Towards Nonlinear (In)stability of Kerr Black Holes with Proca Hair

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Introduction

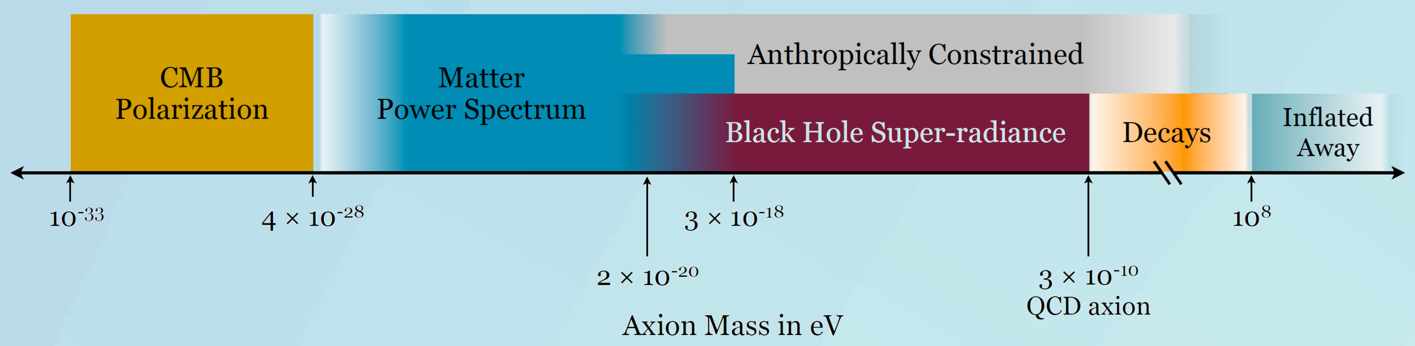
- **Our goal** is to understand the nonlinear stability and dynamics of Kerr black holes with massive vector hair.
- **Our tools are:**
  - Fully nonlinear simulations of the Einstein-Proca system
  - Initial data generated via Herdeiro, Radu, and Runarsson's construction [1].
  - Initial data generated by simulating the superradiant instability to find the end state.

Light, Massive Boson Fields

- **Light massive Boson fields** can emerge from high-energy theory via compactification or as axion-like fields.
- These fields are a candidate for **dark matter.**
- The coupling of massive Bosons to **black holes** is a potential source of **gravitational waves.**
- We study massive *vector* fields, called *Proca* fields.
- **The Einstein-Proca Action**:

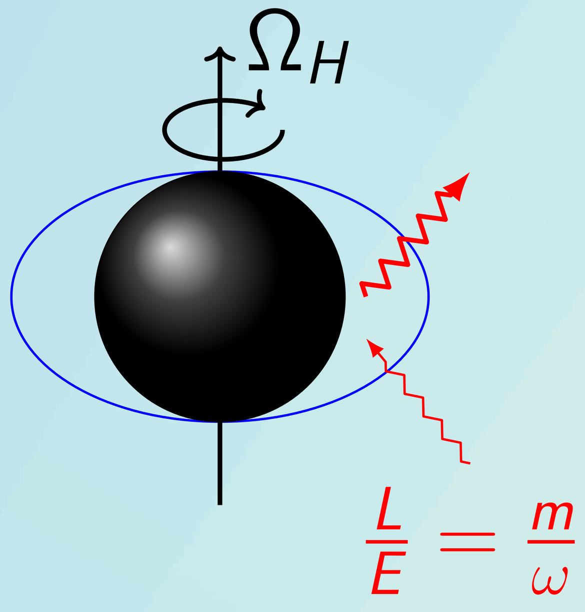
$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi}R - \frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha\right)$$

- Mass can be **constrained** via various types of observations [2]:



Superradiance

- *Superradiance* is the wave-analogue of the *Penrose* process.
- Works only for *boson* fields.
- Can be used to mine energy from a spinning black hole.
- Emerges simply from *black hole thermodynamics:*



$$\begin{cases}\delta M = \frac{k}{8\pi}\delta A_H + \Omega_H\delta J\\ \delta A_h \geq 0\end{cases}$$

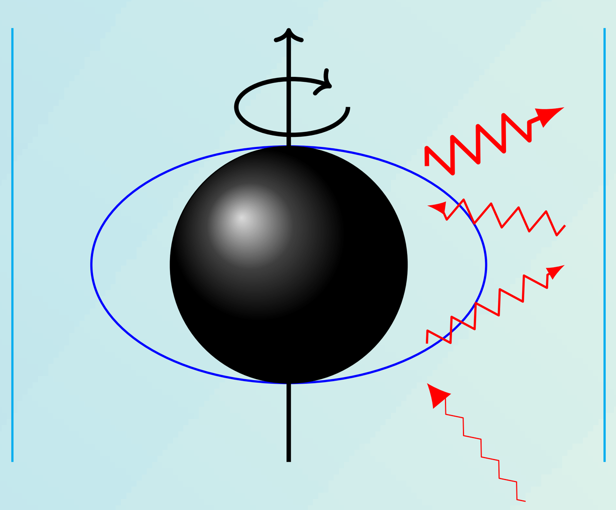
$$\Rightarrow \delta M = \frac{\omega k}{8\pi}\frac{\delta A_H}{\omega - m\Omega_H}$$

$$\frac{L}{E} = \frac{m}{\omega}$$

$$\boxed{\delta M < 0 \Leftrightarrow \omega - m\Omega_H < 0}$$

The Superradiant Instability

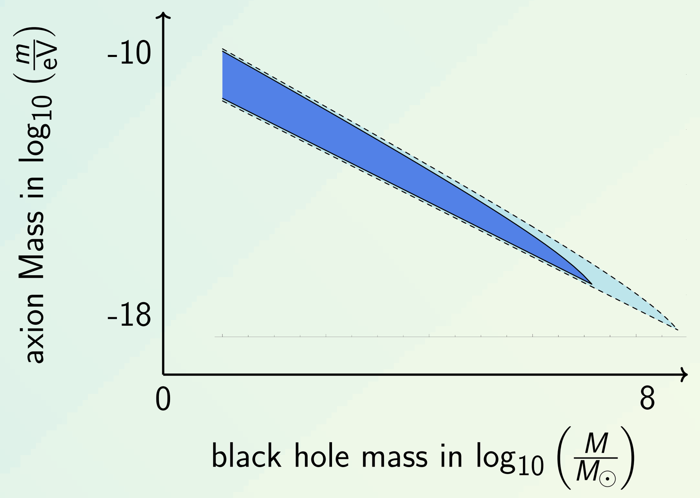
- Superradiance with confinement implies instability—**the black hole bomb**.
- **Mass** provides clear confinement mechanism



Black hole surrounded by mirrors, providing confinement.

- Growth rate (for test-fields) given by hydrogen-like spectrum [3]:

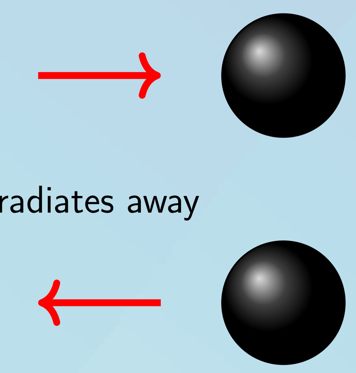
$$\tau \sim (M\mu)^{-(4m+3)}$$



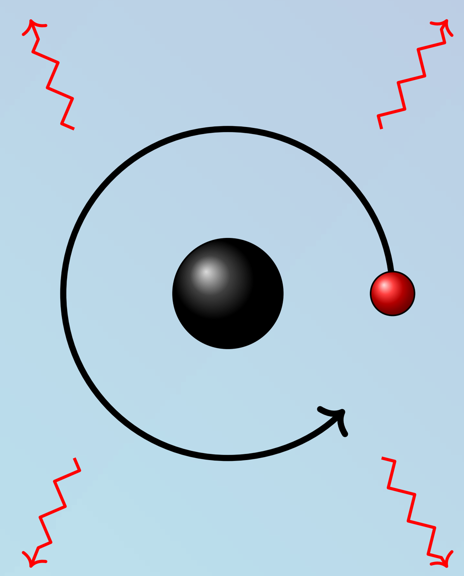
Shaded region indicates BH/Boson mass where timescale observable [2].

The Classic No Hair Theorems

- Original no-hair theorems due to **Bekenstein**:
  All asymptotically flat, stationary black hole solutions that (1) possess no naked singularities and (2) share symmetries between matter and metric fields must be described by mass, spin, and charge.
- **Basic idea**—if you wait long enough, one of several things happens:
  - Matter falls in
  - Matter radiates away

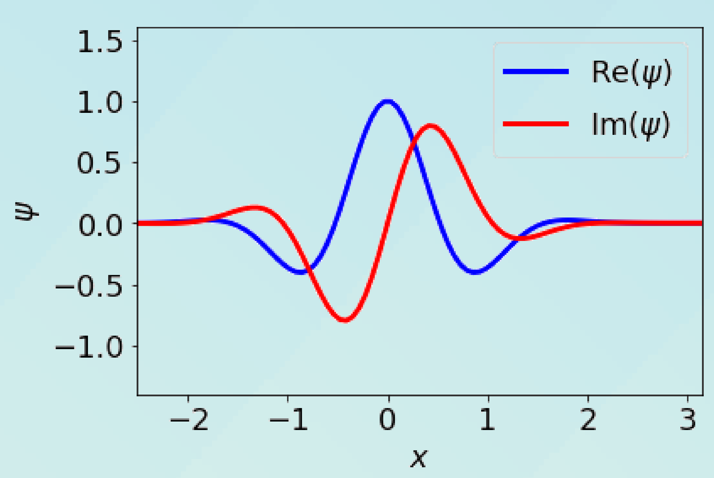


  - Stable orbits decay due to gravitational radiation

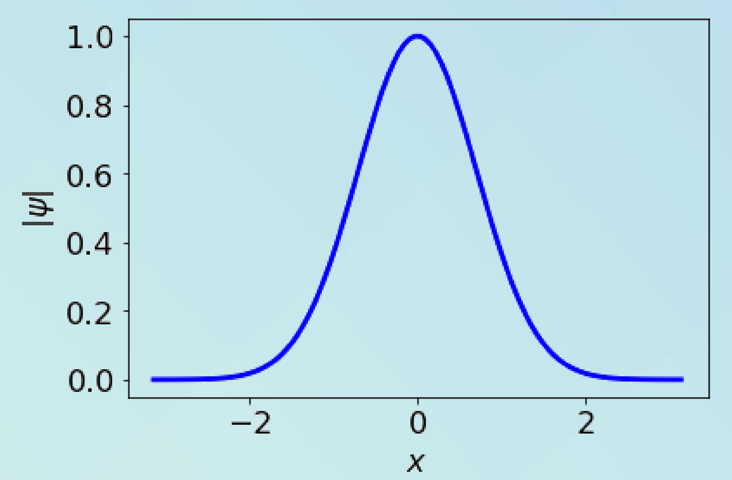


Evading the No Hair Theorems

- **Complex fields** allow for matter and metric to have *different* symmetries.
- Can suppress gravitational waves via *interference*. Exact same as *Boson star* construction.
- **Bound states** exist at *threshold of superradiance*



A time-varying complex field...



...Can have a time-invariant norm, and thus $T_{\mu\nu}$

Stationary, Hairy, Kerr Black Holes

- The end point of superradiance?

Ingredients

- Threshold of superradiance

$$\omega = m\Omega_H$$

- Complex, harmonic azimuthal and time dependence

$$\psi(t,r,\theta,\phi) = e^{i(m\phi-\omega t)}\psi(r,\theta)$$

or

$$\mathcal{L}_\phi\mathcal{A} = im\mathcal{A},\ \mathcal{L}_t\mathcal{A} = -i\omega\mathcal{A}$$

Potential Instabilities

- Multiple winding numbers

$$\mathcal{L}_\phi = im_1\mathcal{A}_1 + im_2\mathcal{A}_2$$

- Frequency perturbation

$$\omega\to\omega+\epsilon$$

- De-phasing

$$\langle \mathrm{Re}(\mathcal{A}), \mathrm{Im}(\mathcal{A})\rangle \neq 0$$

- Stable light rings

The Herdeiro-Radu Ansatz

- **Metric ansatz:**

$$ds^2 = -e^{2F_0}Ndt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2d\theta^2\right) + e^{2F_2}r^2\sin^2\theta\,(d\phi - Wdt)^2$$

- **Field ansatz:**

$$\mathcal{A} = e^{i(m\phi-\omega t)}\left(iVdt + H_1dr + H_2d\theta + iH_3\sin(\theta)d\phi\right)$$

- **For**

$$N = 1 - \frac{r_h}{r}$$

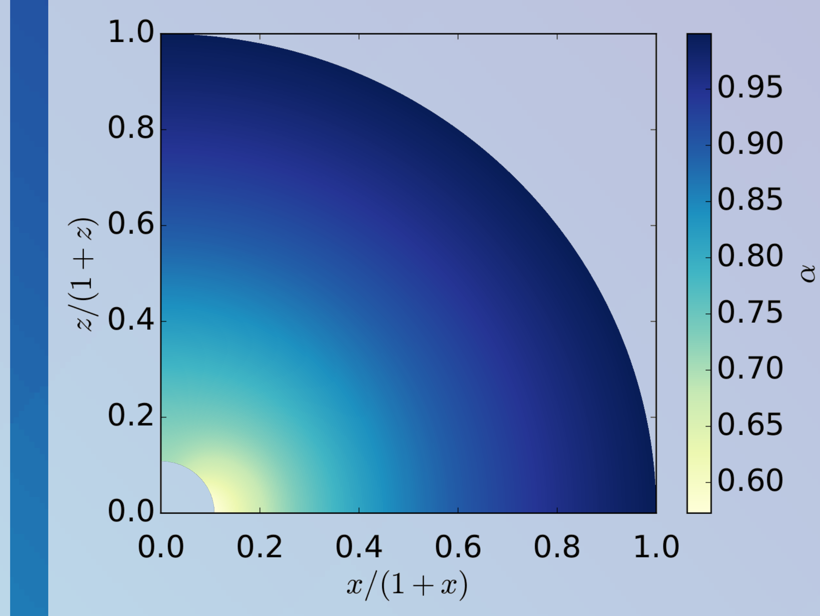
with F_i, W, V, H_j, $i = 0,1,2$ and $j = 1,2,3$ subject to regularity conditions at $r = r_h$ and asymptotically flat conditions at $r = \infty$ [1].

- Solve **constraint equations** for spacetime and **Helmholtz**-like equation for Proca field to find stationary solution.
- Ansatz is *not* **horizon-penetrating**.
- **Coordinate singularities** at the horizon mean it cannot directly applied as initial data to a time-evolution.

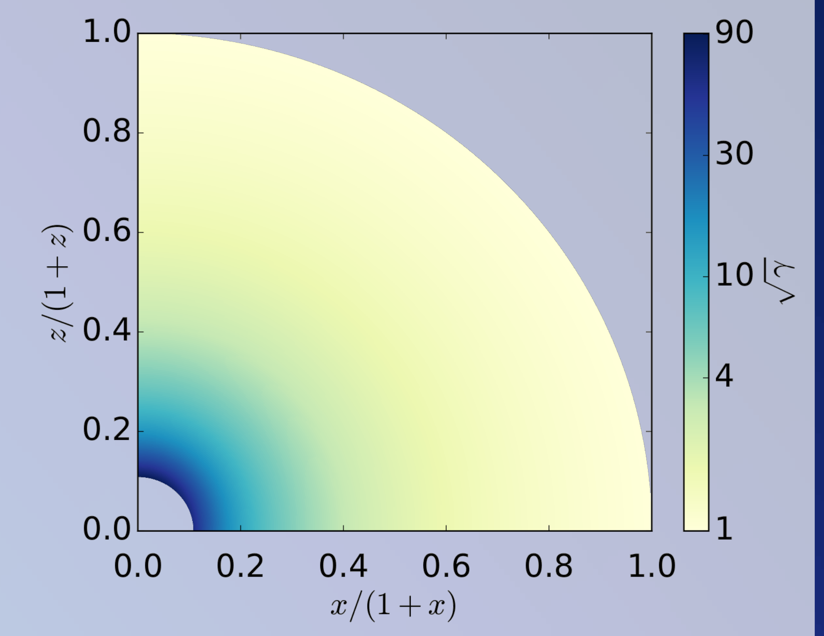
A New Horizon-Penetrating Coordinate System

- Define new mapping which *regularizes* the HR ansatz and respects the unknown ansatz functions:

$$(g_{new})_{\mu'''\nu'''} = (g_{old})_{\mu\nu}\left(\frac{\partial x^\mu}{\partial x^{\mu'}}\right)\left(\frac{\partial x^\nu}{\partial x^{\nu'}}\right)\left(\frac{\partial x^{\mu'}}{\partial x^{\mu''}}\right)\left(\frac{\partial x^{\nu'}}{\partial x^{\nu''}}\right)\left(\frac{\partial x^{\mu''}}{\partial x^{\mu'''}}\right)\left(\frac{\partial x^{\nu''}}{\partial x^{\nu'''}}\right)$$

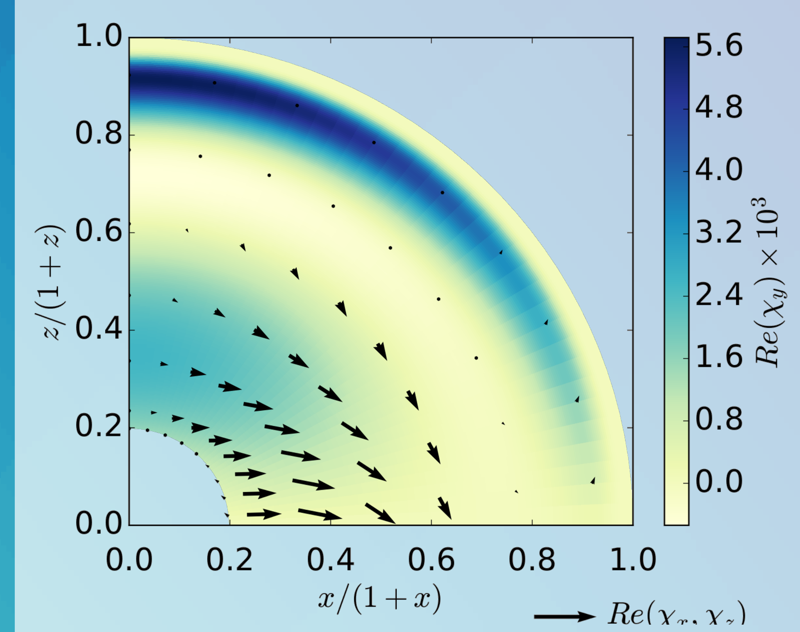


Example lapse in new, regular coordinates

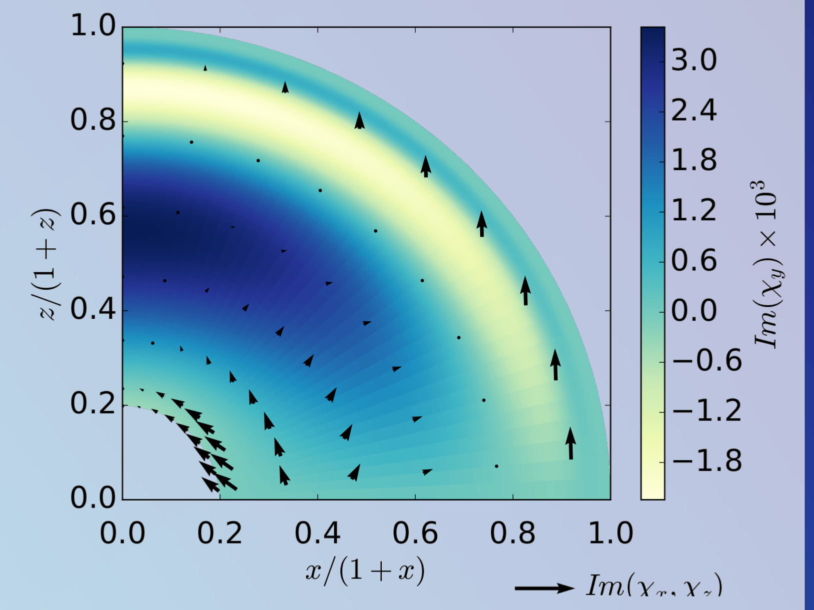


Example volume form in new, regular coordinates

- Perform 3+1 split of Proca field



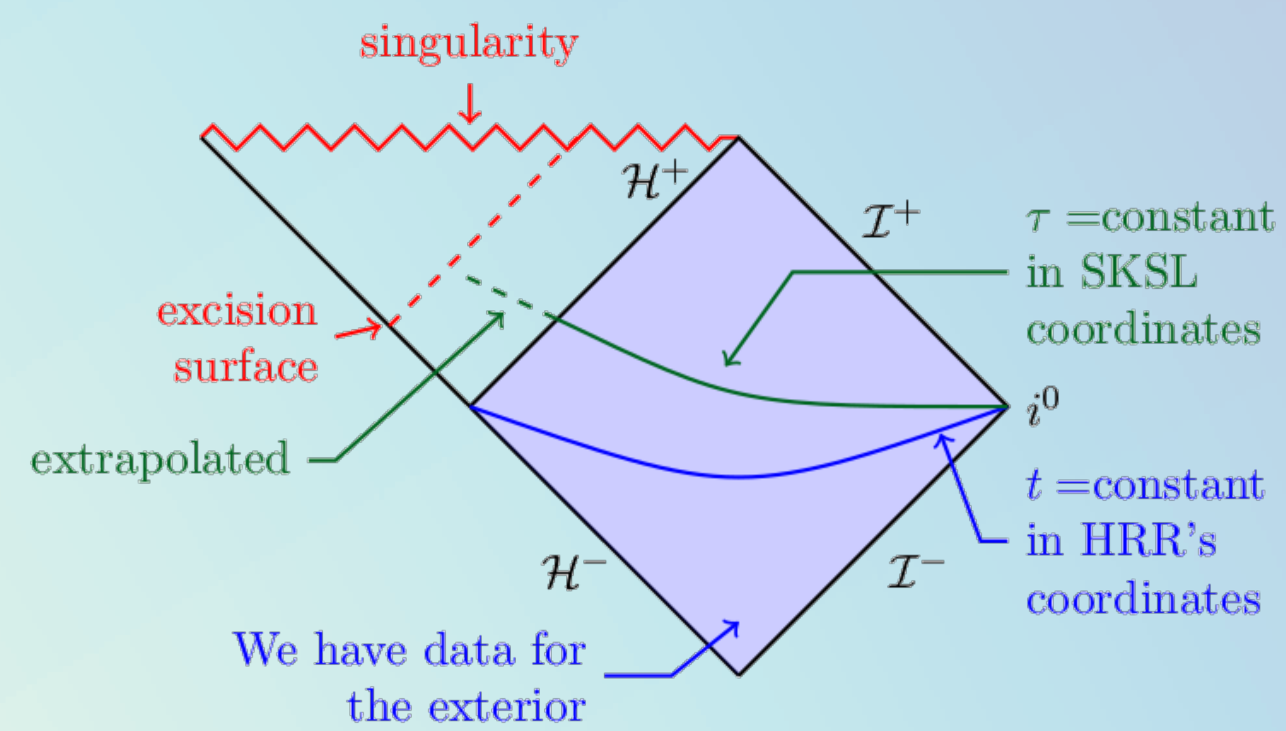
Example real vector potential in new coordinates



Example imaginary vector potential in new coordinates

Preliminary Calculations

- Use *cartoon method* to perform axisymmetric simulations
- Use horizon penetrating coordinates, with *compactification* and *excision*
- Preliminary calculations in-progress.



spacetime diagram of the BH-proca system. SKSL is the new coordinate system, HRR is the old one.

References

[1] C. Herdeiro, E. Radu, and H. Rúnarsson. Kerr black holes with Proca hair. *Classical and Quantum Gravity*, 33(15):154001, 2016.

[2] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper, and J. March-Russell. String axiverse. *Phys. Rev. D*, 81:123530, 2010.

[3] R. Brito, V. Cardoso, and P. Pani. *Superradiance: Energy Extraction, Black-Hole Bombs and Implications for Astrophysics and Particle Physics.* Lecture Notes in Physics. Springer International Publishing, 2015.

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