Thermal Evolution of Neo-Neutron Stars

M. Beznogov, D. Page

Institute of Astronomy, UNAM, Mexico City, Mexico

I. Abstract

A neo-neutron star is the next step after the proto-neutron star phase. It begins 30–60 seconds after the birth of the neutron star when neutrinos are free to escape and the crust of the neutron star is forming. Super-Eddington luminosities are still present for some time. A neo-neutron star produced in a core collapse supernova is not observable but the one produced by a binary merger, likely associated with a short gamma-ray burst, may be observable for some time while the super-massive neutron star is supported by fast rotation. A neutron star envelope can also reach Eddington luminosity during an X-ray burst. We present preliminary results of study of this neo-neutron star phase obtained with a modified version of a "standard" neutron stars' thermal evolution code, which was adapted to handle this regime. We investigated how long the star can have near-Eddington luminosity and demonstrate that this depends greatly on the initial conditions unlike "standard" cooling scenarios in which the initial conditions are quickly forgotten. We also show the importance of positrons and contraction energy during neo-neutron star phase.

II. Introduction

Neutron stars are by far the most intriguing objects in the Universe. They are superdense, superfast rotators, have superstrong magnetic fields, etc. (see, e.g., [1]). They are interesting objects to study on their own. But there is another important reason to study them.

Investigation of the properties of the matter at nuclear ($\rho_0 = 2.8 \times 10^{14}$ g cm^{-3} [1]) and supranuclear densities is a fundamental problem not only for astrophysics but for nuclear and particle physics and condensed matter physics as well. This is a very difficult task for a number of reasons among which are:

- The superdence matter is difficult to study in the laboratories;
- Its properties are problematic to compute due to the absence of a reliable description of the strong baryonic interactions taking into account many-body effects.

Some properties of superdense matter can be inferred from heavy ion collision experiments (e.g., [2]) or measurements of the neutron skin thickness of heavy nuclei (e.g., [3]). At densities $\rho > 10^4\rho_0$ one can employ methods of asymptotic QCD to calculate properties of such matter [1]. But the matter at the densities $\rho \sim 10\rho_0$ is the most mysterious. It cannot be studied reliable neither in experiments, nor by pure theoretical computations. One of the "most wanted" characteristics in such studies is the equation of state (EOS).

And here neutron stars come to the rescue. Neutron stars are unique natural laboratories of superdense matter. Their average density is $\sim 2.5\rho_0$ and the central density of massive neutron stars can reach up to $\sim 10\rho_0$. Exactly the range which is most needed. Of course, this is also not a simple task. To study the properties of the superdense matter one needs to study the interiors of the neutron stars. And from observations we can only infer the information about the surface. The neutrino emission from the core would have given us the information about the interiors directly, but it is far beyond the sensitivity of the current generation of neutrino detectors. The gravitational waves (GWs) so far cannot provide much of an insight about the EOS because the GW waveform depends on it only in the very last moments of binary neutron star merger. Thus, for the time being we have to deal with the electromagnetic observations of the surface.

The question is how. One of the possibilities is to study the thermal evolution of neutron stars (e.g., [1, 4]). This we will discuss further. Among other possibilities are independent measurement of masses and radiuses of neutron stars. Only four relatively precise measurements are enough to reconstruct $M(R)$ (mass–radius) dependence and then the EOS.

III. Physical ingredients of thermal evolution

We consider a spherically symmetric problem and neglect the effects of rotation and magnetic fields. The equations governing the thermal evolution can be divided into two categories: structure equations, which are *almost* temperature independent and thermal evolution equations.

Let us start with defining a space-time metric suitable for spherically symmetric non-rotating neutron stars:

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2\mathrm{e}^{2\Phi} - \mathrm{e}^{2\lambda}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi\right) \tag{1}$$

Here Φ and λ are metric functions,

$$\mathrm{e}^{-2\lambda} = 1 - \frac{2Gm}{rc^2} \tag{2}$$

(m being the gravitational mass enclosed within the radius r).

In 1-dimensional spherically symmetric problem the convenient independent variable is the enclosed baryon number, a. This is a Lagrange variable, which allows one to easily handle the situation where the star's structure changes in time.

$$a = \int_0^r 4\pi r'^2 n\left(r'\right)\mathrm{e}^{\lambda\left(r'\right)}\mathrm{d}r', \tag{3}$$

n is the baryon number density.

The structure equations are:

$$\frac{\partial r}{\partial a} = \frac{1}{4\pi r^2 n\mathrm{e}^{\lambda}}, \tag{4}$$

$$\frac{\partial P}{\partial a} = -\frac{G\left(\rho + U/c^2 + P/c^2\right)\left(m + 4\pi r^3 P/c^2\right)}{4\pi r^4 n}\mathrm{e}^{\lambda}, \tag{5}$$

$$\frac{\partial \Phi}{\partial a} = \frac{G\left(m + 4\pi r^3 P/c^2\right)}{4\pi r^4 n c^2}\mathrm{e}^{\lambda}, \tag{6}$$

$$\frac{\partial m}{\partial a} = \frac{\rho + U/c^2}{n\mathrm{e}^{\lambda}}. \tag{7}$$

P is the pressure, U is the internal energy, ρ is the mass density (not the energy-mass density).

Thermal evolution equations are:

$$\widetilde{L} = -\kappa\left(4\pi r^2\right)^2 n\mathrm{e}^{\Phi}\frac{\partial\widetilde{T}}{\partial a}, \tag{8}$$

$$\mathrm{e}^{\Phi}\frac{\partial\left(\widetilde{T}\mathrm{e}^{-\Phi}\right)}{\partial t} = -\frac{n}{C_\mathrm{V}}\frac{\partial\widetilde{L}}{\partial a} - \mathrm{e}^{2\Phi}\frac{Q_\nu}{C_\mathrm{V}} + \frac{\widetilde{T}}{C_\mathrm{V}}\left(\frac{\partial P}{\partial T}\right)\bigg|_n \frac{\partial \ln n}{\partial t}. \tag{9}$$

Here $\widetilde{L} = L\mathrm{e}^{2\Phi}$ and $\widetilde{T} = T\mathrm{e}^{\Phi}$ are red-shifted luminosity and temperature, respectively. C_V is the heat capacity, κ is the thermal conductivity and Q_ν is the energy loss due to neutrinos minus heat source (if any). The last term in Eq. (9) is the contraction energy. It describes the additional energy release due to the contraction of the star.

These six equations combined with the expressions for the thermal conductivity, heat capacity, neutrino energy losses and EOS constitute the full set of equations to calculate the thermal evolution of the neutron star. These equations are basically just the evolutionary equations for normal stars, but they fully take into account the general relativity (GR) effects.

Boundary conditions at the center are obvious:

$$\widetilde{L}(0) = 0, \quad r(0) = 0 \tag{10}$$

Boundary conditions at the surface are more complicated:

$$L_s = 4\pi\sigma R^2 T_s^4, \quad P_s = \frac{2}{3}\frac{g_s}{\mathrm{K}_s}, \quad m_s = M, \quad \Phi(r > R) = -\lambda(r > R). \tag{11}$$

Subscript "s" refers to the quantities at the surface; g_s is the free-fall acceleration at the surface, σ is the Stephan-Boltzmann constant and K is the opacity. The pressure boundary condition is actually approximate. It assumes that free-fall acceleration in the layers above the surface is constant. In some cases this is not true (when expansion or contraction of the star is significant). It is important to notice that for the calculations of neutron stars' thermal evolution the "surface" might be actually at rather high density, which makes luminosity boundary condition more complicated (see next section).

IV. "Standard" cooling

Let us quickly review the "standard" cooling of neutron stars. The cooling can be divided into three stages [4]:

- Initial relaxation, $0 \le t \lesssim 100$ yr. The core is thermally decoupled from the crust. Surface temperature reflects crust physics.
- Neutrino cooling stage, $100 \lesssim t \lesssim (1-3)\times 10^5$ yr. The core and the crust are thermally coupled. Near isothermal interior. The cooling is regulated by the neutrino emission from the core.
- Photon cooling stage, $t \gtrsim (1-3)\times 10^5$ yr. The core and the crust are thermally coupled. Near isothermal interior. The cooling is regulated by the photon emission from the surface.

During the thermal evolution the neutron star tends to "forget" the initial conditions. Thus, if we are interested in what is happening during the neutrino and/or photon cooling stage, we can have rather approximate treatment of what is happening in the initial relaxation phase and still get accurate solutions for the later phases. This greatly simplifies things. Most of the neutron star interior (except for a thin surface layer which is called the heat blanketing envelope) contains highly degenerate matter. This matter from the point of view of the structure equations can be considered as being at zero temperature. Thus, one can solve structure equations ones and then the cooling calculation will deal only with thermal equations.

So, the "standard" procedure is as following:

- Choose the heat blanketing envelope bottom density. Usually $\rho_b \sim 10^{10}$ g cm^{-3} (the thickness is ~ 100 m). Subscript "b" stands for 'bottom'.
- Compute the heat blanketing envelope profiles to find the relation between the surface temperature T_s and the bottom temperature T_b.
- Calculate the structure of a star starting from the center till ρ_b at zero temperature.
- Compute the cooling of a star by solving only the thermal evolution equations with the fixed structure and using $T_s - T_b$ relation as the boundary condition, i.e.: $L_s = 4\pi\sigma R^2 T_s^4(T_b) = L_b$ (assuming there are no heat sinks or sources in the heat blanketing envelope).

This scheme works well if we are interested in neutrino or photon cooling stages.

Here a couple of questions may arise. First, why not set ρ_b to surface density?

- The scheme with artificial separation of the heat blanketing envelope is much more computationally efficient.
- The very outer parts are not degenerate enough and their structure cannot be computed reasonably at zero temperature.

Second, what governs the choice of ρ_b?

- The balance between the computational efficiency and the heat diffusion timescale through the envelope.
- Presence of strong magnetic fields and some other considerations.

The question about heat diffusion timescale deserves special comment. For a pure iron envelope and $T_s =$ 1 MK the heat diffusion time is:

$$\rho_b = 10^8 \text{ g cm}^{-3}\text{: } t_d \sim 1 \text{ day}$$
$$\rho_b = 10^{10} \text{ g cm}^{-3}\text{: } t_d \sim 1 \text{ yr.}$$
$$\rho_b = 4\times 10^{11} \text{ g cm}^{-3}\text{: } t_d \sim 10 \text{ yrs.}$$

V. Preliminary results for neo-neutron stars

Now we want to focus on what is happening at the initial relaxation stage.

- Proto-neutron star, $0 \le t \lesssim 30 - 60$ s. The star is *opaque* to neutrinos, $T \sim 10^{11}$ K.
- Neo-neutron star, $30 - 60 \lesssim t \lesssim ???$ s. The star becomes transparent to neutrinos, $T < 10^{11}$ K. The crust is being formed.

The neo-neutron stage is important as it may occur not only in the newly born neutron stars (they are obscured from view), but also after a merger of two neutron stars. Unlike newly born stars, the latter situation might be observable. This is one of the main motivations to focus on this evolutionary phase. The other motivation is to study the formation of the crust.

"Standard" approach is inapplicable to neo-neutron stars due to several reasons. Consider one of them: we are now studying much shorter timescales and much faster processes. Thus, we need much lower values of ρ_b. In particular, for $\rho_b = 10^5$ g cm^{-3} t_d is about 1 s. This is fast enough for our purposes. So, we have to start from 10^5 g cm^{-3} and temperatures about 10^{10} K. This leads to the following issues:

- Region with these densities and temperatures is not degenerate enough to calculate its structure at zero temperature.
- Presence of positrons modifies thermal conductivity, heat capacity and equation of state.
- Radiative pressure cannot be neglected.

So, the main issue now (apart from more complicated physics due to the presence of positrons and photons) is that the structure and the thermal evolution equations cannot be separated and have to be solved together at each timestep (in other words the structure now is not constant and evolves in time). However, deeper inside ($\rho \gtrsim 10^{11}$ g cm^{-3}) the structure is again temperature independent. Thus, one can separate the star into three regions:

- heat blanketing envelope ($\rho < 10^5$ g cm^{-3})
- variable structure part ($10^5 < \rho < 10^{11}$ g cm^{-3})
- fixed zero temperature structure part ($\rho > 10^{11}$ g cm^{-3})

Note that the density at the boundary between the heat blanketing envelope and the variable structure part will evolve with time (see Figs. 2 and 4).

The most considerable difficulty now lies in the fact that the outer parts of the variable structure region are completely dominated by photons. Thus, the adiabatic index is close to 4/3 and the system is close to being unstable. From numerical point of view this results in substantial problems with convergence of the Newton-Raphson iterations (evolutionary equations for stars usually require fully implicit solver).

The second numerical complication is the necessity to match initial and boundary conditions. For the "standard" cooling it is possible to start with constant (red-shifted) temperature profile (thus, zero luminosity inside, which is inconsistent with non-zero surface luminosity, i.e. $L_s \neq L_b$) and the matching will occur automatically at the first time step. For neo-neutron stars it is not the case. If one starts with inconsistent luminosity, the first time step will diverge. So, we have developed a special matching procedure for the luminosity to start with the consistent initial and boundary conditions: $T_{t=0}(\rho) = F\big(\rho, \{p_1, p_2, \ldots\}, p_\mathrm{match}\big)$, where $p_1, p_2, \ldots, p_\mathrm{match}$ are free parameters of the parametrization of an arbitrary initial temperature profile. The procedure is as follows: we fix the values of $p_1, p_2, \ldots$ and use Newton-Raphson method to search for the value of p_match until the initial profile satisfies the boundary condition to the desired precision. Typically, this takes 5-6 Newton-Raphson iterations.

Preliminary results are presented on the figures below. On all figures "(N)P" stands for "(no)positrons", "(N)C" – "(no)contraction energy", "HT" – "high temperature", "LD"– "low density". All curves correspond to the initial value $\rho_{b,0} = 10^5$ g cm^{-3} except for the "low density" curves on Fig. 3 and 4 for which the initial value is $\rho_{b,0} = 3\times 10^4$ g cm^{-3}. See some discussion of the results in Section VI.

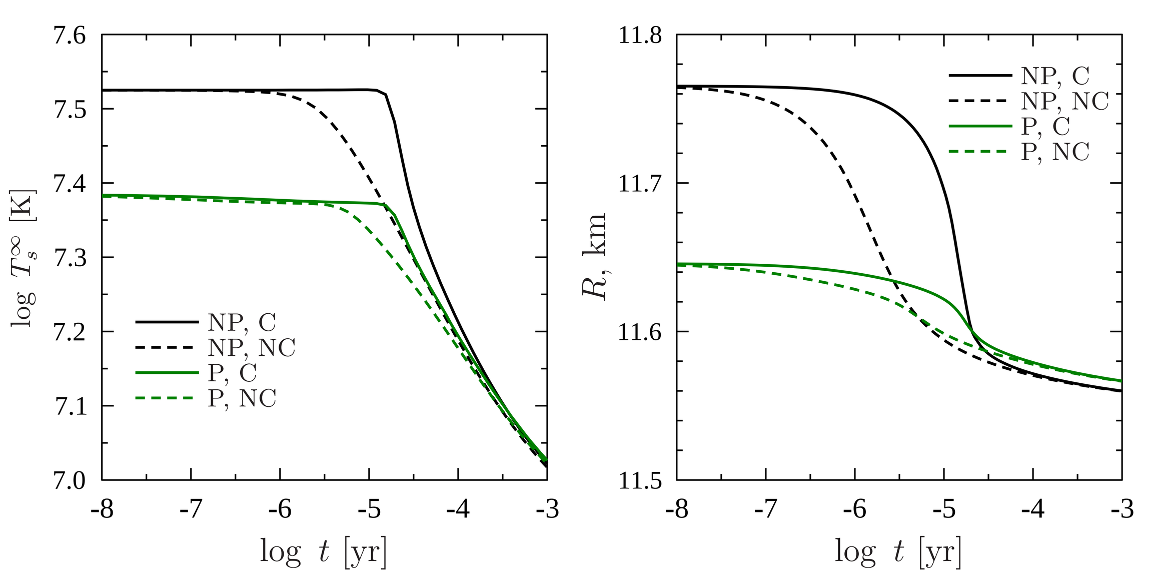

Figure 1: The dependence of the redshifted surface temperature T_s^∞ (left) and the radius of the star R (right) on the star age t. The difference between no-positron (black, NP) and positron (green, P) cases can be clearly seen as well as the impact of the contraction energy term in Eq. (9).

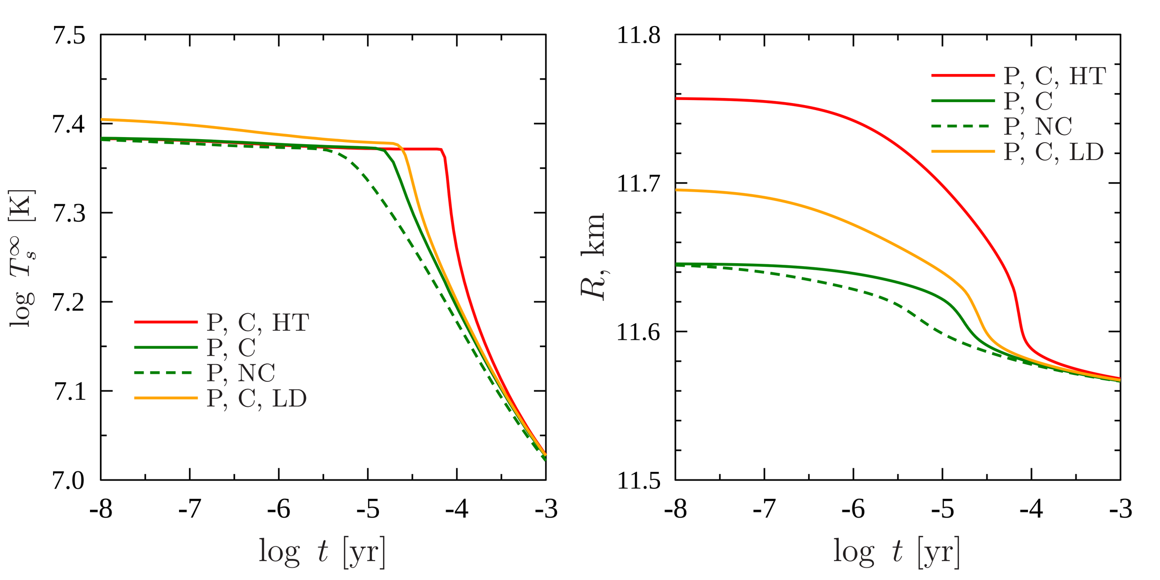

Figure 3: Same as Fig. 1, but here we demonstrate the impact of the initial conditions. Pair production is taken into account. Contraction energy is taken into account except for the dashed green curve. Red curve have different parametrization of the initial temperature profile. Orange curve have lower ρ_b value: $\rho_b = 3\times 10^4$ g cm^{-3} instead of 10^5 g cm^{-3}.

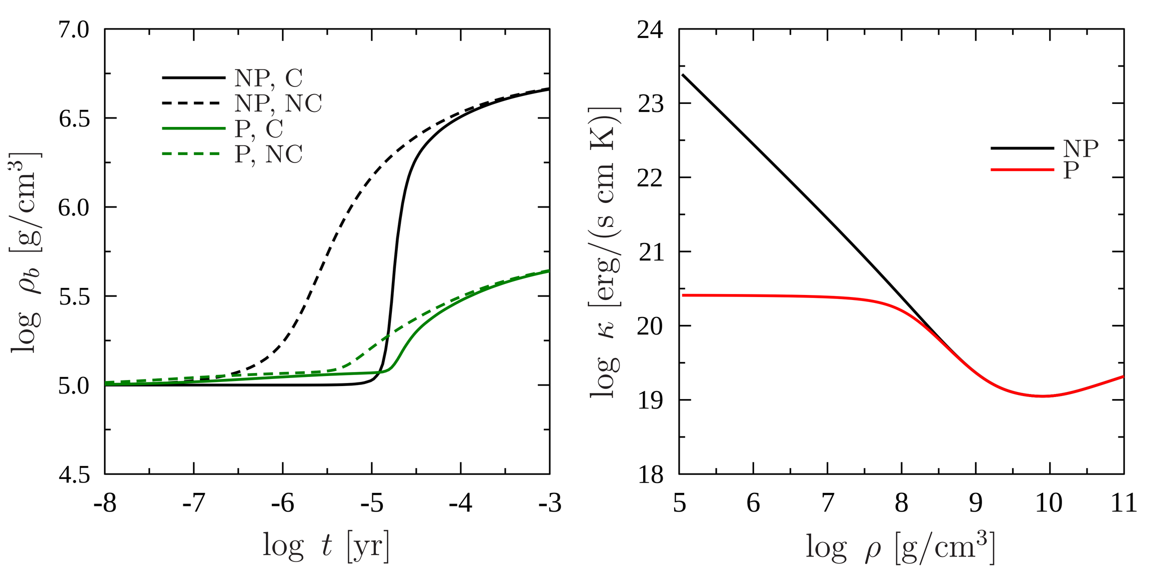

Figure 2: Left: same as Fig. 1, but for the density at the boundary between the heat blanketing envelope and the variable structure part (ρ_b). Right: comparison of the thermal conductivities ignoring positrons (black, NP) and taking into account pair production (red, P). The temperature is constant $T = 10^{10}$ K.

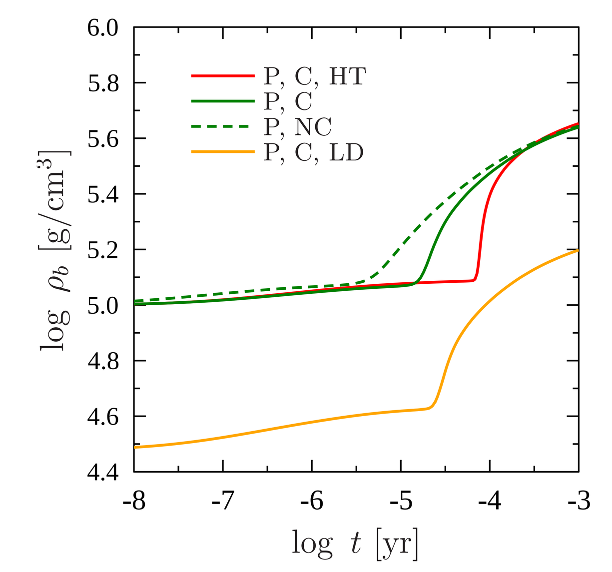

Figure 4: Same as Fig. 3 but for the density at the boundary between the heat blanketing envelope and the variable structure part (ρ_b).

VI. Discussion

From Fig. 1 we can see the impact of positrons and contraction energy. The energy input due to the contraction of the star allows it to maintain maximum temperature for considerably longer time. Positrons effect is also interesting. As it can be seen from right-hand side of Fig. 2, presence of the positrons considerably reduces thermal conductivity. It means that to match the initial profile with the boundary condition one needs higher temperature gradient (to increase L_b) and lower surface luminosity (L_s). Both goals can be achieved by lowering the value of T_b and, thus, T_s (the temperature at $\rho = 10^{11}$ g cm^{-3} is the same). This happens automatically as a result of our matching procedure (see Section V). As the variable structure part with positrons turns out to be overall colder than without them, the star is less "expanded", i.e. its radius is lower. It is important to emphasis that the model without positrons is unphysical and we present it only for comparison. Also, it is interesting to note that even not very large amount of positrons ($n_\mathrm{positrons}/n_\mathrm{protons} \sim 0.5$) is enough to considerably change the thermal conductivity (up to ~ 3 orders of magnitude, see Fig. 2).

Figs. 3 and 4 demonstrate the importance of the initial configuration. The red curve corresponds to the different initial temperature profile but with almost the same initial value of T_b. This profile makes the star more "expanded" (see right-hand side of Fig. 3) from "inside" (i.e. the very outer layers of the variable structure parts are the same because T_b is the same, but inner parts are hotter and, thus, are more "expanded"). As a result, during the contraction of the star more energy is released and this energy keeps the star surface at maximum temperature for about 2000 s. From Fig. 4 we can also see that the star is contracting non-uniformly and in the case of such "expanded from inside" configuration it contracts also mainly from "inside" (because the density at the envelope-variable structure part boundary stays close to its initial value for almost 2000 s). The orange curve corresponds to lower initial value of ρ_b: $\rho_{b,0} = 3\times 10^4$ g cm^{-3} (see Fig. 4). From Fig. 3 one can see that lower value of $\rho_{b,0}$ produces a star with slightly larger radius which can stay at maximum temperature a bit longer than "reference" configuration (solid green curve). But the 3 times change in $\rho_{b,0}$ is not that significant as slight change in temperature profile (red curve).

Figs. 1 and 3 clearly prove what was stated in Sections IV and V: after the initial relaxation stage neutron star "forgets" its initial conditions. We can see that at $t \sim 10^{-3}$ yr all curves converge (except, maybe, for ρ_b, which is somewhat artificial quantity).

VII. Conclusions

We have modified "standard" cooling calculation to handle the thermal evolution of neo-neutron stars. Positrons' and photons' contributions to the EOS, thermal conductivity and heat capacity were taken into account.

The results clearly demonstrate that the initial configuration is very important for the neo-neutron stars' thermal evolution (unlike "standard" cooling). Thus, a question of finding the proper initial conditions arises. With certain initial conditions the star can stay at near-Eddington luminosity for ~ 2000 s. This implies possibility of some noticeable mass loss.

Variable structure region of a neo-neutron star is dominated by photons. As a result the system is close to being unstable. This causes considerable numerical difficulties. So far solutions are possible only within rather limited range of the initial conditions. Solver is currently being improved to handle wider range of the initial conditions.

VIII. Acknowledgements

This work was supported by a Conacyt grant (CB-2014-1, #240512).

IX. References

[1] P. Haensel, A. Y. Potekhin, and D. G. Yakovlev, *Neutron Stars. 1. Equation of State and Structure*, vol. 326 of *Astrophysics and Space Science Library* (Springer, New York, 2007).

[2] A. Dainese, Nuovo Cimento C **33**, 115 (2010), 1012.4038.

[3] C. Mondal, B. K. Agrawal, M. Centelles, G. Colò, X. Roca-Maza, N. Paar, X. Viñas, S. K. Singh, and S. K. Patra, Phys. Rev. C **93**, 064303 (2016).

[4] D. G. Yakovlev and C. J. Pethick, Annu. Rev. Astron. Astrophys. **42**, 169 (2004).