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A model for confinement improvement in presence of a stationary MARFE

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Abstract

A model for the observed improvement in confinement when a stationary MARFE develops is proposed. It is based on the fact that field-aligned flows are naturally created in association to a MARFE, which, when coupled to the field line curvature, give rise to plasma spin-up. The resulting radially-sheared poloidal rotation about the layer affected by the MARFE at the edge may suppress turbulence and reduce transport losses.

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1. Introduction

Multifaceted Asymmetric Radiation from the Edge (MARFE) in tokamaks is a radiative phenomenon that takes place mostly at the high field side (HFS) plasma edge and appears at relatively high densities. It is usually observed as the density limit is approached just before a disruption, but it has also been possible to produce long-living stationary MARFEs [1], which has provided the possibility to study their properties. The MARFE appears when an initial density increase, due to gas feeding, produces an increment in plasma radiation at the HFS of the tokamak near the edge, which in turn cools down the plasma. As a result there is a pressure drop in that region, and then a particle flow is driven along the field lines, which brings in more newly injected particles, from the low field side (LFS) of the plasma flux surface, thus accumulating more particles and radiating more. Hence, the MARFE works as a kind of plasma pump. This effect reinforces another unstable process that occurs when the temperature lies in a part of the radiation curve L(T), where the radiation increases as the temperature decreases (negative slope). The whole process is known as the thermal condensation instability, and it arises if the radiation increase is larger than the heat influx coming

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from perpendicular diffusion, which apparently is the case in a MARFE [2]. The reason for the appearance of MARFEs at the HFS is that the radial heat transport has a minimum there, and thus a temperature drop cannot be compensated by heat diffusing from the core. However, the average density over a flux surface is unchanged: the density is just redistributed.

Recently, it has been observed in the HT-7 tokamak a regime of improved confinement when a stationary MARFE is created [3]. This is evidenced by: (1) a density profile peaking, (2) a reduction of the D_α line emission and (3) an increase in line-averaged electron density. While the MARFE-related confinement improvement is associated with edge radiation, it seems that its characteristics are different from the RI (radiative improved) mode found in TEXTOR [4]. In the latter, the reason for the transport reduction is apparently related to the suppression of the ITG mode due to a reduction of the ion temperature gradient parameter $\eta_i = d\log T_i/d\log n$ [5]. Although it has also been claimed that in HT-7 an internal transport barrier (ITB) is formed [6], the evidence is not convincing. Hence, it is most likely that the improved confinement in HT-7 originate in the region where the MARFE is localized, as a result of the formation of an edge transport barrier (ETB). Stationary MARFEs have also been produced in TEXTOR [7] and Tore Supra [8] experiments but not associated with confinement improvement. Thus, it should be remarked that the experimental evidence in favor of an improvement in confinement during stationary MARFEs is not conclusive. It is also worth mentioning that a similar particle confinement improvement has been observed in detached plasmas [9,10], producing an increase in the central plasma density. The increased density has been attributed to a displacement of the particle source towards the plasma core due to the decrease of the edge temperature, but also to a reduction of the transport due to the dissipative trapped electron mode. Therefore, an analogous mechanism might be in effect in the case of a stationary MARFE, thus producing the observed density rise in HT-7. However, the concomitant increase in the energy confinement, also reported for HT-7, would require an altenative explanation. Thus, eventhough the experimental data are still not complete, lacking reliable temperature measurements and any information on plasma rotation, it is worthwhile to investigate from a theoretical point of view, the possibility that another process be responsible for the creation of a thermal barrier, that improves both particle and thermal confinement.

In this Letter we present a mechanism that can explain the improved confinement at the plasma edge due to a MARFE, based on the formation of an ETB. The physical picture proposed is the following. As mentioned before, the MARFE gives rise to a field-aligned plasma flow in order to keep pressure balance, from the LFS to the HFS. If gas-puffing is maintained at a high rate the MARFE density increases indefinitely, but if it is low enough, cross-field particle transport can balance mass input and thus keep the density at a stationary value. Radial particle diffusion out of the MARFE is large due to the increased density gradient that is created there; the particles recycle at the wall but since they are neutralized, re-enter the plasma in a wider poloidal range, adding up to gas-puffing. Now, since the high-density MARFE is localized in the HFS, the density on a given flux surface is a function of the poloidal angle, θ, having its maximum at $\theta = \pi$, i.e., at the plasma inboard side. The poloidal anisotropy of the equilibrium makes it susceptible to the Stringer spin-up instability [11], that is driven by the magnetic field line curvature. The equilibrium parallel-flow is shown in Fig. 1(a), and it gets modified when a perturbed poloidal rotation sets in. In this case, as shown in Fig. 1(b), the flows convergence point is displaced in the direction of the rotation (counterclockwise in the case of the figure), thus creating a density increase at this point. Due to the presence of magnetic curvature, which plays the role of an effective gravity, this high

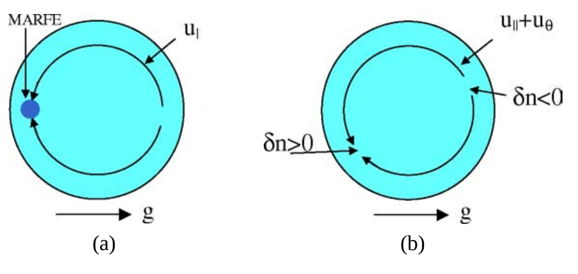

Fig. 1. Poloidal cross section of toroidal plasma, with the symmetry axis to the left, showing the effective gravity associated to the magnetic field line curvature and the plasma flows; (a) the equilibrium state in a stationary MARFE, with field-aligned flows converging at the MARFE position, (b) perturbed state by a poloidal velocity, displaces the converging point and modifies the initial density.

density region tends to "sink" moving in the direction of "gravity", that is, towards the LFS. This makes the plasma to move counterclockwise, which reinforces the initial rotation, and the process continues, spinning up the plasma. The result is a sheared poloidal rotation at the edge which can suppress turbulence and reduce anomalous transport.

2. Plasma flows and spin-up

The plasma flow along the magnetic field lines, that is established when the MARFE structures set in, create a state that is subject to the spin-up mechanism as described in [12,13]. We first recall the physics of the spin-up for a poloidally asymmetric radial particle flow [13], and then we develop the corresponding formal model. In the case of a MARFE, the asymmetry is due to the poloidal dependence of both the density and the diffusion coefficient near the edge. For the magnetic field the usual axisymmetric representation is used: $\mathbf{B} = \nabla\psi \times \nabla\zeta + I(\psi)\nabla\zeta$, in terms of the flux function ψ. In the analysis, we will at times make use of a toroidal geometry with circular cross section and a coordinate system (r, θ, ζ), where the major radius is $R = R_0(1 + \epsilon\cos\theta)$, so that the magnetic field can be approximated by $\mathbf{B} = (B_0/R)(\hat{\zeta} + \epsilon\hat{\theta})$, and ψ is identified with r. We will explicitly separate the poloidally asymmetric part of any quantity (like a source S) by writing $\delta S = S - \langle S\rangle$, where the angular brackets denote flux-surface average $\langle S\rangle = \oint S(1 + \epsilon\cos\theta)\,d\theta/2\pi$. To heuristically understand the spin-up, following [13], let us consider the radial particle flow Γ to have an asymmetric part, $\delta\Gamma = -\delta(D\partial n/\partial r)$, that consequently drives a parallel flow given by the continuity equation,

$$\nabla_\parallel \delta u_\parallel = -\frac{1}{nr}\frac{\partial}{\partial r} r\delta\Gamma, \tag{1}$$

where $\nabla_\parallel = \hat{b}\cdot\nabla$, with $\hat{b} = \mathbf{B}/B$, and $u_\parallel$ is a function of the poloidal angle θ. Then if a seed poloidal rotation, u_p, is applied, an advective force would act upon the parallel flow, which should be offset by pressure; so the parallel momentum balance gives, assuming $T = \text{const}$,

$$n\frac{u_p}{r}\frac{\partial\delta u_\parallel}{\partial\theta} = -\frac{c_s^2}{qR}\frac{\partial\delta n}{\partial\theta}, \tag{2}$$

where $c_s^2 = T/m_i$ and $q = rB_\phi/RB_p$. According to this, the initial poloidally dependent density gets modified by $\delta n(\theta)$. In addition, in the presence of magnetic line curvature, there is an effective gravity, $\mathbf{g}_{\text{eff}} = -c_s^2\kappa$ ($\kappa = \hat{b}\cdot\nabla\hat{b} \sim -\hat{R}/R$), that produces a "buoyancy" force which moves the higher density plasma region, $n + \delta n(\theta)$, towards larger R. This force accelerates the initial plasma rotation, since it can only move on flux surfaces. This is seen from the poloidal equation of motion averaged over a flux surface (the curvature contribution comes from the convective term),

$$n\frac{\partial u_p}{\partial t} = \oint \delta n g_{\text{eff},\theta}\frac{d\theta}{2\pi}. \tag{3}$$

Now, in a MARFE the particles are sucked in at the inboard side (a plasma pump [1]), which initially increases the density, but when the stationary state is established the incoming particles have to be diffused out. So the radial flow is peaked at $\theta = \pi$, which means a dependence of the type $\delta\Gamma \propto -\cos\theta$. Then, combining Eqs. (1)–(3), one finds that $\delta u_\parallel \propto \sin\theta$ and $\delta n \propto -u_p\sin\theta$. Thus, for $u_p > 0$ the density fluctuations reach their maximum amplitude at the bottom of the poloidal cross section. Moreover, since $g_{\text{eff},\theta} \propto -\sin\theta$, the evolution equation for the poloidal flow will have the form $\partial u_p/\partial t = \Gamma u_p$ with $\Gamma > 0$ and therefore u_p will tend to grow, spinning up the plasma. We point out that this effect has to compete with the viscous damping that comes from rotating the plasma through regions of different magnetic field magnitude, the so-called magnetic pumping [14]. Thus, the spin-up can occur only if it can overcome magnetic pumping.

Having established the nature of the spin-up, we now consider more specifically the situation in the presence of a MARFE. As mentioned in the Introduction, the MARFE can be described as a thermal condensation instability. In order to keep the MARFE stationary, a feedback gas puffing system is needed, which stabilizes this instability. Thus, while the plasma still radiates profusely from the MARFE region, the density is kept at a steady value instead of growing continuously. To study the dynamical behavior we consider the fluid equations obtained from Braginskii's equations, in the superdiamagnetic and subsonic limit ($U_d < u_p < B_p c_s/B$) and for time scales faster than the ion transport time ($t < \tau_{iD} \sim$

$(L/q\rho_i)^2/\nu_{ii})$, as given in Ref. [14]. Here, u_p is the plasma poloidal velocity, U_d is the diamagnetic speed, L is the characteristic scale length, ρ_i the ion gyroradius and ν_{ii} the ion collision frequency. The equations, valid for the edge plasma region, are,

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}_\perp) + \mathbf{B} \cdot \nabla n \frac{u_\parallel}{B} = S, \tag{4}$$

$$m_i\left(\frac{\partial}{\partial t}(n\mathbf{u}) + \nabla \cdot (n\mathbf{uu})\right) = -\nabla p - \nabla \cdot \mathbf{\Pi} + \frac{1}{c}\mathbf{J} \times \mathbf{B}, \tag{5}$$

$$-\nabla\phi + \frac{1}{c}\mathbf{u} \times \mathbf{B} = \mathbf{R}_{ei}, \tag{6}$$

$$\frac{3}{2}n\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right)T = \nabla_\parallel \kappa_\parallel \nabla_\parallel T + \nabla_\perp \kappa_\perp \nabla_\perp T - nT\nabla \cdot \mathbf{u} - L(n,T), \tag{7}$$

where the particle source is S (i.e., gas puffing) and the radiation function is $L(n,T)$. The plasma velocity $\mathbf{u}$ is the ion velocity and the ion and electron temperatures are equal $T_i = T_e = T$. In the electron momentum equation (6), the term $\mathbf{R}_{ei}$ represents a general electron–ion momentum exchange (collisional and anomalous), which will be responsible for the diffusive radial transport. In the total momentum balance equation (5), the stress tensor $\mathbf{\Pi}$ is responsible for damping by magnetic pumping. The parallel and perpendicular thermal conductivities, $\kappa_\parallel, \kappa_\perp$, in the total energy equation (7), determine the appearance of the MARFE. No heating sources are assumed since ohmic heating is negligible near the edge.

First, to simplify the analysis, we will assume that the equilibrium density function in the stationary MARFE is determined by the energy equation (7) only. That is, the energy balance established in the MARFE, defined by the equation,

$$\nabla_\parallel \kappa_\parallel \nabla_\parallel T + \nabla_\perp \kappa_\perp \nabla_\perp T - \frac{3}{2}nu_\parallel \nabla_\parallel T - nT\nabla_\parallel u_\parallel - L(n,T) = 0 \tag{8}$$

produces temperature and density distributions that are minimum and maximum, respectively, at the HFS, where the MARFE is located. Thus, instead of solving Eq. (8), we propose an equlibrium density function with these feature, expressed as,

$$n(r,\theta) \approx n_0(r)\big(1 - \alpha(r)\cos\theta\big), \tag{9}$$

with the function $\alpha(r)$ determining the radial extent of the MARFE, whose value should be related to the function $L(n,T)$, while $n_0(r)$ corresponds to the equilibrium density profile. Note that the function $\alpha(r)$ is such that $|\alpha(r)| < 1$, it peaks at $r = r_M$, the MARFE's radial position, and vanishes within the plasma core.

Now we can study the dynamical evolution of the plasma described by Eqs. (4)–(6), for the poloidally varying density given in (9). A low-β plasma will be considered, so that we can neglect the time variations of the magnetic field. For the vectors representation, we use the contravariant components in the orthogonal coordinates (ψ,θ,ζ) defined by, $u^\gamma = u \cdot \nabla\gamma$, where $\gamma = \psi,\theta,\zeta$, as well as the covariant components: $u_\gamma = |\nabla\gamma|^2 u^\gamma$ (toroidal and poloidal components are $u_T = |u_\zeta \nabla\zeta|$ and $u_p = |u_\theta \nabla\theta|$, respectively). From Eq. (6) we get,

$$\mathbf{u}_\perp = \frac{c}{B^2}(\mathbf{B} \times \nabla\phi + \mathbf{R}_{ei} \times \mathbf{B}),$$
$$\mathbf{B} \cdot \nabla\phi = \mathbf{B} \cdot \mathbf{R}_{ei} = 0, \tag{10}$$

for the last equality we assumed, for simplicity, that $\mathbf{R}_{ei}$ is perpendicular to $\mathbf{B}$, which gives the realtionship, $R_{ei}^\theta = R_{ei}^\zeta B_T^2/qB_p^2$. Eq. (10) implies that the electrostatic potential is a flux function $\phi = \phi(\psi)$. Adding a parallel component to the velocity, $\mathbf{u}_\parallel = u_\parallel \hat{b}$, the total velocity can be written as,

$$\mathbf{u} = \mathbf{u}_\parallel + \mathbf{u}_\perp = \frac{B}{B_p}u_p\hat{b} + Rc\phi'\hat{\zeta} + \mathbf{R}_{ei} \times \frac{\hat{b}}{B}, \tag{11}$$

where $u_p = (B_p/B)(u_\parallel - cI\phi'/B)$. This is not a flux function and therefore it varies along the poloidal direction; but its flux-surface average describes the bulk plasma flow. In order to study the evolution of the averaged plasma velocity, we first take the flux-surface average of the toroidal component of Eq. (5) to obtain,

$$m_i\frac{\partial}{\partial t}\langle nu_\zeta\rangle = -\frac{\partial}{\partial\psi}\big\langle nu_\zeta u^\psi\big\rangle, \tag{12}$$

which gives the conservation of toroidal angular momentum. Here the contravariant radial velocity $u^\psi = \mathbf{u} \cdot \nabla\psi = R^2 R_{ei}^\zeta$, is proportional to the toroidal friction force. In deriving (12) use was made of axisymmetry and of charge conservation, $\nabla \cdot \mathbf{j} = 0$, which implies the ambipolarity condition: $\langle \mathbf{j} \cdot \nabla\psi\rangle = 0$. This is certainly valid for collisional transport, but for anomalous transport due to electrostatic fluctuations it also holds, since they approximately preserve quasineutrality. In any case, the non-ambipolar contribution would be of order $(v_{A\theta}/c)^2$, as it was shown in Ref. [16] (where $v_{A\theta}$ is the poloidal Alfvén speed), which is very small and therefore it will be neglected. Additionally, we need the parallel momentum equation from Eq. (5),

$$m_i\left(\frac{\partial}{\partial t}(n\mathbf{B}\cdot\mathbf{u})+\mathbf{B}\cdot\nabla\cdot(n\mathbf{uu})\right) = -\mathbf{B}\cdot\nabla p+\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}. \tag{13}$$

For the solution of Eqs. (4), (12) and (13), we will assume all relevant time scales are of higher order than the sound wave transit time, qR/c_s, so we will solve the equations expanding in this parameter. To lowest order, Eq. (13) gives $\mathbf{B}\cdot\nabla p = 0$, so $p = p(\psi)$. Therefore the temperature must vary in inverse proportion to the density (9) on a flux surface,

$$T(\psi,\theta) = p(\psi)/n(\psi,\theta). \tag{14}$$

Now, we can obtain an equation for the average density by flux-surface averaging Eq. (4),

$$\frac{\partial}{\partial t}\langle n\rangle + \big\langle\nabla\cdot(nu_\perp)\big\rangle = \langle S\rangle. \tag{15}$$

However, for a stationary MARFE, density evolves according to the transport time scale which is very long in relation to our relevant scales. Thus,

$$\frac{\partial}{\partial t}\delta n = \frac{\partial}{\partial t}\big(n-\langle n\rangle\big)\sim\epsilon^2 n$$

is of higher order. Inside the MARFE region the equilibrium is given by $\langle\nabla\cdot nu_\perp\rangle = \langle S\rangle$. The poloidally dependent part of the continuity equation is obtained by substracting Eq. (15) from (4). We will assume a poloidally uniform gas puffing, so that $S = \langle S\rangle$, and thus,

$$\nabla\cdot(n\mathbf{u}) - \langle\nabla\cdot nu_\perp\rangle = 0 \tag{16}$$

or equivalently,

$$\mathbf{B}\cdot\nabla\left(n\frac{u_p}{B_p}\right) = \left\langle\nabla\cdot\left(\frac{n}{B}\mathbf{R}_e\times\hat{b}\right)\right\rangle - \nabla\cdot\left(\frac{n}{B}\mathbf{R}_e\times\hat{b}\right). \tag{17}$$

The right-hand side contains the poloidally asymmetric radial diffusion, given by the quantity $\mathbf{R}_{ei}$. In fact, for collisional transport, $\mathbf{R}_{ei}\times\hat{b}/B = -D_\perp(\nabla_\perp n + n\nabla_\perp \log T_e^{-1/2})$, with $D_\perp = \rho_e^2\nu_{ei}$. For anomalous transport, an analogous expression could be written, proportional to the effective diffusion coefficient and the density gradient, and possibly including a convective term. It is mainly the poloidal dependence of the density, $n(\theta)$, what makes the RHS of Eq. (17) nonzero in our case, and this is the cause of the spin-up. There is an additional contribution due to the poloidal dependence of the diffusion coefficient which was considered in [15], and both are of the same order, thus enhancing the resulting spin-up. We are interested in obtaining an equation for the average poloidal plasma velocity in order to elucidate its evolution; for this we use the flux function $\lambda(\psi) = \langle u_p/B_p\rangle$, which is proportional to the average poloidal angular momentum. Therefore, in terms of this parameter, the complete poloidal velocity is written as,

$$\frac{u_p}{B_p} = \lambda(\psi)+\delta\lambda, \tag{18}$$

the latter containing the poloidal dependence. It is possible to write $\lambda(\psi)$ in terms of the averaged toroidal and parallel components of the flow, by eliminating ϕ' in Eq. (11) from the two respective components, after being averaged over a flux surface. The result is,

$$\lambda = \frac{1}{\Theta\langle nB^2\rangle}\left(\langle n\mathbf{B}\cdot\mathbf{u}\rangle - \langle nu_\zeta\rangle\frac{\langle n\rangle I}{\langle nR^2\rangle} - \big\langle nB^2\delta\lambda\big\rangle + \langle n\delta\lambda\rangle\frac{\langle n\rangle I^2}{\langle nR^2\rangle}\right), \tag{19}$$

where,

$$\Theta = 1-\frac{I^2\langle n\rangle^2}{\langle nB^2\rangle\langle nR^2\rangle}\sim\frac{B_p^2}{B^2}.$$

The time derivative of this equation will give the evolution of the poloidal velocity. We already have Eq. (12) for the average toroidal angular momentum, but still have to get the equation for the surface-averaged parallel flow. By averaging Eq. (13) one obtains,

$$\frac{\partial}{\partial t}\langle n\mathbf{B}\cdot\mathbf{u}\rangle = \big\langle\mathbf{B}\cdot\mathbf{u}(\partial n/\partial t - S)\big\rangle + \langle\mathbf{R}_{ei}\cdot\nabla\times n\mathbf{u}\rangle + \left\langle\left(\mathbf{B}\cdot\mathbf{uu}-\frac{u^2}{2}\mathbf{B}\right)\cdot\nabla n\right\rangle - \frac{1}{m_i}\langle\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}\rangle, \tag{20}$$

where use has been made of Eq. (16), Faraday's law and our assumption of quasi-static magnetic field. The next to the last term is nonlinear ($\sim u^2$) and may be neglected for small rotation speeds. The term involving the friction, R_{ei}, will be responsible for the poloidal spin-up; it contains a term proportional to λ. In order to express the averages of velocities involved in Eqs. (12) and (20), in terms of our variables of interest λ and $\langle nu_\zeta\rangle$, we make use of the following relations, derived by combining components of Eq. (11),

$$u_\zeta = -\lambda I\Delta_R + \frac{R^2}{\langle nR^2\rangle}\langle nu_\zeta\rangle + I\delta\lambda, \tag{21}$$

$$B\cdot u = \lambda\left(B^2 - \frac{nI^2}{\langle nR^2\rangle}\right) + \frac{nI}{\langle nR^2\rangle}\langle nu_\zeta\rangle + B^2\delta\lambda, \tag{22}$$

where $\Delta_R = (nR^2/\langle nR^2\rangle) - 1$ and we assumed the radial component $R_{ei}^\psi = 0$. The evolution equation for the poloidal average velocity is obtained from Eq. (19) using the forementioned assumption that the density varies in a slower time scale, thus treating it here as a constant. Keeping dominant terms (neglecting terms of order $\sim B_p/B_T$) the result is,

$$\begin{aligned}\frac{\partial\lambda}{\partial t} = \frac{1}{\Theta\langle nB^2\rangle}\Bigg[& I\left\langle\frac{u^\psi}{R^2}\frac{\partial}{\partial\psi}nI\lambda\Delta_R\right\rangle \\ & - \frac{I\langle n\rangle}{\langle nR^2\rangle}\frac{\partial}{\partial\psi}I\lambda\langle nu^\psi\Delta_R\rangle \\ & + \lambda\left\langle\left(B^2 - \frac{nI^2}{\langle nR^2\rangle}\right)u^\psi\frac{\partial n}{\partial\psi}\right\rangle \\ & - \lambda\left(\langle B^2\rangle - \frac{\langle n\rangle I^2}{\langle nR^2\rangle}\right)\frac{\partial}{\partial\psi}\langle nu^\psi\rangle \\ & - I\left\langle\frac{u^\psi}{R^2}\frac{\partial}{\partial\psi}\left(\frac{nR^2}{\langle nR^2\rangle}\langle nu_\zeta\rangle\right)\right\rangle \\ & - \frac{I\langle n\rangle}{\langle nR^2\rangle}\frac{\partial}{\partial\psi}\left(\frac{\langle nR^2u^\psi\rangle}{\langle nR^2\rangle}\langle nu_\zeta\rangle\right) \\ & - \frac{I\langle nu_\zeta\rangle}{\langle nR^2\rangle}\left(\left\langle u^\psi\frac{\partial n}{\partial\psi}\right\rangle - \frac{\partial}{\partial\psi}\langle nu^\psi\rangle\right) \\ & + F(\delta\lambda) - \frac{\langle\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}\rangle}{m_i}\Bigg],\end{aligned} \tag{23}$$

where $F(\delta\lambda)$ is a function that contains all terms that depend on $\delta\lambda$ which is not important to us. The reason is that we will not try to solve Eq. (23) but use it to determine the conditions for poloidal velocity growth. It turns out that this equation has a weaker dependence on $\langle nu_\zeta\rangle$, so that the RHS is just a function of λ to lowest order in ϵ. Before we make this reduction, in Eq. (23) we have to evaluate the viscous stress term, giving the magnetic pumping, which is only determined by the parallel viscosity [14], and is also proportional to λ. Using the result obtained in Ref. [17] it is found,

$$\langle\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}\rangle = \mu_0\lambda + \kappa, \tag{24}$$

where,

$$\mu_0 = (4.095p_i/\nu_{ii})\big\langle(\hat{b}\cdot\nabla B)^2\big\rangle,$$
$$\kappa = (4.095p_i/\nu_{ii})\big\langle(\hat{b}\cdot\nabla B)^2\delta\lambda\big\rangle.$$

Eq. (23) is valid for arbitrary flux surface geometry, but it is difficult to extract the dominant terms is this form. To proceed further we take the limit of circular flux surface mentioned above and use the density function (9). Then, the replacements $\partial/\partial\psi \to (1/r)(\partial/\partial r)$ and $u^\psi \to ru_r$ are made and the averages are taken, and the terms are ordered according to the small parameter ϵ. It is found that all terms proportional to $\langle nu_\zeta\rangle$ cancel out to lowest order, and taking into account that $\langle nu_\zeta\rangle/\lambda n_0 I \sim \epsilon$, only the terms proportional to λ remain. The evolution of the poloidal velocity to lowest order is finally cast as,

$$\frac{\partial\lambda}{\partial t} = \Gamma\lambda + H, \tag{25}$$

with

$$\begin{aligned}\Gamma = \frac{-1}{\sigma}\Bigg(&\left(\left(1+\frac{\alpha}{2}(1-\alpha)\right)\frac{1}{r}\frac{\partial}{\partial r}ru_{r1} + \frac{u_{r1}}{n_0}\frac{\partial n_0}{\partial r}\right. \\ &\left. - \frac{1}{n_0 r}\frac{\partial}{\partial r}\big(rn_0\alpha[u_{r0}+u_{r2}]\big) + \frac{4.095p_i\epsilon}{2m_in_0\nu_{ii}q^2R^2}\right),\end{aligned}$$
$$H = F_0(\delta\lambda) + \kappa/m_1n_0\epsilon\sigma, \tag{26}$$

where, $\sigma = (2 - 9\alpha/4 + 3\alpha^2/2 + q^{-2})\epsilon$, $u_{rn} = \langle u_r\cos(n\theta)\rangle$ and $F_0(\delta\lambda)$ is the order-ϵ contribution to $F(\delta\lambda)$. The average u_{r1} measures the poloidally asymmetric component of the radial velocity. The parameter Γ includes both the spin-up drive and the damping by magnetic pumping, but the latter is an order ϵ smaller than the spin-up and it will be easy to overcome the damping; the instability condition is $\Gamma > 0$. It is clear that the sign of Γ depends on the sign of the derivatives of the weighted averages of the radial velocity, especially u_{r1}. First of all, we point out that when $\alpha = 0$, the equation reduces to the usual spin-up found in [15]. In that case the spin-up condition on the asymmetrical velocity is simply $(d/dr)(rnu_{r1}) < 0$. In our case, where the MARFE produces the asymmetrical density (9), the function $\alpha(r)$ modifies the stability criterium substantialy. For definitness we assume the radial flow is due to a diffusive and a convective component, so that $nu_r = -Ddn/dr - nu_0(r)$, with $u_0(r)$ independent of θ, but the diffusion coefficient is allowed to be poloidally varying as $D = D_0(1 + \Delta\cos\theta)$ in addition to n. Two limits may be considered:

(a) When D is poloidally dependent but α is constant, the condition $\Gamma > 0$ gives,

$$|\Sigma| \equiv \left|(rn_0'' + n_0')\left(\Delta - \alpha + \Delta\frac{\alpha}{2}(1-\alpha)\right) - \frac{n_0'^2}{n_0} r\Delta\frac{\alpha}{2}(1-\alpha) - \frac{(r\alpha n_0 u_0)'}{D_0}\right| > \frac{\eta_0\epsilon r}{2m_i D_0 q^2 R^2}, \tag{27}$$

with $\eta_0 \equiv 4.095 p_i/\nu_{ii}$ and the prime denotes radial derivative, while the constraint on the density profile is $\Sigma > 0$. When $\alpha = 0$ the conditions from [15] are recovered, as mentioned above, which requires the density profile to be concave-up near the edge. There is, however, another possibility for spin-up when the diffusion coefficient is symmetric ($\Delta = 0$) and just n is poloidally dependent; for $\alpha = \text{const}$ and $u_0 = 0$, the conditions for this are,

$$|rn_0'' + n_0'| > \frac{\eta_0\epsilon r}{2m_i D_0 \alpha q^2 R^2}, \tag{28}$$

and $rn_0'' + n_0' < 0$. This constraint is the opposite to the case with $\alpha = 0$ and a poloidally dependent D, and it means that the density profile should be concave-down at the edge, which is the usual profile shape. Condition (28) could be satisfied for a sufficiently asymmetric density profile ($\alpha \approx 1$).

(b) For the case of variable $\alpha(r)$ the conditions are more complicated and depend on the precise radial dependence. As an example, for $\alpha(r) = \alpha_0 \exp[-(r - r_0)^2/2w^2]$ and constant D, the conditions at $r = r_0$ become,

$$rn_0'' + n_0' - rn_0 F(\alpha_0)/w^2 < 0,$$
$$\left|rn_0'' + n_0' - rn_0 F(\alpha_0)/w^2\right| > \frac{\eta_0\epsilon r}{2m_i D_0 \alpha_0 q^2 R^2},$$

with $F(\alpha) = 2 - (1 - \sqrt{(1-\alpha)/(1+\alpha)})/\alpha > 0$, which can be satisfied by a wider range of density profiles than the case for $\alpha = \text{const}$.

3. Conclusions

We have shown that when a stationary MARFE is produced in a tokamak, the flow pattern thus created is unstable to a poloidal rotation perturbation in the region where the MARFE is located, i.e., near the edge region. Our treatment of the spin-up is made for a poloidally dependent density, in contrast to previous treatments where the MARFE was not present. The spin-up is excited by the asymmetric density associated with the MARFE, even if the radial diffusion is symmetric, especially for a strong MARFE (large α). The ensuing rotation would have an intrinsic radial shear thus creating a zonal flow. Although we did not study the turbulence, we invoke the well-known relationship between zonal flows and turbulence suppression [18], which will produce an ETB. This would explain the observed improved confinement in HT-7. A direct comparison with the experiment could be done in order to check if the unequality $\Gamma > 0$ is actually satisfied. However, we expect it to hold in most practical cases since the drive for the spin-up is an order ϵ larger than the damping, which makes the proposed mechanism plausible. It is quite likely to have plasma spin-up during stationary MARFEs, and the results of HT-7 might already be an evidence of it, as measured by the change in confinement properties.

Although there are no other reports of improved confinement during stationary MARFEs beside that of HT-7, it is interesting to mention that a similar process may occur just before a density-limit disruption. There are indications that the confinement time tends to increase previous to the disruption [19], possibly implying that a thermal barrier starts forming when density and the associated radiation increase. Poloidal asymmetries in these parameters would lead to a spin-up mechanism as the one described here.

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