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The effect of a sheared flow on magnetic islands in plasmas with non-axisymetric geometry

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ABSTRACT

The stability of a magnetic island in a toroidal magnetic confinement device depends on various factors besides the usual tearing-mode stability parameter Δ', determined by the local current profile.The presence of a sheared flow in the vicinity of the rational surface that supports the island is one of the factors that affects its stability since it can give rise to a polarization current around the island position. The contribution of the polarization current to the stability has been computed for a tokamak geometry. Here, we consider the case of magnetic islands with a shear flow in a stellarator which has a non-axisymmetric magnetic geometry. The main difference is a contribution to the polarization current from the toroidal electrostatic oscillation. A correction due to the global toroidal magnetic geometry is also present. It is found that the regime where the stability is affected corresponds to the large island width relative to the ion gyroradius. Thus, the contribution is relevant for low-temperature regimes. In that case, the polarization current is destabilizing for frequencies larger than the ion diamagnetic frequency. Our results imply that the sheared flow can produce a growth of the magnetic island in a cold plasma but it can become narrower as the temperature rises.

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1. Introduction

The plasma in a toroidal confinement device has nested magnetic surfaces that have constant pressure which hold the high-temperature plasma in the core. When a helical magnetic mode perturbs the magnetic surfaces a reconnection process takes place at rational surfaces, where the mode resonates, and creates magnetic islands that connect inner-to-outer plasma regions, deteriorating the confinement. Thus, the study of magnetic islands has received much attention from their formation to the subsequent evolution. Usually, the islands are formed by tearing modes that are unstable when the parallel current profile has a negative gradient around the rational surface, which is the common case in tokamaks, but if non-axisymmetric fields are present such as in stellarators or when the magnetic field ripple is large, the islands can appear because of the symmetry breaking, even without plasma. Once formed, the stability of the island is determined by the plasma conditions in its neighborhood (*1*). If the islands grow in size they can deteriorate the confinement or even lead to disruptions.

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Several effects enter in the island evolution. One of them is the presence of plasma flows which normally inhibit magnetic reconnection (2, 3). But when there is a relative flow between the island and the surrounding plasma, a friction force arises which tends to reduce the flow and when it stops the island can grow. This is what happens with the externally imposed resonant magnetic perturbations at the time of the so-called mode penetration (4). In the linear theory of tearing modes, pressure gradients impose a rotation velocity to the reconnecting structures of the order of the ion diamagnetic frequency (5). In the nonlinear theory, it is found that the diamagnetic effects give rise to the formation of magnetic islands, even in regimes where the modes are linearly stable (6). Depending on the island width size, w, relative to the ion gyroradius ρ_i, the drive for island formation can be the polarization current (for $w/\rho_i > 1$) or the perpendicular velocity of the unmagnetized ions (for $w/\rho_i \ll 1$). The role of the polarization current was analyzed in (7) for both large and small islands establishing the effects that have to be included in each regime. These studies have made clear that plasma flow relative to the island contributes to the island stability through the polarization current. This current is due to the polarization drift associated with the plasma flow along the island separatrix which is subject to an acceleration when passing by the nozzle formed by the flux surfaces of the periodic island profile. Since the drift velocity is not divergenceless, a parallel return current should appear to maintain quasi-neutrality.

Most works on the role of the polarization current in islands with plasma flow have been done for tokamaks, which have an inherent current to drive the tearing modes. In stellarators, where there is not a significant current, the islands are normally formed in the vacuum field produced by the non-axisymmetric external coils which are in turn modified by the plasma dynamics. In that case the islands are initially locked by the coils but there is a rotational plasma flow due to the ambipolar radial electric field, which is quite important in stellarators. The polarization current associated with the velocity along the island separtrix should then play a relevant role in the island dynamics. For locked islands, it has been found (8) that this current has a destabilizing effect on the island. In this work we analyze the island stability including the polarization current for a toroidal non-axisymmetric plasma. The goal is to determine whether this contribution has a stabilizing or destabilizing effect for both regimes where the island width is larger or smaller than the ion gyroradius. This is of interest for the observations regarding the presence of islands in stellarators, in particular, the association of rational magnetic surfaces with transport barriers observed in TJ-II and LHD (9, 10). The observations suggest that the rational surfaces give rise to magnetic islands and at the same location there can be sheared plasma flows that inhibit turbulent transport, then producing barriers. To explore this possible scenario, it would be necessary to study first behavior of the islands when the shear flow is present. Although the effect of the polarization current has been studied in different situations in tokamaks, as described in (11, 12) relation to the excitation of neoclassical tearing modes, in stellarators there has been not so much work. A possible reason is that magnetic island formation, in general, and the flow-related polarization current, in particular, are not so pervasive as in tokamaks. The study performed here shows that the flow provides a destabilizing contribution when the island width is large relative to the ion Larmor radius.

This paper is organized as follows. In the next section, we make a short description of the treatment of magnetic islands introducing the role of the polarization current. The analysis for stellarators is done in magnetic coordinates so we describe the Boozer representation which is the one used here. In Section 3, we compute the contribution of the polarization current to the stability of the island in three regimes: the MHD limit, and the small and large island width. In the last section, we discuss the results and give the conclusions.

2. Magnetic islands and geometry

Formation of magnetic islands takes place at rational surfaces, where $q = 1/\iota = m/n$, due to reconnection of magnetic field lines mainly caused by tearing-mode instability. In the vicinity of the rational surface, there is a layer where a component of the electric field parallel to the magnetic field decouples the magnetic field lines from the fluid, so that a magnetic perturbation grows leading to the a change of topology. Since the perturbations are periodic a chain of islands is formed repeating m times in the poloidal direction and n times in the toroidal direction. The linear theory applies when the island is smaller than the layer width and predicts exponential growth. For larger islands, a nonlinear theory has to be used in which the growth is proportional to time (13) and eventually leads to saturation. Even in this case, the island width is assumed to be smaller than the equilibrium scale-lengths $w < L$. The width of the islands is determined by the amplitude of the perturbation field and the equilibrium magnetic field shear. In order to express this in the toroidal non-axisymmetric geometry of a stellarator, it is convenient to use Boozer coordinates, so we write

$$\mathbf{B}_0 = \nabla\psi \times \nabla(\theta - \iota\phi) = G\nabla\phi + I\nabla\theta + h\nabla\psi, \tag{1}$$

$$\mathbf{B}_1 = \nabla A \times \nabla\zeta + \nabla\vartheta \times \nabla\alpha \tag{2}$$

for the equilibrium and the perturbed field, respectively. Here, θ and ϕ are the poloidal and toroidal angles and G, I and the rotational transform ι are flux functions depending of ψ only, while $h = h(\psi, \theta, \phi)$. The relevant coordinate is the helical angle at the rational surface $\alpha = \theta - \iota_0\phi$. The perturbation describes the island through the symmetry-breaking component A as proposed in (14). This component can arise from the coil errors in stellarators, producing vacuum magnetic islands, or from plasma produced fluctuations. In both cases, we can adopt a single harmonic approximation for which $A(\psi_0, \alpha) = A_0 \cos m\alpha$. Then, the flux function that describes the magnetic surfaces including the island is (14)

$$\Psi = \frac{1}{2}\iota' x^2 - \frac{A_0}{\psi'}\cos\xi \equiv \frac{A_0}{\psi'}(2\chi^2 - 1) \tag{3}$$

with $\xi = m\alpha$ being the helical coordinate, $\iota'_0 = \mathrm{d}\iota/\mathrm{d}\psi(\psi = \psi_0)$ and $x = (\psi - \psi_0)^{1/2}$. The last part defines a normalized flux function χ to label the magnetic surfaces. The X-points of the magnetic island are given by $\cos\xi = -1$ and $\chi = 1$ ($\Psi\psi'/A_0 = 1$) is the separatrix, while O-points have $\cos\xi = 1$ and $\chi = 0$ ($\Psi\psi'/A_0 = -1$).

The island half-widths with plasma and in vacuum are as follows:

$$w = 2\sqrt{\left|\frac{A_0}{\iota'\psi'}\right|}, \quad w^v = 2\sqrt{\left|\frac{A_0^v}{\iota'\psi'}\right|} \tag{4}$$

with $\psi' \equiv \mathrm{d}\psi/\mathrm{d}\rho = B_0/L_\alpha, L_\alpha \equiv |\nabla\alpha|$ and $\iota' = \iota/L_s; L_s = Rq/\hat{s}$ is the magnetic shear length, R is the major radius and $\hat{s} = (r/q)\,\mathrm{d}q/\mathrm{d}r$ is the magnetic shear.

2.1. Evolution of magnetic islands

In the linear theory, the tearing mode instability is treated in two regions: the MHD outer region and the non-ideal small inner region centered at the rational resonant surface. Then the two solutions are matched through the derivative jump parameter Δ' defined as follows:

$$\Delta' = \frac{1}{A(x=0)} \frac{dA}{dx}\bigg|_{x=0^-}^{x=0^+}. \tag{5}$$

When $\Delta' > 0$ the configuration is unstable and reconnection ensues. The nonlinear description provides a relation for the matching condition between the stability parameter Δ' and the cosine component of the parallel current obtained from Ampere's law which in our coordinates is

$$\Delta' A_0 = -\frac{4\pi}{c} \int_{-\infty}^{\infty} dx \oint d\xi \cos\xi \frac{\gamma}{B^2} \mathbf{J}_1 \cdot \mathbf{B}_0 \tag{6}$$

where $\gamma = G + ιI = \mathbf{B}_0 \cdot (\hat{\phi} + (ι - ι_0)\hat{\alpha})$. In addition, the sine component is related to torque balance

$$\int_{-\infty}^{\infty} dx \oint d\xi \sin\xi \frac{\gamma}{B^2} \mathbf{J}_1 \cdot \mathbf{B}_0 = 0 \tag{7}$$

and these coupled equations determine the island evolution. When Equation (6) is combined with Ohm's law and Faraday's law one obtains the Rutherford equation (*13*) for the island width

$$\frac{dw}{dt} = D_\eta \Delta', \tag{8}$$

where D_η is of the order of the magnetic diffusion coefficient $(4\pi\eta/c^2)$; Δ' represents the free energy available for the instability and it has to satisfy $\Delta' > 0$ to destabilize islands as well as the tearing mode. When there are additional contributions to the current they are included in the right-hand side of Equation (8). For instance, in toroidal devices there is a contribution from the bootstrap current Δ_{BS} which gives rise to the neoclassical tearing modes. It turns out that an island can be unstable even if it is linearly stable ($\Delta' < 0$) as long as Δ_{BS} is large enough (*15*). In our case we are interested in the polarization current J_{pol}, which is the fluctuating part of the parallel current, and can be included in the evolution equation of the magnetic island as follows:

$$\frac{dw}{dt} = D_\eta(\Delta' + \Delta_{pol}) \tag{9}$$

the polarization current contribution Δ_{pol} is given by

$$\Delta_{\mathrm{pol}} = -\frac{16 L_s}{c B_0 w} \oint d\xi \int_{-\infty}^{\infty} dx J_{\mathrm{pol}} \cos(\xi). \tag{10}$$

It is worth mentioning that if one is interested in the evolution of the vacuum island in a stellarator it is necessary to include the phase difference between the island with plasma relative to the vacuum island $\Delta\phi$. In this case, Δ' can also be divided into cosine and sine components involving the phase difference

$$\Delta'_c = \Delta'_0\left[1 - \nu\frac{A^V}{A_0}\cos(\Delta\phi)\right] \quad \Delta'_s = -\Delta'_0\nu\frac{A^V}{A_0}\sin(\Delta\phi) \tag{11}$$

where ν is an order-unity parameter (*14*). The cosine component Δ'_c will contribute to the island width evolution, while the sine component Δ'_s will contribute to the torque balance, entering in the right-hand-side of Equation (7), which will determine the electromagnetic torque N_{EM} (*16*). Torque balance determines the relative phase of the island which also includes a viscous torque N_V,

$$\frac{d^2\Delta\phi}{dt^2} = N_{EM} + N_V. \tag{12}$$

The goal is to compute the polarization current in order to obtain Δ_{pol}. If $\Delta_{pol} < 0$ then J_{pol} has a stabilizing effect in Equation (9) while it will be destabilizing if $\Delta_{pol} > 0$. Δ_{pol} can be parametrized as (*7*)

$$\Delta_{pol} = 4g\frac{L_s^2}{w^3}\frac{\omega(\omega - \omega_{*i})}{k^2 v_A^2}, \tag{13}$$

where ω is the rotation frequency of the island, $\omega_{*i} = k(T_i/q_iBL_n)$ the ion diamagnetic frequency and g will determine the sign of Δ_{pol}. Combining Equations (10) and (13), we obtain and expression for g in the limit $T_i \rightarrow 0$ that will be considered in the next section:

$$g = -\frac{4k^2 v_A^2 w^2}{cB_0L_s\omega^2}\oint d\xi \int_{-\infty}^{\infty} dx J_{pol}\cos\xi. \tag{14}$$

It can be proved (*1*) that boundary conditions in the island lead to a relation for the island rotation frequency given by $\omega = f\omega_{*e} + (1-f)\omega_{*i}$, with f being the flattening factor inside the island of the ion temperature profile. Wide islands have profile flattening so $f = 0$, meanwhile for narrow islands there are gradients and $f = 1$.

3. Polarization current

The polarization current is defined as the oscillating part of the parallel current, $\mathbf{J}_{pol} \equiv \mathbf{J}_{\parallel} - \langle \mathbf{J}_{\parallel} \rangle$. We follow (*7*) in deriving the parallel current but using the geometry of the stellarator in the coordinates of last section. The transport in a stellarator depends on this special geometry (*17*) and it is reflected in the calculations of J_{pol}. The model uses the drift-MHD equations with cold ions, retaining the nonlinearity in the ion response. The width of the layer where the velocity changes rapidly is of the order of the ion-sound Larmor radius, ρ_s, so that, finite ion Larmor radius effects and ion diamagnetic drift can be neglected. Thus, only the electron diamagnetic frequency will be of importance. The relevant equations are particle conservation, quasi-neutrality ($\nabla \cdot \mathbf{J} = 0$), Ohm's law and electron energy conservation, which for an equilibrium reduce to (*7*)

$$\vec{v}_E \cdot \nabla n = \frac{1}{e}\nabla_{\parallel} J_{\parallel}, \tag{15}$$

$$\nabla_{\parallel} J_{\parallel} = \frac{c^2}{4\pi v_A^2}\vec{v}_E \cdot \nabla U, \tag{16}$$

$$\frac{1}{n}\nabla_{\parallel}n - \frac{e}{T_e}\nabla_{\parallel}\varphi = -(1+a)\frac{\nabla_{\parallel}T_e}{T_e}, \tag{17}$$

$$T_e = T_\sigma(\chi) \tag{18}$$

the last equation representing the simplest energy conservation when the parallel transport dominates and $\sigma = \pm$ denotes the profiles on each side of the island. Combining the last two equations one obtains the normalized linear Boltzmann relation

$$\tilde{n} = \tilde{\varphi} + H_\sigma(\chi) \tag{19}$$

where $H_\sigma(\chi)$ is an integration constant, $\tilde{n} = (n - n_0)/n_0'$ and $\tilde{\varphi} = (e\varphi/T_e)L_n/w$, with n_0 the density at the O-point. We will consider cases with $H_\sigma(\chi) = \sigma H(\chi)$ and $H_\sigma(\chi) = 0$ inside the island. Combining Equations (15) and (16), the later can be replaced by

$$\rho_s^2\nabla_\perp^2\tilde{\varphi} - K(\tilde{\varphi}) = H_\sigma(\chi) \tag{20}$$

where $K(\tilde{\varphi})$ is another integration constant. This is like the Grad–Shafranov equation for the island and its solution would give the potential $\tilde{\varphi}$ entering in the polarization current derived below. The relevant equations are then Equations (15), (19) and (20).

The $\mathbf{E} \times \mathbf{B}$ velocity, $\vec{v}_E$, has a toroidal component which affects the polarization current in stellarators through the first term of Equation (15). The radial component of $\vec{v}_E$ is found to be

$$\mathbf{v}_E \cdot \nabla\psi = -\frac{G}{\iota B_0}\nabla_{\parallel}\varphi - \frac{\partial\varphi}{\partial\alpha}. \tag{21}$$

Using this in Equations (15) and (19), the resulting parallel current is

$$J_{\parallel} = \frac{e\sigma n_0'}{4\chi A}\frac{\mathrm{d}H}{\mathrm{d}\chi}\frac{G}{\iota B_0}\varphi + \frac{e\sigma n_0'}{4\chi A}\frac{\mathrm{d}H}{\mathrm{d}\chi}\frac{\partial\varphi}{\partial\alpha}s + I(\chi), \tag{22}$$

where $I(\chi)$ is an integration constant and s is the magnetic line length. The function $H_\sigma(\chi)$ as well as $K(\chi)$ have to be determined by transport analysis. From the equation for $J_{\parallel}$, the polarization current is obtained by subtracting its average over the island flux surface, which leads to the following expression in which the last term is a new contribution not appearing in the axisymmetric case,

$$J_{\mathrm{pol}} = n_0' e c_s \frac{\rho_s}{w}\frac{L_s}{L_n L_\alpha}\frac{\sigma}{\chi}\frac{G}{\iota^2 B_0}\frac{\mathrm{d}H}{\mathrm{d}\chi}\left(\tilde{\varphi} - \frac{\langle\tilde{\varphi}\rangle}{\langle 1\rangle} + \frac{\iota B_0 s}{G}\frac{\partial\tilde{\varphi}}{\partial\alpha}\right) \tag{23}$$

or, in terms of g,

$$g = -\left(\frac{\omega_{*e}w}{\omega\rho_s}\right)^2 \frac{G}{\iota^2 B_0 L_\alpha}\oint\frac{\mathrm{d}\xi}{\pi}\int_{-\infty}^{\infty}\mathrm{d}x\frac{\sigma}{\chi}\frac{\mathrm{d}H}{\mathrm{d}\chi}\left[\tilde{\varphi}\left(\cos\xi - \frac{\langle\cos\xi\rangle_\chi}{\langle 1\rangle_\chi}\right) + \iota m\frac{B_0}{G}s\frac{\partial\tilde{\varphi}}{\partial\xi}\cos\xi\right]. \tag{24}$$

The last term ($\equiv g_\alpha$) gives the non-axisymmetric effect. We next analyze the effect of this extra term for wide and narrow islands. It will have a contribution to Δ_{pol} only for small ρ_s/w, coming from the region outside the separatrix layer. The type of flow profile should be inserted through Equation (7) in which the torques transmit the information to the inner region, thus determining $H(\chi)$. In the case of locked islands the equivalent Equation (12) would give this information. However, we will not consider this part here, assuming $H(\chi)$ is given, and concentrate on the effect of the non-axisymmetric contribution of the polarization current. Following Connor et al. (7), we consider three different regimes.

3.1. MHD limit

For the large scales relevant for the MHD description, we can take the limit $\rho_s \to 0$ and there is no distinction between electrons and ions. The electric potential can be expanded in powers of ρ_s/w and the lowest term turns out to be a flux function: $\tilde{\varphi}_0(\chi)$. Therefore, it does not depend on the helical coordinate and consequently the new term is $g_\alpha = 0$ for this case. Thus, g will be essentially the same as in the case for slab geometry, modified only by the scale factors $mG/\iota^2 B_0 L_\alpha$ (7),

$$g = \frac{mG}{\iota^2 B_0 L_\alpha} \int_0^\infty \frac{\mathrm{d}\chi}{\chi} \frac{\mathrm{d}}{\mathrm{d}\chi} \left(\frac{1}{\chi} \frac{\mathrm{d}\hat{\Phi}}{\mathrm{d}\chi} \right)^2 \left(\langle \cos^2(m\alpha) \rangle_\chi - \frac{\langle \cos(m\alpha) \rangle_\chi^2}{\langle 1 \rangle_\chi} \right). \tag{25}$$

It can be seen that $g > 0$ and therefore the polarization current has a destabilizing effect in the MHD limit. It has been shown with numerical calculations, however, that the destabilization is overestimated with this approximation (7).

3.2. The large ρ_s/w limit

When $\rho_s \gg w$ the vorticity term is dominant, as the island is thin; then its contributions, on H and K, are negligible. Therefore, the solution in this limit does not contribute to g_α either. The only modification relative to the result in (7) is the scale factors, like the MHD limit, obtaining

$$g = \mathcal{I} \frac{G}{\iota^2 B_0 L_\alpha} \frac{w^2}{\rho^2} = \mathcal{I} \frac{G}{\iota^2 B_0 L_\alpha} \frac{w^2}{\rho_s^2} \left(1 - \frac{\omega_{*e}}{\omega} \right) \tag{26}$$

with

$$\mathcal{I} = -4 \int_1^\infty \mathrm{d}\chi \frac{\mathrm{d}\hat{H}}{\mathrm{d}\chi} \frac{\langle \cos\xi \rangle_\chi}{\langle 1 \rangle_\chi} > 0 \quad \hat{H}(\chi) = \frac{H(\chi)}{1 - \frac{\omega}{\omega_{*e}}}$$

In this limit, Δ_{pol} is also destabilizing in the fast rotation regime $\omega > \omega_{*e}$ since $g > 0$, but its magnitude is smaller than in the MHD limit.

3.3. The small ρ_s/w limit

For the opposite limit, $\rho_s \ll w$, the potential can again be expanded in powers of ρ_s/w keeping the first two terms. This provides a correction to the MHD solution. Solving

Equation (16), the resulting electric potential is

$$\tilde{\varphi} = \tilde{\varphi}_0 + \frac{\rho}{w}\tilde{\varphi}_1 \tag{27}$$

with

$$\tilde{\varphi}_0 = \tilde{\varphi}_{\text{ext}}(x,\alpha) = a[|x| - \cos(m\alpha/2)]\Theta(|x| - \cos(m\alpha/2)), \tag{28}$$

$$\tilde{\varphi}_1(x,\alpha) = \frac{a}{2}[\mathrm{e}^{-w||x|-|\cos(m\alpha/2)||/\rho} - \mathrm{e}^{-w(|x|+|\cos(m\alpha/2)|)/\rho}], \tag{29}$$

where $a = \tilde{\varphi}_0'(1)\cos\xi/2$. Since there is now a dependence on the helical coordinate α, the new term g_α would be non-zero. There two regions:

(A) Inside the separatrix $|x| < \cos((m/2)\alpha)$, $\Theta(|x| - \cos((m/2)\alpha)) = 0$
(B) Outside the separatrix $|x| > \cos((m/2)\alpha)$, $\Theta(|x| - \cos((m/2)\alpha)) = 1$

have to be considered separately to solve for g_α. Inside the separatrix, the profiles are uniform ($\mathrm{d}H/\mathrm{d}\chi = 0$). Meanwhile, in the outside, they have gradients. The derivatives in the helical coordinate of the two terms of the electric potential $\tilde{\varphi}$ are as follows:

$$\begin{aligned}\frac{\partial\tilde{\varphi}_1}{\partial\alpha} = &-\frac{m\tilde{\varphi}_0'(1)}{4}\sin\frac{\xi}{2}\left[\left(1 + \frac{\frac{w}{\rho}|\cos\frac{\xi}{2}|}{\mathrm{sgn}(|x| - |\cos\frac{\xi}{2}|)}\right)\mathrm{e}^{-w||x|-|\cos\frac{\xi}{2}||/\rho}\right.\\ &\left. -\left(1 - \frac{w}{\rho}\left|\cos\frac{\xi}{2}\right|\right)\mathrm{e}^{-w(|x|+|\cos(\xi/2)|)/\rho}\right] < 0\end{aligned} \tag{30}$$

and

$$\frac{\partial\tilde{\varphi}_{\text{ext}}}{\partial\alpha} = \frac{\tilde{\varphi}_0'(1)m}{2}\left(\sin\xi - |x|\sin\frac{\xi}{2}\right). \tag{31}$$

The term given in Equation (30) is negative and it would be expected to have positive contribution to g_α, as long as $dH/d\chi > 0$. On the other hand, the term in Equation (31) is odd in ξ and when integrated with the $\cos\xi$ in g_α, which is even, cancels out giving $g_\alpha^{\text{ext}} = 0$ Then the contribution of the polarization current for non-axisymmetric geometry is found to be destabilizing, $\Delta_{\text{pol}} > 0$, having the value

$$\begin{aligned}g_\alpha = &\left(\frac{\omega_{*e}w}{\omega\rho_s}\right)^2\frac{m\sigma s\tilde{\varphi}_0'(1)}{2\iota L_\alpha}\int_1^\infty\frac{\mathrm{d}\chi}{x}\frac{\mathrm{d}H}{\mathrm{d}\chi}\\ &\times\oint\frac{\mathrm{d}\xi}{2\pi}\cos\xi\sin\frac{\xi}{2}[\mathrm{e}^{-w||x|-|\cos(\xi/2)||/\rho} - \mathrm{e}^{-w(|x|+|\cos(\xi/2)|)/\rho}]\end{aligned} \tag{32}$$

The result of the integral in g will depend on the expression for H (actually, it is more convenient to use the normalized $\hat{H}(\chi) = H(\chi)/(1 - \omega/\omega_{*e})$) which will be related to the plasma flow profile. That integral will have a numerical value whose sign will be the sign of $dH/d\chi$ since, for the region outside the separatrix layer, the term in brackets will be positive. Since for physical profiles the derivative is positive, the polarization current contribution is destabilizing. In (7) it was found that there is also a correction to the MHD value in this small ρ_s/w regime that is stabilizing. For a given island width, it is expected these contributions to be a small for fusion plasmas since it applies at $\rho_s \ll w$, i.e. low temperatures.

4. Conclusions

The effect of the ion polarization current due a plasma flow on the stability of an island is considered for a non-axisymmetric device. Cases with the island initially formed by the tearing mode or produced by symmetry-breaking external coils are discussed. The later represents a locked island which has the plasma flow around it. Our emphasis is about the influence the polarization current on the island evolution for a given velocity profile but we do not attempt to derive this profile. The results, the implications and the further work needed are the following.

- The main effect of Δ_{pol} is the same as in the case of axisymmetry which is generally destabilizing in the MHD limit and in the limit of small island relative to the ion Larmor radius. For large relative size of the island the non-axisymmetric contribution is important and has two parts coming from the solution outside the separatrix layer and an internal term which is second order in ρ_s/w.
- The toroidal geometry in flux coordinates provides a correction with a scale of the order of $mG/\iota^2 B_0 L_\alpha$ which would be less than one. Thus, the effect is to decrease the contribution of the polarization current.
- The new non-axisymmetric contribution is found to be destabilizing in the regime of high velocity $\omega > \omega_*$.
- A vacuum island that is locked to the external fields would tend to grow provided the effect of Δ_{pol} is large enough, which will happen for steep velocity profiles. This might be related to the interplay observed between MHD activity and transport barriers around rational surfaces in stellarators (*10*).
- Here, there is no consideration of the torque balance equation or of the transport analysis, so it should be included in a more complete study. Transport is expected to provide a better information of the coupling with the external plasma and the torques would determine the influence of the external flow profiles.

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