogenesis at GUT scales

Low-scale seesaws (Mohapatra-Valle'86; Bernabeu et al.'87; Malinsky-Romao-Valle'05) have an additional parameter regulating LNV, are less prone to GUT, **but** yield measurable cLFV observables.

MOTIVATION

(Mu2e@FNAL & Comet@J-Parc)



Small neutrino masses

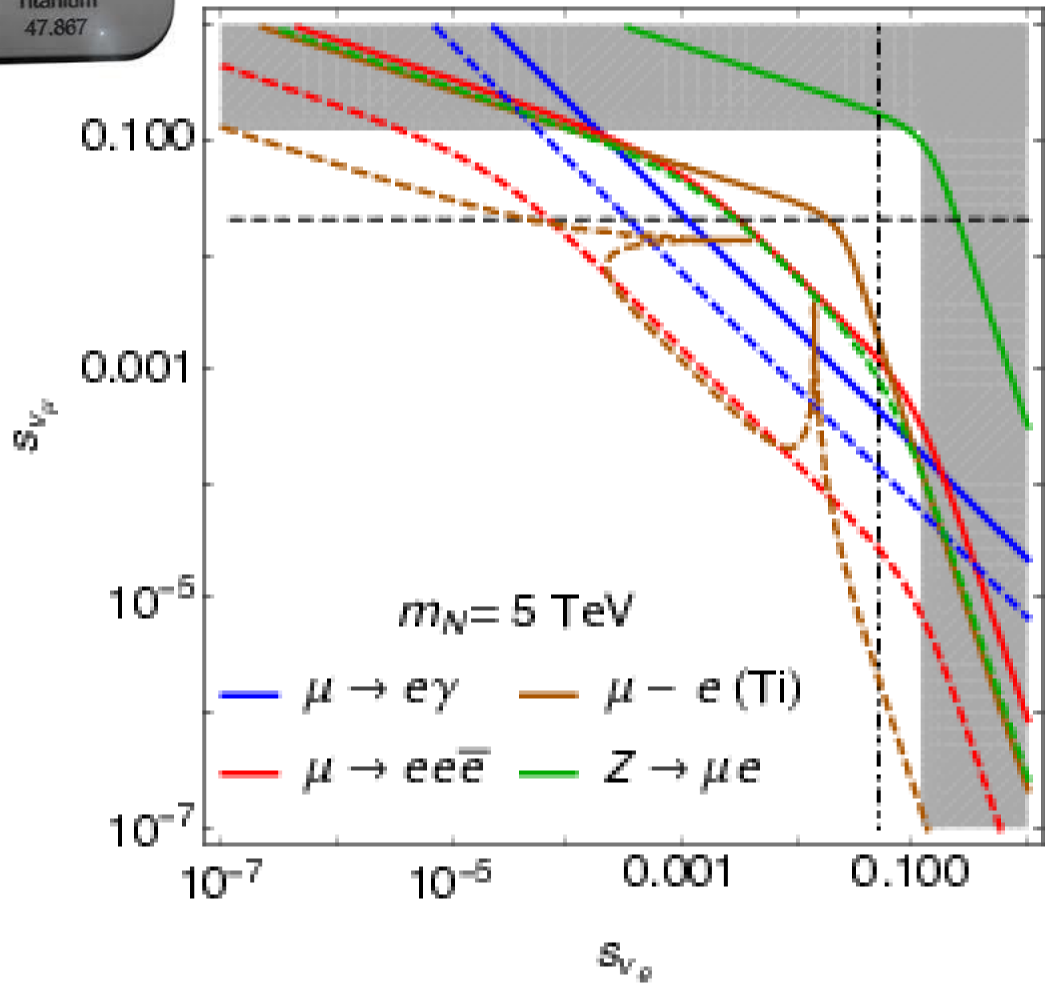
$BR(\mu \to e\gamma) \sim (\alpha/\pi)\, \Sigma_i [U_{ei}^* U_{\mu i} (m_{\nu i}/M_W)^2]^2 \sim 10^{-50}$

(Cheng-Li '77; Petcov '77; Marciano-Sanda'77; Shrock-Lee '77) With similar suppression in H/Z -> l l', L -> 3l, l -> l' in nuclei, etc. (Illana-Riemann'01, Arganda et al. '05, Hernández-Tomé—López-Castro—Roig'20, Blackstone-Fael-Passemar'20)

Low-scale seesaws (Mohapatra-Valle'86; Bernabeu et al.'87; Malinsky-Romao-Valle'05) have an additional parameter regulating LNV, are less prone to GUT, **but** yield measurable cLFV observables.

Just as an example we will show a result from Hernandez-Tomé—Illana—Masip—López-Castro—Roig'20, which only includes a pair of additional heavy (O (TeV)) ν_R's, simple enough to get the relevant H-L mixings driving LFV processes.

Solid (Dashed) is current (future) limit:
mu-e (Ti) will lead!!



MOTIVATION

(Mu2e@FNAL & PRISM/Comet@J-Parc)

Small neutrino masses

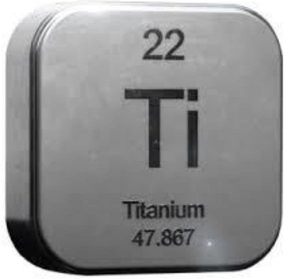
$BR(\mu \to e\,\gamma) \sim (\alpha/\pi)\, \Sigma_i\, [U_{ei}^*\, U_{\mu i}\, (m_{\nu i}/M_W)^2]^2 \sim 10^{-50}$

(Cheng-Li '77; Petcov '77; Marciano-Sanda'77; Shrock-Lee '77)
With similar suppression in H/Z -> l l', L -> 3l, l -> l' in nuclei, etc. (Illana-Riemann'01, Arganda et al. '05, Hernández-Tomé—López-Castro—Roig'20, Blackstone-Fael-Passemar'20)

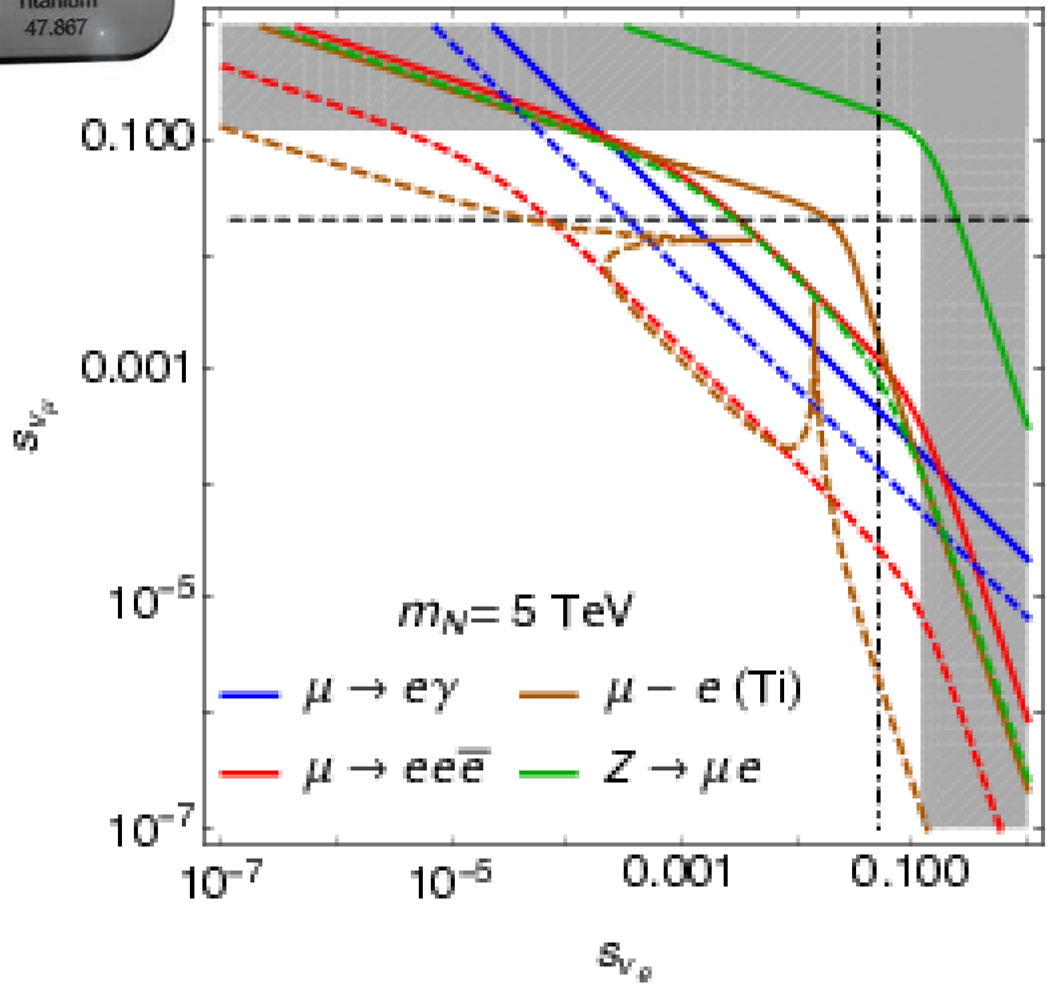
Low-scale seesaws (Mohapatra-Valle'86; Bernabeu et al.'87; Malinsky-Romao-Valle'05) have an additional parameter regulating LNV, are less prone to GUT, **but** yield measurable cLFV observables.

Just as an example we will show a result from Hernandez-Tomé—Illana—Masip—López-Castro—Roig'20, which only includes a pair of additional heavy (O (TeV)) ν_R's, simple enough to get the relevant H-L mixings driving LFV processes.

$\mu/e \to \tau$ conversion in nuclei has been revived recently (Gninenko et al.'01 & '18, Husek—Monsálvez-Pozo—Portolés 21', Ramírez—Roig'22, Fortuna—Marcano—Marín—Roig'23, etc.) and will be studied at NA62, EIC, ILC, LHeC...



Solid (Dashed) is current (future) limit:
mu-e (Ti) will lead!!



NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

(Bars, +h.c., etc. to be understood where approppriate, see additional material for full expressions)

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

The effective Lagrangian for μ->e conversion in nuclei contains dipole operators (μ->e γ) and (e Γ μ) (q Γ q) structures, with Γ=S, P, V, A, T.

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

The effective Lagrangian for μ->e conversion in nuclei contains dipole operators (μ->e γ) and (e Γ μ) (q Γ q) structures, with Γ=S, P, V, A, T.

|i> is 1s of muonic atom, with BE=ε_b & |f> is E eigenstate with E=m_μ-ε_b. Both wf are determined by solving Dirac eq. In presence of the E field of the nucleus.

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

The effective Lagrangian for μ->e conversion in nuclei contains dipole operators (μ->e γ) and (e Γ μ) (q Γ q) structures, with Γ=S, P, V, A, T.

|i> is 1s of muonic atom, with BE=ε_b & |f> is E eigenstate with E=m_μ-ε_b. Both wf are determined by solving Dirac eq. In presence of the E field of the nucleus.

We are generally most interested in the coherent case <=> $|i>_{Nucleus}=|f>_{Nucleus}$, which dominates (Feinberg&Weinberg'59)

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

The effective Lagrangian for μ->e conversion in nuclei contains dipole operators (μ->e γ) and (e Γ μ) (q Γ q) structures, with Γ=S, P, V, A, T.

|i> is 1s of muonic atom, with BE=ε_b & |f> is E eigenstate with E=m_μ-ε_b. Both wf are determined by solving Dirac eq. In presence of the E field of the nucleus.

We are generally most interested in the coherent case <=> $|i\rangle_{Nucleus}=|f\rangle_{Nucleus}$, which dominates (Feinberg&Weinberg'59)

Thus, the only non-vanishing relevant m.e. are <N|qq|N> & $\langle N|q\gamma^0 q|N\rangle$, which are straightforwardly related to the p/n densities in nuclei, $\rho^{(p/n)}$:

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

The effective Lagrangian for μ->e conversion in nuclei contains dipole operators (μ->e γ) and (e Γ μ) (q Γ q) structures, with Γ=S, P, V, A, T.

$|i\rangle$ is 1s of muonic atom, with BE=ε_b & $|f\rangle$ is E eigenstate with E=$m_\mu-\varepsilon_b$. Both wf are determined by solving Dirac eq. In presence of the E field of the nucleus.

We are generally most interested in the coherent case <=> $|i\rangle_{\text{Nucleus}}=|f\rangle_{\text{Nucleus}}$, which dominates (Feinberg&Weinberg'59)

Thus, the only non-vanishing relevant m.e. are $\langle N|qq|N\rangle$ & $\langle N|q\gamma^0 q|N\rangle$, which are straightforwardly related to the p/n densities in nuclei, $\rho^{(p/n)}$: $\langle N|qq|N\rangle = Z\, G_S^{(q,p)}\rho^{(p)}+(A-Z)G_S^{(q,n)}\rho^{(n)}$, $\langle N|q\gamma^0 q|N\rangle=2Z\,\rho^{(p)}+(A-Z)\rho^{(n)}$, for q=u with Z<->A-Z for q=d.

$G_S^{(q,p/n)}$ are O(5), Kosmas et al. '93, ...

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

The effective Lagrangian for μ->e conversion in nuclei contains dipole operators (μ->e γ) and (e Γ μ) (q Γ q) structures, with Γ=S, P, V, A, T.

$|i\rangle$ is 1s of muonic atom, with BE=ε_b & $|f\rangle$ is E eigenstate with $E=m_\mu-\varepsilon_b$. Both wf are determined by solving Dirac eq. In presence of the E field of the nucleus.

We are generally most interested in the coherent case <=> $|i\rangle_{Nucleus}=|f\rangle_{Nucleus}$, which dominates (Feinberg&Weinberg'59)

Thus, the only non-vanishing relevant m.e. are $\langle N|qq|N\rangle$ & $\langle N|q\gamma^0 q|N\rangle$, which are straightforwardly related to the p/n densities in nuclei, $\rho^{(p/n)}$: $\langle N|qq|N\rangle= Z\, G_S^{(q,p)}\rho^{(p)}+(A-Z)G_S^{(q,n)}\rho^{(n)}$, $\langle N|q\gamma^0 q|N\rangle=2Z\,\rho^{(p)}+(A-Z)\rho^{(n)}$, for q=u with Z<->A-Z for q=d.

The conversion probability depends on the effective Lagrangian couplings, $G_S^{(q,p/n)}$ and the overlap integrals: D, $S/V^{(p,n)}$, which depend on the μ/e wf, $\rho^{(p/n)}$(which determine E in the nucleus, that is also needed) and A&Z. For instance, $w_{conv}/(2G_F^2)=|...+\tilde{g}_{LS}^{(p)}S^{(p)}+...|^2+|L\leftrightarrow R|^2$.

D integrates the nucleus E, while S&V $\rho^{(p/n)}$ times wfs weighted by Z (A-Z).

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

Proton densities are extracted model-independently from e-p scattering experiments.

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

Proton densities are extracted model-independently from e-p scattering experiments.

Neutron densities are much more difficult. Info on them comes from π-atoms, scatt. exps. with p,α,$\pi^{\pm}$, exps. with anti-proton atoms.

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

Proton densities are extracted model-independently from e-p scattering experiments.

Neutron densities are much more difficult. Info on them comes from π-atoms, scatt. exps. with p,α,$\pi^{\pm}$, exps. with anti-proton atoms.

For light nuclei, assuming $\rho^{(n)}=\rho^{(p)}$ works at the percent level. For heavy nuclei it is better to use info from π-atoms or from polarized p scatt. Exps. (the former tend to give 10-20% smaller integrals).

NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

Proton densities are extracted model-independently from e-p scattering experiments.

Neutron densities are much more difficult. Info on them comes from π-atoms, scatt. exps. with p,α,$\pi^{\pm}$, exps. with anti-proton atoms.

For light nuclei, assuming $\rho^{(n)}=\rho^{(p)}$ works at the percent level. For heavy nuclei it is better to use info from π-atoms or from polarized p scatt. Exps. (the former tend to give 10-20% smaller integrals).

In any case, **conversion rates increase as Z for Z $\leq$30, are largest for 30$\leq$Z$\leq$60, and then decrease**.

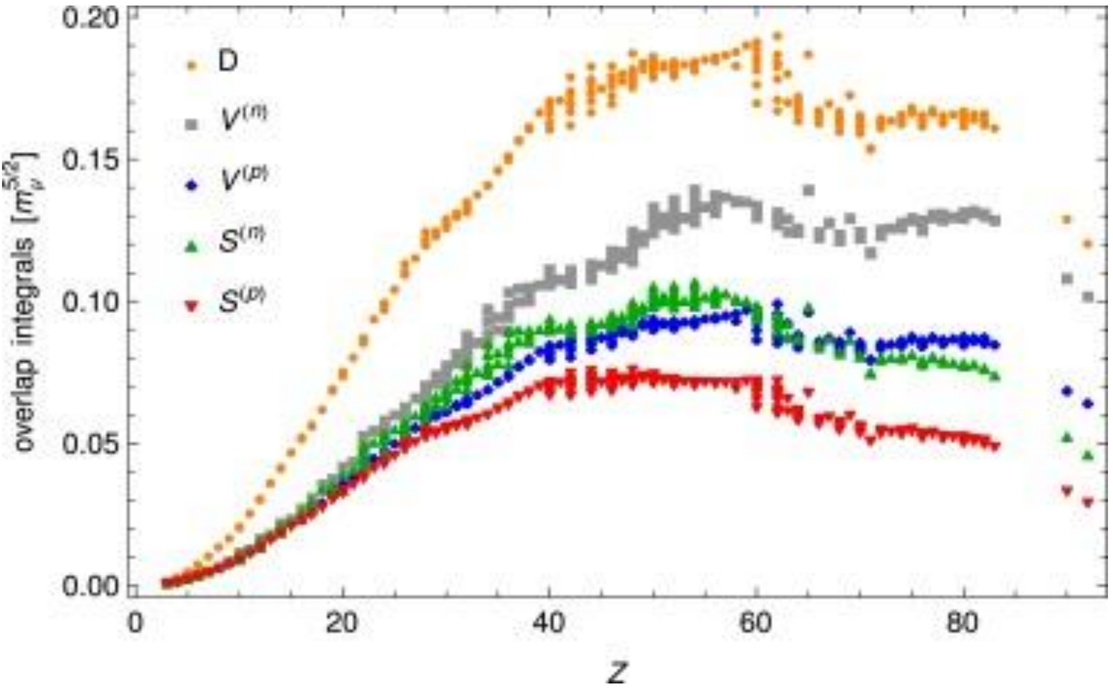
NUCLEAR PHYSICS INPUT (Kitano-Koide-Okada'02,...)

Proton densities are extracted model-independently from e-p scattering experiments.

Neutron densities are much more difficult. Info on them comes from π-atoms, scatt. exps. with p,α,$\pi^{\pm}$, exps. with anti-proton atoms.

For light nuclei, assuming $\rho^{(n)}=\rho^{(p)}$ works at the percent level. For heavy nuclei it is better to use info from π-atoms or from polarized p scatt. Exps. (the former tend to give 10-20% smaller integrals).

In any case, **conversion rates increase as Z for Z ≤30, are largest for 30≤Z≤60, and then decrease.**



Uncertainties are relatively small for Ti (Z=22), but not for Au (Z=79) or Pb (Z=82).

NUCLEAR PHYSICS INPUT (For e/μ->τ conversions...)

NUCLEAR PHYSICS INPUT (For $e/\mu \to \tau$ conversions...)

These are DIS experiments with an O(100)GeV e/μ beam hits a nucleus (fixed target, Fe/Pb). In this case the parton level process can be described in perturbative QCD and the rest depends on the nuclear PDFs (nCTEQ15 project, incorporated within the ManeParse Mathematica package).

NUCLEAR PHYSICS INPUT (For e/μ->τ conversions...)

These are DIS experiments with an O(100)GeV e/μ beam hits a nucleus (fixed target, Fe/Pb). In this case the parton level process can be described in perturbative QCD and the rest depends on the nuclear PDFs (nCTEQ15 project, incorporated within the ManeParse Mathematica package).

Until a ratio between the conversion and capture rates of ~10^{-15} is reached (~100 times larger at NA62), these processes are not competitive with the bounds coming from BaBar/Belle (which are/will be superseded by Belle-II).

CONCLUSIONS

- The generation of neutrino masses will generalize and extend the SM (Weinberg operator or fundamental LN conservation).

CONCLUSIONS

- The generation of neutrino masses will generalize and extend the SM (Weinberg operator or fundamental LN conservation).

- In the most beautiful theoretical realization, it will be linked to both Grand Unification and baryogenesis through leptogenesis. Unfortunately, even $0\nu2\beta$ may escape detection, forbiding its experimental verification.

CONCLUSIONS

- The generation of neutrino masses will generalize and extend the SM (Weinberg operator or fundamental LN conservation).

- In the most beautiful theoretical realization, it will be linked to both Grand Unification and baryogenesis through leptogenesis. Unfortunately, even $0\nu2\beta$ may escape detection, forbiding its experimental verification.

- Low-scale seesaws have been proposed as alternatives. These are less simple models, without straightorward GU but yielding O(TeV) states potentially observable at present/future colliders and cLFV signatures in the reach of current/forthcoming facilities. Within them, mu->e conversion in nuclei will be giving the strongest limits thanks to Mu2e & PRISM/Comet.

CONCLUSIONS

- The generation of neutrino masses will generalize and extend the SM (Weinberg operator or fundamental LN conservation).

- In the most beautiful theoretical realization, it will be linked to both Grand Unification and baryogenesis through leptogenesis. Unfortunately, even $0\nu2\beta$ may escape detection, forbiding its experimental verification.

- Low-scale seesaws have been proposed as alternatives. These are less simple models, without straightorward GU but yielding O(TeV) states potentially observable at present/future colliders and cLFV signatures in the reach of current/forthcoming facilities. Within them, mu->e conversion in nuclei will be giving the strongest limits thanks to Mu2e & PRISM/Comet. The interplay of these with mu->3e (also mu-> eγ, ...) will be key in identifying the underlying new physics. This task will be cleaner if the uncertainties on the overlap integrals get reduced [Nuclear physics input]

CONCLUSIONS

- The generation of neutrino masses will generalize and extend the SM (Weinberg operator or fundamental LN conservation).

- In the most beautiful theoretical realization, it will be linked to both Grand Unification and baryogenesis through leptogenesis. Unfortunately, even $0\nu2\beta$ may escape detection, forbiding its experimental verification.

- Low-scale seesaws have been proposed as alternatives. These are less simple models, without straightorward GU but yielding O(TeV) states potentially observable at present/future colliders and cLFV signatures in the reach of current/forthcoming facilities. Within them, mu->e conversion in nuclei will be giving the strongest limits thanks to Mu2e & PRISM/Comet. The interplay of these with mu->3e (also mu-> eγ, ...) will be key in identifying the underlying new physics. This task will be cleaner if the uncertainties on the overlap integrals get reduced [Nuclear physics input].

- The tau sector is in principle more sensitive to heavy new physics. Conversions in nuclei depend in this case on NPDFs, but we will need better sensitivity than NA62 (by ~ 2 orders of magnitude) so that they are at the level of other cLFV processes involving τ leptons.

CONCLUSIONS

- The generation of neutrino masses will generalize and extend the SM (Weinberg operator or fundamental LN conservation).

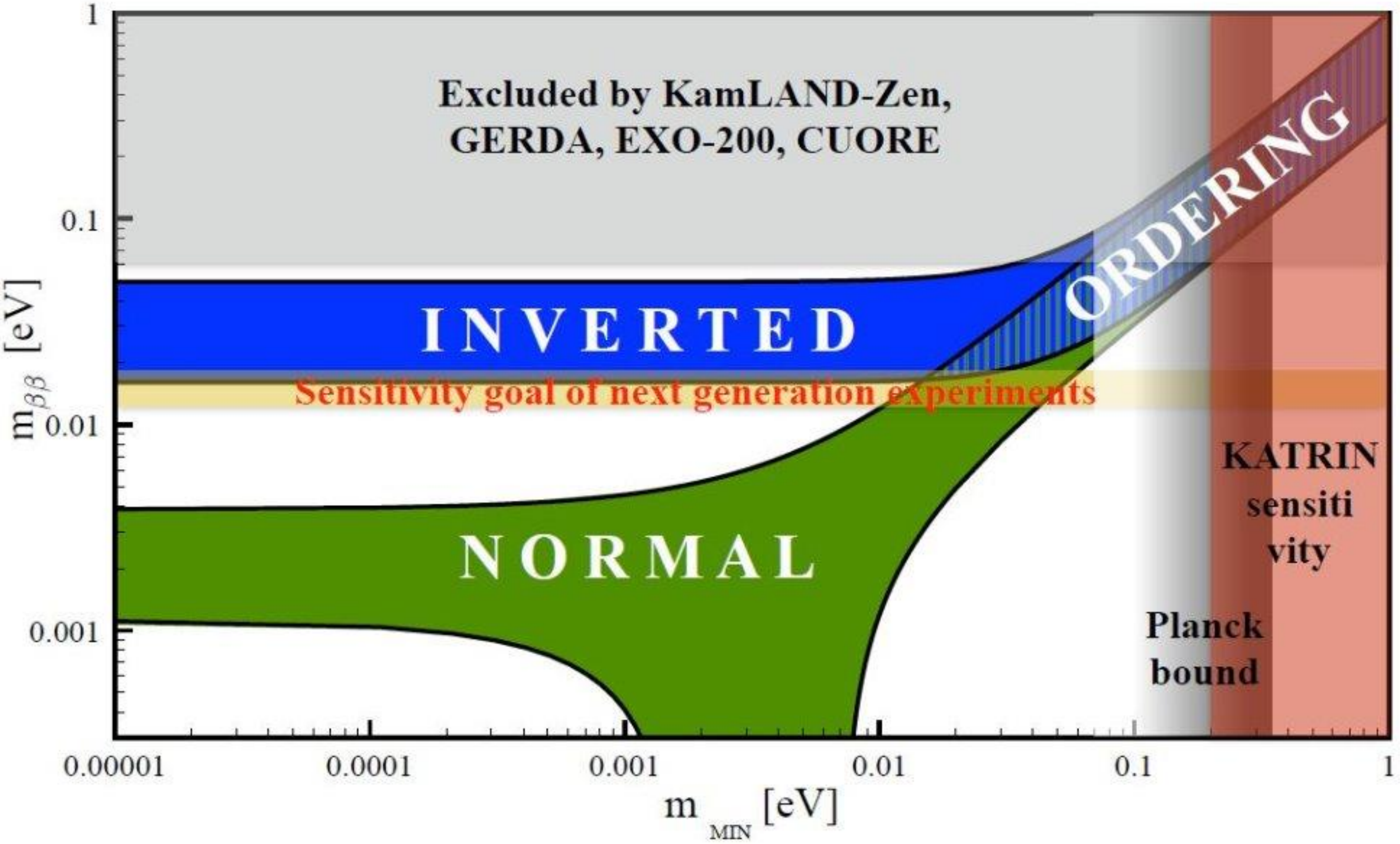
- In the most beautiful theoretical realization, it will be linked to both Grand Unification and baryogenesis through leptogenesis. Unfortunately, even $0\nu2\beta$ may escape detection, forbiding its experimental verification.

- Low-scale seesaws have been proposed as alternatives. These are less simple models, without straightorward GU but yielding O(TeV) states potentially observable at present/future colliders and cLFV signatures in the reach of current/forthcoming facilities. Within them, mu->e conversion in nuclei will be giving the strongest limits thanks to Mu2e & PRISM/Comet. The interplay of these with mu->3e (also mu-> eγ, ...) will be key in identifying the underlying new physics. This task will be cleaner if the uncertainties on the overlap integrals get reduced [Nuclear physics input].

- The tau sector is in principle more sensitive to heavy new physics. Conversions in nuclei depend in this case on NPDFs, but we will need better sensitivity than NA62 (by ~ 2 orders of magnitude) so that they are at the level of other cLFV processes involving τ leptons.

THANK YOU!

ADDITIONAL MATERIAL



Excluded by KamLAND-Zen,
GERDA, EXO-200, CUORE
INVERTED
ORDERING
Sensitivity goal of next generation experiments
NORMAL
KATRIN
sensiti
vity
Planck
bound
$m_{\beta\beta}$ [eV]
m_{MIN} [eV]
1
0.1
0.01
0.001
0.00001
0.0001
0.001
0.01
0.1
1

EFFECTIVE LAGRANGIAN FOR μ->e CONVERSION IN NUCLEI

$$
\begin{aligned}
\mathcal{L}_{\rm int} \;\; = \;\; & -\frac{4G_{\rm F}}{\sqrt{2}}\left(m_\mu A_R \bar{\mu}\sigma^{\mu\nu}P_L e F_{\mu\nu} + m_\mu A_L \bar{\mu}\sigma^{\mu\nu}P_R e F_{\mu\nu} + \text{h.c.}\right) \\
& -\frac{G_{\rm F}}{\sqrt{2}} \sum_{q=u,d,s} \Bigg[\left(g_{LS(q)}\bar{e}P_R\mu + g_{RS(q)}\bar{e}P_L\mu\right)\bar{q}q \\
& \qquad + \left(g_{LP(q)}\bar{e}P_R\mu + g_{RP(q)}\bar{e}P_L\mu\right)\bar{q}\gamma_5 q \\
& \qquad + \left(g_{LV(q)}\bar{e}\gamma^\mu P_L\mu + g_{RV(q)}\bar{e}\gamma^\mu P_R\mu\right)\bar{q}\gamma_\mu q \\
& \qquad + \left(g_{LA(q)}\bar{e}\gamma^\mu P_L\mu + g_{RA(q)}\bar{e}\gamma^\mu P_R\mu\right)\bar{q}\gamma_\mu\gamma_5 q \\
& \qquad + \frac{1}{2}\left(g_{LT(q)}\bar{e}\sigma^{\mu\nu}P_R\mu + g_{RT(q)}\bar{e}\sigma^{\mu\nu}P_L\mu\right)\bar{q}\sigma_{\mu\nu}q + \text{h.c.} \Bigg]
\end{aligned}
$$

AMPLITUDE FOR μ->e CONVERSION IN NUCLEI

$$
\begin{aligned}
M \;=\; & \frac{4G_\mathrm{F}}{\sqrt{2}} \int d^3x \left(m_\mu A_R^* \bar{\psi}^{\mu(e)}_{\kappa,W} \sigma^{\alpha\beta} P_R \psi^{(\mu)}_{1s} + m_\mu A_L^* \bar{\psi}^{\mu(e)}_{\kappa,W} \sigma^{\alpha\beta} P_L \psi^{(\mu)}_{1s} \right) \langle N'|F_{\alpha\beta}|N\rangle \\
& + \frac{G_\mathrm{F}}{\sqrt{2}} \sum_{q=u,d,s} \int d^3x \Bigg[\left(g_{LS(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} P_R \psi^{(\mu)}_{1s} + g_{RS(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} P_L \psi^{(\mu)}_{1s} \right) \langle N'|\bar{q}q|N\rangle \\
& \qquad + \left(g_{LP(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} P_R \psi^{(\mu)}_{1s} + g_{RP(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} P_L \psi^{(\mu)}_{1s} \right) \langle N'|\bar{q}\gamma_5 q|N\rangle \\
& \qquad + \left(g_{LV(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} \gamma^\alpha P_L \psi^{(\mu)}_{1s} + g_{RV(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} \gamma^\alpha P_R \psi^{(\mu)}_{1s} \right) \langle N'|\bar{q}\gamma_\alpha q|N\rangle \\
& \qquad + \left(g_{LA(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} \gamma^\alpha P_L \psi^{(\mu)}_{1s} + g_{RA(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} \gamma^\alpha P_R \psi^{(\mu)}_{1s} \right) \langle N'|\bar{q}\gamma_\alpha \gamma_5 q|N\rangle \\
& \qquad + \frac{1}{2} \left(g_{LT(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} \sigma^{\alpha\beta} P_R \psi^{(\mu)}_{1s} + g_{RT(q)} \bar{\psi}^{\mu(e)}_{\kappa,W} \sigma^{\alpha\beta} P_L \psi^{(\mu)}_{1s} \right) \langle N'|\bar{q}\sigma_{\alpha\beta} q|N\rangle \Bigg]
\end{aligned}
$$

μ->e CONVERSION RATE (IN NUCLEI)

$$\omega_{\rm conv} = 2G_{\rm F}^2 \left| A_R^* D + \tilde{g}_{LS}^{(p)} S^{(p)} + \tilde{g}_{LS}^{(n)} S^{(n)} + \tilde{g}_{LV}^{(p)} V^{(p)} + \tilde{g}_{LV}^{(n)} V^{(n)} \right|^2$$
$$+ \; 2G_{\rm F}^2 \left| A_L^* D + \tilde{g}_{RS}^{(p)} S^{(p)} + \tilde{g}_{RS}^{(n)} S^{(n)} + \tilde{g}_{RV}^{(p)} V^{(p)} + \tilde{g}_{RV}^{(n)} V^{(n)} \right|^2$$

Couplings redefinition:

$$\tilde{g}_{LS,RS}^{(p)} = \sum_q G_S^{(q,p)} \; g_{LS,RS(q)} \; ,$$
$$\tilde{g}_{LS,RS}^{(n)} = \sum_q G_S^{(q,n)} \; g_{LS,RS(q)} \; ,$$
$$\tilde{g}_{LV,RV}^{(p)} = 2g_{LV,RV(u)} + g_{LV,RV(d)} \; ,$$
$$\tilde{g}_{LV,RV}^{(n)} = g_{LV,RV(u)} + 2g_{LV,R(d)} \; .$$

μ->e CONVERSION RATE (IN NUCLEI)

$$\omega_{\rm conv} = 2G_{\rm F}^2 \left| A_R^* D + \tilde{g}_{LS}^{(p)} S^{(p)} + \tilde{g}_{LS}^{(n)} S^{(n)} + \tilde{g}_{LV}^{(p)} V^{(p)} + \tilde{g}_{LV}^{(n)} V^{(n)} \right|^2$$
$$+ 2G_{\rm F}^2 \left| A_L^* D + \tilde{g}_{RS}^{(p)} S^{(p)} + \tilde{g}_{RS}^{(n)} S^{(n)} + \tilde{g}_{RV}^{(p)} V^{(p)} + \tilde{g}_{RV}^{(n)} V^{(n)} \right|^2$$

Overlap integrals:

$$D = \frac{4}{\sqrt{2}} m_\mu \int_0^\infty dr r^2 [-E(r)] (g_e^- f_\mu^- + f_e^- g_\mu^-) ,$$

$$S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 Z \rho^{(p)} (g_e^- g_\mu^- - f_e^- f_\mu^-) ,$$

$$S^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 (A-Z) \rho^{(n)} (g_e^- g_\mu^- - f_e^- f_\mu^-)$$

$$V^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 Z \rho^{(p)} (g_e^- g_\mu^- + f_e^- f_\mu^-) ,$$

$$V^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 (A-Z) \rho^{(n)} (g_e^- g_\mu^- + f_e^- f_\mu^-)$$

μ->e CONVERSION RATE (IN NUCLEI)

$$\omega_{\rm conv} = 2G_{\rm F}^2 \left| A_R^* D + \tilde{g}_{LS}^{(p)} S^{(p)} + \tilde{g}_{LS}^{(n)} S^{(n)} + \tilde{g}_{LV}^{(p)} V^{(p)} + \tilde{g}_{LV}^{(n)} V^{(n)} \right|^2 + 2G_{\rm F}^2 \left| A_L^* D + \tilde{g}_{RS}^{(p)} S^{(p)} + \tilde{g}_{RS}^{(n)} S^{(n)} + \tilde{g}_{RV}^{(p)} V^{(p)} + \tilde{g}_{RV}^{(n)} V^{(n)} \right|^2$$

Wave functions:

$$\psi_{1s}^{(\mu)}(r,\theta,\phi) = \begin{pmatrix} g_\mu^-(r)\chi_{-1}^{\pm 1/2}(\theta,\phi) \\ i f_\mu^-(r)\chi_{1}^{\pm 1/2}(\theta,\phi) \end{pmatrix},$$

$$\psi_{\kappa=-1,W}^{\mu=\pm 1/2(e)}(r,\theta,\phi) = \begin{pmatrix} g_e^-(r)\chi_{-1}^{\pm 1/2}(\theta,\phi) \\ i f_e^-(r)\chi_{1}^{\pm 1/2}(\theta,\phi) \end{pmatrix},$$

$$\psi_{\kappa=1,W}^{\mu=\pm 1/2(e)}(r,\theta,\phi) = \begin{pmatrix} g_e^+(r)\chi_{1}^{\pm 1/2}(\theta,\phi) \\ i f_e^+(r)\chi_{-1}^{\pm 1/2}(\theta,\phi) \end{pmatrix}.$$

m_e->0 => $g_e^+ = i f_e^-$ and $i f_e^+ = g_e^-$.