Matching effective models to lattice QCD: few-body and many-body physics

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Acknowledgements

FAPESP

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Financial Support

Outline (Few-Body)

Dedicated to Prof. Manuel Malheiro

* Lattice QCD predictions
* Two-body potentials
* Three-body bindings

Outline (Many-Body)

Dedicated to Prof. Manuel Malheiro

* Thermo-magnetic NJL coupling
* Thermodynamics and meson properties
* Magnetization

Introduction / Motivation (Few-Body)

Verify if lattice QCD predictions for two and three nucleons
are supported by effective calculations with separable potentials

for a review of 3N system, see works
from Bochum, Krakow and Ohio groups

Lattice QCD	Separable 2N potentials
Lüscher formula	Faddeev equation

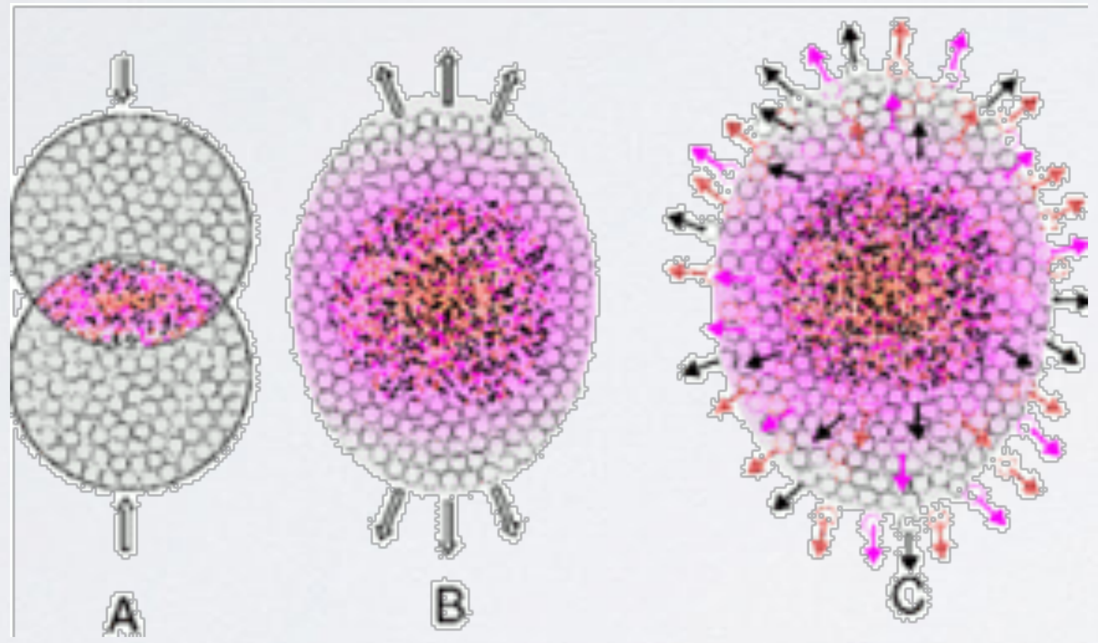
S.R.Beane *et al*. [NPLQCD], Phys. Rev. C 88 (2013) 024003 (two-nucleons)

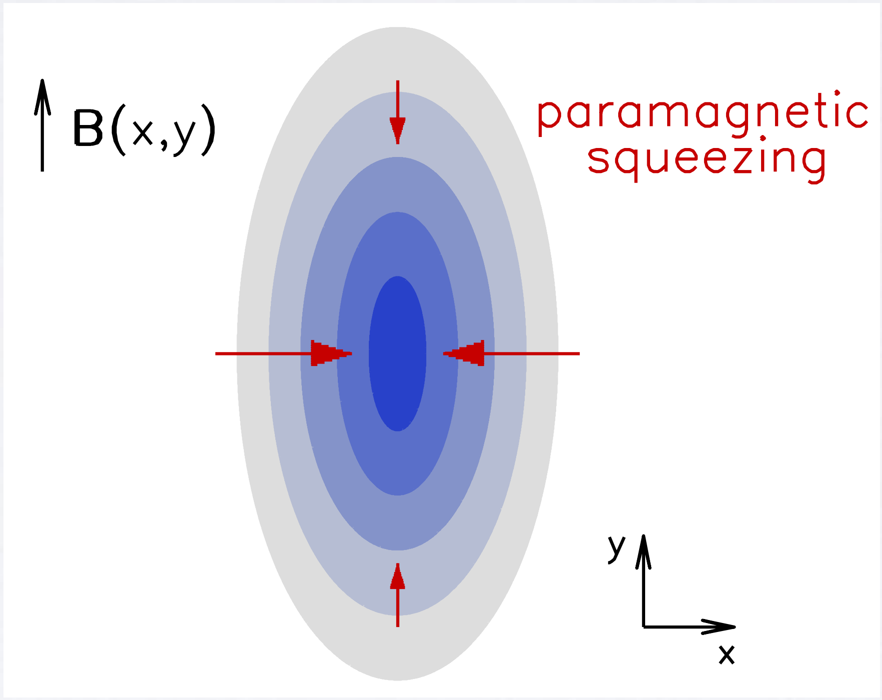
S.R.Beane *et al*. [NPLQCD], Phys. Rev. D 87 (2013) 034506 (three-nucleons)

Introduction / Motivation (Many-Body)

Verify if lattice QCD predictions for hot and magnetized quark matter are supported by calculations with effective NJL models

LHC / RHIC





Phys. Rev. Lett. 112 (2014) 042301

G. S. Bali, F. Bruckmann, G. Endrődi, and A. Schäfer

Magnetars

neutron stars

hadronic matter

Lattice QCD predictions

$m_\pi = 806$ MeV $\qquad$ $m_N = 1.634\ (0)\ (0)\ (18)$ GeV

triplet: $a_{21} = 1.82^{+0.14+0.17}_{-0.13-0.12}$ fm, $\quad r_{21} = 0.906^{+0.068+0.068}_{-0.075-0.084}$ fm

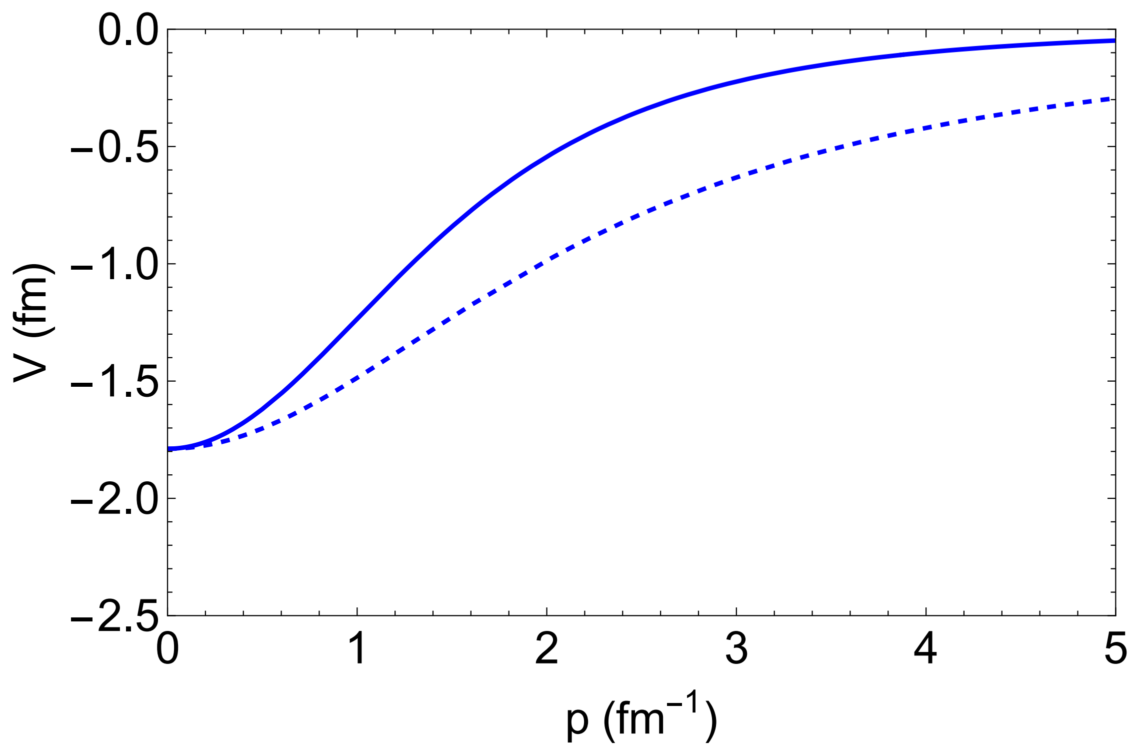
singlet: $a_{20} = 2.33^{+0.19+0.27}_{-0.17-0.20}$ fm, $\quad r_{20} = 1.130^{+0.071+0.059}_{-0.077-0.063}$ fm

$$a \sim 2\, r$$

S.R.Beane *et al*. [NPLQCD], Phys. Rev. C 88 (2013) 024003

2N separable potentials

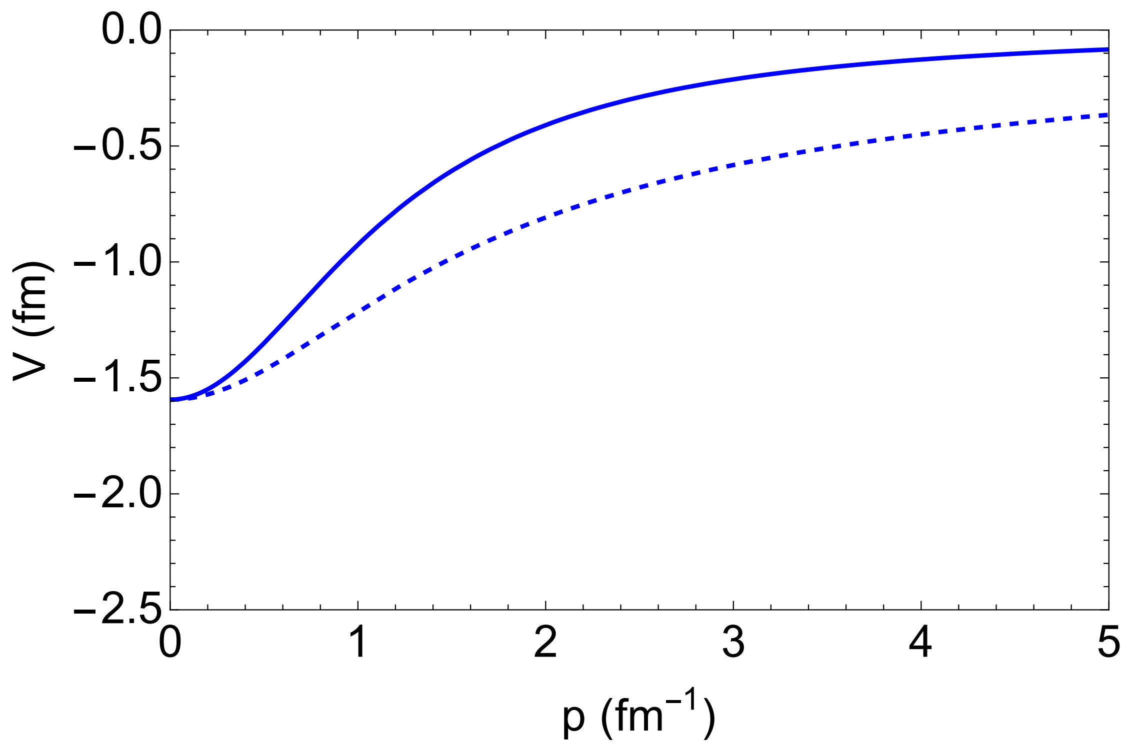
$$g(p) = \left(1 + \frac{p^2}{\alpha^2}\right)^{-1}$$



Separable two-nucleon potentials

$$V_2(p', p) = \frac{4\pi}{m} \lambda \, g(p') \, g(p)$$

$$g(p) = \left(1 + \frac{p^2}{\alpha^2}\right)^{-1/2}$$

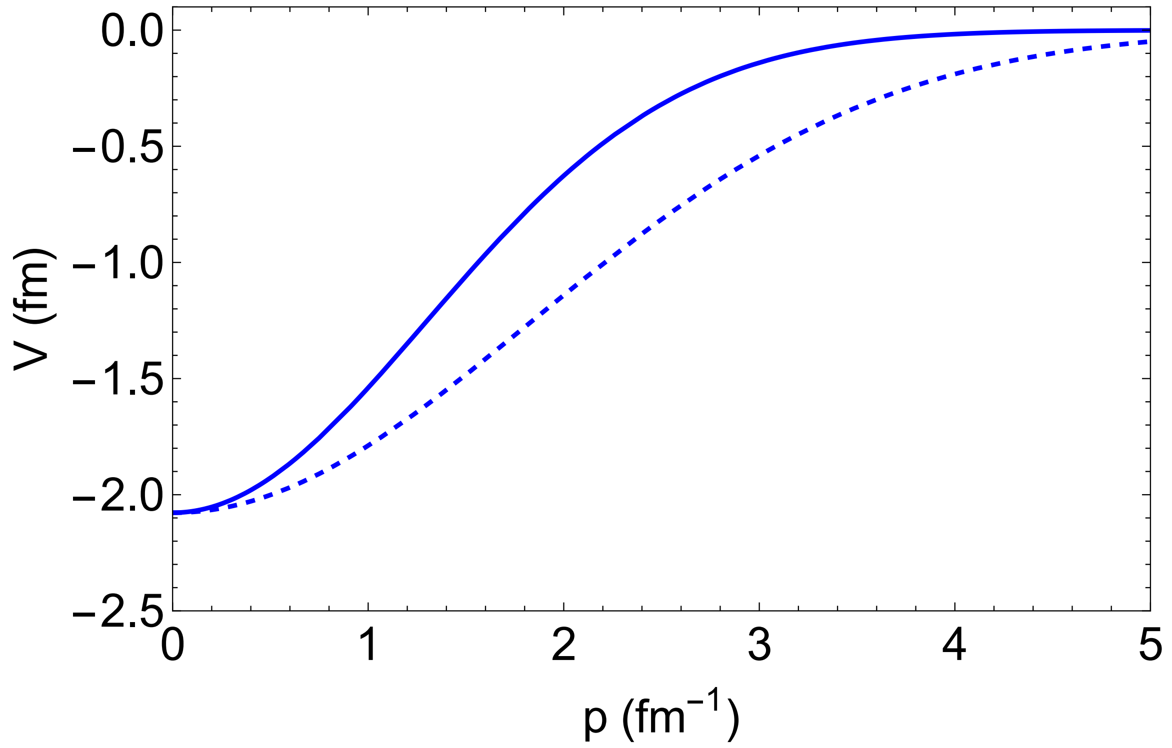


α , λ

$a \; = \; 2\, r$ (LQCD)

$a_{s,t}$, $r_{s,t}$ (empirical)

$$g(p) = e^{-p^2/\alpha^2}$$



Y. Yamaguchi, Phys. Rev. 95 (1954) 1628

2N T-matrix

Analytical T-matrix:

$$T_2(p',p;k) = \frac{4\pi}{m}\frac{g(p')g(p)}{\Lambda^{-1}(-ik)-\lambda^{-1}} = \frac{4\pi}{m}\frac{g(p')g(p)}{g^2(k)}\left[-ik-\frac{1}{a_2}+R(k)k^2\right]^{-1}$$

with

$$\frac{1}{a_2} = \frac{1}{\lambda}+\frac{2}{\pi}\int_0^\infty dl\, g^2(l) \qquad R(k) = \frac{1}{a_2k^2}\left(g^{-2}(k)-1\right)+\frac{i}{k}-\frac{2}{\pi}g^{-2}(k)\int_0^\infty dl\,\frac{g^2(l)}{l^2-k^2-i\varepsilon}$$

$$\Lambda^{-1}(-ik) = -\frac{2}{\pi}\int_0^\infty dl\, g^2(l) - ik + R(k)k^2$$

Pole:

$$k = i\,\kappa_2 \qquad \lambda = \Lambda(\kappa_2)$$

Double pole

$$g(p) = \left(1 + \frac{p^2}{\alpha^2}\right)^{-1/2} \qquad \frac{1}{a_2} = \frac{1}{\lambda} + \alpha \qquad 2R(k) = r_2 = -\frac{2}{\lambda\alpha^2}$$

For $\lambda < -\frac{1}{\alpha}$ the separable potential with the above g(p) generates two poles:

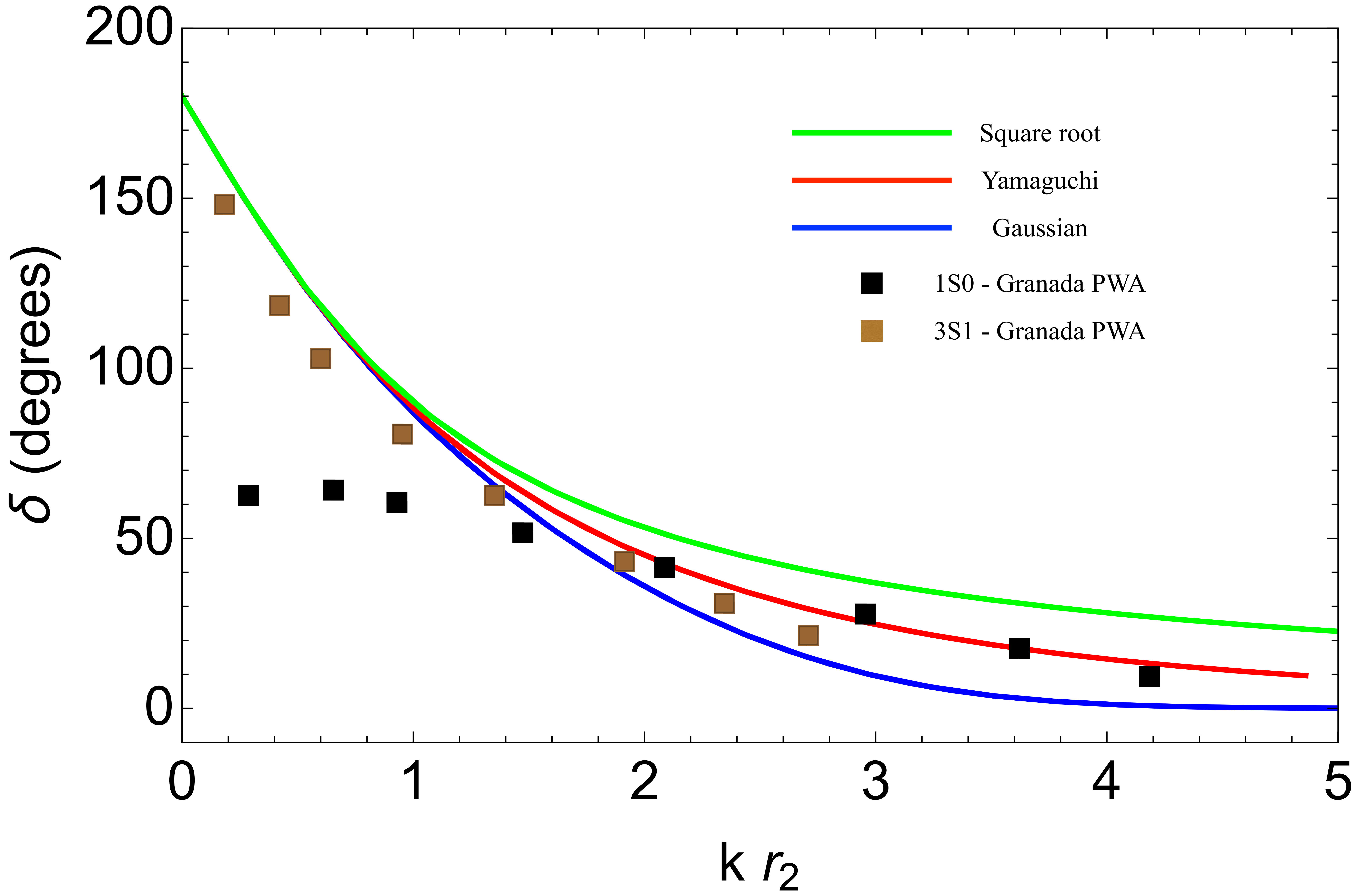
$$\kappa_2 = \alpha$$

(independent of λ)

$$\frac{1}{\lambda} = -\frac{\alpha^2}{\kappa_2 + \alpha}$$

2N phase-shifts

$a \sim 2r$



Spinless three-body system

A. G. Sitenko and V. F. Kharchenko, Nucl. Phys. 49 (1963) 15

V. S. Timóteo, Ann. Phys. 432 (2021) 168573

Jacobi momenta

$$\vec{k}_{ij} = \frac{1}{2}(\vec{p}_i - \vec{p}_j), \quad \vec{k}_i = \frac{1}{3}(2\vec{p}_i - \vec{p}_j - \vec{p}_k)$$

Wave function

$$\psi(p,q) = -\frac{\lambda g(p)}{\kappa_3^2 + p^2 + 3q^2/4} a(q)$$

$$\det\left[\delta_{ij} - \bar{\mathscr{K}}(q_i, q_j'; \kappa_3)\right] = 0$$

Profile function

$$a(q) = \frac{2}{\pi}\int_0^\infty dq'\, q'^2\, \mathscr{K}(q,q';\kappa_3)\, a(q')$$

Kernel

$$\mathscr{K}(q,q';\kappa_3) = \left[\Lambda^{-1}\left(\sqrt{\kappa_3^2 + \frac{3q^2}{4}}\right) - \lambda^{-1}\right]^{-1} \int_{-1}^{1} dy\, \frac{g(\pi_2)\, g(\pi_1)}{\kappa_3^2 + q^2 + q'^2 + qq'y}$$

$$\pi_1 = \sqrt{q^2/4 + q'^2 + qq'y}, \quad \pi_2 = \sqrt{q^2 + q'^2/4 + qq'y}$$

Three-nucleon system

Wave functions

$$\begin{pmatrix} \psi_1 \\ \psi_0 \end{pmatrix} = -\frac{1}{\kappa_3^2 + p^2 + 3q^2/4} \begin{pmatrix} \lambda_1 g_1(p) a(q) \\ \lambda_0 g_0(p) b(q) \end{pmatrix}$$

Profile functions

$$\begin{pmatrix} a(q) \\ b(q) \end{pmatrix} = \frac{1}{2\pi} \int_0^\infty dq' q'^2 \begin{pmatrix} \mathcal{K}_{11}(q,q';\kappa_3) & 3\mathcal{K}_{10}(q,q';\kappa_3) \\ 3\mathcal{K}_{01}(q,q';\kappa_3) & \mathcal{K}_{00}(q,q';\kappa_3) \end{pmatrix} \begin{pmatrix} a(q') \\ b(q') \end{pmatrix}$$

Kernel

$$\mathcal{K}_{ss'}(q,q';\kappa_3) = \left[\Lambda_s^{-1}\left(\sqrt{\kappa_3^2 + 3q^2/4}\right) - \lambda_s^{-1}\right]^{-1} \int_{-1}^{1} dy \, \frac{g_s(\pi_2) g_{s'}(\pi_1)}{\kappa_3^2 + p^2 + q'^2 + qq'y}$$

$$\det \begin{pmatrix} \mathbf{1} - \bar{\mathcal{K}}_{11}(q,q';\kappa_3) & -3\bar{\mathcal{K}}_{10}(q,q';\kappa_3) \\ -3\bar{\mathcal{K}}_{01}(q,q';\kappa_3) & \mathbf{1} - \bar{\mathcal{K}}_{00}(q,q';\kappa_3) \end{pmatrix} = 0$$

A. G. Sitenko and V. F. Kharchenko, Nucl. Phys. 49 (1963) 15

V. S. Timóteo, Ann. Phys. 432 (2021) 168573

Two-Nucleon Binding Energies

Lattice QCD ER parameters, unphysical pion mass

	Square-Root	Yamaguchi	Gaussian	LQCD
B_{21}/MeV	25.3	19.5	18.4	19.5 (3.6) (3.1) (0.2)
B_{20}/MeV	12.7	11.1	10.7	15.9 (2.7) (2.7) (0.2)

Empirical ER parameters, unphysical pion mass

	Square-Root	Yamaguchi	Gaussian	experiment
B_{21}/MeV	13.4949	9.26145	8.69714	2.224575(9)
B_{20}/MeV	5.66342	3.88675	3.64993	—

Three-Nucleon Binding Energies

Lattice QCD ER parameters, unphysical pion mass

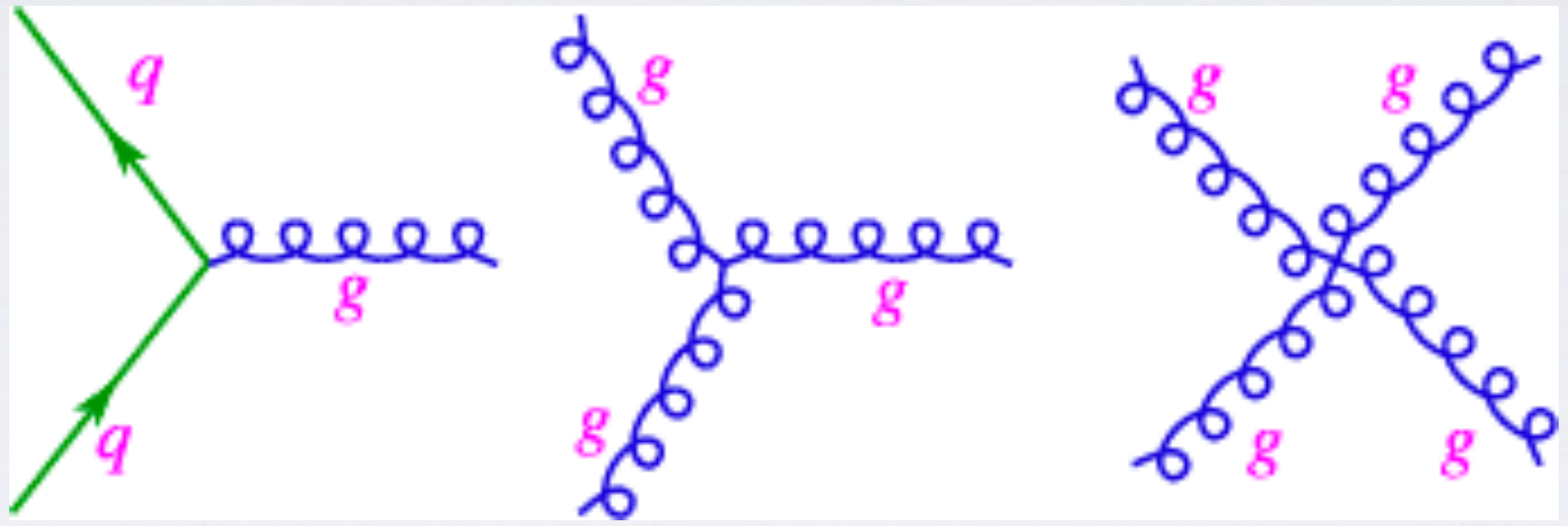
	Square-Root	Yamaguchi	Gaussian	LQCD
B_3/MeV	56.5	56.6	56.5	53.9 (7.1) (8.0) (0.6)

Empirical ER parameters, physical pion mass

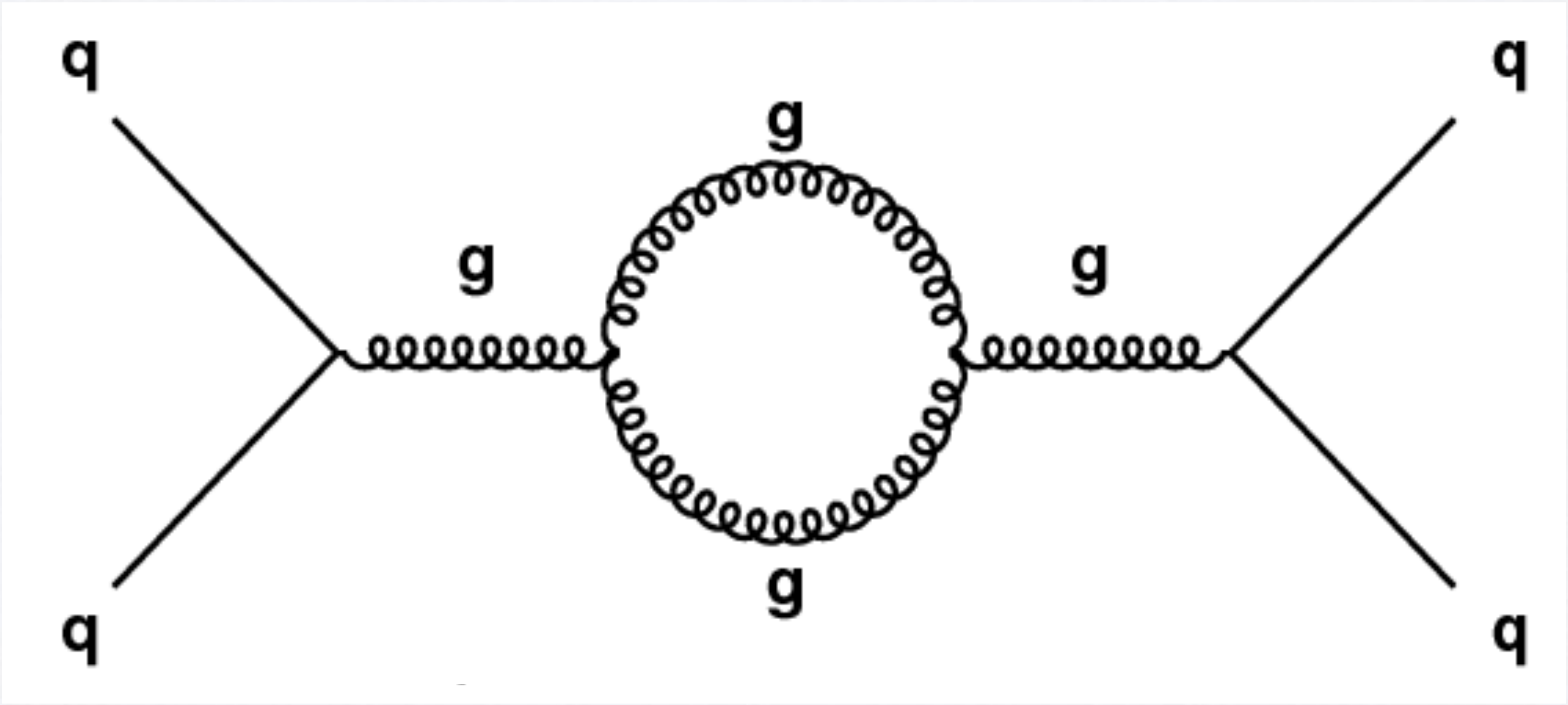
	Square-Root	Yamaguchi	Gaussian	experiment
B_3/MeV	7.496939	8.945608	8.397675	8.481798(2)

QCD complications

Force carriers interact directly (increases the number of possible processes)



Couplings are strong (high order processes are not necessarily less important)



Effective Models

Models are less powerful than theories

They are used when the fundamental theory is too complicated

Many examples: meson exchange, van der Waals, quark-meson coupling, …

Built to explain part of the features of a complex theory

QCD	NJL
Confinement	No confinement
Asymptotic freedom	No asymptotic freedom
Symmetry breaking	Symmetry breaking
Mass generation	Mass generation

Nambu–Jona-Lasinio Models

Buballa, Bernard, Klevansky, Ratti, Weise,...

SU(2)

$$\mathcal{L}_{\mathrm{NJL}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left(\not{D} - m\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right]$$

SU(3) + PL

$$\mathcal{L} = \bar{q}\left[i\gamma_\mu D^\mu - \hat{m}_c\right]q \;+\; \mathcal{L}_{sym} \;+\; \mathcal{L}_{det}$$

$$+\,\mathcal{U}\left(\Phi, \bar{\Phi}; T\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{sym} = \frac{G_s}{2}\sum_{a=0}^{8}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right]$$

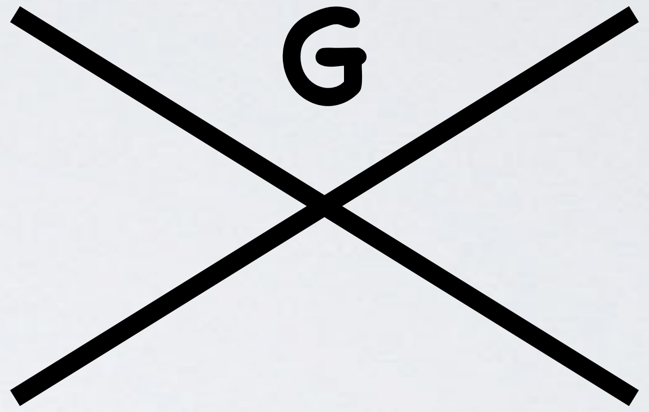
$$\mathcal{L}_{det} = -K\left\{\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right\}$$

NJL gap equation: simple view

see review by Weise

$$\mathcal{L}_D = \bar{\psi}\, i\gamma^\mu \partial_\mu\, \psi \;-\; m\, (\bar{\psi}\psi)$$

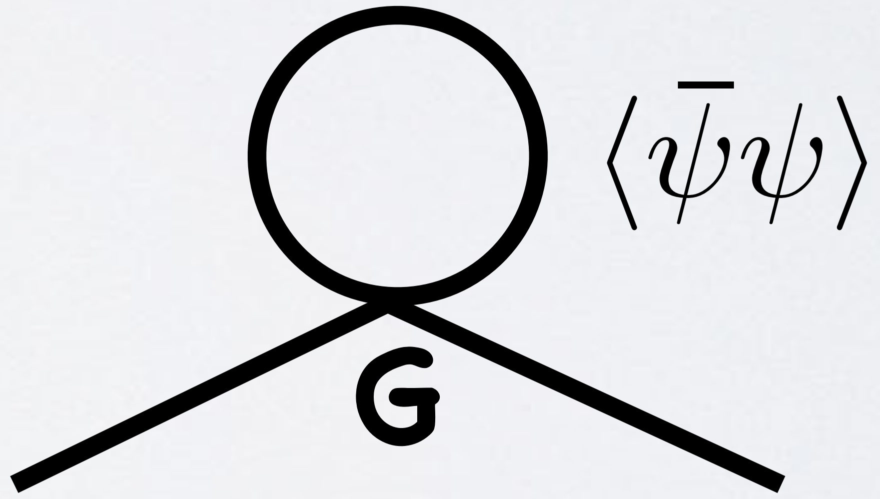
$$(\, i\gamma^\mu \partial_\mu - m \,)\, \psi \;=\; 0$$



$$\mathcal{L}_{\rm NJL} = \bar{\psi}\, i\gamma^\mu \partial_\mu\, \psi \;+\; G\, (\bar{\psi}\psi)^2$$

$$(\, i\gamma^\mu \partial_\mu + 2G\langle\bar{\psi}\psi\rangle \,)\, \psi \;=\; 0$$

$$m \;=\; -\,2G\, \langle\bar{\psi}\psi\rangle$$



$$\langle\bar{\psi}\psi\rangle \sim \int d^4p \frac{1}{p^2+m^2} \sim \int^\Lambda dp\; p^2 \frac{m}{\sqrt{p^2+m^2}}$$

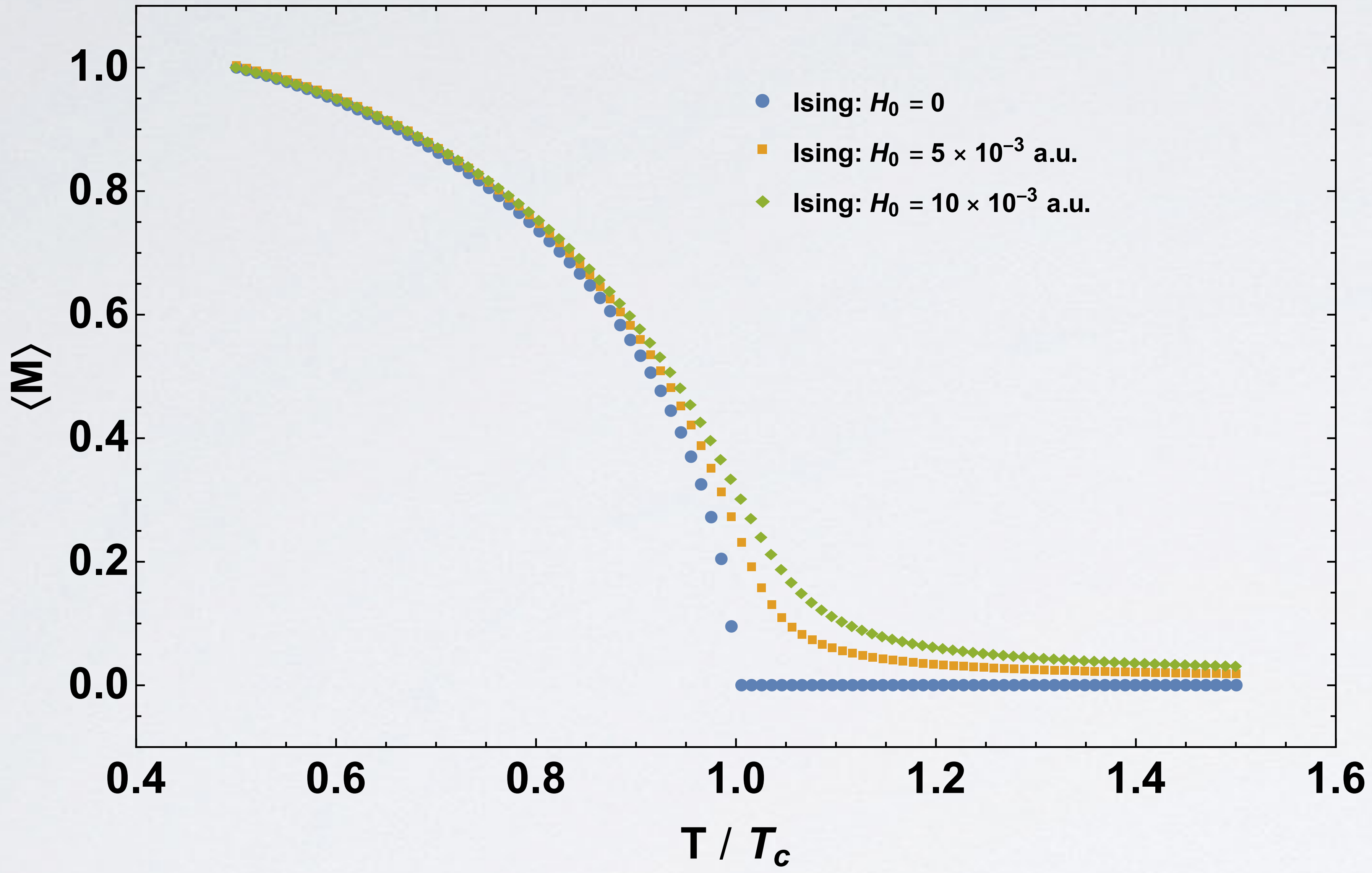
<M> and <$\bar{q}q$>

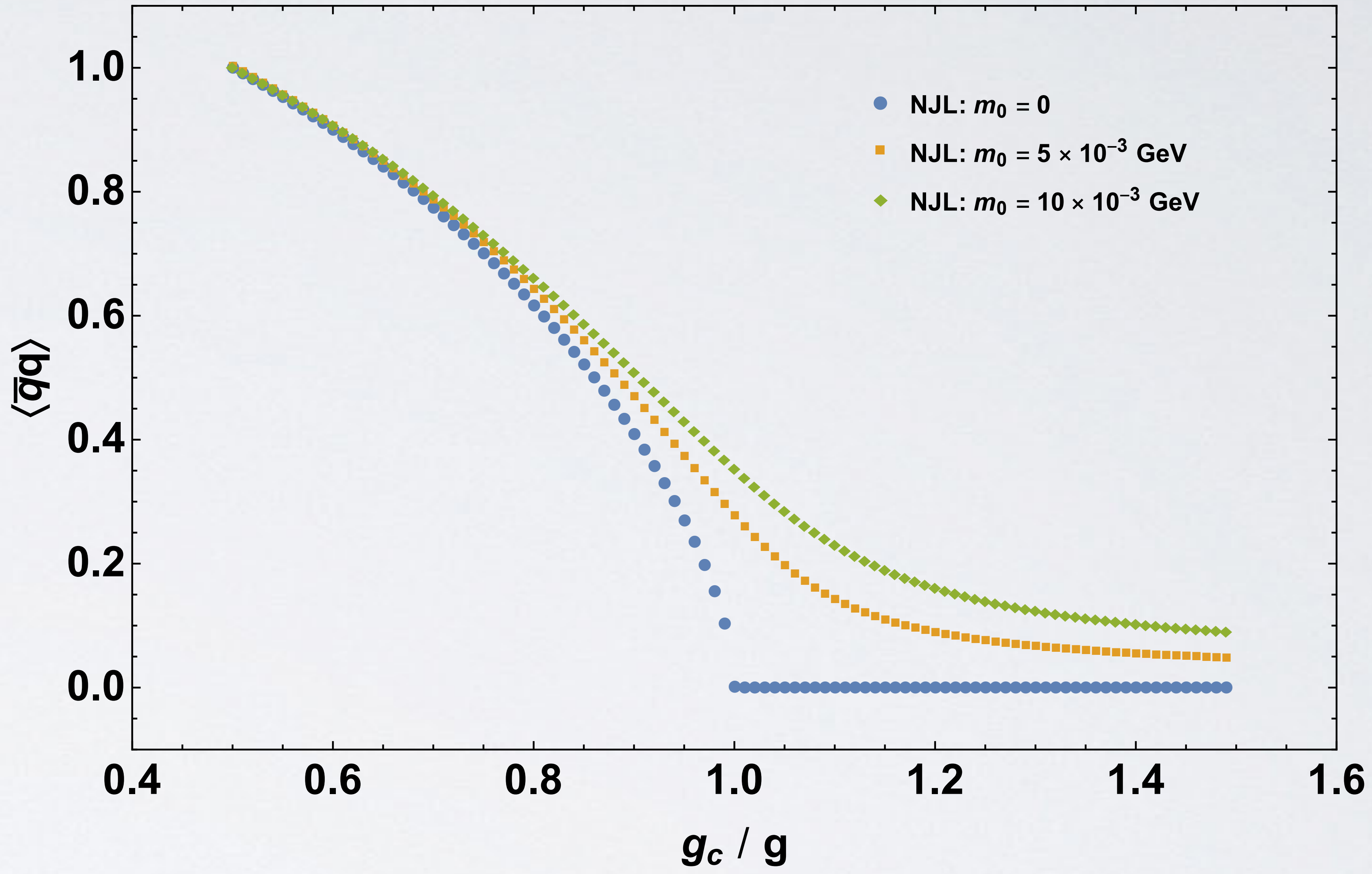
2D Ising model

Magnetization

2F NJL model

Chiral condensate





Rotational symmetry is broken

Chiral symmetry is broken

Collective Goldstone mode: spin waves

Collective Goldstone mode: pions

External field H_0 (explicit breaking)

Current quark mass m_0 (explicit breaking)

Gap equations at finite temperature and magnetic field

$$M_u = m_u - 2G\langle \bar{u}u \rangle - 2K\langle \bar{d}d \rangle \langle \bar{s}s \rangle$$

$$M_d = m_d - 2G\langle \bar{d}d \rangle - 2K\langle \bar{s}s \rangle \langle \bar{u}u \rangle$$

$$M_s = m_s - 2G\langle \bar{s}s \rangle - 2K\langle \bar{u}u \rangle \langle \bar{d}d \rangle$$

$$\langle \bar{q}q \rangle \to \langle \bar{q}q \rangle_{\mathrm{vac}} + \langle \bar{q}q \rangle_{\mathrm{mag}} + \langle \bar{q}q \rangle_{\mathrm{Tmag}}$$

Condensates

$$\langle\bar{\psi}_f\psi_f\rangle^{vac} = -\frac{MN_c}{2\pi^2}\left[\Lambda\,\epsilon_\Lambda - M^2\ln\left(\frac{\Lambda+\epsilon_\Lambda}{M}\right)\right]$$

$$\langle\bar{\psi}_f\psi_f\rangle^{mag} = -\frac{M|q_f|BN_c}{2\pi^2}\left[\ln\Gamma(x_f) - \frac{1}{2}\ln(2\pi) + x_f - \frac{1}{2}(2x_f-1)\ln(x_f)\right]$$

$$\langle\bar{\psi}_f\psi_f\rangle^{Tmag} = \sum_{k=0}^{\infty}\alpha_k\frac{M|q_f|BN_c}{2\pi^2}\int_{-\infty}^{+\infty}\mathrm{d}p\,\frac{n(E_f)}{E_f}$$

$$\epsilon_\Lambda = \left(\Lambda^2+M^2\right)^{1/2}$$

$$E_f = \left(p^2+M^2+2|q_f|Bk\right)^{1/2}$$

$$x_f = \frac{M^2}{2|q_f|B}$$

$$n(E_f) = \frac{1}{1+\exp(E_f/T)}$$

$$\mu = 0$$

Grand canonical potential in MFA

$$\Omega = -\, T \; ln \; \mathcal{Z} \qquad\qquad \mathcal{Z} = Tr \; e^{-\beta(H-\mu N)}$$

$$\Omega(T,\mu) = G_s \sum_{f=u,d,s} \langle \bar{q}_f q_f \rangle^2 + 4K \langle \bar{q}_u q_u \rangle \langle \bar{q}_d q_d \rangle \langle \bar{q}_s q_s \rangle$$
$$+\mathcal{U}(\Phi,\bar{\Phi},T) + \sum_{f=u,d,s} \left(\Omega^f_{\rm vac} + \Omega^f_{\rm med} + \Omega^f_{\rm mag} \right)$$

$$\Omega^f_{\rm vac} = -6 \int_\Lambda \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + M_f^2}$$

$$\zeta'(-1, x_f) = \left. \frac{\mathrm{d}\zeta(z, x_f)}{\mathrm{d}z} \right|_{z=-1}$$

$$\Omega^f_{\rm mag} = -\frac{3(|q_f|B)^2}{2\pi^2} \left[\zeta'(-1, x_f) - \frac{1}{2}(x_f^2 - x_f) \ln x_f + \frac{x_f^2}{4} \right]$$

$$\Omega^f_{\rm T,\,B} = -T \frac{|q_f B|}{2\pi} \sum_{k=0} \alpha_k \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} \ln\{1 + \exp[-(E_f/T)]\}$$

$$\mu = 0$$

Thermodynamics

$$\Omega = -\,T\;ln\;\mathcal{Z}$$

$$\mathcal{Z} = Tr\;e^{-\beta(H-\mu N)}$$

$$\epsilon = \Omega + T\;s + \mu\;\rho$$

$$p = -\Omega$$

$$s = -\frac{\partial \Omega}{\partial T}$$

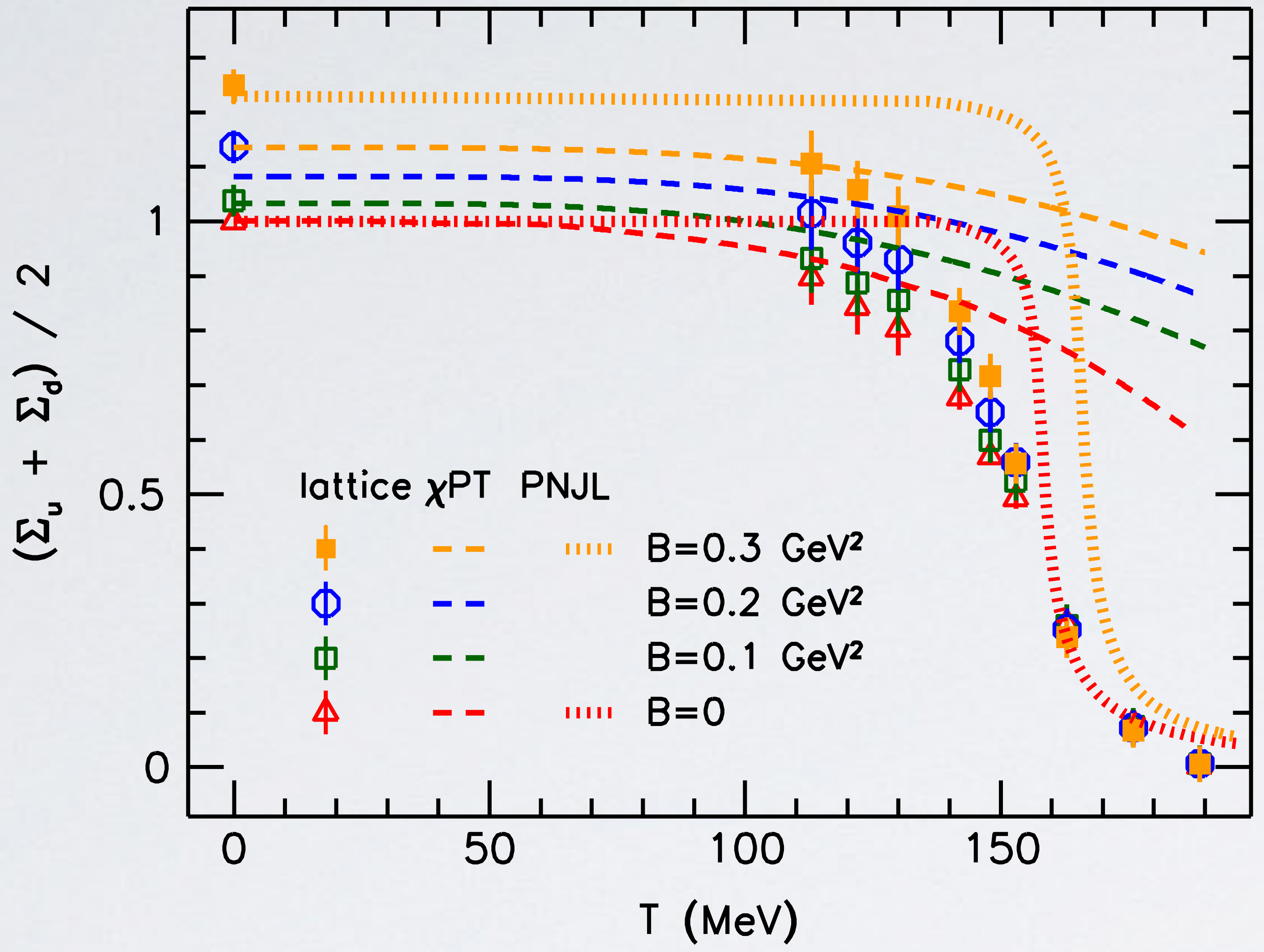
$$c_v = T\frac{dS}{dT}$$

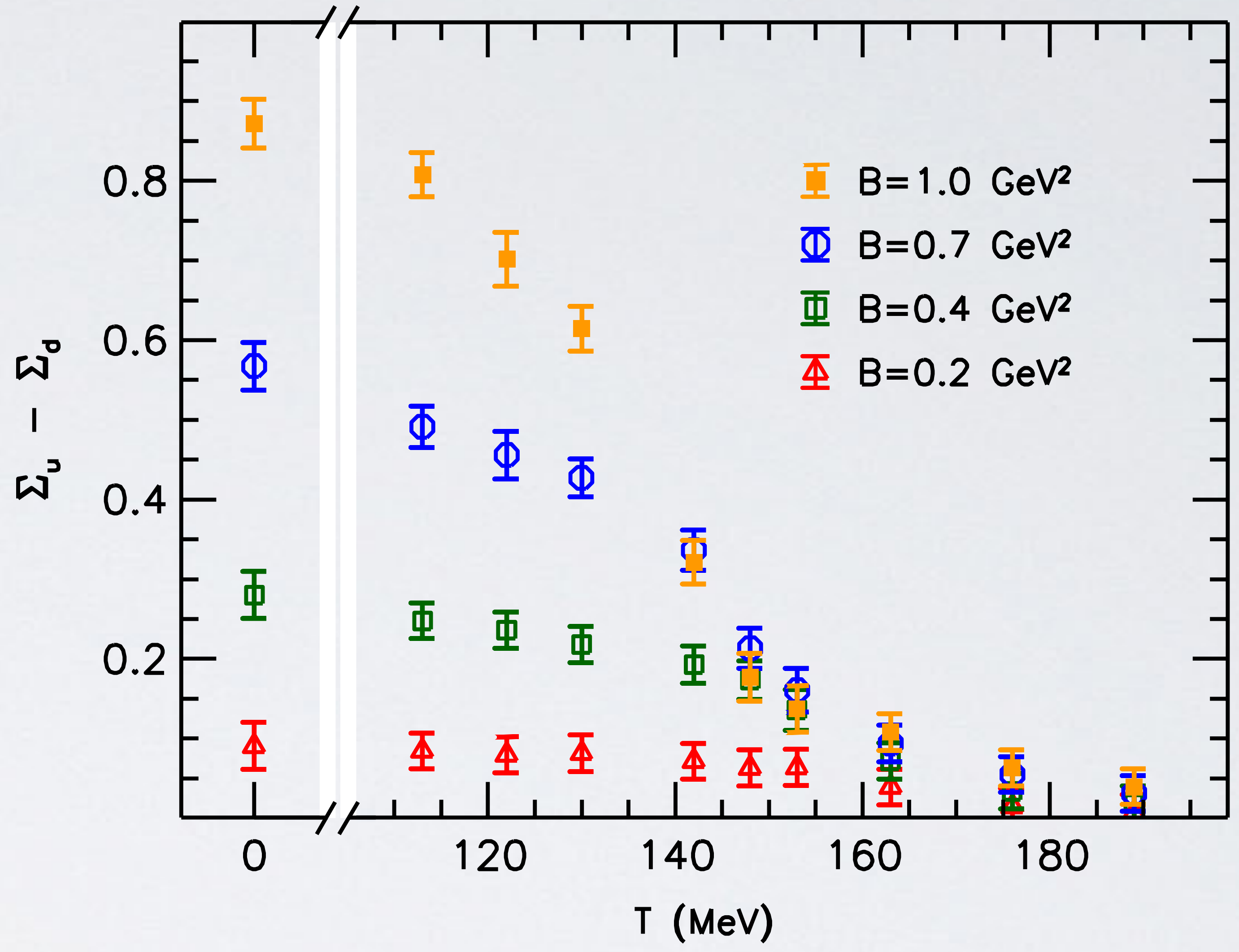
$$\Delta = \epsilon - 3\;p$$

$$v_s^2 = \frac{dp}{d\epsilon}$$

$$\mathcal{M} = -\frac{\partial \Omega}{\partial B}$$

Some lattice QCD results



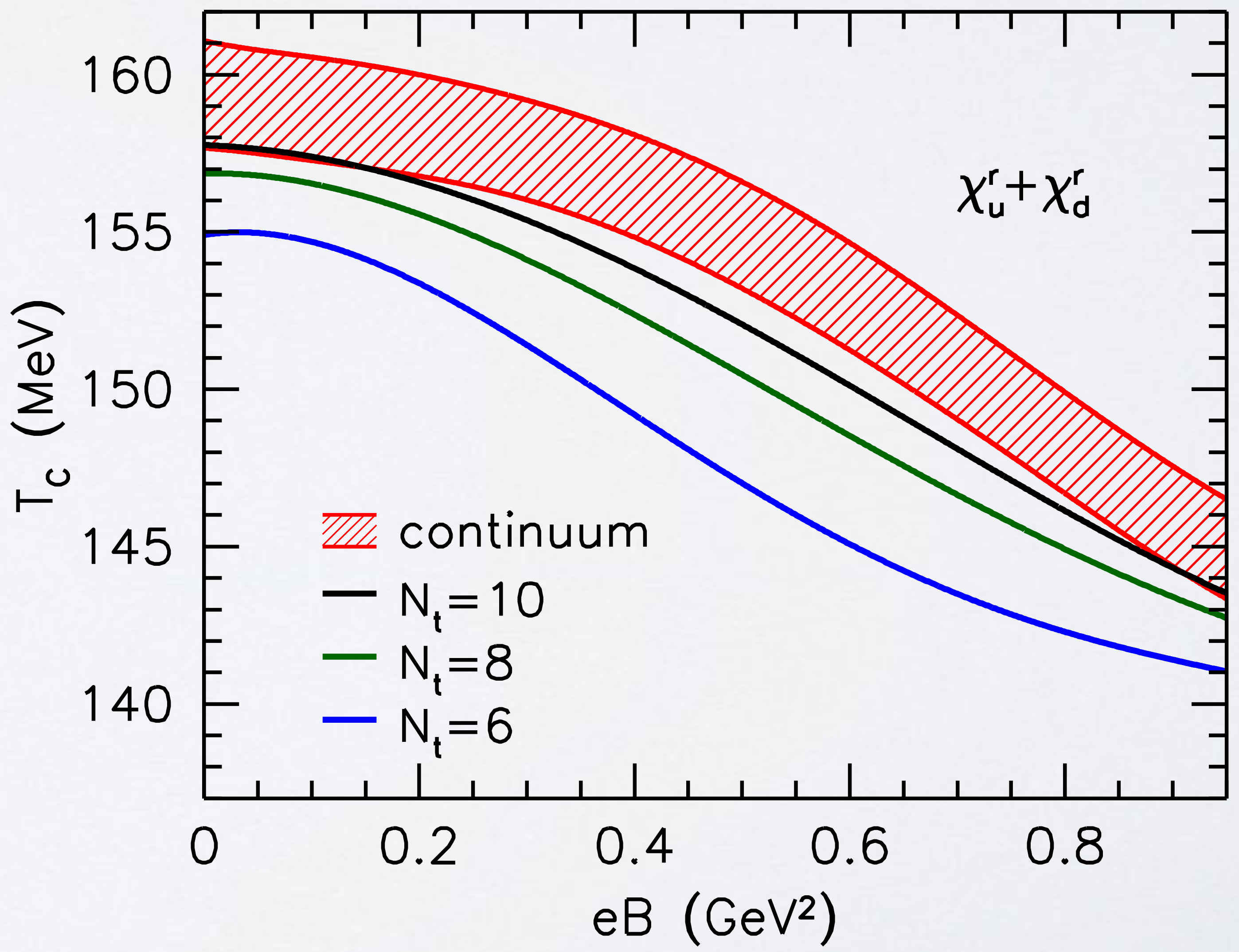


G. Bali et al.

JHEP 02 (2012) 044

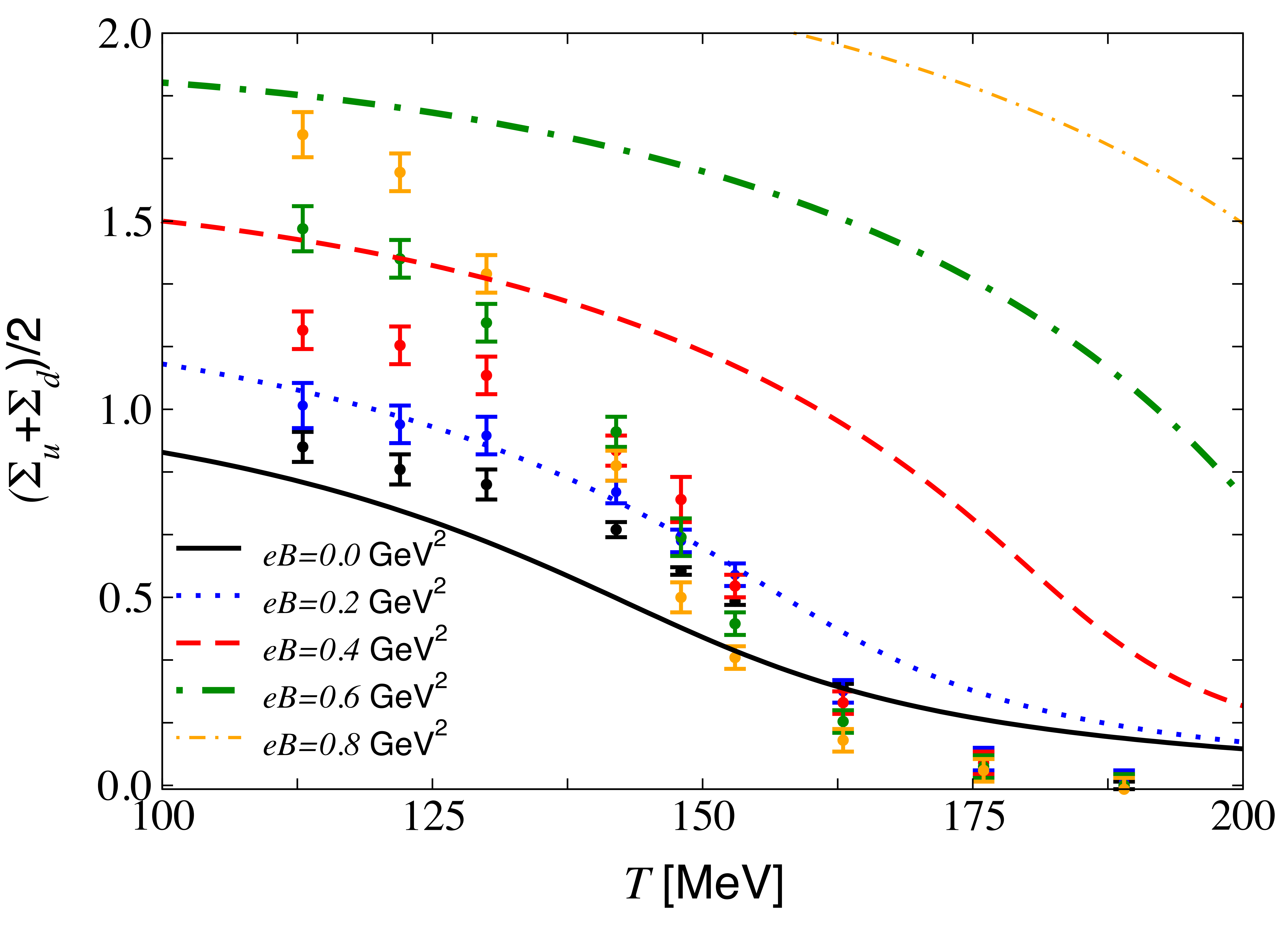
Phys. Rev. D 86 (2012) 071502

JHEP 08 (2014) 177



Constant coupling: SU(2) NJL model

G



Thermo-magnetic coupling: prototype

$$eB >> \Lambda_{\rm QCD}^2$$

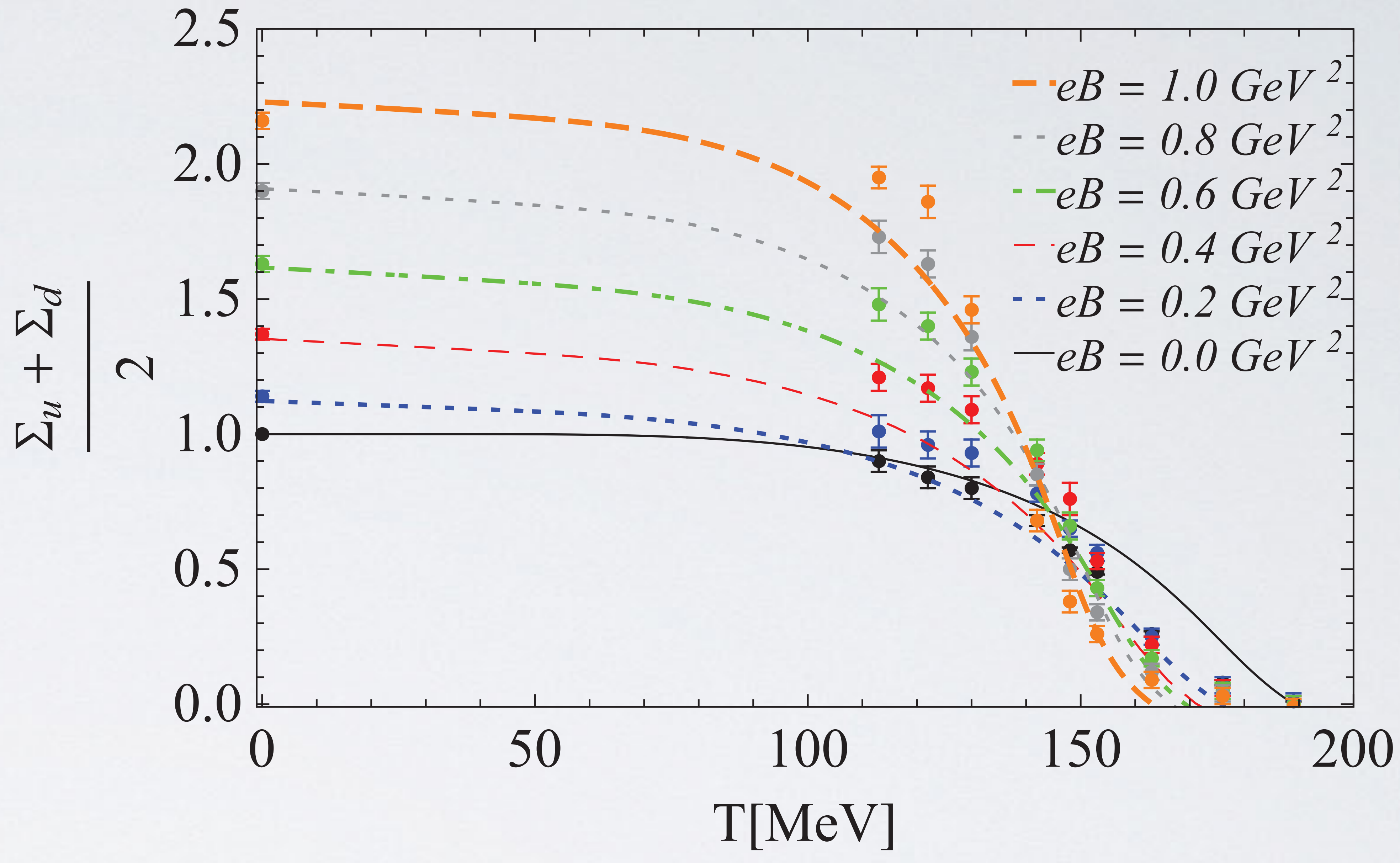
$$\alpha_s \sim \frac{1}{b\ln(eB/\Lambda_{\rm QCD}^2)}$$

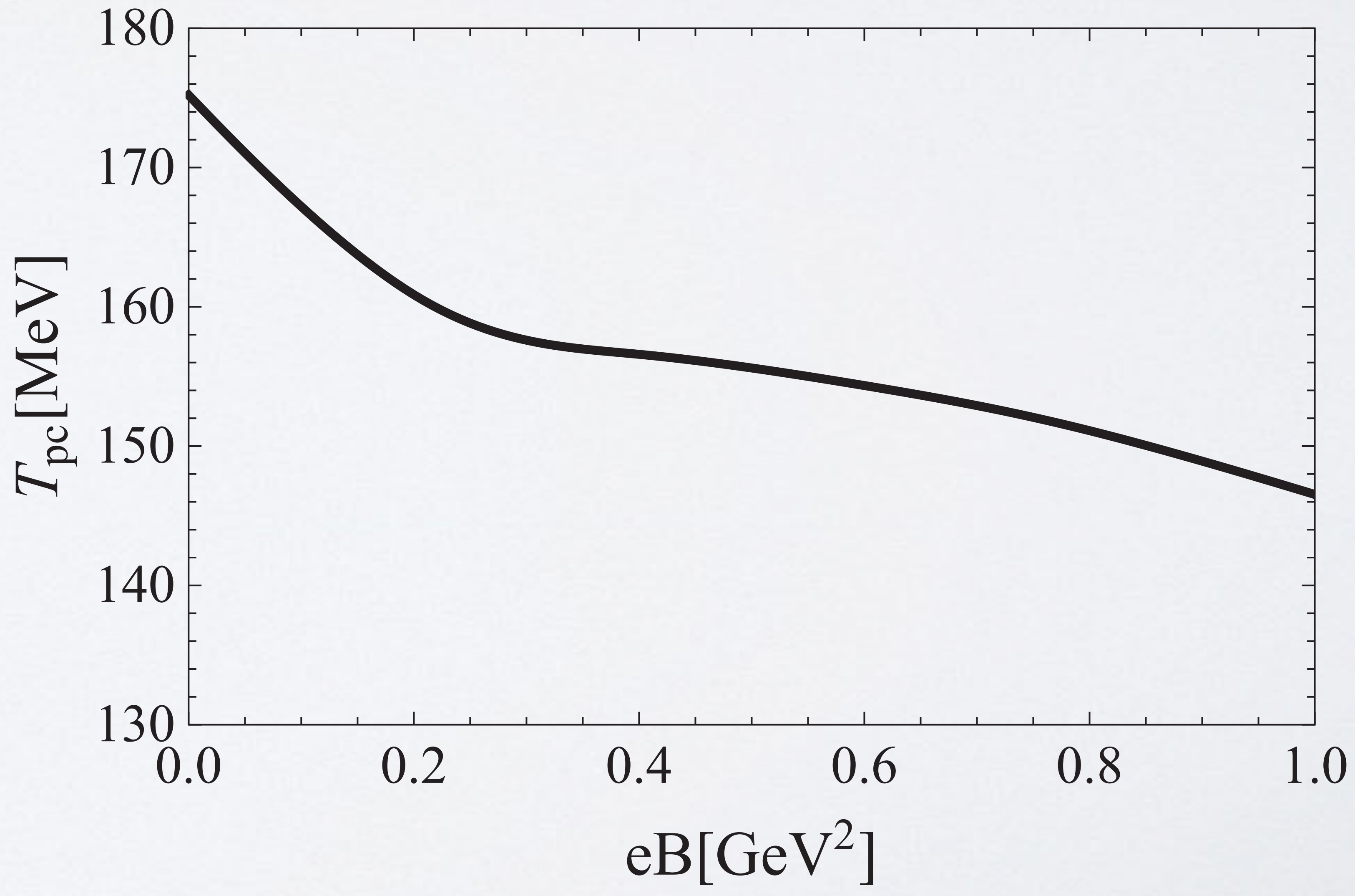
$T = 0$

$$G(B) = \frac{G_0}{1+\alpha\ln\left(1+\beta\frac{eB}{\Lambda_{\rm QCD}^2}\right)}$$

$T > 0$

$$G(B,T) = G(B)\left(1-\gamma\frac{|eB|}{\Lambda_{\rm QCD}^2}\frac{T}{\Lambda_{\rm QCD}}\right)$$





Farias, Gomes, Krein, Pinto
Phys. Rev. C **90**, 025203 (2014)

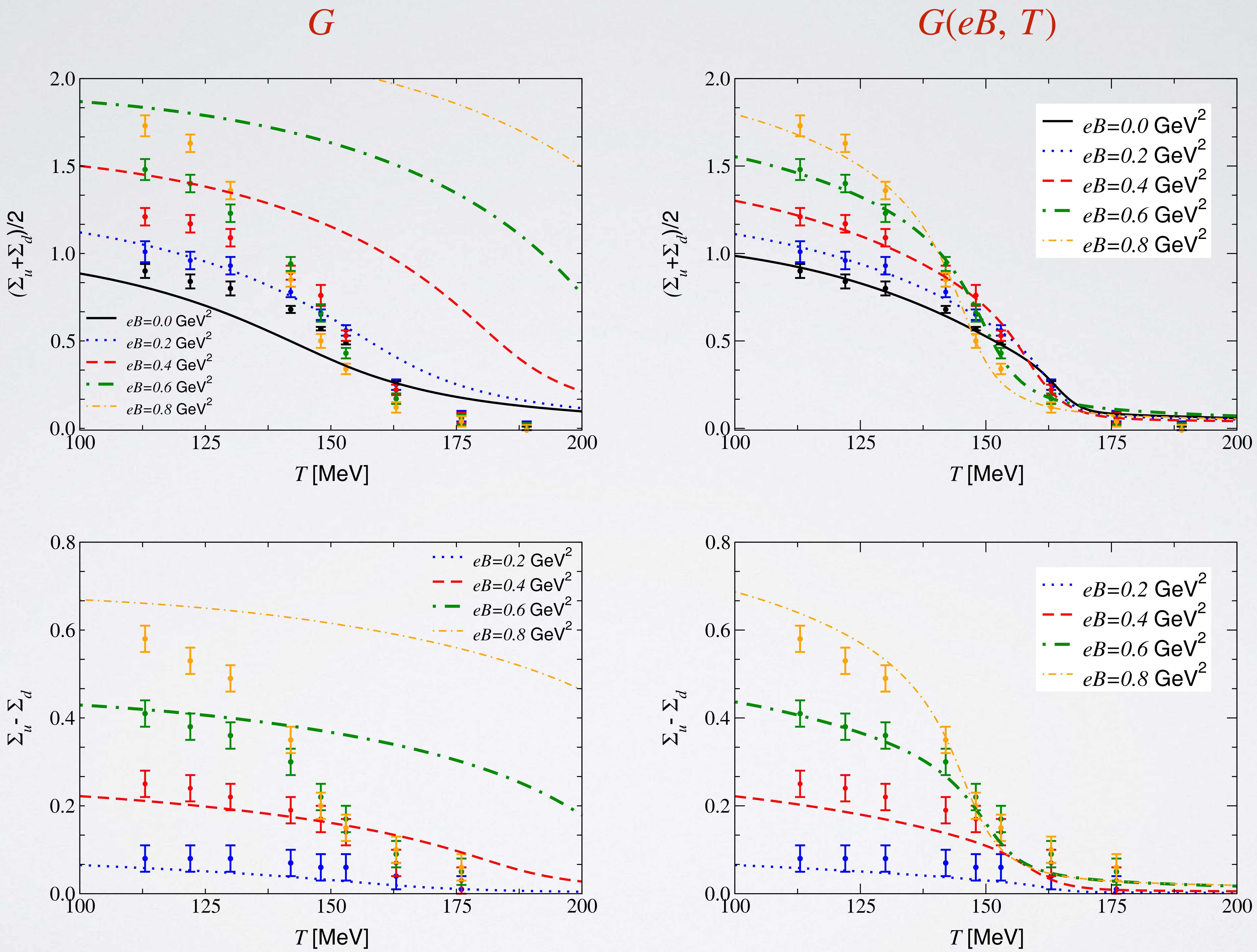
Matching the NJL model to lattice QCD

Build a thermo-magnetic coupling for the NJL model from lattice QCD results

For given values of T and eB:

- start with an initial attempt for $G(T, eB)$
- for this G, make an initial guess for M
- solve the gap equation
- with M, compute the condensate averages
- compare to lattice QCD result for that T and eB
- repeat until the best $G(T, eB)$ is found

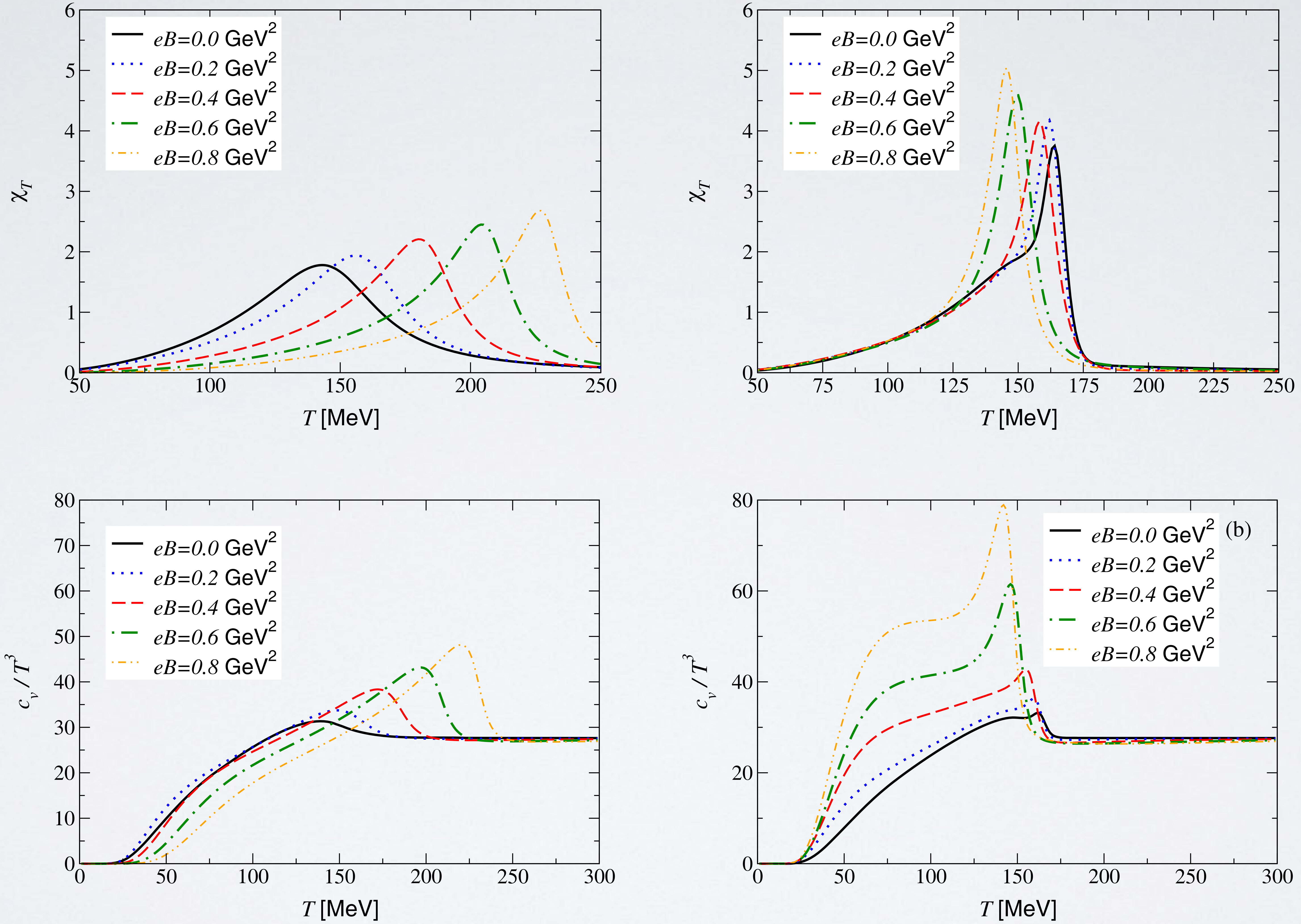
Thermo-magnetic dependent coupling



Eur. Phys. J. A 53 (2017) 101

Thermal Susceptibilities and Specific Heat

G | $G(eB, T)$



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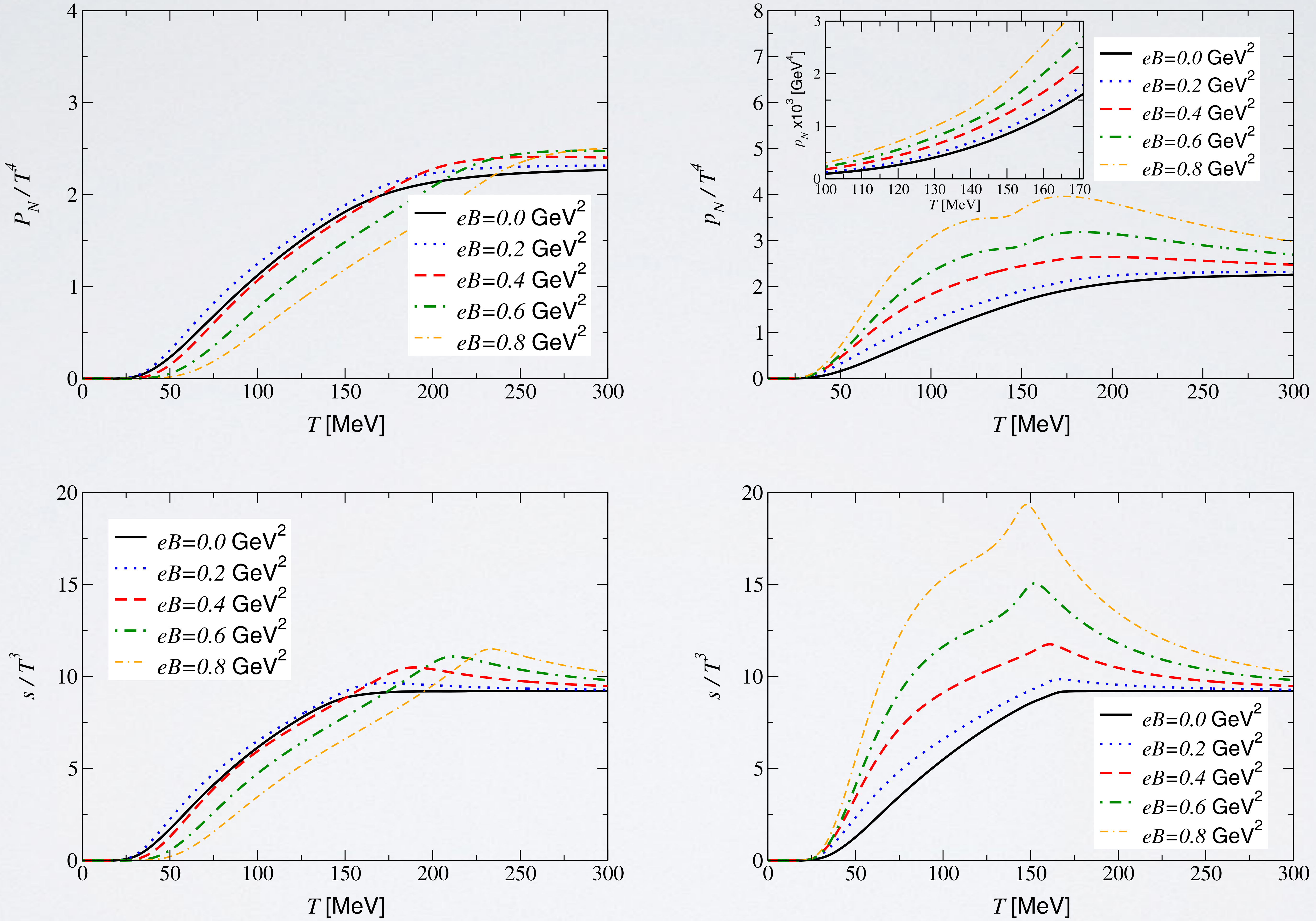
Pressure and Entropy

G $G(eB, T)$



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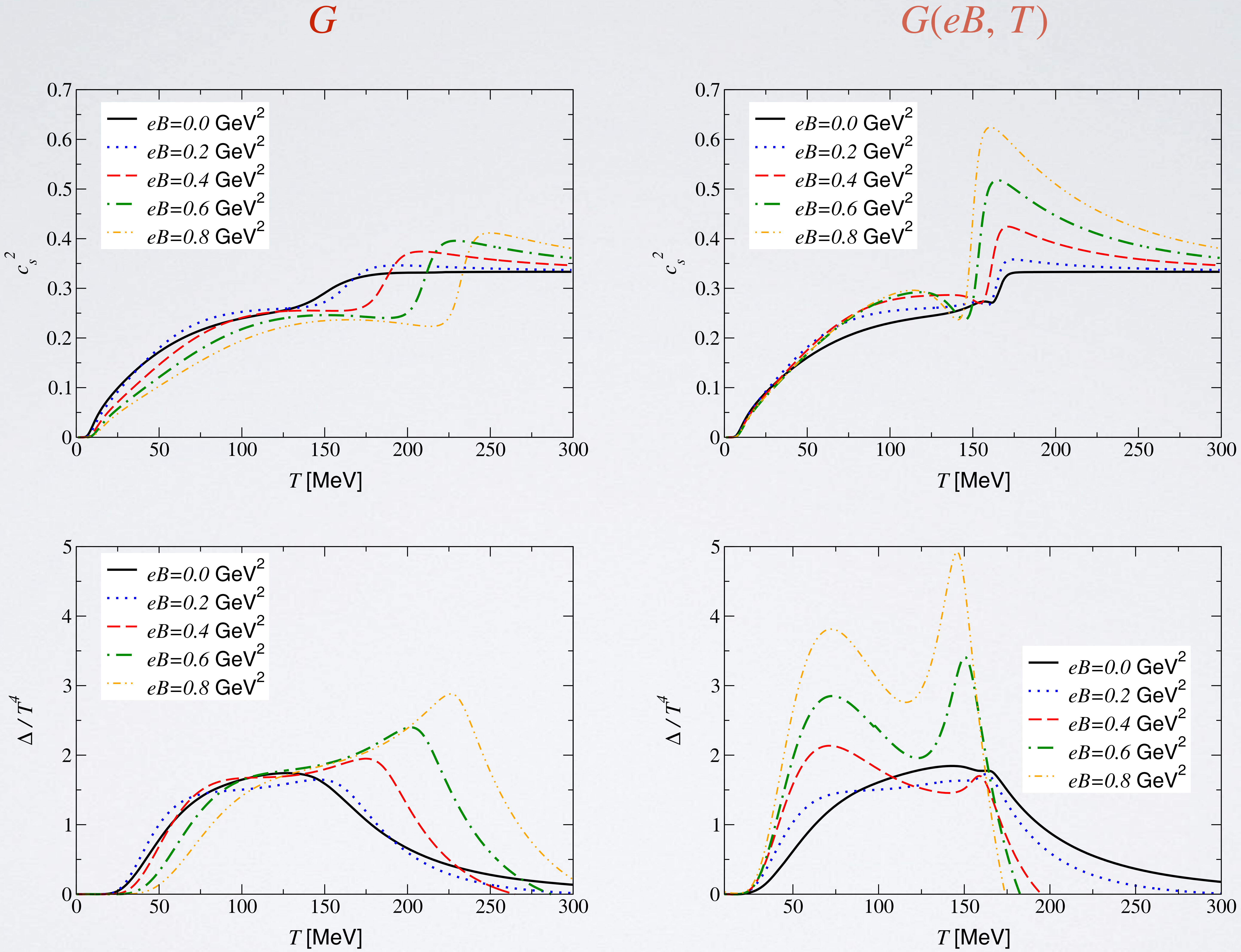
Sound Velocity and Interaction Measure



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Pseudo-critical temperature

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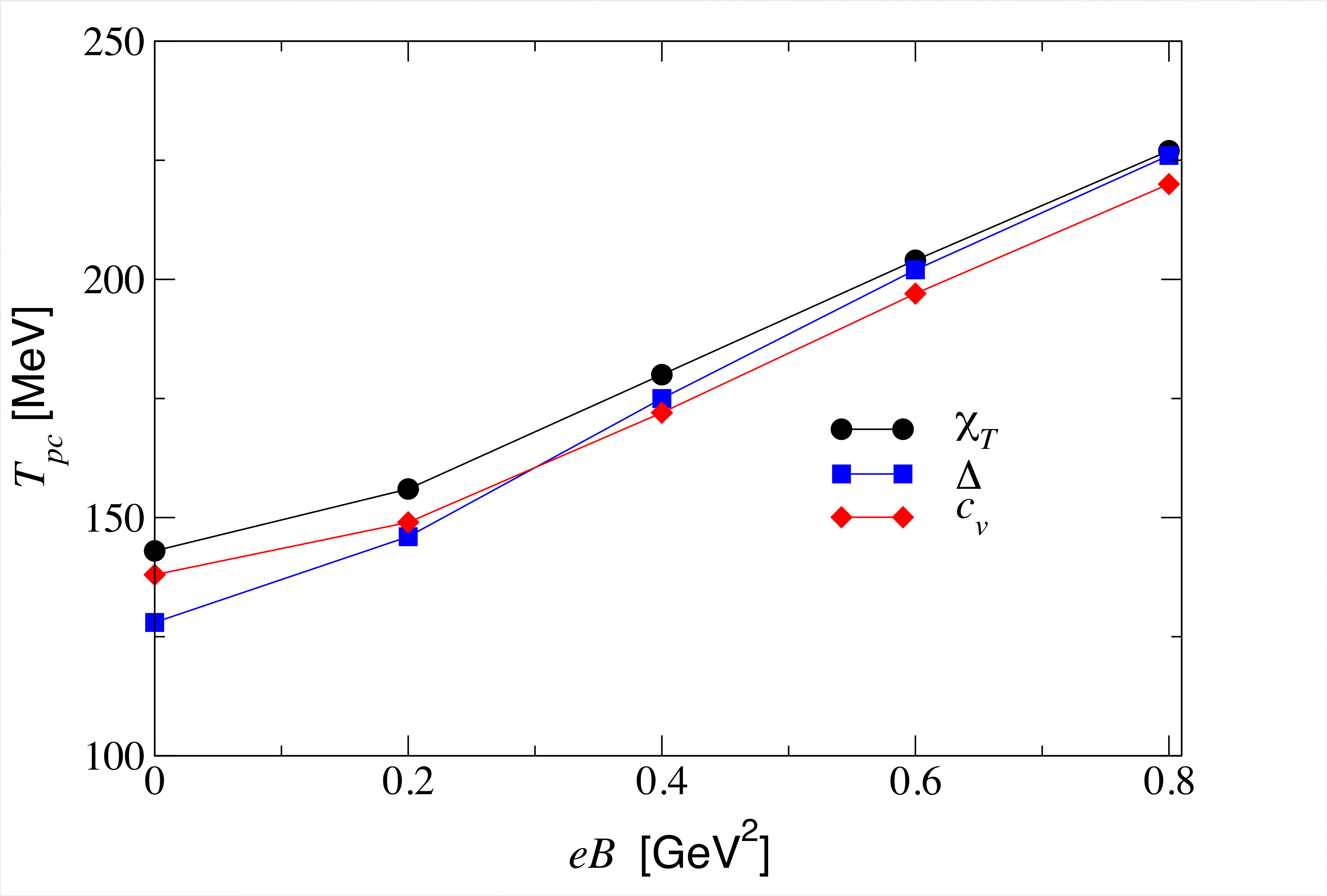
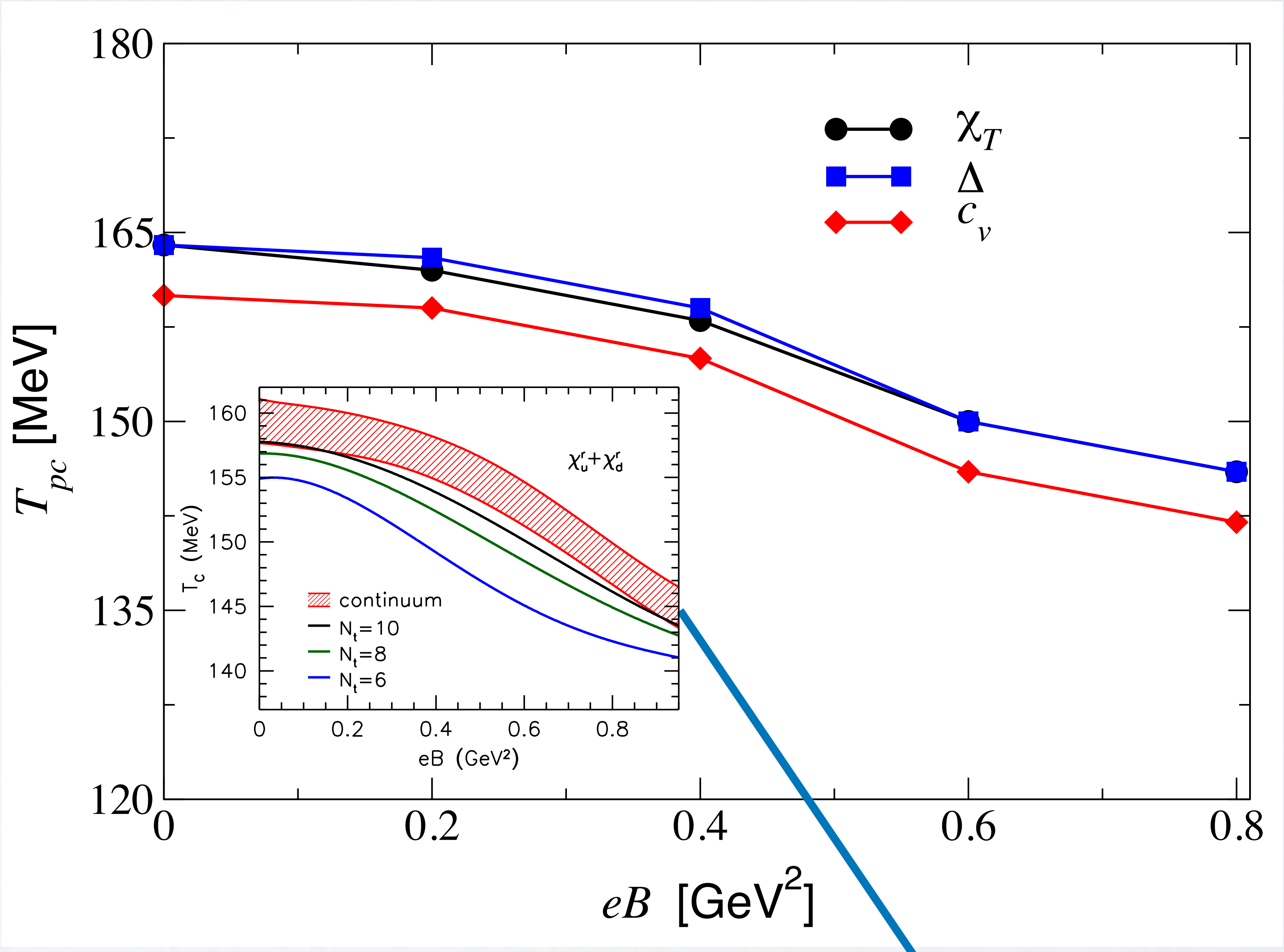


$G(eB, T)$



Bali et al.
JHEP 02 (2012) 044

Magnetization

$$\mathcal{M} = -\left.\frac{\partial \Omega}{\partial B}\right|_{\{\phi_f\},\rho} - \frac{\partial \Omega}{\partial \phi_f}\frac{\partial \phi_f}{\partial B} - \frac{\partial \Omega}{\partial \rho}\frac{\partial \rho}{\partial B}$$

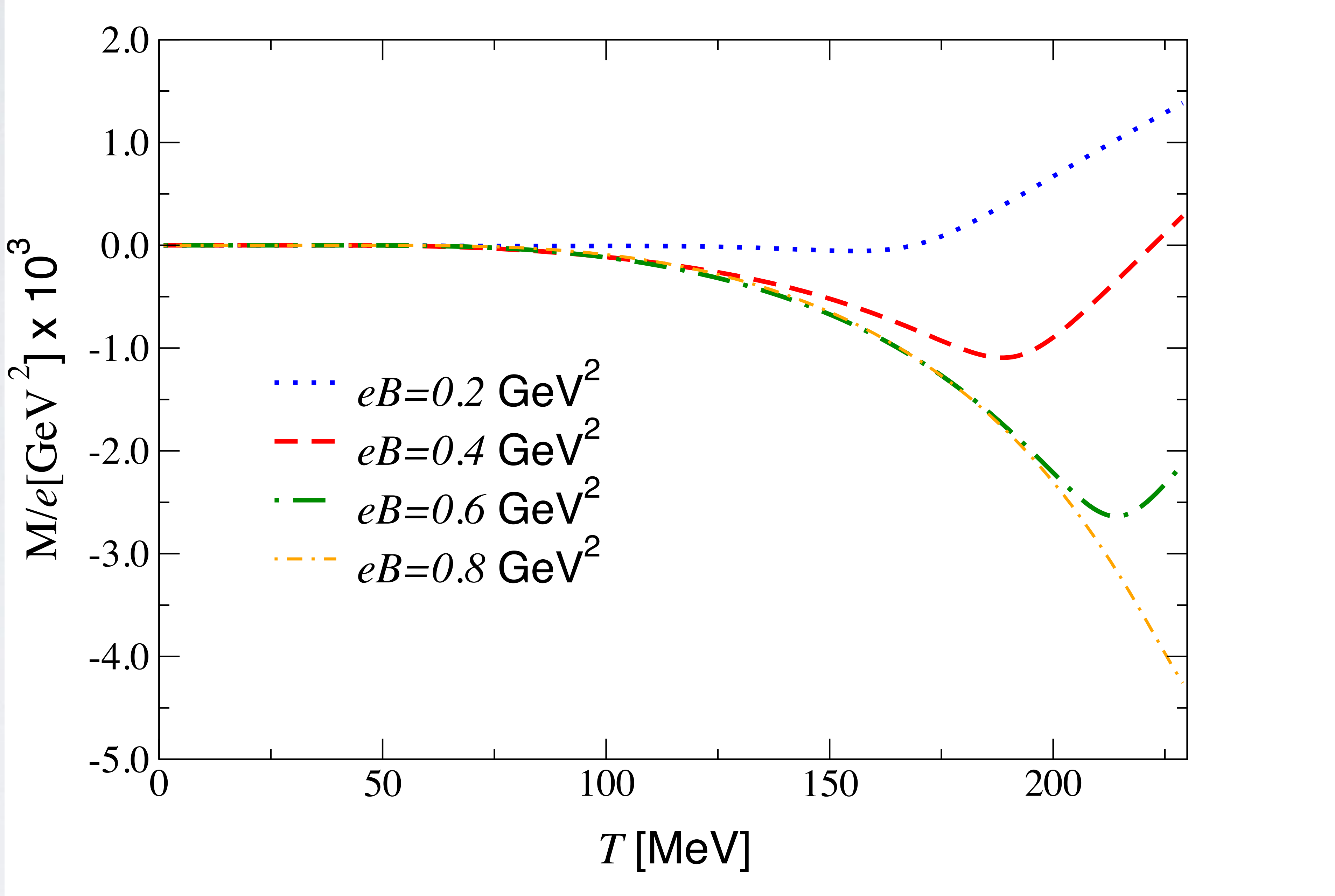
$$\frac{\partial \Omega}{\partial \phi_f} = 0 \qquad \frac{\partial \Omega}{\partial \rho} = 0$$

$$\mathcal{M} = \sum_f \left(\frac{\partial P_f^{\mathrm{mag}}}{\partial B} + \frac{\partial P_f^{\mathrm{Tmag}}}{\partial B} \right)$$

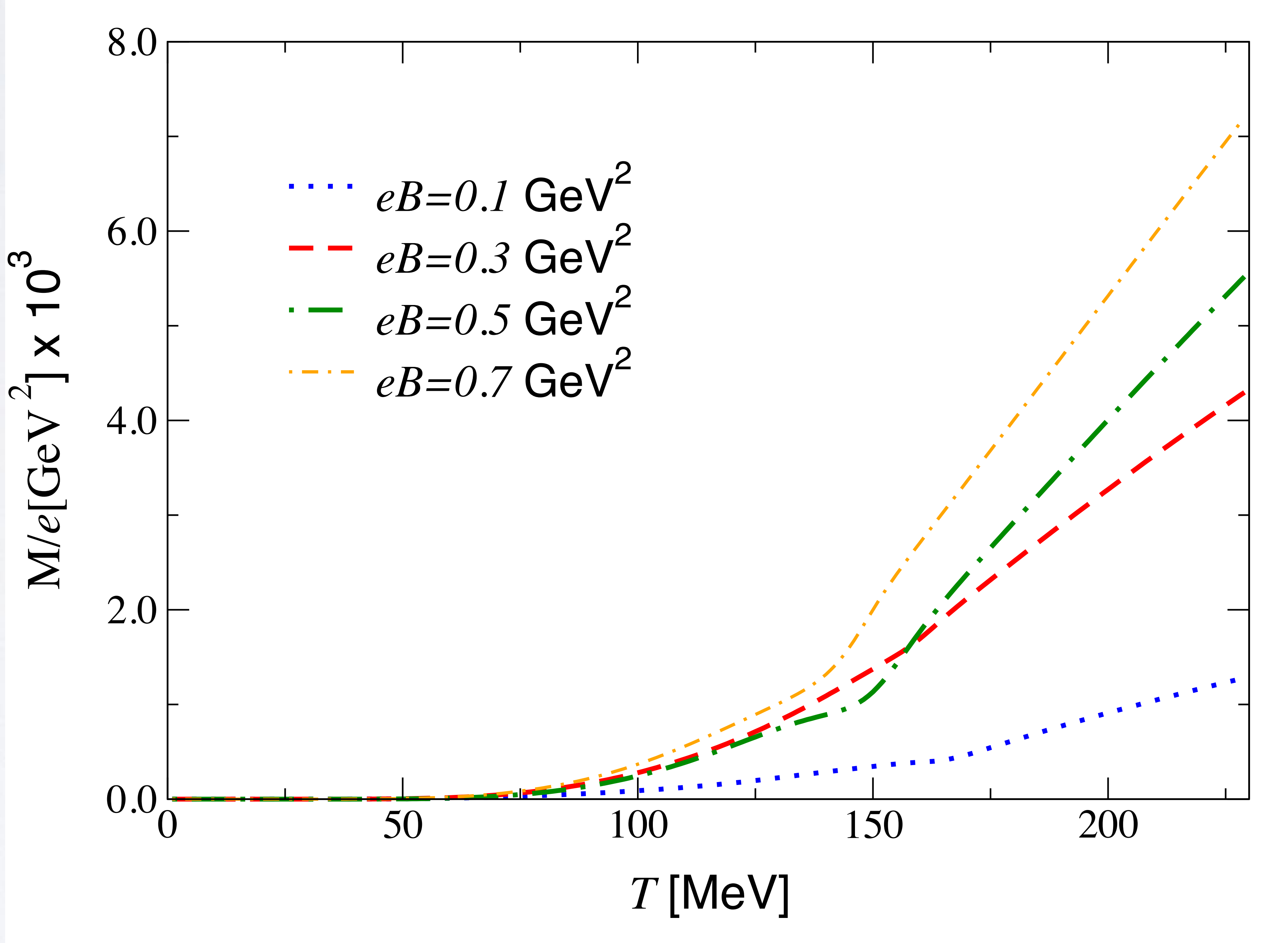
$$P = -\Omega$$

Magnetization

G



$G(eB, T)$



Meson properties under strong magnetic fields

$T = 0$

$$(ig_{\pi_0 qq})^2 \, iD_{\pi_0}(k^2) = \frac{2iG}{1 - 2G\Pi_{PS}(k^2)} \qquad D_{\pi_0}(k^2) = \frac{1}{k^2 - m_{\pi_0}^2}$$

$$\mathcal{L}_{\pi qq} = ig_{\pi qq}\bar{\psi}\gamma_5 \vec{\tau}\cdot\vec{\pi}\,\psi \qquad S_q(x,x') = e^{i\Phi_q(x,x')}\sum_{n=0}^{\infty} S_{q,n}(x-x')\,,\ q = u,d$$

$\beta_q = |q_q|B$

$k_\parallel = k_0 - k_3$

$g_n = 2 - \delta_{n0}$

$$\frac{1}{i}\Pi_{PS}(k^2) = -\sum_{q=u,d}\int \frac{d^4p}{(2\pi)^4} Tr\left[i\gamma_5 iS_q\left(p+\frac{k}{2}\right) i\gamma_5\, iS_q\left(p-\frac{k}{2}\right)\right]$$

$$\frac{1}{i}\Pi_{PS}(k_\parallel^2) = -i\left(\frac{M-m}{2MG}\right) - \sum_{q=u,d}\beta_q N_c \frac{k_\parallel^2}{(2\pi)^3}\sum_{n=0}^{\infty} g_n I_{q,n}(k_\parallel^2) \qquad I_{q,n}(k_\parallel^2) = \int d^2p_\parallel \frac{1}{[p_\parallel^2 - M^2 - 2\beta_q n][(p+k)_\parallel^2 - M^2 - 2\beta_q n]}$$

$$1 - 2G\,\Pi_{PS}(k^2)|_{k^2 = m_{\pi_0}^2} = 0 \qquad I({k_\parallel}^2, B) = I_{vac}(k_\parallel^2) + I(k_\parallel^2, B)$$

$$m_{\pi_0}^2(B) = -\frac{m}{M(B)}\frac{1}{4iGN_cN_f I(m_{\pi_0}^2, B)} \qquad I(m_{\pi_0}^2, B) = \frac{1}{4(2\pi)^3}\sum_{q=u,d}\beta_q\sum_{n=0}^{\infty} g_n I_{q,n}(k_\parallel^2 = m_{\pi_0}^2)$$

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Simple $G(eB)$ at $T = 0$

- fit to lattice QCD condensates (few values of eB)
- interpolation to generate a larger set
- fit of the larger set to a shifted gaussian

$$G(eB) = \alpha + \beta\, e^{-\gamma\, (eB)^2}$$

$$\alpha = 1.44373 \text{ GeV}^{-2},\ \beta = 3.06 \text{ GeV}^{-2} \text{ and } \gamma = 1.31 \text{ GeV}^{-4}$$

$$G(0) = \alpha + \beta$$

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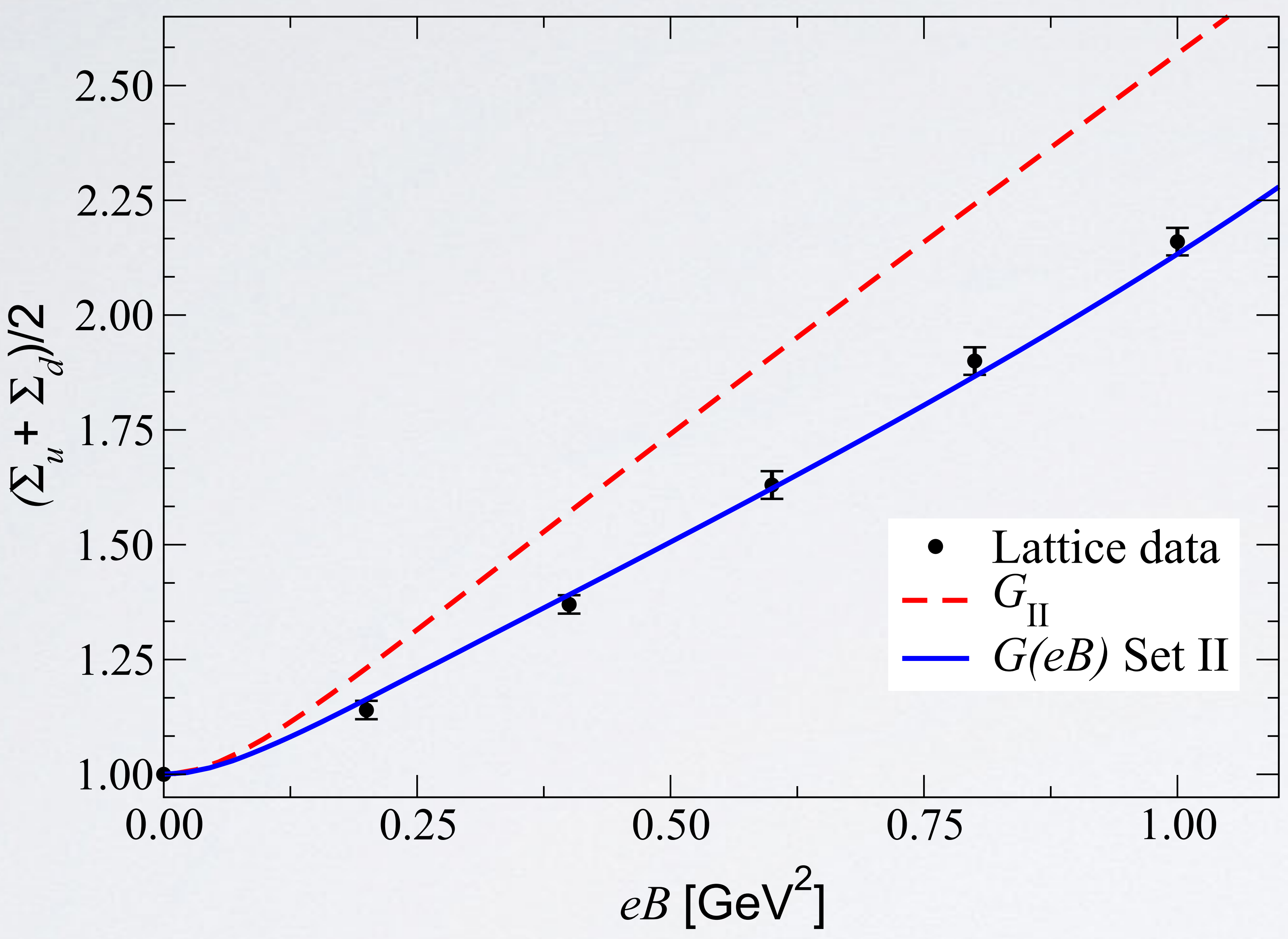
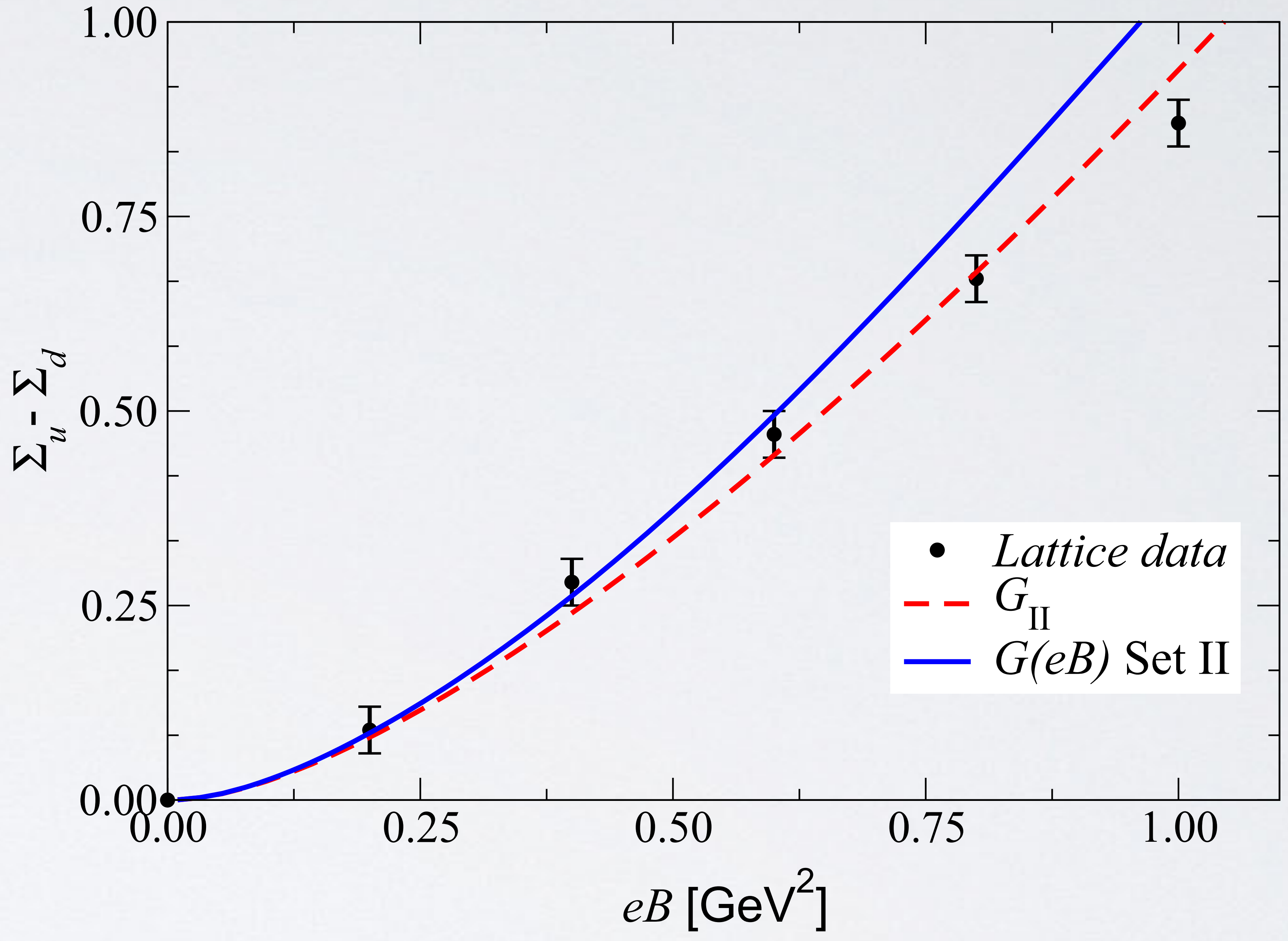
Condensates at $T = 0$





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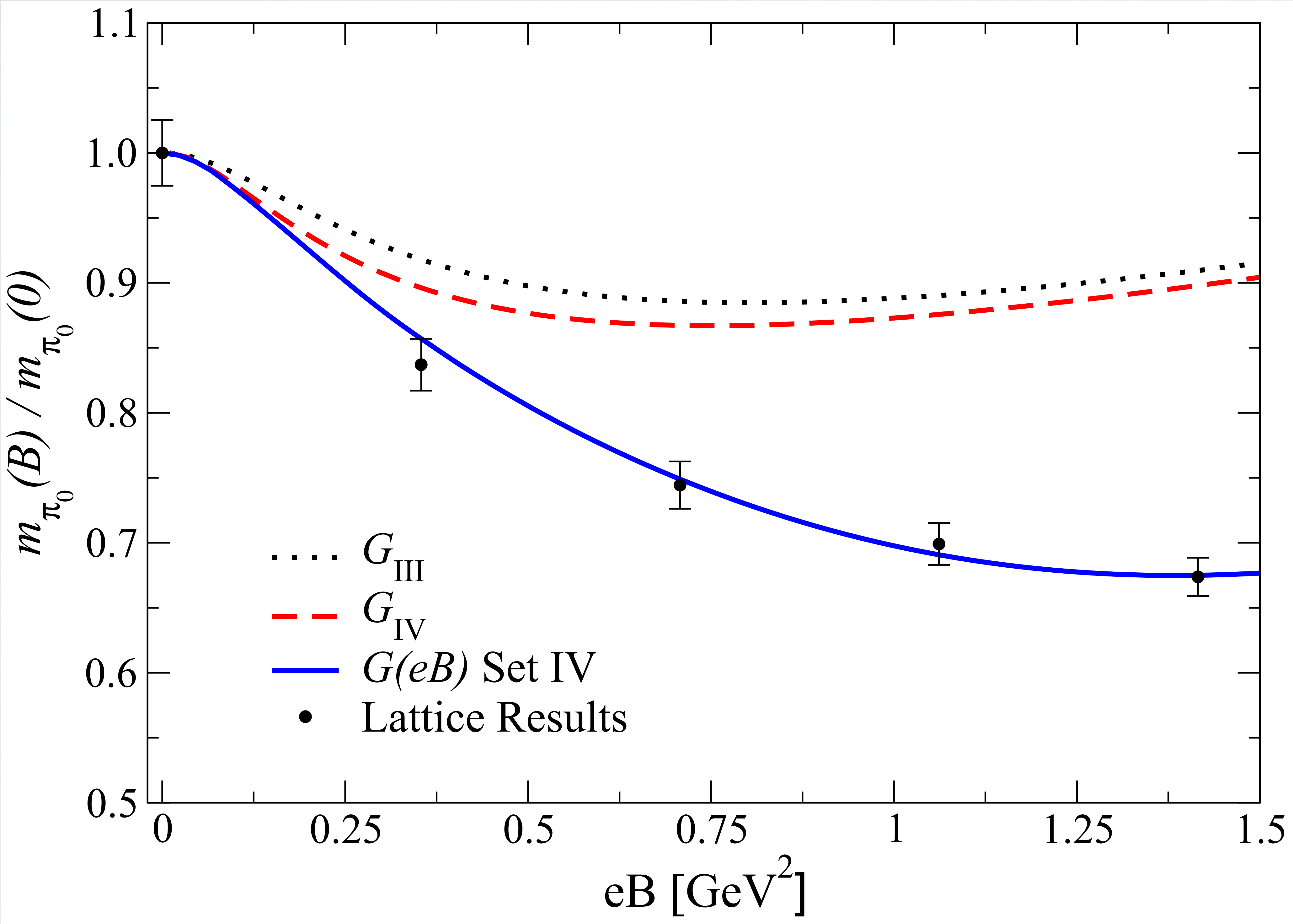
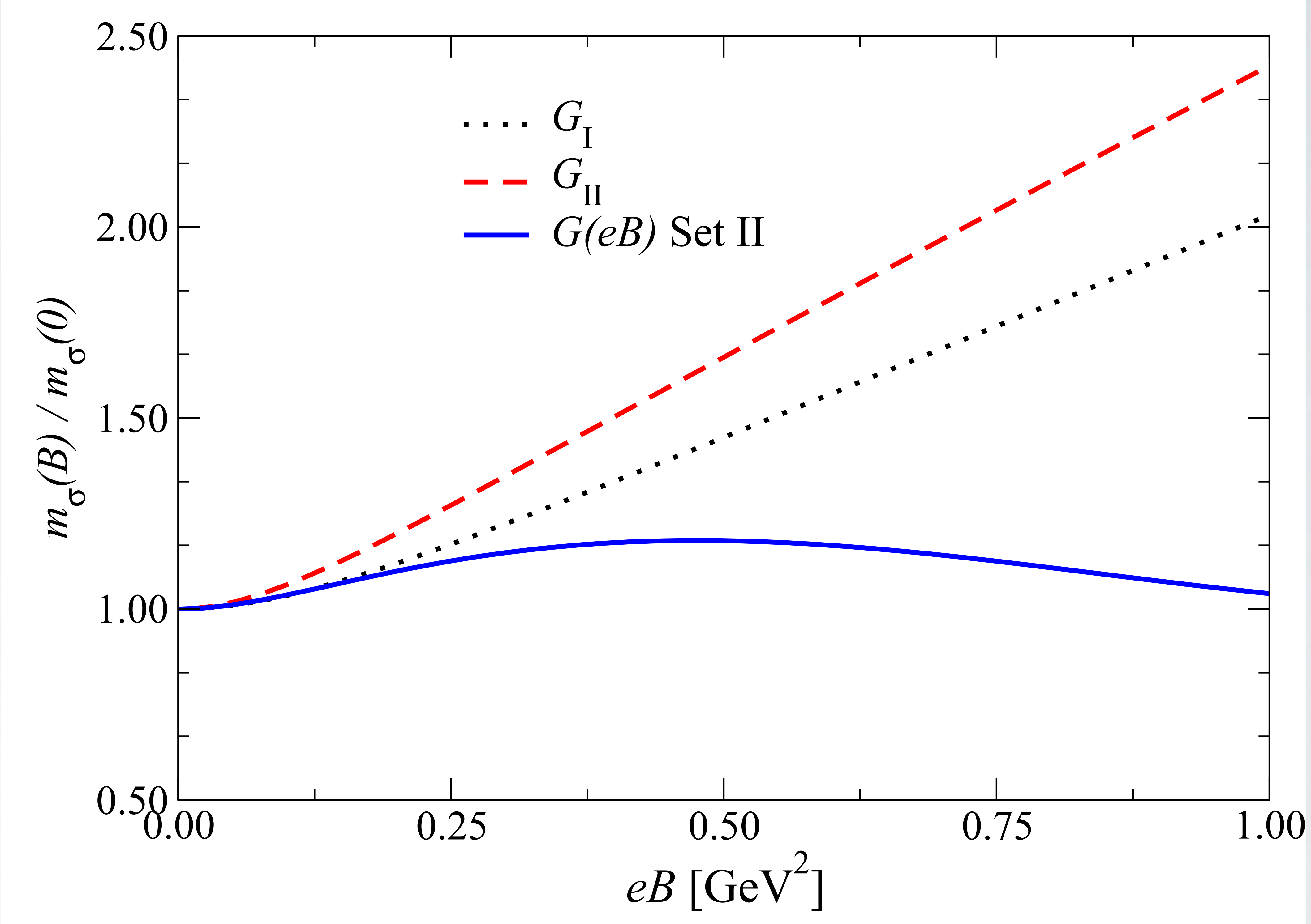
π_0 and σ masses at $T = 0$





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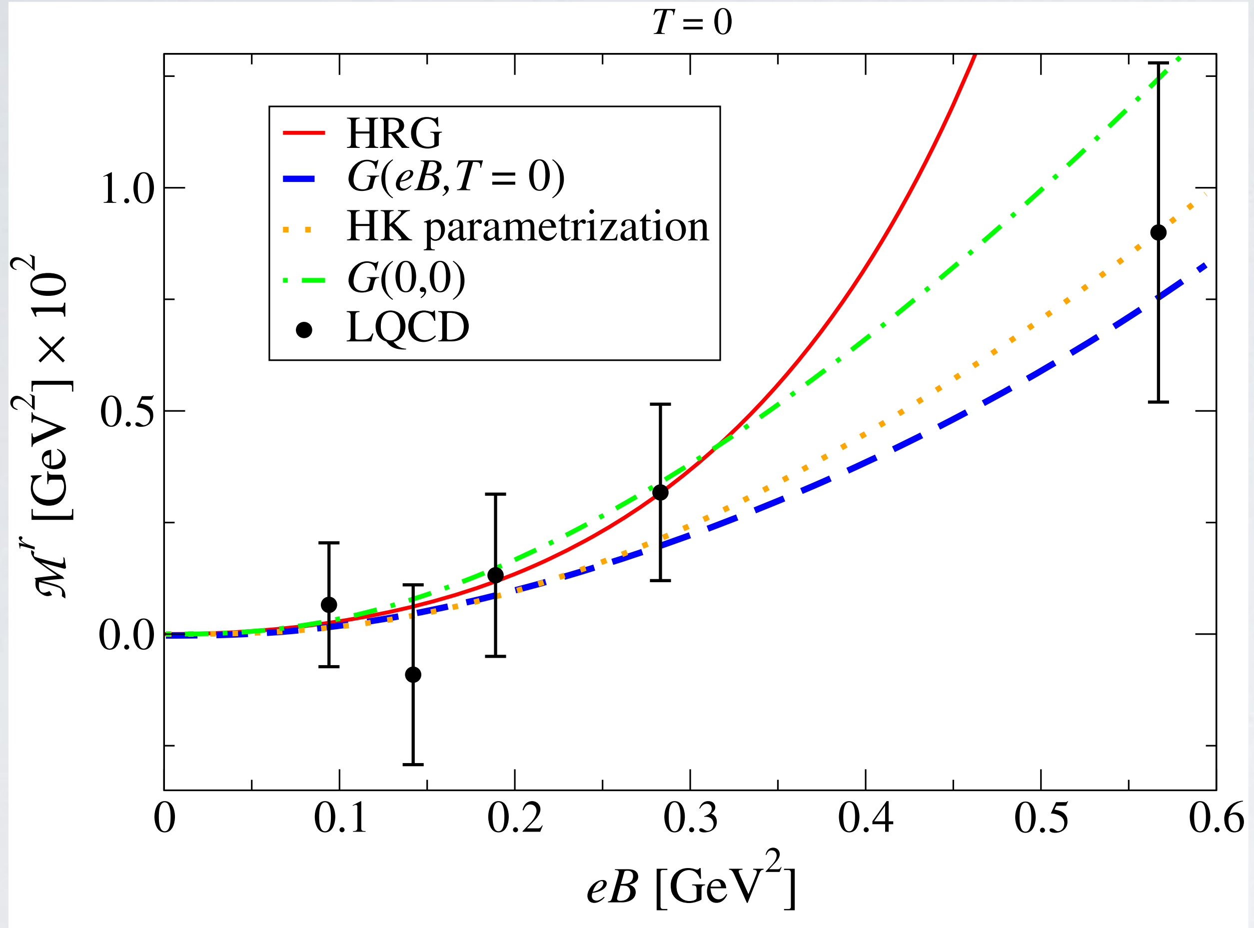
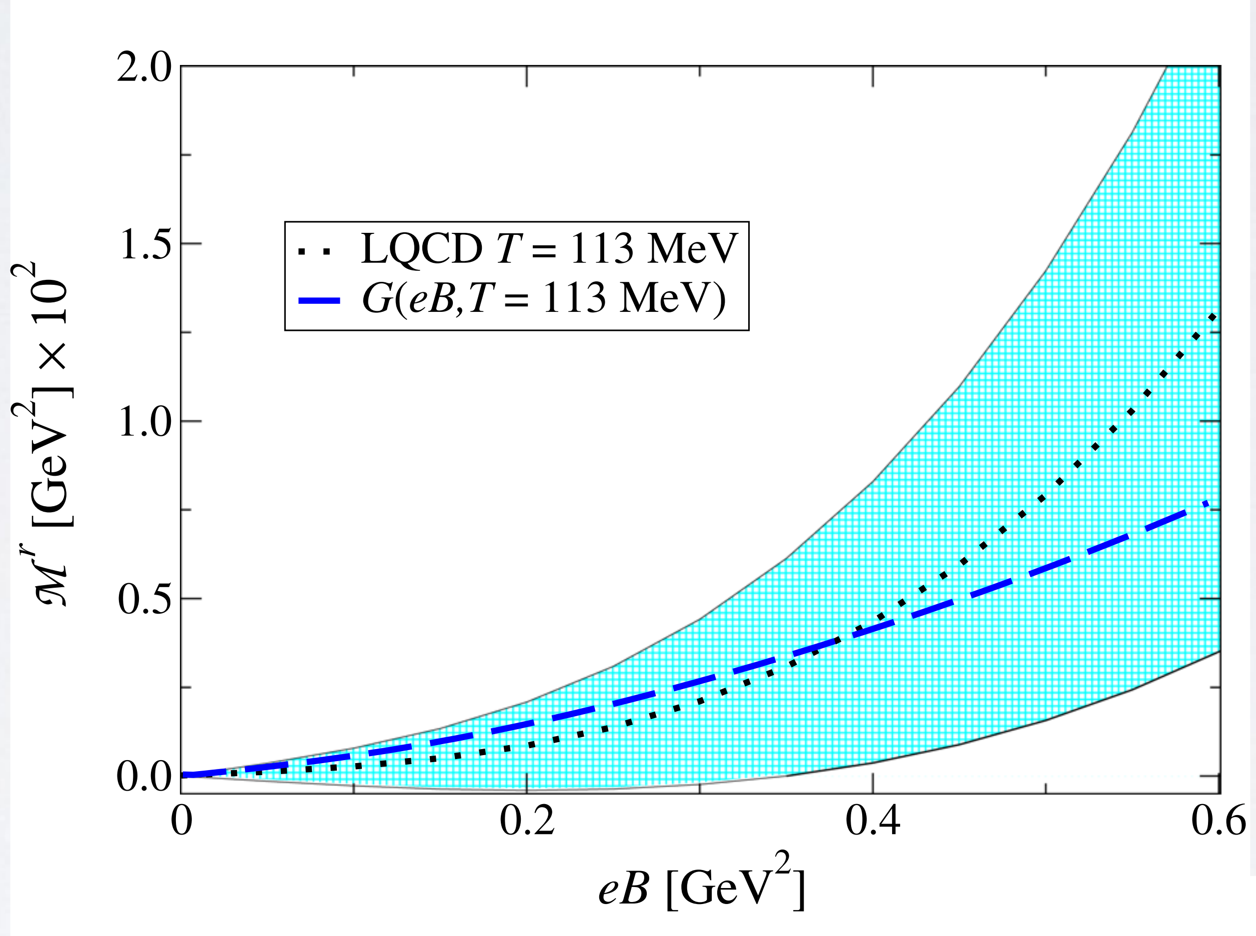
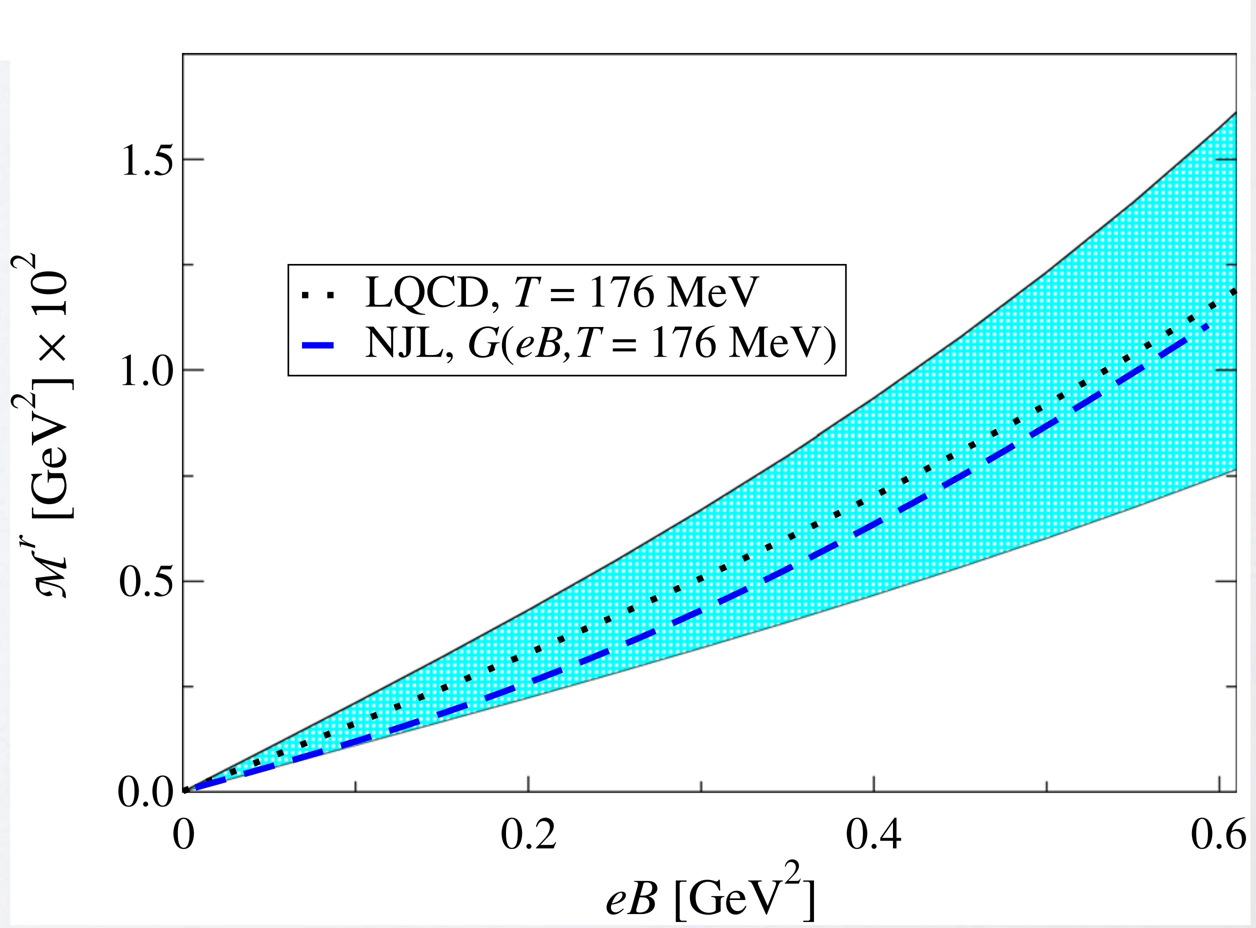
Renormalized Magnetization



$$\mathcal{M}^r \cdot eB = \mathcal{M} \cdot eB - (eB)^2 \lim_{eB\to 0} \frac{\mathcal{M}\cdot eB}{(eB)^2}\bigg|_{T=0}$$



SU(3)



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Final Remarks

Few-Body

* The results with unphysical masses are close to the lattice QCD calculations
* The results with physical masses are close to the experimental values
* Our calculations with effective potentials support the lattice QCD results

Many-Body

* NJL models with fixed coupling fails to describe lattice QCD calculations
* Thermo-magnetic coupling seems to be adequate to improve NJL results
* Thermodynamic quantities are all affected by the variation of the coupling
* Sign of magnetization changes when $G \to G(eB, T)$
* Pion mass at $T = 0$ matches lattice QCD calculations with $G(eB)$