Induction of poloidal rotation by a radial ponderomotive force of electron cyclotron waves

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The nonlinear ponderomotive (PM) force due to high-power rf waves is studied as a drive for plasma rotation. It is shown that poloidal rotation may be driven by a radial PM force, in addition to the usual mechanism of direct angular momentum transfer from a poloidal PM force. Here, the effect of a radial PM force producing a radial plasma flow in the presence of viscous damping and neutral collisions is considered. The PM force is produced around an electron cyclotron resonant surface at a specific poloidal location, which naturally creates a poloidally asymmetric steady radial flow, when friction is present. The flow can also arise as a result of poloidal or toroidal PM force components, even in the absence of friction. In toroidal geometry this situation is unstable due to the Stringer spin-up mechanism, for a high enough power of the rf waves. This process is most important near the outer regions of the plasma, where it can then give rise to a high confinement mode (*H* mode), once a sheared poloidal flow is established. The advantage of this method of driving rotation is that the wave can propagate radially and when it is absorbed at the resonant surface the radial PM force is produced, instead of launching a wave in the poloidal direction. It is shown that this effect may be large enough for electron-cyclotron resonance heating, due to the small width of the resonant surface.
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I. INTRODUCTION

The ponderomotive (PM) force associated with rf waves is more prominent in high amplitude electromagnetic waves, such as those presently used in experimental fusion devices. Typical power requirements for rf heating and current drive in those machines are of several megawatts, which can make the nonlinear wave effects important. In most of the magnetic fusion experiments with radio frequency heating it has been possible to measure significant plasma rotation, which is usually associated with the establishment of the high confinement mode (*H* mode). It is now clear that poloidal plasma rotation at the edge region has a major role in the transition from the *L* to the *H* mode, where the associated sheared radial electric field suppresses turbulent fluctuations and establishes a transport barrier. This indicates that rf waves are in some cases responsible for plasma rotation near the edge, as they are also for rotation of electrons in rf wave current drive. The models for current drive assume a preferential momentum transfer in one direction, from the wave to the electrons at a selected radial location. With the same reasoning, it is possible to think of a scenario in which rf waves can be used to produce transport barriers, in the same way current drive is used to modify current profiles, i.e., by controlling the spatial region where the particle flow is generated. However, this spatial control is more easily accomplished if a nonresonant mechanism is employed for transferring momentum to the plasma, such as the ponderomotive force.

There is experimental evidence that rf waves, in particular lower-hybrid waves, can influence the dynamics of a tokamak plasma as a whole, by producing bulk rotation.[1] It has also been observed in stellarators with central electron cyclotron heating.[2] This rotation may well be driven by nonlinear forces giving momentum to the plasma ions in a preferential direction. If one could develop a mechanism that has the ability to establish at will a transport barrier at any desired location in a controlled way, from the outside of the machine, it would allow one to improve plasma confinement in many ways, as has occurred with the identification of internal transport barriers (ITB) in several experiments like the Doublet III-D (DIII-D),[3] Tokamak Fusion Test Reactor (TFTR),[4] Joint European Torus (JET),[5] and Alcator C Mod.[6] This could be done by injection of rf waves, and the simplest way of injection should be chosen.

The idea of driving rotation by a PM force has already been analyzed by a number of authors.[7–12] They have considered the basic expression of the PM force obtained originally by Klima,[13] or variations thereof. The way this ponderomotive force can drive rotation varies in the different models proposed. In one mechanism the PM force is in the radial direction and the resulting drift velocity is mainly directed in the poloidal direction, which makes the ions move poloidally.[7] This has been studied for cylindrical geometry, neglecting the important effect of viscosity, and it is difficult

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that this process occurs when magnetic pumping in a toroidal device is taken into account. The other mechanism, which has received a great deal of attention, considers the angular (toroidal or poloidal) component of the PM force and relies on a radial convection of plasma which is gaining momentum from the PM force, to produce a sheared plasma rotation.[8] According to these works only the angular components of the PM force can produce rotation.[11,12]

We show here that a radial PM force that is poloidally asymmetric can drive poloidal rotation, in the presence of particle collisions. It is based on the Stringer spin-up mechanism. Recently, another mechanism for plasma rotation based on the Stringer spin-up has been proposed,[14] which relies on the inhomogeneous ion accumulation during electron-cyclotron resonance heating (ECRH). Our model also depends on the natural poloidal asymmetry that is inherent in an off-center rf heating scheme; the wave absorption takes place on the side where the resonant surface is. We propose the use of electron cyclotron waves since the resonant region is very localized. The mechanism is the following: As the wave propagates radially inwards, the strong absorption near the resonance creates a large electric field gradient, which in turn gives rise to a significant ponderomotive force. The force $\mathbf{F}$ can drive a radial flow of ions in a diffusive-like way, as the $\mathbf{F}\times\mathbf{B}$ drift velocity finds a frictional resistance from either electrons or neutrals. This poloidally asymmetric radial flow is unstable, resulting in a plasma spin-up.[15] This effect has to compete with viscous damping, but as we show they can be comparable due to the fact that the important parameter is the force radial gradient rather than just the force itself. The plasma spin-up is produced in a narrow region due to the localized wave absorption, and thus a strong sheared rotation would be established, with the subsequent suppression of turbulence. With this mechanism it would be possible to produce transport barriers at selected locations in the plasma, although not very much to the inside in order to maintain enough collisions.

This would also have practical advantages over other schemes which require complicated antenna phasing procedures, since the waves are launched radially inwards perpendicular to the magnetic field, allowing a simpler operation. A spin-up mechanism driven by external momentum sources was previously considered in Ref. 16 but only for angular forces, not radial. With the inclusion of collisional friction we extend the analysis to allow radial momentum sources to produce a spin-up.

This paper is organized as follows. In Sec. II we make an analysis of the relevant dynamical system describing the plasma under the conditions produced by a poloidally asymmetrical ponderomotive force (which at that point could be any external force). It is shown that, due to the Stringer mechanism, an initial perturbation in the poloidal velocity is amplified in the presence of the radial flow driven by the radial PM force, and establish the conditions under which the spin-up would occur. In Sec. III we take the particular case of a PM force from electron cyclotron waves, and show that the X mode is the most convenient to produce rotation. Finally, in Sec. IV we make estimates of the magnitude of the PM force and give the conclusions and expectations for the proposed mechanism.

II. PLASMA DYNAMICAL ANALYSIS

Using the two-fluid description for a plasma, the ion particle conservation and momentum balance equations, averaged over the fast time scale of a high frequency oscillating field, are written as[17]

$$\frac{\partial n_i}{\partial t}+\nabla\cdot(n_i\mathbf{u}_i)=0, \tag{1}$$

$$m_in_i\left(\frac{\partial \mathbf{u}_i}{\partial t}+\mathbf{u}_i\cdot\nabla\mathbf{u}_i\right)$$
$$=n_iq_i\left(\mathbf{E}+\frac{1}{c}\mathbf{u}_i\times\mathbf{B}\right)-\nabla p_i-\nabla\cdot\mathbf{\Pi}_i+\mathbf{R}_i+\mathbf{F}_{pi}, \tag{2}$$

where $\mathbf{\Pi}_i$ is the ion viscosity tensor, p_i the total pressure taken in the center-of-mass frame, $\mathbf{u}_i$ the ion fluid velocity, $\mathbf{F}_{pi}$ the ponderomotive force on the ions, $\mathbf{R}_i=\mathbf{R}_{ie}-n_im_i\nu_{in}(\mathbf{u}_i-\mathbf{u}_n)\approx\mathbf{R}_{ie}-n_im_i\nu_{in}\mathbf{u}_i$ the friction force with electrons ($\mathbf{R}_{ie}$) and cold neutrals, and all other quantities have their usual meaning. For the magnetic field we take the axisymmetric representation,

$$\mathbf{B}=\nabla\psi\times\nabla\zeta+I\nabla\zeta, \tag{3}$$

where ψ is the magnetic flux function and ζ the toroidal coordinate with $\nabla\zeta=\hat{\zeta}/R$.

We are interested in the slow time evolution of the flow, so that $\partial/\partial t\sim\delta^2\nu_{ii}$, where $\delta=\rho_i/a$, the ratio of the ion gyroradius to the minor radius, is the small expansion parameter. The ordering we consider is $u_i/v_{\mathrm{th}i}\sim|\nabla\cdot\mathbf{\Pi}_i|/|\nabla p_i|\sim\delta$, the drift velocity $v_E\sim u_i$, and $|\mathbf{F}_{pi}|$ second order in δ. Then, separating the lowest order terms from the others by writing

$$\mathbf{u}_i=\mathbf{u}_i^0+\tilde{\mathbf{v}}_\mathbf{i},\quad n_i=n_i^0+\tilde{n}_i,\quad \text{etc.},$$

the equations for the lowest order quantities are

$$\nabla\cdot(n_i^0\mathbf{u}_i^0)=0, \tag{4}$$

$$n_i^0q_i\left(\mathbf{E}^0+\frac{1}{c}\mathbf{u}_i^0\times\mathbf{B}\right)-\nabla p_i^0=0. \tag{5}$$

Writing the solution to Eq. (5) as $\mathbf{u}_i^0=\lambda_i\mathbf{B}+\varpi\hat{\zeta}$, one can find ϖ by noting that $\nabla\times\mathbf{E}^0=0$, which implies $\mathbf{E}^0=-\nabla\phi^0$, and thus[18]

$$\mathbf{u}_i^0=\lambda_i\mathbf{B}-cR\left(\frac{p'^0}{q_in_i^0}+\phi'^0\right)\hat{\zeta}, \tag{6}$$

where the prime denotes derivative with respect to ψ, and the function λ_i is still undetermined. The fact that $\phi^0=\phi^0(\psi,t)$ and $p_i^0=p_i^0(\psi,t)$ comes from subtracting the electron's momentum equation from the ion's to get the lowest order Ohm's law, and from the parallel component of Eq. (5). We notice that $u_{\theta i}^0=\lambda_iB_\theta$, and thus λ_i is a measure of the poloidal velocity. Inserting Eq. (6) into Eq. (4) we find that $n_i^0\lambda_i$ is a flux function, and if we assume constant temperature this means $\lambda_i=\lambda_i(\psi,t)$.

Now we get momentum equations for the higher order velocity. Taking the component $R\hat{\zeta}$ of Eq. (2) and averaging over a flux surface (denoted by angular brackets), one eliminates the lowest order terms leaving

$$m_i\left\langle n_i R\hat{\zeta}\cdot\left(\frac{\partial \mathbf{u}_i}{\partial t}+\mathbf{u}_i\cdot\nabla\mathbf{u}_i\right)\right\rangle = q_i\left\langle n_i R\hat{\zeta}\cdot\left(\tilde{\mathbf{E}}+\frac{1}{c}\tilde{\mathbf{v}}_i\times\mathbf{B}\right)\right\rangle + \langle R\hat{\zeta}\cdot\mathbf{R}_{ie}\rangle - m_i\langle n_i\nu_{in}R\hat{\zeta}\cdot\mathbf{u}_i\rangle + \langle R\hat{\zeta}\cdot\mathbf{F}_{pi}\rangle, \tag{7}$$

where use has been made of $\langle R\hat{\zeta}\cdot\nabla\tilde{p}_i\rangle = \langle R\hat{\zeta}\cdot\nabla\cdot\mathbf{\Pi}_i\rangle = 0$ due to axisymmetry and since we consider only the parallel viscosity. Similarly, an equation for the parallel velocity is obtained by subtracting Eq. (5) from Eq. (2) and applying the operator $\langle\mathbf{B}\cdot\rangle$,

$$m_i\left\langle n_i\mathbf{B}\cdot\left(\frac{\partial \mathbf{u}_i}{\partial t}+(\mathbf{u}_i\cdot\nabla)\mathbf{u}_i\right)\right\rangle = -\langle\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i\rangle + q_i\langle n_i\mathbf{B}\cdot\tilde{\mathbf{E}}\rangle + \langle\mathbf{B}\cdot\mathbf{R}_{ie}\rangle - m_i\langle n_i\nu_{in}\mathbf{B}\cdot\mathbf{u}_i\rangle + \langle\mathbf{B}\cdot\mathbf{F}_{pi}\rangle, \tag{8}$$

since $\langle\mathbf{B}\cdot\nabla\tilde{p}_i\rangle = 0$.

Now, according to our ordering, the time derivative of $\mathbf{u}_i^0$ is second order in δ, and then the other terms have to be evaluated to this order. The convective terms can then be computed to lowest nonvanishing order, recalling that $n_i^0 = n_i^0(\psi,t)$, $\nabla(R\hat{\zeta}) = (\nabla R)\hat{\zeta} - \hat{\zeta}\nabla R$, using Eq. (1) and combining Faraday's law with the curl of Eq. (5). The magnetic term can be cast as $-\langle n_i^0\tilde{\mathbf{v}}_i\cdot\nabla\psi\rangle$, while for the electric field we have an inductive and an electrostatic component $\tilde{\mathbf{E}} = -\nabla\tilde{\phi} + \mathbf{E}_I$. Equations (7) and (8) thus become

$$m_i\left\langle R\hat{\zeta}\cdot\frac{\partial n_i\mathbf{u}_i}{\partial t}\right\rangle + m_i\frac{d}{d\psi}\langle n_i R\hat{\zeta}\cdot\mathbf{u}_i\mathbf{u}_i\cdot\nabla\psi\rangle = q_i\left\langle n_i R\hat{\zeta}\cdot\mathbf{E}_I - \frac{n_i}{c}\tilde{\mathbf{v}}_i\cdot\nabla\psi\right\rangle + \langle R\hat{\zeta}\cdot\mathbf{R}_{ie}\rangle - m_i\langle n_i\nu_{in}R\hat{\zeta}\cdot\mathbf{u}_i\rangle + \langle R\hat{\zeta}\cdot\mathbf{F}_{pi}\rangle, \tag{9}$$

$$m_i\left\langle n_i\frac{\partial\mathbf{B}\cdot\mathbf{u}_i}{\partial t}\right\rangle + m_i n_i\frac{d}{d\psi}\langle\mathbf{u}_i\cdot\mathbf{B}\mathbf{u}_i\cdot\nabla\psi\rangle = -\langle\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i\rangle + q_i\langle n_i\mathbf{B}\cdot\mathbf{E}_I\rangle + \langle\mathbf{B}\cdot\mathbf{R}_{ie}\rangle - m_i\langle n_i\nu_{in}\mathbf{B}\cdot\mathbf{u}_i\rangle + \langle\mathbf{B}\cdot\mathbf{F}_{pi}\rangle. \tag{10}$$

It is more convenient to write momentum equations for one fluid in order to eliminate the inductive field. In order to do this, we use the equivalent to Eqs. (9) and (10) for electrons, obtained by neglecting electron inertia and viscosity,

$$q_e\left\langle n_e R\hat{\zeta}\cdot\mathbf{E}_I - \frac{R}{q_e}\hat{\zeta}\cdot\mathbf{R}_{ie} - \frac{\dot{n}_e}{c}\tilde{\mathbf{v}}_e\cdot\nabla\psi\right\rangle - m_e\langle n_e\nu_{en}R\hat{\zeta}\cdot\mathbf{u}_e\rangle + \langle R\hat{\zeta}\cdot\mathbf{F}_{pe}\rangle = 0, \tag{11}$$

$$q_e\langle n_e\mathbf{B}\cdot\mathbf{E}_I\rangle - \langle\mathbf{B}\cdot\mathbf{R}_{ie}\rangle - m_e\langle n_e\nu_{en}\mathbf{B}\cdot\mathbf{u}_e\rangle + \langle\mathbf{B}\cdot\mathbf{F}_{pe}\rangle = 0. \tag{12}$$

Then, we can find the toroidal and parallel momentum equations for the fluid plasma by adding Eqs. (9) and (10) and Eqs. (11) and (12) and defining $\mathbf{j} = n_e q_e\tilde{\mathbf{v}}_e + n_i q_i\tilde{\mathbf{v}}_i$, $\mathbf{F}_p = \mathbf{F}_{pe} + \mathbf{F}_{pi}$. The resulting term $\langle\mathbf{j}\cdot\nabla\psi\rangle$ which would be zero in a stationary situation due to ambipolarity, can be computed from the flux-surface average of Ampere's law to be $-(\partial/\partial t)\langle\mathbf{E}^0\cdot\nabla\psi\rangle/4\pi$; however, we anticipate that it will turn out to be of order $v_{\rm A}^2/c^2$ (with $v_{\rm A}$ the Alfvén speed in the poloidal field). The resulting equations for the lowest order terms are

$$m_i\left\langle R\hat{\zeta}\cdot\frac{\partial n_i\mathbf{u}_i^0}{\partial t}\right\rangle + m_i\frac{d}{d\psi}\langle n_i R\hat{\zeta}\cdot\mathbf{u}_i^0\tilde{\mathbf{v}}_i\cdot\nabla\psi\rangle = \frac{1}{4\pi c}\frac{\partial}{\partial t}\langle\mathbf{E}^0\cdot\nabla\psi\rangle - \sum_{j=e,i} m_j\langle n_j\nu_{jn}R\hat{\zeta}\cdot\mathbf{u}_j\rangle + \langle R\hat{\zeta}\cdot\mathbf{F}_p\rangle, \tag{13}$$

$$m_i\left\langle n_i\frac{\partial\mathbf{B}\cdot\mathbf{u}_i^0}{\partial t}\right\rangle + m_i n_i\frac{d}{d\psi}\langle\mathbf{u}_i^0\cdot\mathbf{B}\tilde{\mathbf{v}}_i\cdot\nabla\psi\rangle = -\langle\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i\rangle - \sum_{j=e,i} m_j\langle n_j\nu_{jn}\mathbf{B}\cdot\mathbf{u}_j\rangle + \langle\mathbf{B}\cdot\mathbf{F}_p\rangle, \tag{14}$$

where we used $\mathbf{u}_i\cdot\nabla\psi = \tilde{\mathbf{v}}_i\cdot\nabla\psi$ since $\mathbf{u}_i^0\cdot\nabla\psi = 0$; $\tilde{\mathbf{v}}_i\cdot\nabla\psi$ has to be computed to order δ^2.

Now, since we are interested in the time evolution of the poloidal velocity, we want to express toroidal and parallel velocities in terms of λ_i. Using Eq. (3) it is straightforward to find

$$\mathbf{B}\cdot\mathbf{u}_i^0 = \frac{I}{\langle R^2\rangle}\langle R\hat{\zeta}\cdot\mathbf{u}_i^0\rangle + \langle B^2\rangle(\Theta^2+\Delta_B)\lambda_i, \tag{15}$$

$$R\hat{\zeta}\cdot\mathbf{u}_i^0 = (1+\Delta_R)\langle R\hat{\zeta}\cdot\mathbf{u}_i^0\rangle - I\Delta_R\lambda_i, \tag{16}$$

where

$$\Theta^2 = 1 - \frac{I^2}{\langle R^2\rangle\langle B^2\rangle}, \quad \Delta_B = \frac{B^2}{\langle B^2\rangle} - 1, \quad \Delta_R = \frac{R^2}{\langle R^2\rangle} - 1.$$

Similarly, the averaged radial electric field can be written in terms of λ_i and the averaged toroidal velocity $\langle R\mathbf{u}_i^0\cdot\hat{\zeta}\rangle$, using Eq. (16), as

$$\langle\mathbf{E}^0\cdot\nabla\psi\rangle = \frac{1}{c}\left\langle\frac{|\nabla\psi|^2}{R^2}(1+\Delta_R)\right\rangle\langle R\mathbf{u}_i^0\cdot\hat{\zeta}\rangle - \frac{I}{c}\frac{\langle|\nabla\psi|^2\rangle}{\langle R^2\rangle}\lambda_i, \tag{17}$$

where a term depending on the ion pressure in the toroidal ion velocity was neglected since $R\mathbf{u}_i^0\cdot\hat{\zeta} \approx R\mathbf{u}^0\cdot\hat{\zeta} \approx \lambda_i I - cR^2\phi'^0$, as obtained from the lowest order Ohm's law: $\mathbf{E}^0 + \mathbf{u}^0/c\times\mathbf{B} = 0$. In this way, the evolution equations (13) and (14) can be written for the poloidal and toroidal velocities, λ_i and $\langle R\mathbf{u}_i^0\cdot\hat{\zeta}\rangle$.

In order to do this we need to express the parallel viscosity term as a linear function of velocity, as[19]

$$\langle\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i\rangle = \mu u_{\theta i}^0 + k. \tag{18}$$

The coefficients μ and k should be calculated from the full expression of the viscosity tensor,[16] but we will treat them as known parameters. In addition, the electron inertia in the neutral collisions is neglected, and time derivatives of n_i^0 are replaced, using the continuity equation, by $-(d/d\psi)\times\langle n_i^0\tilde{\mathbf{v}}_i\cdot\nabla\psi\rangle$.

The two relevant equations (13) and (14) can finally be written, with the help of Eqs. (15)–(18) as

$$C_1\frac{\partial}{\partial t}\langle R\hat{\zeta}\cdot\mathbf{u}_i^0\rangle+C_2\frac{\partial\lambda_i(\psi)}{\partial t}=\frac{\partial}{\partial\psi}C_3\langle R\hat{\zeta}\cdot\mathbf{u}_i^0\rangle+\frac{\partial}{\partial\psi}C_4\lambda_i(\psi)+C_5\langle R\hat{\zeta}\cdot\mathbf{u}_i^0\rangle+C_6\lambda_i(\psi)+C_7, \tag{19}$$

$$D_1\frac{\partial}{\partial t}\langle R\hat{\zeta}\cdot\mathbf{u}_i^0\rangle+D_2\frac{\partial\lambda_i(\psi)}{\partial t}=n_i^0\frac{\partial}{\partial\psi}D_3\langle R\hat{\zeta}\cdot\mathbf{u}_i^0\rangle+n_i^0\frac{\partial}{\partial\psi}D_4\lambda_i(\psi)+D_5\langle R\hat{\zeta}\cdot\mathbf{u}_i^0\rangle+D_6\lambda_i(\psi)+D_7, \tag{20}$$

with

$$C_1=m_in_i^0\left(1-\frac{1}{4\pi m_in_i^0c^2}\left\langle\frac{|\nabla\psi|^2(1+\Delta_R)}{R^2}\right\rangle\right),$$

$$C_2=\frac{I}{4\pi c^2}\frac{\langle|\nabla\psi|^2\rangle}{\langle R^2\rangle},$$

$$C_3=-m_in_i^0\frac{\langle R^2\tilde{\mathbf{v}}_i\cdot\nabla\psi\rangle}{\langle R^2\rangle},\quad C_4=m_in_i^0I\langle\tilde{\mathbf{v}}_i\cdot\nabla\psi\Delta_R\rangle,$$

$$C_5=m_i\frac{\partial}{\partial\psi}(n_i^0\langle\tilde{\mathbf{v}}_i\cdot\nabla\psi\rangle)-\sum_{j=e,i}m_jn_j^0\langle\nu_{jn}(1+\Delta_R)\rangle,$$

$$C_6=I\sum_{j=e,i}m_jn_j^0\langle\nu_{jn}\Delta_R\rangle,\quad C_7=\langle R\hat{\zeta}\cdot\mathbf{F}_p\rangle,$$

$$D_1=m_in_i^0\frac{I}{\langle R^2\rangle},\quad D_2=m_in_i^0\Theta^2\langle B^2\rangle,$$

$$D_3=-m_i\langle\tilde{\mathbf{v}}_i\cdot\nabla\psi\rangle\frac{I}{\langle R^2\rangle},$$

$$D_4=-m_i\langle B^2\rangle(\Theta^2\langle\tilde{\mathbf{v}}_i\cdot\nabla\psi\rangle+\langle\Delta_B\tilde{\mathbf{v}}_i\nabla\psi\rangle),$$

$$D_5=-\frac{I}{\langle R^2\rangle}\sum_{j=e,i}m_jn_j^0\langle\nu_{jn}\rangle,$$

$$D_6=-\mu B_\theta-\sum_{j=e,i}m_jn_j^0\langle\nu_{jn}(\Theta^2+\Delta_B)\rangle\langle B^2\rangle,$$

$$D_7=-k+\langle\mathbf{B}\cdot\mathbf{F}_p\rangle.$$

The coupled equations (19) and (20) have to be simultaneously solved. Notice that the coefficient C_2 is of order v_A^2/c^2 smaller and that C_3, C_4, and D_3, D_4 contain the radial flux, which we are interested in here, since the radial PM force can produce this flux, as we will see later. The solution of these equations may be formally written in terms of the inverse Laplace transform for a vector variable $\mathbf{U}(\psi,t)=(\langle R\hat{\zeta}\cdot\mathbf{u}_i^0\rangle,\lambda_i)$ using a matrix representation (when the coefficients are time independent) as

$$\mathbf{U}(\psi,t)=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}ds\,\exp\left(st+\int_0^\psi\mathbf{A}_2^{-1}\cdot(\mathbf{A}_3-s\mathbf{A}_1)d\psi'\right)\times\left[\mathbf{U}_1+\int_0^\psi\exp\left(-\int_0^{\psi'}\mathbf{A}_2^{-1}\cdot(\mathbf{A}_3-s\mathbf{A}_1)d\psi''\right)\cdot\mathbf{A}_2^{-1}\cdot\left(\frac{\mathbf{A}_4}{s}+\mathbf{A}_1\cdot\mathbf{U}_0\right)d\psi'\right], \tag{21}$$

where

$$\mathbf{A}_1=\begin{pmatrix}C_1 & C_2\\ D_1 & D_2\end{pmatrix},\quad \mathbf{A}_2=-\begin{pmatrix}C_3 & C_4\\ n_i^0D_3 & n_i^0D_4\end{pmatrix},$$

$$\mathbf{A}_3=\begin{pmatrix}C_5+C_3' & C_6+C_4'\\ D_5+n_i^0D_3' & D_6+n_i^0D_4'\end{pmatrix},\quad \mathbf{A}_4=\begin{pmatrix}C_7\\ D_7\end{pmatrix},$$

$\mathbf{U}_0(\psi)$ is the initial velocity vector, $\mathbf{U}_1$ is a constant related to the velocities at the center $(\psi=0)$. In order to be able to interpret Eq. (21) and gain some insight for the conditions for the spin-up, we consider the case in which all coefficients in the matrices $\mathbf{A}_j$ are not dependent on ψ. Then, the integrals over ψ can be performed and the complex integral is evaluated by adding the residues at the poles. The result is

$$\mathbf{U}(\psi,t)=(\chi_+e^{s_+t}-\chi_-e^{s_-t})\frac{\mathbf{A}_1\cdot\mathbf{U}_0}{s_+-s_-}+\left(\frac{\chi_+e^{s_+t}}{s_+}-\frac{\chi_-e^{s_-t}}{s_-}\right)\frac{\mathbf{A}_4}{s_+-s_-}+\frac{\chi_0\mathbf{A}_4}{s_+s_-}, \tag{22}$$

where $\chi_\pm$, χ_0 are the residue matrices of $[\mathbf{A}_1s-\mathbf{A}_3]^{-1}$ at the poles $s=s_\pm,0$, which are the roots of its determinant, $\Delta_{13}=\Delta_1(s-s_+)(s-s_-)$, Δ_1 being the determinant of $\mathbf{A}_1$. These are

$$s_\pm=s_1\pm[s_1^2+s_2]^{1/2}, \tag{23}$$

$$s_1\equiv\frac{C_1(D_6+n_i^0D_4')+D_2(C_5+C_3')-C_2(D_5+n_i^0D_3')-D_1(C_6+C_4')}{2\Delta_1},$$

$$s_2\equiv\frac{(C_6+C_4')(D_5+n_i^0D_3')-(C_5+C_3')(D_6+n_i^0D_4')}{\Delta_1}.$$

In that form, Eq. (22) shows that the velocities can result either from a steady process, given by the last term, or from an exponential growth/damping at the rates $s_{\pm}^{-1}$. For instability, the real part of either of the roots $s_{\pm}$ has to be positive. From the point of view of the ponderomotive force, which is our present interest, the first case is the one that has been considered in previous works,[8–12] represented by the direct momentum transfer of toroidal and poloidal PM forces contained in $\mathbf{A}_4$. The spin-up, given by the exponentials, may arise from a radial flow produced by the PM force, if it is large enough to overcome the damping due to collisions and viscosity. Equation (23) shows that these dissipative terms contribute negatively to $s_{\pm}$, since Δ_1 is positive definite, while the terms involving the radial flow may give positive contributions. In particular, when $D_4'>0$, a positive feedback to poloidal rotation would be expected, as we can see also from Eq. (20), which is the poloidal angular momentum balance equation. It is interesting to see the requirements imposed to the flow by this term, for a circular cross-section, large aspect-ratio tokamak. By looking at the definition of the coefficient D_4', which depends on the poloidal asymmetry of the flow through Δ_B, we notice that

$$\frac{d}{d\psi}\langle \tilde{\mathbf{v}}_i \cdot \nabla \psi \Delta_B \rangle < 0. \tag{24}$$

Assuming the flow is a delta function in the poloidal angle $v_{\psi} = v_0 \delta(\theta - \theta_0)$, and $\Delta_B \approx -2\epsilon \cos\theta$ (with ϵ the inverse aspect ratio) then Eq. (24) implies that the ψ derivative of $2 v_0 \epsilon \cos\theta_0$ has to be positive, which means that the flow has to be in the direction of the major radius, $\hat{\mathbf{R}}$ (where $\hat{\mathbf{R}}$ is a unit vector in the direction of the major radius), as depicted in Fig. 1, if the ψ gradient is positive.

Now we consider the possible ways of producing the radial flow. We analyze two different cases in which the PM force can give rise to this flow: (a) for a PM force with a component on the magnetic surface and (b) for a radial PM force.

(a) In this case the spin-up would be due to a poloidal or toroidal PM force. We recall that $\tilde{\mathbf{v}}_i \cdot \nabla \psi$ has to computed to second order. By taking the component $\nabla \psi \cdot \mathbf{B} \times$ of Eq. (2), we can find for the radial velocity

$$\tilde{\mathbf{v}}_i^{(2)} \cdot \nabla \psi = \frac{c}{n_i q_i} \nabla \psi \cdot \frac{\mathbf{F}_{pi} \times \mathbf{B}}{B^2} + H, \tag{25}$$

where H represents all the terms not involving the PM force. From here we note that $|\mathbf{F}_{pi}|$ can be of second order to contribute in this order to $\tilde{\mathbf{v}}_i \cdot \nabla \psi$, so a relatively weak force would be of importance.

(b) In this second case, a radial force produces the spin-up, but the presence of a collisional force is required, as in a diffusive flux. To obtain the radial flow it is more convenient to use one-fluid equations by subtracting from Eq. (2) its equivalent for electrons, to get the generalized Ohm's law, and adding them to get the total momentum balance. Taking again the component $\nabla \psi \cdot \mathbf{B} \times$ of Ohm's law,

$$\tilde{\mathbf{v}}^{(2)} \cdot \nabla \psi = c\,\eta \nabla \psi \cdot \frac{\mathbf{B} \times \mathbf{j}^{(1)}}{B^2} + G^{(2)}, \tag{26}$$

where $\eta = (\nu_{ei} + \nu_{en}) m_e / n_e e^2$ is an effective resistivity including collisions with neutrals, which is of first order, and

$$G^{(2)} = \frac{c}{B^2} \mathbf{E}^{(2)} \times \mathbf{B} \cdot \nabla \psi + \frac{\nabla \psi}{ne} \cdot \mathbf{j}^{(2)} + \frac{c}{neB^2} \nabla \psi \cdot \nabla p^{(2)} \times \mathbf{B}$$

involves only second-order variables. The first-order current is obtained from the total momentum balance equation; focusing only on the contribution from the PM force, we have

$$\mathbf{j}_{\perp}^{(1)} = \frac{c}{B^2} [\mathbf{F}_p^{(1)} \times \mathbf{B} - \nabla p^{(1)} \times \mathbf{B}],$$

which upon substitution in Eq. (26) gives

$$\tilde{\mathbf{v}}^{(2)} \cdot \nabla \psi = \frac{c^2 \eta}{B^2} \nabla \psi \cdot \mathbf{F}_p^{(1)} + H_2, \tag{27}$$

and H_2 involves all other terms unrelated to $\mathbf{F}_p$, not of importance for the present discussion. Unlike Eq. (25), the PM force has to be of first order to produce the desired flow, so a stronger radial force is required to spin-up the plasma. In Sec. III we will calculate the radial PM force for electron-cyclotron waves, which are the ones that could give a large enough magnitude for it.

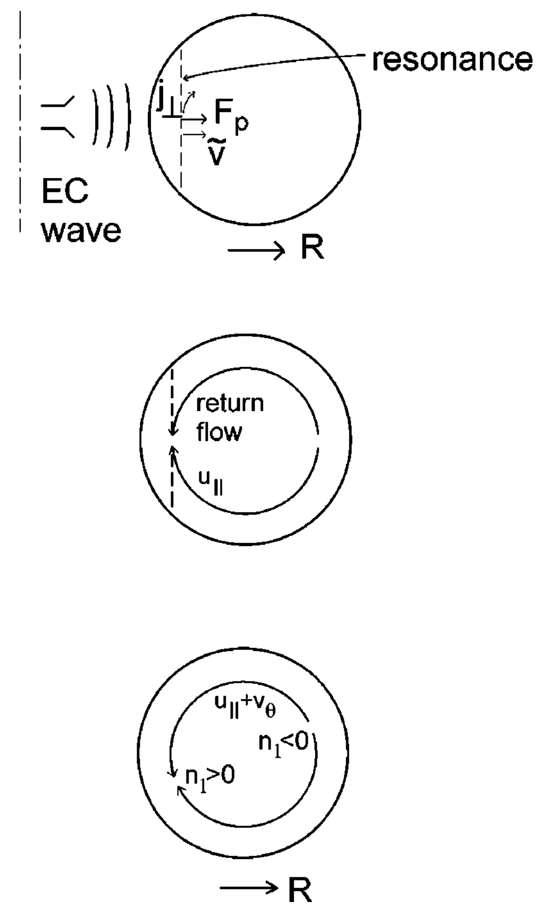

FIG. 1. Spin-up mechanism driven by the poloidal asymmetry of the wave absorption. The radial ponderomotive force creates a poloidal current $\mathbf{j}_{\perp} \sim \mathbf{F}_p \times \mathbf{B}$ which produces a radial flow $\tilde{\mathbf{v}} \sim \mathbf{j}_{\perp} \times \mathbf{B}$ in the inboard side. To maintain mass continuity a field-aligned return flow is driven. The field line curvature, acting as a gravity along $\mathbf{R}$, makes an initial poloidal rotation to grow, since local density is perturbed (after Ref. 15).

III. DRIVING RF WAVE PROPERTIES

The ponderomotive force has different representations depending on the way it is derived, including a possible time dependence. Ignoring this possibility, the PM force of an electromagnetic field oscillating at frequency ω can be written, for the species α, as[13]

$$\mathbf{F}_{p\alpha}=\frac{1}{2}\,\mathrm{Re}\left\{\frac{i}{\omega}\nabla\mathbf{E}_{\omega}^{*}\cdot\mathbf{j}_{\alpha\omega}-\nabla\cdot\left[\mathbf{j}_{\alpha\omega}\left(\frac{i}{\omega}\mathbf{E}_{\omega}^{*}+\frac{4\pi\mathbf{j}_{\alpha\omega}^{*}}{\omega_{p\alpha}^{2}}\right)\right]\right\}, \tag{28}$$

where $\mathbf{j}_{\alpha}$ is the current of species α induced by the wave electric field $\mathbf{E}$, and $\omega_{p\alpha}^{2}=4\pi n q_j^2/m_j$. The total current $\mathbf{j}=\mathbf{j}_i+\mathbf{j}_e$ is related with $\mathbf{E}$ through the conductivity tensor σ_{ik}, which depends on the special kind of waves being considered,

$$j_j=\sigma_{jk}E_k. \tag{29}$$

In order for the PM force to have an important effect on momentum transfer it is necessary to have a region of large gradient for E. This happens at a resonant surface, where the electric field magnitude decreases to almost zero within the absorption region. The smaller the absorption region the larger the gradient. So, it seems the most convenient option is to use electron-cyclotron (EC) waves, since the small gyroradius of electrons produces a thin resonant surface. In this section we will focus on the analysis of EC waves as the cause of the required PM force. Lower hybrid waves are also a possible candidate but their effect would probably be less important.

With the purpose of estimating the PM force we use the cold plasma dielectric tensor for high frequencies, which is related to the conductivity tensor by $\epsilon_{jk}=\delta_{jk}+4\pi i\sigma_{jk}/\omega$. For this calculation we use slab geometry with a local coordinate system ($\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_e$), with the $\mathbf{e}_3$ axis along the magnetic field, and $\mathbf{e}_1$ along the direction of $\nabla\psi$. Since we consider waves propagating perpendicularly to the magnetic field, we can have the ordinary (O mode) and the extraordinary (X mode) modes. For the latter only the σ_{jk} components transverse to the magnetic field are needed, which are

$$\sigma_{11}=\sigma_{22}=\frac{i\omega}{4\pi}\frac{v}{1-u},\quad \sigma_{12}=-\sigma_{21}=-\frac{\omega}{4\pi}\frac{\sqrt{u}v}{1-u}, \tag{30}$$

where $v=\omega_{pe}^2/\omega^2$, $u=\omega_B^2/\omega^2$, and $\omega_B=eB/m_e c$ is the electron cyclotron frequency. The PM force produced by the X mode is obtained from Eqs. (28) to (30). What we need is the radial component of this force, which in our local coordinates is identified with $\mathbf{e}_1$. The result is

$$F_{p1}=\frac{1}{8\pi}\frac{v\,\mathrm{Im}\,k_1}{(1-u)^2}\left((1+u)|E|^2+4\sqrt{u}\,\mathrm{Im}(E_1E_2^*)\right)+\frac{1}{8\pi}\left(|E|^2\partial_1\frac{uv}{(1-u)^2}+\mathrm{Im}(E_1E_2^*)\partial_1\frac{\sqrt{u}v(1+u)}{(1-u)^2}\right), \tag{31}$$

where ∂_1 indicates derivative with respect to the radial coordinate.

Since the O mode behaves as if there was no magnetic field, the PM force produced by it may be obtained from Eq. (31) by just making $u=0$. Thus, it simplifies to

$$F_{p1}=\frac{v}{8\pi}\,\mathrm{Im}\,k_1|E|^2. \tag{32}$$

In both cases, the PM force is determined by the absorption coefficient of the wave, represented by $\mathrm{Im}\,k_1$. This can be obtained from the dispersion equation of the specific mode, and it depends on the particular harmonic one chooses to use.[20] It turns out that the direction of the PM force needed for the establishment of the Stringer mechanism does not depend on the harmonic used but on the wave injection site.

Physically, the driving force has to point in the positive direction of the major radius, as mentioned in Sec. II, in order to produce a return flow from the outboard side of the torus to the inboard side. This can be seen in Fig. 1, drawn after Hassam and Drake.[15] If the wave is injected from the low field side (LFS) then the PM force is in the direction of the minor radius ($F_{p\psi}>0$), but if it is injected from the high field side (HFS), then it has to be in a direction toward the plasma column center ($F_{p\psi}<0$). In this situation, the unbalanced radial flow created by $F_{p\psi}$ has to be drained by a field-aligned return flow, to maintain continuity, which goes from the LFS to the HFS. If this flow is in the opposite direction, then there is no instability. The instability may be visualized when the effect of the field-line curvature is considered as an effective gravity along the major radius. As soon as a small poloidal plasma rotation arises, the region of mass depletion is displaced poloidally, creating a region of lower density, which tends to move up the gravity. This amplifies the initial rotation, producing a rotational instability. Mathematically, the requirement is that, in order to have a positive contribution in $s_{\pm}$, it is necessary that $\mathbf{F}_{p\psi}\sim\hat{\mathbf{R}}$.

The direction of the PM force is given mainly by the sign of $\mathrm{Im}\,k_1$, according to Eq. (32) and the first term of Eq. (31). For absorption, $\mathrm{Im}\,\mathbf{k}\cdot\mathbf{r}>0$, so when the injection is from the LFS $\mathrm{Im}\,\mathbf{k}\sim\hat{\mathbf{r}}\sim-\hat{\mathbf{R}}$, which is unfavorable for Stringer spin-up. On the other hand, for HFS injection $\mathrm{Im}\,\mathbf{k}\sim\hat{\mathbf{r}}\sim\hat{\mathbf{R}}$, allowing the Stringer mechanism. Thus, regardless of the EC mode used, the wave injection has to be from the HFS, which may be achieved with appropriate wave guides. Both, O and X modes can be used, and one could choose the one with the highest absorption. For normal propagation, at the fundamental harmonic the O mode is the most strongly absorbed, with an absorption coefficient given by[20]

$$\frac{\mathrm{Im}\,k_1}{k}=\frac{2\sqrt{\pi}q}{15}z_1^{5/2}e^{-z_1}, \tag{33}$$

while at the second harmonic the X mode has a quite high absorption, according to[20]

$$\frac{\mathrm{Im}\,k_1}{k}=\frac{2\sqrt{\pi}q}{15}(1+\Xi)^2 z_2^{5/2}e^{-z_2}. \tag{34}$$

In these equations $z_s=mc^2(s\omega_B-\omega)/s\omega_B T_e$, $\Xi=q/(3-q)$, $q=\omega_{pe}^2/\omega_B^2$, and $z_s>0$ is needed for a nonzero absorption, which again means injection from the HFS.

The ponderomotive force could now be estimated from Eq. (31) or Eq. (32) and the result can be used in Eq. (27) to compute the contribution of the radial flow to the roots $s_{\pm}$ given in Eq. (23) As mentioned before, at least one of these has to be positive including the dissipative terms in order to produce the spin-up. As we can see from Eq. (31) the X mode produces a PM force with a resonant denominator, which is not present in the O mode, and therefore it is the most convenient to get a large force. However, since the X mode presents a strong absorption at the second harmonic, the resonant effect is not so prominent. Thus, it would be advisable to use the most appropriate wave to produce rotation which is the X mode at the second harmonic and it has to be tuned in a way that finds the resonant surface close to the edge in order to have the required friction.

IV. DISCUSSION AND CONCLUSIONS

The results of the previous two sections can be now put together to estimate the required magnitude of the PM force and the wave power. We notice that the dominant term in Eq. (23), determining the sign of the exponential, is $D_6 + n_i^0 D_4'$; for the spin-up to take place this has to be positive. For a circular cross-section tokamak and keeping the lowest order terms in the inverse aspect-ratio ϵ, this can be evaluated to get the condition

$$-2m_i n_i \nu_{in}\epsilon^2 B_0^2 - \mu B_\theta + 2m_i c^2 \frac{d}{dr}(n_i \eta \epsilon F_{pr}) \geqslant 0.$$

The value of μ can be obtained from the parallel viscosity tensor to be[21] $\mu B_\theta = \eta_0 \langle (\nabla_\parallel B)^2 \rangle \approx \eta_0 B_\theta^2 / 2R_0^2$, where $\eta_0 = 0.96 n_i T_i / \nu_{ii}$. This term is usually dominant over the neutral collisions. Note that the fastest varying quantity in the radial derivative is the PM force, so neglecting the variation of the others we find

$$\frac{dF_{pr}}{dr} \geqslant \frac{B_\theta^2}{m_i n_i c^2 \eta \epsilon}\left(\frac{\eta_0}{4R_0^2} + m_i n_i q^2 \nu_{in}\right). \tag{35}$$

For the edge region of a tokamak of the type of the Swiss tokamak, TCV,[22] having enough dissipation (i.e., $\eta \sim 10^{-15}$ s given by both ion–electron and ion–neutral collisions), the condition for the radial force is $dF_{pr}/dr \gtrsim 10^6$ dyn/cm^4, or if we assume that the PM force appears within a region of the order of the electron gyroradius, $\rho_e \sim 10^{-2}$ cm then $F_{pr} \gtrsim 10^4$ dyn/cm^3. The corresponding absorbed wave electric field can be estimated from Eqs. (31) and (34) for the X mode as $E \sim 10$ kV/cm, or equivalently, an absorbed power density $P \sim 5k$ W/cm^2. Although this means a quite intense wave is necessary, it should be possible to get this power with an array of several gyrotrons like those used in TCV.[22]

There are several experiments where this rotation mechanism could be applied, that already possess facilities for ECRH, in addition to TCV. In the Dutch Rijnhuizen Tokamak Project, RTP,[23] the effect of ECRH in creating ITB has been proven, using modest amounts of rf power. Increasing the gyrotron power over 1 MW and doing off-axis heating, but with radial injection, would allow one to test the spin-up effect of the radial PM force. This could also be detected by the establishment of ITB by measuring the changes in the temperature profiles. In DIII-D, interesting work is being performed to produce reverse magnetic shear near the center using ECH and study electron transport barriers,[24] with moderate (0.5 MW) rf powers. It should be possible to increase the power to produce plasma rotation with radial injection of EC waves. Other tokamaks that are doing important work with ECRH in relation to the establishment of transport barriers are the Frascati Torus Upgrade[25] (FTU) and the axisymmetric divertor experiment[26] (ASDEX Upgrade). The close-to-the-axis heating used in these experiments to take advantage of the negative central shear could be easily modified to produce plasma spin-up at specific radial locations based on the technique proposed here, provided powers in excess of 1 MW are available.

To conclude, we have shown that it is possible to implement a convenient way of rotating a plasma near the edge region, where significant friction (resistivity or neutral collisions) is present, using rf waves. This is based on the Stringer spin-up mechanism, initiated by the radial particle flow produced by a radial PM force, which affects only a localized region in poloidal angles. An important fact that makes this nonlinear mechanism a plausible one is that it depends on the radial gradient of the PM force, and this can be significantly large for narrow absorption layers.

EC waves are especially appropriate for producing the necessary PM force drive, due to their small absorption region. Injection has to be from the HFS of the torus in order to have the required direction of the radial flow, using either O mode at the fundamental resonance or the X mode at the second harmonic, although the latter is more favorable. However, other wave frequencies could be used, provided the resulting PM force gradient is large enough. In this scheme there is no need for antenna phasing in order to direct the wave launching. Perpendicular propagation would give the desired effect. It is important to note that, even though we are proposing the use of resonant wave absorption to produce a large wave-amplitude gradient, the mechanism for momentum transfer itself is nonresonant. As mentioned in Sec. I, other nonresonant, spin-up mechanisms for rotating the plasma have been proposed,[14,16] which could be complementary to the one presented here, depending on the way the wave is injected.

Finally, it would be highly desirable to test this method in experimental settings in order to confirm the theoretical predictions. This would also provide a means to confirm the Stringer spin-up mechanism, which, although it has been invoked in several theoretical works, it does not count with a direct experimental evidence of its operability. If this rotation scheme can be adequately implemented, it promises to be a very simple one, and future experiments could benefit from a convenient way to create regimes of high confinement.

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