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Effect of induced toroidal rotation on poloidal rotation and ion heat conductivity of tokamak edge plasmas

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Abstract. The transport processes in edge (collisional) plasmas of tokamaks with smooth profiles of macroscopic plasma parameters and induced poloidal and toroidal plasma flows are considered. The toroidal and poloidal velocities of particles, the radial electric field and the ion heat flux are derived. It is shown that forces, induced by radio frequency waves, plasma turbulence or neutral beam injection, can be used to control the poloidal and toroidal plasma velocities, as well as ion heat conductivity, in a wide range of these values.

1. Introduction

The possibility of controlling plasma poloidal flows is very attractive, as the low-to-high confinement transition (L–H transition) in tokamaks depends on sheared plasma poloidal flows [1]. External radio frequency waves [2, 3] plasma turbulence [4–6] or biased electrodes [7] are usually considered to control the plasma poloidal flows. Neutral beam injection can also strongly affect poloidal plasma velocities and transport processes in tokamaks, as has been shown both for collisional plasmas, as well as for the weakly collisional tokamak plasma [8–21].

Since we are interested in generalizing the results in [8–21] to the case in which external forces affect the tokamak plasma, we shall briefly review them. Hazeltine's expression [22] for poloidal velocity $U_{i\theta} = kU_{Ti}$ (residual rotation, where $k = -2.1$, $U_{Ti} = 1/(M_i\omega_{ci})\partial T_i/\partial r$, M_i is the ion mass, ω_{ci} is the ion cyclotron frequency, T_i is the ion temperature, r is the torus minor radius) was generalized in [8] for the case of large Mach numbers $M = U_{i\zeta}/c_s \geqslant 1$, where $U_{i\zeta}$ is the plasma velocity along the torus, $c_s = (T_e + T_i)/M_i$ is the sound velocity, and T_e is the electron temperature. In [8], the viscosity obtained in [23] and [24] was used for calculations. This viscosity is of the Burnett kind [25], which contains the spatial derivatives of ion heat fluxes, except for derivatives of ion velocities, in contrast with the Braginskii expression [26] for ion viscosity (which is of the Navier–Stokes type). A dependence of the coefficient k on the square of the Mach number $\alpha = M^2$, $k = k(\alpha)$, was obtained [8].

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It was also shown both for collisional plasma [27] and for weakly collisional plasma [10] with sonic toroidal rotation velocities that, in the absence of external forces affecting the plasma, the inequality $U_{i\theta} \ll c_s$ is fulfilled. We consider in this paper the case in which the external forces are small enough. Thus, the plasma poloidal velocity is also under the condition $U_{i\theta} \ll c_s$.

Analogous calculations for a weakly collisional plasma in the plateau regime of tokamaks were made in [9], and for all collisional regimes in [10]. The radial particle and heat fluxes were also analysed in this paper, and then these analyses were extended for the case of the collisional plasma [11]. However, the viscosity tensor dependence on the ion heat fluxes was omitted in [11]. Results of special interest for collisional tokamak plasmas were obtained in [8] and [11]. It was found that, already under the condition $\alpha \ll 1$, the poloidal velocity changes sign and achieves values that are much greater than the drift velocities [8], and the ion heat conductivity diminishes to the classical level [11]. In [8] and [11], the importance of the collisional parameter $b = \nu_i^{*2}$ for the investigation of plasma dynamics in edge (collisional) plasmas of tokamaks was underlined. Here $\nu_i^* = qR/\lambda_i$, q is the safety factor, R is the torus major radius, $\lambda_i = v_{Ti}/\nu_i$ is the ion mean free path, $v_{Ti} = \sqrt{2T_i/M_i}$ is the ion thermal velocity, and ν_i is the ion–ion collisional frequency.

However, in [8] and [11], the parameter b was practically supposed to satisfy the condition $1 \ll b \ll \sqrt{M_i/M_e}$. In [15], it was noted that this range of b is very narrow, and it is necessary to take into account the ion–electron heat exchange, which gives the possibility of widening this interval to $1 \ll b \ll M_i/M_e$ (see also [20]). On the other hand, when $b \geqslant M_i/M_e$, it is necessary to take into account the toroidal perturbations of the electron temperature [15, 20]. The poloidal plasma velocities and ion radial heat fluxes were found in [15] for such a case. We point out that the collisional regime is relevant for the conditions at the edge of the confined region in modern tokamaks, and the parameter b can achieve large values, $b \leqslant M_i/M_e$. These data can be found from analyses of experimental results in some tokamaks. For the Phaedrus-T edge plasma parameters [28]: $T_i < 20$ eV, $n_0 > 10^{12}$ cm^{-3}, $z_{\rm eff} \approx 2$, $q > 3$, $R = 92$ cm, $r = 26$ cm, we find the inequality $b > 15$. The ratio of the ion poloidal Larmour radius $\rho_{i\theta}$ to the inverse gradient r of plasma macroscopic parameters is small in experiments [28], $\rho_{i\theta}/r \approx 0.04$. For DIII-D [29] and Tokamak Experiment for Technology Oriented Research (TEXTOR) [30], profiles of ν_i^* (and, consequently, of b) were analysed, for example, in [20], where it was shown that ν_i^* can be much greater than one in these tokamaks taking into account the enhancement of ν_i^* by a factor one to four in the presence of an axisymmetric magnetic separatrix.

In [8–11] and [15], only the parallel viscosity was taken into account. Then, a number of papers, considering additionally the gyro- and perpendicular viscosities [12–14, 16, 17, 19, 20], impurity dynamics [18] and turbulent processes [21], were also published to explain the anomalous relaxation times of the toroidal velocities and to improve the neoclassical theory of transport processes in tokamaks. In [14] and [16] it was confirmed that there is no Pfirsch–Schlüter enhancement in the perpendicular viscosity in axially-symmetric tokamaks with sonic toroidal plasma flows. In [16] it was also shown that gyro-viscosity effects are negligibly small, at least for circular cross section tokamaks with up–down symmetry and smooth radial profiles of macroscopic parameters (see, also, [4]). Again, the temperature gradient dependence of viscosities was omitted in [14] and [16], and finite values of the Mach number M were neglected in [18].

The poloidal angle asymmetry of plasma macroscopic parameters, connected with taking into account the plasma gyro-viscosity in plasma dynamics equations, was noted in [12, 13] and [17]. The neoclassical theory of transport processes in tokamak edge plasmas with gradient scale lengths of macroscopic plasma parameters comparable to the ion poloidal gyro-radius was developed in [19] and [20]. It has also been obtained that the poloidal viscosity has a nonlinear dependence on $U_{i\theta}$ for large flows [31].

It has been recently shown that the kinetic [32, 33] or slow [34] Alfvén waves (AWs) can be used to create the localized sheared poloidal flows in the edge plasmas of tokamaks. These wave techniques use well elaborated, long-lived and simple devices [28], which allows AWs to be considered as a suitable method of plasma heating and current drive in tokamaks [28, 35, 36]. Plasma rotation, induced by AWs, was found experimentally in the Phaedrus-T tokamak [37]. Calculations, performed in [30] and [31], are in satisfactory agreement with the experimental results [37]. We find the main agreement. (i) The upshift Δf in frequency (connected with induced plasma flows) of the tearing mode for current drive antenna phasing was linear with rf power, and (ii) the experimental toroidal velocity $V_{\zeta i} \approx (3 \pm 1.2) \times 10^5$ cm s^{-1} is approximately the same as calculated in [30] and [31]. (iii) The variation of the poloidal and toroidal rotation velocities induced by rf fields linearly depends on dissipated power and does not depend on toroidal magnetic field, as follows from experiments [37]. This scaling can be found from figure 1 of [37]. The frequency upshift Δf was flat for $0.65 \leqslant B_0 \leqslant 0.8$ T, and the average $\Delta f = 0.92 \pm 19$ kHz and $\Delta f = -0.79 \pm 0.16$ kHz for the phase $\phi = \pm\pi/2$, respectively, between the currents in the two antenna straps [1]. This indicates that the viscous force, balancing the rf force, can be connected with the parallel viscosity.

The present work is intended to extend the results of [33] and [34], where only the parallel viscosity was taken into account in the viscous forces. This is supported by the agreement found with the experimental results of [37], where the macroscopic parameter profiles were smooth (scale length is much greater than the ion poloidal gyro-radius). Therefore, we take the viscous forces in the same approximation, and according to the results of [16], not consider gyro-viscosity, assuming also an up–down symmetric configuration. Any modification of the viscosity that might arise at very large rotation velocities is not considered, either. We generalize the previous results to make them dependent on the Mach number, for slightly rippled tokamaks, with external forces affecting the plasma, where the collisional parameter b is large enough for the ion–electron heat exchange and toroidal perturbations of the ion and electron temperatures to play an essential role in the ambipolarity condition. We analyse the possibility of controlling the plasma poloidal and toroidal velocities as well as ion heat fluxes by external forces.

2. Ambipolarity condition

Let us consider a slightly rippled tokamak, in which the angle dependence of the magnetic field is given by $B = B_0(1 + \epsilon\cos\theta + \delta\cos N\zeta)$, where θ and ζ are the poloidal and toroidal angles, ϵ is the inverse aspect ratio, δ is the ζ-modulation depth of the magnetic field, and N is the number of ripples.

To find the poloidal and toroidal plasma velocities, we use the ambipolarity condition, in the well known form that follows from the current continuity equation $\nabla \cdot \boldsymbol{j} = 0$,

$$\langle j^r \rangle = 0 \tag{1}$$

where

$$\langle \ldots \rangle = \int_0^{2\pi}\int_0^{2\pi} (\ldots)\sqrt{g}\,\mathrm{d}\theta\,\mathrm{d}\zeta \bigg/ \int_0^{2\pi}\int_0^{2\pi} \sqrt{g}\,\mathrm{d}\theta\,\mathrm{d}\zeta$$

g is the metric tensor determinant. From the MHD equations [26] we find

$$\left(M_i n \frac{\mathrm{d}\boldsymbol{V}_\mathrm{i}}{\mathrm{d}t} + \nabla p + \nabla\cdot\hat{\boldsymbol{\pi}} - \boldsymbol{F}^{\mathrm{an}} - \boldsymbol{F}^h\right)_\theta = -\frac{\sqrt{g}}{c} j^r B^\zeta \tag{2}$$

$$\left(M_i n \frac{\mathrm{d}\boldsymbol{V}_\mathrm{i}}{\mathrm{d}t} + \nabla p + \nabla\cdot\hat{\boldsymbol{\pi}} - \boldsymbol{F}^{\mathrm{an}} - \boldsymbol{F}^h\right)_\zeta = \frac{\sqrt{g}}{c} j^r B^\theta \tag{3}$$

where $\boldsymbol{F}^{\mathrm{an}}$ is an anomalous viscosity which should be taken into account if magnetic field ripples are very small, and $\boldsymbol{F}^h$ is an external force being exerted on the plasma. In the general case, this force can originate from any kind of rf wave, plasma turbulence or neutral beam injection. In particular, this force has been calculated for the case of slow Alfvén waves [32,34], where its poloidal and toroidal physical components F_θ^h and F_ζ^h are respectively [34]

$$F_\theta^h \approx \frac{h_\zeta k_b}{\omega} P_w \qquad F_\zeta^h \approx -\frac{h_\theta k_b}{\omega} P_w \qquad k_b = k_\theta h_\zeta - k_\zeta h_\theta. \tag{4}$$

Here $\boldsymbol{h} = \boldsymbol{B}/B$, $k_\theta = m/r$ and $k_\zeta = n/R$ are the components of the propagation vector $\boldsymbol{k}$, m and n are the poloidal and toroidal wavenumbers, r is the torus minor radius, and P_w is the absorbed power

$$P_w = \tfrac{1}{4}(\boldsymbol{E}\cdot\boldsymbol{j}^* + \mathrm{CC}). \tag{5}$$

We assume that the magnetic field can be written in the form

$$\boldsymbol{B}_0 = \{0;\ \chi'/2\pi\sqrt{g};\ \phi'/2\pi\sqrt{g}\}.$$

Here, χ and ϕ are the poloidal and toroidal magnetic fluxes, respectively, and the prime means the radial derivative. Thus, we can find the contravariant radial component of the current from equations (2) and (3).

A very important problem in the investigation of the collisional plasma rotation in tokamaks is which kind of the viscosity (parallel, gyro or perpendicular) [26] should be taken in the magnetohydrodynamic equations. We consider the case when the radial profiles of the plasma macroscopic parameters are smooth enough so that we can use, in the motion equations, only the parallel viscosity $\pi_\parallel$. Thus, we take the plasma viscosity in the form [23, 24, 26, 38]

$$\hat{\boldsymbol{\pi}} = \tfrac{3}{2}(\boldsymbol{h}\cdot\boldsymbol{h} - \tfrac{1}{3}\hat{\boldsymbol{g}})\pi_\parallel \tag{6}$$

where $\hat{\boldsymbol{g}}$ is the metric tensor. From equation (6), we get the expression for $\nabla\cdot\hat{\boldsymbol{\pi}}$

$$\nabla\cdot\hat{\boldsymbol{\pi}} = \tfrac{3}{2}\{[\boldsymbol{h}\nabla\cdot\boldsymbol{h} + (\boldsymbol{h}\cdot\nabla)\boldsymbol{h}]\pi_\parallel + \boldsymbol{h}(\boldsymbol{h}\cdot\nabla)\pi_\parallel - \tfrac{1}{3}\nabla\pi_\parallel\}. \tag{7}$$

Then, the covariant components of this vector are given by

$$(\nabla\cdot\hat{\boldsymbol{\pi}})_\theta = \frac{3}{2}\pi_\parallel \frac{\partial}{\partial\theta}\ln B - \frac{1}{2}\frac{\partial\pi_\parallel}{\partial\theta} \tag{8}$$

$$(\nabla\cdot\hat{\boldsymbol{\pi}})_\zeta = -\frac{3}{2}\pi_\parallel \frac{\partial}{\partial\zeta}\ln B + \frac{\partial\pi_\parallel}{\partial\zeta}. \tag{9}$$

As was noted in the introduction to this paper, here we consider the plasma poloidal velocity to be under the condition $U_{\mathrm{i}\theta} \ll c_\mathrm{s}$ (we define the 'physical' components of particle velocities as U_{jk}, $j = \mathrm{i}, \mathrm{e}$, $k = r, \theta, \zeta$; for example, we have the relation between 'physical' and contravariant components $U_{jk} \approx V_j^k\sqrt{g_{kk}}$). We find from the ion momentum equation [26]

$$U_{\mathrm{i}\theta} \approx U_{\mathrm{i}\zeta}h_\theta + U_{p\mathrm{i}} - \frac{c}{B}\bar{E}_r \qquad U_{p\mathrm{i}} = \frac{c}{e_\mathrm{i} n_0 B}\frac{\partial p_\mathrm{i}}{\partial r} \tag{10}$$

where p_i is the ion pressure. As we assume the toroidal velocity is under the condition $h_\theta U_{\mathrm{i}\zeta} \sim h_\theta c_\mathrm{s} \gg U_{\mathrm{i}\theta}$, we can further put $c\bar{E}_r/B \approx U_{\mathrm{i}\zeta} h_\theta$, where it is applicable.

Using the momentum equation for ions [26], we obtain for the part oscillating with the angle θ of the θ-covariant component of the momentum equation

$$M_\mathrm{i} n_0 \left(\frac{\mathrm{d}\boldsymbol{V}_\mathrm{i}}{\mathrm{d}t}\right)_\theta = -M_\mathrm{i} n_0 U_\zeta^2 \epsilon \sin\theta. \tag{11}$$

The quantities $\partial p/\partial\theta$ and $\partial p/\partial\zeta$ can be derived from the parallel component of the summed electron and ion motion equations [26]

$$\frac{\partial \langle p\rangle_\zeta}{\partial\theta} = -\frac{\partial\langle\pi_\parallel\rangle_\zeta}{\partial\theta} + M_\mathrm{i} n_0 U_\zeta^2 \epsilon \sin\theta \tag{12}$$

$$\frac{\partial \langle p\rangle_\theta}{\partial\zeta} = -\frac{\partial\langle\pi_\parallel\rangle_\theta}{\partial\zeta} \tag{13}$$

where

$$\langle\ldots\rangle_\theta = \frac{1}{2\pi}\int_0^{2\pi} (\ldots)\,\mathrm{d}\theta \qquad \langle\ldots\rangle_\zeta = \frac{1}{2\pi}\int_0^{2\pi} (\ldots)\,\mathrm{d}\zeta.$$

The external forces are omitted in equations (11)–(13) as we suppose that they are taken in the quasicylindrical approximation. Accounting for toroidal distortion of these forces in equations (11)–(13) has given terms of order ϵ^2 in angle-averaged equations.

Then, from equation (13) we get

$$\tilde{n} = -\frac{n_0(\langle\tilde{T}_\mathrm{i} + \tilde{T}_\mathrm{e}\rangle_\zeta) + \langle\pi_\parallel\rangle_\zeta}{T_\mathrm{i} + T_\mathrm{e}} - \alpha\epsilon n_0 \cos\theta \tag{14}$$

where $\langle\tilde{n}\rangle_\zeta$ and $\langle\tilde{T}_{\mathrm{i,e}}\rangle_\zeta$ are the oscillating parts (in terms of angle θ) of the density and ion and electron temperatures.

Now the ambipolarity condition, equation (1), takes the form

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\theta \left\{\frac{3}{2r}\langle\pi_\parallel\rangle_\zeta \frac{\partial}{\partial\theta}\ln B - \frac{\alpha\epsilon}{R}[n_0(\langle\tilde{T}_\mathrm{e} + \tilde{T}_\mathrm{i}\rangle_\zeta) + \langle\pi_\parallel\rangle_\zeta]\sin\theta\right\} + F_\theta^h = 0 \tag{15}$$

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\zeta \left(\frac{3}{2R}\langle\pi_\parallel\rangle_\theta \frac{\partial}{\partial\zeta}\ln B\right) + F_\zeta^{\mathrm{an}} + F_\zeta^h = 0. \tag{16}$$

3. Perturbed quantities and ion heat flux

As one can see from equations (15) and (16), we need to calculate the perturbed particle temperatures and ion viscosity. To find the particle temperatures we proceed from equations [26]

$$-T_\mathrm{i}\frac{U_{\mathrm{i}\theta}}{r}\frac{\partial\tilde{n}}{\partial\theta} + \frac{1}{qR}\frac{\partial q_{\mathrm{i}\parallel}}{\partial\theta} + \nabla\cdot\boldsymbol{q}_{\mathrm{i}\perp} - \frac{3M_\mathrm{e} n_0 \nu_\mathrm{e}}{M_\mathrm{i}}(\tilde{T}_\mathrm{e} - \tilde{T}_\mathrm{i}) = 0 \tag{17}$$

$$-T_\mathrm{e}\frac{U_{\mathrm{e}\theta}}{r}\frac{\partial\tilde{n}}{\partial\theta} + \frac{1}{qR}\frac{\partial q_{\mathrm{e}\parallel}}{\partial\theta} + \nabla\cdot\boldsymbol{q}_{\mathrm{e}\perp} + \frac{3M_\mathrm{e} n_0 \nu_\mathrm{e}}{M_\mathrm{i}}(\tilde{T}_\mathrm{e} - \tilde{T}_\mathrm{i}) = 0 \tag{18}$$

where [26]

$$q_{\mathrm{i}\parallel} = -3.91\frac{n_0 T_\mathrm{i}}{M_\mathrm{i}\nu_\mathrm{i} qR}\frac{\partial T_\mathrm{i}}{\partial\theta} \qquad q_{\mathrm{e}\parallel} = -3.16\frac{n_0 T_\mathrm{e}}{M_\mathrm{e}\nu_\mathrm{e} qR}\frac{\partial T_\mathrm{e}}{\partial\theta} \qquad \boldsymbol{q}_{k\perp} = \frac{5}{2}\frac{cn_0 T_k}{\mathrm{e}_k B}[\boldsymbol{h}\times\nabla T_k].$$

The terms with the poloidal velocities $U_{\mathrm{i}\theta}$ and $U_{\mathrm{e}\theta}$ in equations (17) and (18) can be obtained using the quasistationary continuity equations

$$n_j \nabla\cdot\boldsymbol{V}_j + \boldsymbol{V}_j\cdot\nabla n_j = 0.$$

Using equation (14) and solving the system of equations (17) and (18), we find the perturbed particle temperatures

$$\tilde{T}_{\mathrm{i}} = 0.51\frac{bT_0}{R\nu_{\mathrm{i}}d(b)}\left\{\alpha U_{\mathrm{i}\theta}\left(1+7.6b\frac{M_{\mathrm{e}}}{M_{\mathrm{i}}}\right) - 5U_{T\mathrm{i}} - 3.8b\frac{M_{\mathrm{e}}}{M_{\mathrm{i}}}\left(\frac{\epsilon}{q}\frac{j_{\|}}{e_{\mathrm{i}}n_0} + U_p\right)\right\}\sin\theta \tag{19}$$

$$\tilde{T}_{\mathrm{i}} + \tilde{T}_{\mathrm{e}} = 0.51\frac{bT_0}{R\nu_{\mathrm{i}}d(b)}\left\{\alpha U_{\mathrm{i}\theta}\left(1+15.2b\frac{M_{\mathrm{e}}}{M_{\mathrm{i}}}\right) - 5U_{T\mathrm{i}} - 7.6b\frac{M_{\mathrm{e}}}{M_{\mathrm{i}}}\left(\frac{\epsilon}{q}\frac{j_{\|}}{e_{\mathrm{i}}n_0} + U_p\right)\right\}\sin\theta \tag{20}$$

where $d(b) = 1 + 2.2b\sqrt{M_{\mathrm{e}}/M_{\mathrm{i}}}$, $b = q^2R^2/\lambda_{\mathrm{i}}^2$. We have used

$$U_{\mathrm{e}\theta} = U_{\mathrm{i}\theta} - \frac{\epsilon}{q}\frac{j_{\|}}{e_{\mathrm{i}}n_0} - U_p \qquad U_p = \frac{1}{M_{\mathrm{i}}n_0\omega_{\mathrm{ci}}}\frac{\partial p}{\partial r} \qquad p = p_{\mathrm{i}} + p_{\mathrm{e}} \qquad \nu_{\mathrm{e}} = \nu_{\mathrm{i}}\sqrt{\frac{2M_{\mathrm{i}}}{M_{\mathrm{e}}}}$$

with $Z_{\mathrm{eff}} \approx 1$ and $T_{\mathrm{e}0} \approx T_{\mathrm{i}0}$.

We take the parallel viscosity tensor $\pi_{\|}$ in the form derived in [8, 23, 24], since for the problem under consideration it is important that the viscosity tensor depends on the ion thermal fluxes $\boldsymbol{q}_{\mathrm{i}}$ (the Burnett type of the viscosity [25]). Using the usual Braginskii viscosity expression [26] (the Navier–Stokes type of the viscosity), it would lead to the conclusion that the equilibrium poloidal velocity of plasma rotation (residual rotation) is zero, as has been discussed repeatedly [8, 22–24, 39]. Thus, we have [8, 23, 24]

$$\pi_{\|} = -\frac{2}{3}\frac{p_{\mathrm{i}}}{\nu_{\mathrm{i}}}(0.96\beta - 0.59\gamma) \tag{21}$$

where

$$\beta = 3\left\{\boldsymbol{h}(\boldsymbol{h}\cdot\nabla)\boldsymbol{V}_{\mathrm{i}} + \frac{2}{5p_{\mathrm{i}}}\boldsymbol{h}(\boldsymbol{h}\cdot\nabla)\boldsymbol{q}_{\mathrm{i}} - \frac{1}{3}\left(\nabla\cdot\boldsymbol{V}_{\mathrm{i}} + \frac{2}{5p_{\mathrm{i}}}\nabla\cdot\boldsymbol{q}_{\mathrm{i}}\right)\right\} \tag{22}$$

$$\gamma = -\frac{6}{5}\left\{\boldsymbol{h}(\boldsymbol{h}\cdot\nabla)(\boldsymbol{q}_{\mathrm{i}} + 0.27\boldsymbol{q}_{\mathrm{i}\|}) + \frac{1}{3}(\nabla\ln p_{\mathrm{i}}\cdot\boldsymbol{q}_{\mathrm{i}} - \nabla\cdot(\boldsymbol{q}_{\mathrm{i}} + 0.27\boldsymbol{q}_{\mathrm{i}\|}))\right\}. \tag{23}$$

The origin of the terms with the heat flux derivatives (with the temperature gradients) in equations (22) and (23) can be understood analogously to the similar terms in the electron momentum equation [26]. The electron–ion friction term $\boldsymbol{R}_{\mathrm{e}}$ in the electron momentum equation, contains two contributions: the friction of the electron–ion particle fluxes $\boldsymbol{R}_u \sim M_{\mathrm{e}}n_{\mathrm{e}}(\boldsymbol{V}_{\mathrm{e}} - \boldsymbol{V}_{\mathrm{i}})$, and the friction of the electron–ion heat fluxes (the thermal force) $\boldsymbol{R}_T \sim M_{\mathrm{e}}(\boldsymbol{q}_{\mathrm{e}} - \boldsymbol{q}_{\mathrm{i}})/T_{\mathrm{e}}$. The last term can be expressed via the temperature gradients, as has been demonstrated in [26]. The terms with the velocity derivatives in equations (22) and (23), (i.e. the friction between the adjacent ion velocity fluxes) can be considered analogous to the term $\boldsymbol{R}_u$. Finally, the terms with the ion heat flux derivatives are analogous to the thermal force $\boldsymbol{R}_T$.

In order to find the ion perturbed velocities in equations (22) and (23) as functions of the angles θ and ζ, we proceed from the frozen-in condition

$$\nabla\times[\boldsymbol{V}_{\mathrm{i}}\times\boldsymbol{B}] \approx 0 \tag{24}$$

and the continuity equation [26]. Thus, we have

$$\tilde{V}_{\mathrm{i}}^{\zeta} = q\tilde{V}_{\mathrm{i}}^{\theta} \qquad \frac{\partial\tilde{V}_{\mathrm{i}}^{\theta}}{\partial\theta} = -\frac{U_{\theta\mathrm{i}}}{r}\frac{\partial}{\partial\theta}\ln(n\sqrt{g}) \qquad \langle\tilde{V}_{\|}\rangle_{\theta} \approx U_{\mathrm{i}\zeta}\frac{(\langle B\rangle_{\theta} - B_s)}{B_s} \tag{25}$$

for the case $U_{\mathrm{i}\zeta} > U_{\mathrm{i}\theta}$. Here, B_s is the magnetic field on the tokamak axis.

Furthermore, using equations (21)-(25), we find

$$\beta = \frac{3}{r}\left\{\epsilon U_{\zeta\mathrm{i}}\frac{\partial}{\partial\zeta}\ln B - U_{\theta\mathrm{i}}\frac{\partial}{\partial\theta}\ln(\sqrt{g}n^{2/3}B) + U_{T\mathrm{i}}\frac{\partial}{\partial\theta}\ln\frac{B}{n}\right\} \tag{26}$$

$$\gamma = -\frac{3}{r}\left\{0.34U_{\theta\mathrm{i}}\frac{\partial}{\partial\theta}\ln n + U_{T\mathrm{i}}\left(1.36\frac{\partial}{\partial\theta}\ln B - 0.84\frac{\partial}{\partial\theta}\ln n\right)\right\}. \tag{27}$$

Then, the ion heat flux $\Gamma_{T\mathrm{i}}$ can be derived by integrating the temperature evolution equation over the plasma volume [26]

$$\Gamma_{T\mathrm{i}} = \langle q_\mathrm{i}^r\rangle \tag{28}$$

where the heat flux radial contravariant component is given by [26]

$$q_\mathrm{i}^r = -\frac{2p_\mathrm{i}\nu_\mathrm{i}}{M_\mathrm{i}\omega_\mathrm{ci}^2}\frac{\partial T_\mathrm{i}}{\partial r} - \frac{5}{2}\frac{p_\mathrm{i}g_{33}h^\zeta}{M_\mathrm{i}\omega_\mathrm{ci}\sqrt{g}}\frac{\partial T_\mathrm{i}}{\partial\theta}. \tag{29}$$

The ion temperature poloidal dependence can thus be found from equation (19).

After the integration over the angle θ, we obtain from (28)

$$\Gamma_{T\mathrm{i}} = -\frac{2nT_\mathrm{i}\nu_\mathrm{i}}{M_\mathrm{i}\omega_\mathrm{ci}^2}\frac{\partial T_\mathrm{i}}{\partial r} - \frac{5cB_\mathrm{s}}{4\pi\epsilon e_\mathrm{i}R}\int_0^{2\pi}\mathrm{d}\theta\,\frac{p_\mathrm{i}}{B^2}\frac{\partial T_\mathrm{i}}{\partial\theta}. \tag{30}$$

As can be seen from (29), it is necessary to calculate the ion poloidal velocities $U_{\mathrm{i}\theta}$ and $U_{\mathrm{e}\theta}$.

4. Flux analysis

Now we can find the plasma rotation velocities using the ambipolarity conditions equations (15) and (16), the expressions for perturbed temperatures (19) and (20), and the ion viscosity (21), (26) and (27):

$$U_{\mathrm{i}\zeta} = 1.39\frac{\nu_\mathrm{i}R^2}{M_\mathrm{i}n_0v_\mathrm{Ti}^2(N\delta)^2}F_\zeta^h \tag{31}$$

$$U_{\mathrm{i}\theta} = G_{u1}(\alpha,b)U_{T\mathrm{i}} + G_{u2}(\alpha,b)\left(\frac{\epsilon}{q}\frac{j_\parallel}{e_\mathrm{i}n_0} + U_p\right) + 1.39\frac{\nu_\mathrm{i}R^2}{M_\mathrm{i}n_0v_\mathrm{Ti}^2}G_{u3}(\alpha,b)F_\theta^h \tag{32}$$

where

$$G_{u1}(\alpha,b) = -\frac{f_2(\alpha,b)}{f_1(\alpha,b)} \qquad G_{u2}(\alpha,b) = 1.35\frac{M_\mathrm{e}}{M_\mathrm{i}}\frac{\alpha b^2}{f_1(\alpha,b)} \qquad G_{u3}(\alpha,b) = \frac{d(b)}{f_1(\alpha,b)}$$

$$f_1(\alpha,b) = d(b)\left(1+\frac{2}{3}\alpha\right)(1+0.19\alpha) + 0.18\alpha^2b\left(1+15.2b\frac{M_\mathrm{e}}{M_\mathrm{i}}\right)$$

$$f_2(\alpha,b) = d(b)\left(1+\frac{2}{3}\alpha\right)(1.83+1.52\alpha) - 0.9\alpha b \qquad d(b) = 1+2.2b\sqrt{M_\mathrm{e}/M_\mathrm{i}}.$$

Functions $G_{u1}(\alpha,b)$, $G_{u2}(\alpha,b)$, $G_{u3}(\alpha,b)$ are plotted for the deuterium plasma case in figure 1. To find the toroidal velocity $U_{\mathrm{i}\zeta}$, equation (31), we confined ourselves to the rippled magnetic field case. If ripples are small it is necessary to take account of anomalous viscosity.

It can be observed that function $G_{u1}(\alpha,b)$ (figure 1) changes sign at $\alpha_0 \approx 2d(b)/b$, which, if we omit the ion–electron heat exchange, coincides with the previously known result [8]. When the collisional parameter b satisfies the inequality $b \geqslant (M_\mathrm{i}/M_\mathrm{e})^{0.5}$, the

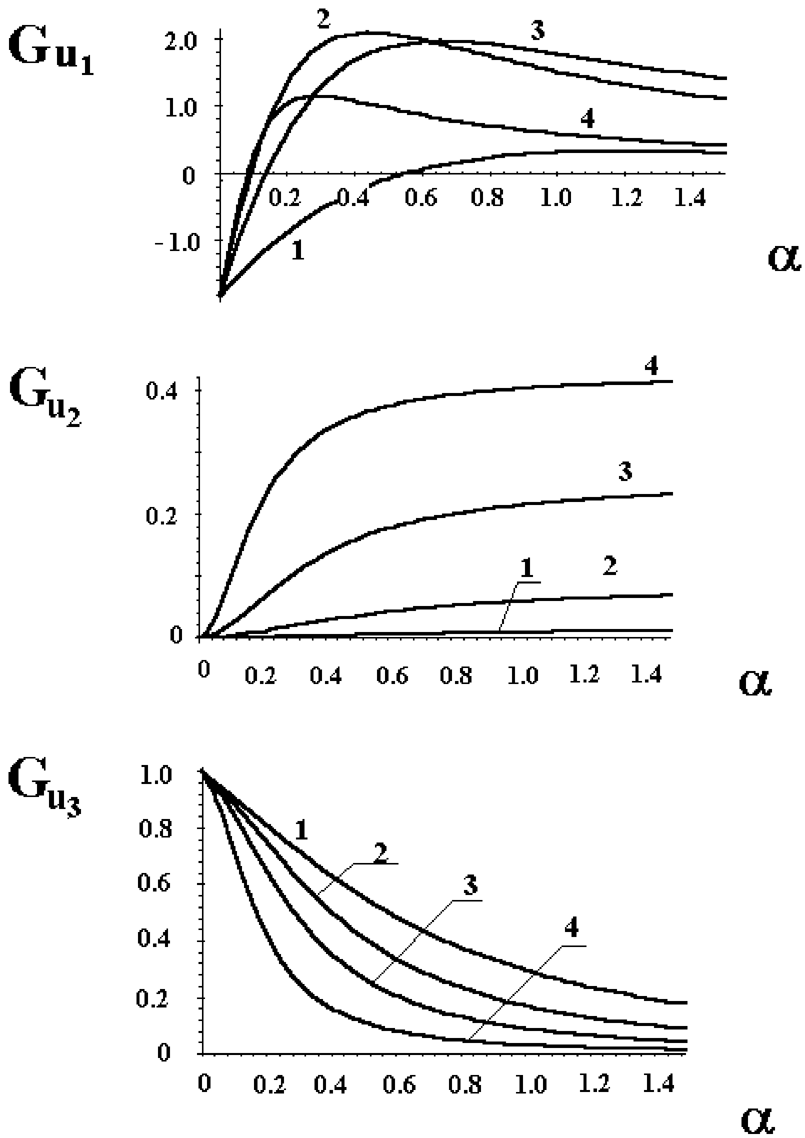

Figure 1. The dependence of the functions $G_{u1}(\alpha, b)$, $G_{u2}(\alpha, b)$, and $G_{u3}(\alpha, b)$ on α for different magnitudes of the parameter $b : b_1 = 10$, $b_2 = 50$, $b_3 = 250$, $b_4 = 1250$.

quantity α is equal to $\alpha_0 \approx 0.1$. The maximum of function $G_{u1}(\alpha, b)$ is achieved when $b_1 \approx 50$, $\alpha_1 \approx 0.5$, and is $G_{u1}(\alpha, b1) \approx 2$.

The radial electric field can be found using the expression which is defined via the ion velocity from the ion motion equation, equation (10),

$$\bar{E}_{\mathrm{r}} \approx \frac{B}{c}(U_{\mathrm{i}\zeta} h_\theta + U_{p\mathrm{i}} - U_{\mathrm{i}\theta}) \qquad U_{p\mathrm{i}} = \frac{c}{e_{\mathrm{i}} n_0 B} \frac{\partial p_{\mathrm{i}}}{\partial r}. \tag{33}$$

We should substitute (31) and (32) into (33). Thus we have

$$\bar{E}_{\mathrm{r}} \approx \frac{B}{c}\left\{1.39\frac{\nu_{\mathrm{i}}R^2}{M_{\mathrm{i}}n_0v_{\mathrm{Ti}}^2}\left[\frac{h_\theta}{(N\delta)^2}F_\zeta^h - G_{u3}(\alpha,b)F_\theta^h\right]\right.$$
$$\left. +U_{p\mathrm{i}} - G_{u1}(\alpha,b)U_{T\mathrm{i}} - G_{u2}(\alpha,b)\left(\frac{\epsilon}{q}\frac{j_\parallel}{e_{\mathrm{i}}n_0} + U_p\right)\right\}. \qquad (34)$$

Equation (34) can be simplified if we consider slow Alfvén waves as the source of external forces. In this case, we have from (4) $F_\zeta^h \approx -h_\theta F_\theta^h$ and from (34)

$$\bar{E}_{\mathrm{r}} \approx \frac{B}{c}\left\{-1.39\frac{\nu_{\mathrm{i}}R^2F_\theta^h}{M_{\mathrm{i}}n_0v_{\mathrm{Ti}}^2}\left[\frac{h_\theta^2}{(N\delta)^2} + G_{u3}(\alpha,b)\right]\right.$$
$$\left. +U_{p\mathrm{i}} - G_{u1}(\alpha,b)U_{T\mathrm{i}} - G_{u2}(\alpha,b)\left(\frac{\epsilon}{q}\frac{j_\parallel}{e_{\mathrm{i}}n_0} + U_p\right)\right\}. \qquad (35)$$

Function $G_{u2}(\alpha, b)$ grows with α and b, and can only contribute to the poloidal velocity at very large values of b, when toroidal perturbations of the electron temperature are essential. The function $G_{u3}(\alpha, b)$, on the other hand, drops quickly with α and b (figure 1). Thus, we can conclude that toroidal rotation does not allow strong variations of poloidal rotation, when forces induced by radio frequency waves, plasma turbulence or neutral beam injection are present. The relevant regime for L–H transition, when the poloidal rotation is sufficient, occurs when $\alpha \ll 1$.

From (19), (20), (30) and (32) we find the radial ion heat flux. We write this equation in the Shafranov form [40, 41]

$$\Gamma_{T\mathrm{i}} = -\frac{2nT_{\mathrm{i}}\nu_{\mathrm{i}}}{M_{\mathrm{i}}\omega_{\mathrm{ci}}^2}\frac{\partial T_{\mathrm{i}}}{\partial r}\left\{1 + 1.6q^2\left[G_{T1}(\alpha,b) + G_{T2}(\alpha,b)\frac{1}{U_{T\mathrm{i}}}\left(\frac{\epsilon}{q}\frac{j_\parallel}{e_{\mathrm{i}}n_0} + U_p\right)\right.\right.$$
$$\left.\left. -1.39\frac{\nu_{\mathrm{i}}R^2}{U_{T\mathrm{i}}M_{\mathrm{i}}n_0v_{\mathrm{Ti}}^2}G_{T3}(\alpha,b)F_\theta^h\right]\right\} \qquad (36)$$

where

$$G_{T1}(\alpha,b) = \frac{(1+\alpha/2)f_3(\alpha,b)}{f_1(\alpha,b)}$$
$$G_{T2}(\alpha,b) = 0.76\frac{M_{\mathrm{e}}}{M_{\mathrm{i}}}\frac{\alpha b(1+\alpha/2)}{f_1(\alpha,b)}\left[\left(1+\frac{2}{3}\alpha\right)(1+0.19\alpha) - \frac{0.18\alpha^2 b}{d(b)}\right]$$
$$G_{T3}(\alpha,b) = \frac{\alpha(1+7.6bM_{\mathrm{e}}/M_{\mathrm{i}})(1+\alpha/2)}{5f_1(\alpha,b)}$$
$$f_3(\alpha,b) = \left(1+\frac{2}{3}\alpha\right)\left[1+0.19\alpha+\frac{\alpha}{5}(1.83+1.52\alpha)\left(1+7.6b\frac{M_{\mathrm{e}}}{M_{\mathrm{i}}}\right)\right] + 1.4\frac{M_{\mathrm{e}}}{M_{\mathrm{i}}}\frac{\alpha^2b^2}{d(b)}.$$

If we suppose that b satisfies the inequality $1 < b < \sqrt{M_{\mathrm{i}}/M_{\mathrm{e}}}$, we obtain the previously known expression for the radial ion heat flux [9, 10].

As proper analysis shows, the quantity $G_{T2}(\alpha, b)$ is negligible under the condition $b \leqslant M_{\mathrm{i}}/M_{\mathrm{e}}$. The quantities $G_{T1}(\alpha, b)$ and $G_{T3}(\alpha, b)$ are plotted in figure 2. As can be seen from figure 2, the neoclassical contribution in the radial ion heat flux, as a function of α, is approximately of the same order for $b \geqslant \sqrt{M_{\mathrm{i}}/M_{\mathrm{e}}}$, dropping with increasing b. The influence of induced forces on the radial heat flux is usually small for $b \geqslant \sqrt{M_{\mathrm{i}}/M_{\mathrm{e}}}$.

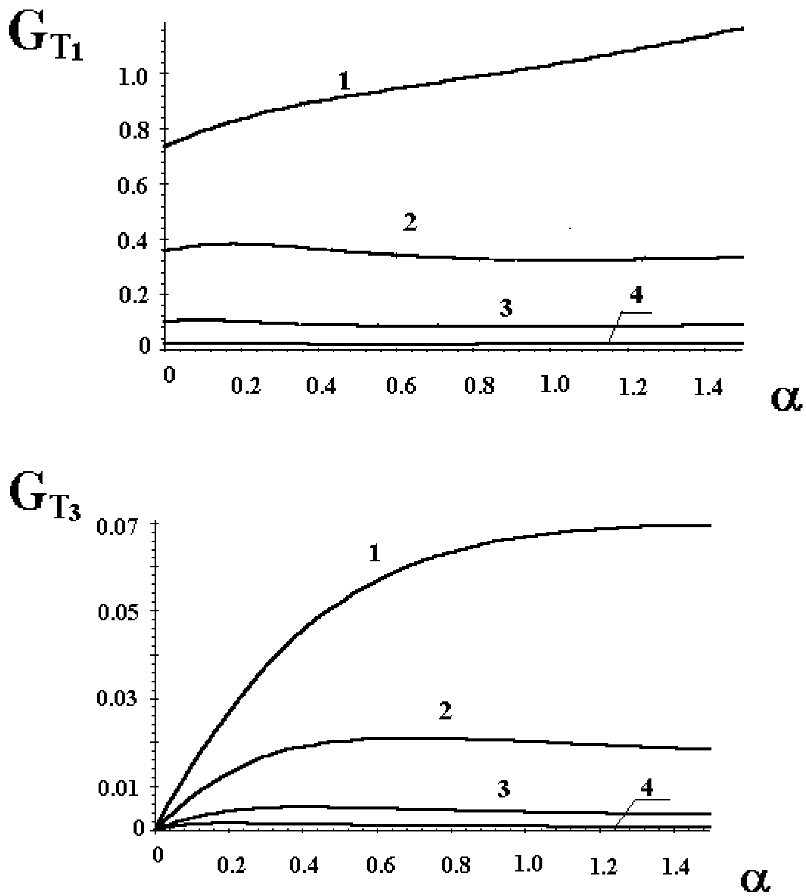

Figure 2. The dependence of the functions $G_{T1}(\alpha, b)$ and $G_{T3}(\alpha, b)$ on α for different magnitudes of the parameter b : $b_1 = 10$, $b_2 = 50$, $b_3 = 250$, $b_4 = 1250$.

5. Conclusion

We have considered the transport process in edge (collisional) plasmas of slightly rippled tokamaks, in the presence of external forces, and obtained the poloidal and toroidal plasma flows, induced by radio frequency waves, plasma turbulence or neutral beam injection. The dependence of the radial ion heat flux and the poloidal plasma velocity on the induced toroidal plasma velocity has been studied. It has been shown that the coefficient $G_{u1}(\alpha, b)$ of the ion temperature gradient in the poloidal velocity, changes sign with growing toroidal plasma velocity (or, of the parameter α), achieving its maximum, approximately equal to 2, at $\alpha < 1$. The input of the plasma current and pressure gradient into the plasma poloidal velocity can be essential at very large values of the collisional parameter b when $b \geqslant M_{\rm i}/M_{\rm e}$. The influence of forces acting on the plasma drops quickly with the growth of α and b. The radial ion heat flux depends very weakly on α for $b \geqslant \sqrt{M_{\rm i}/M_{\rm e}}$, and drops quickly with the growth of b. We can conclude that the relevant regime for the operation of L–H transition in the edge plasmas of tokamaks, by means of inducing strongly sheared strong poloidal plasma flows, occurs for $\alpha \ll 1$ regime. For example, the slow or kinetic Alfvén waves satisfy this condition.

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