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Burn control of an ITER-like fusion reactor using fuzzy logic

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ABSTRACT

An operating fusion reactor should have a fuel burning rate nearly constant in order to have a steady power exhaust. This could be achieved with a control system that monitors the produced power and modifies the fuel intake accordingly. In this work we develop such a system based on a fuzzy logic controller which adjusts the external parameters to keep the plasma temperature and density at the design values of a reactor of the characteristics of ITER. The control parameters chosen are the D-T refueling rate, the auxiliary heating power and a neutral Helium source. We use a fuzzy controller of the Mamdani type that uses a number of membership functions appropriate to produce a response to parameter deviations that minimizes the response time. The inference rules are determined in a way to provide stabilization to all initial perturbations of the temperature, density and alpha particle fraction. The dynamical response of the reactor is simulated with a zero-dimensional model that uses confinement times provided by the ITER scaling. We show how the system is feedback stabilized for a large range of parameters comprising ± 25% around the nominal values. The recovery time after an initial perturbation from the steady state is within the range of one to tens of seconds depending on the type of initial perturbations applied, even with a noise term added to the energy confinement time. Furthermore the results of this Fuzzy Control System are compared with another control system based on neural networks that was previously developed.

1. Introduction

A thermonuclear fusion reactor will very likely be based on a magnetically confined plasma with toroidal geometry such as the tokamak or the stellarator, since these have produced the best results so far. The International Thermonuclear Experimental Reactor (ITER) project actually aims to demonstrate that a large tokamak is able to sustain a burning plasma producing ten times more energy than the input energy but still far from ignition. Eventually, a reactor should operate in an ignited state, where there is no need for external energy injection and the heating is maintained by the fusion reactions themselves. Once a fusion reactor can be built it will be necessary to assure that the energy output is maintained at a constant level which should be done with an appropriate control system. Such a system should also be able to stabilize any disturbances to the nominal operational state that may arise, in order to avoid disruptions or plasma quenching.

Essentially, what has to be done to achieve a stable, continuous operation is to control the fusion fuel burn. Several control systems can be devised. Control theory is growing continuously to this day employing various advanced mathematics tools, or applying recent artificial intelligence methods such as machine learning or deep neural networks. However, it would be desirable to have a controller that is simple and inexpensive. One possibility is to use two-layered neural networks as control systems which is what was done in [1,2] for the same problem addressed here. A simpler alternative is the use of fuzzy logic which is the approach followed in this work. The physical system considered is a fusion reactor containing a burning plasma in a sub-ignited state, meaning that it operates in a marginal state exactly sustained with no external power. For definiteness, the reactor parameters used are similar to those of ITER. The variables chosen to control the reactor are the fuel injection and the auxiliary heating power as well a as a monitored injection of impurities as in [3]. But in order not to increment the radiation losses by increasing the effective charge, the injected impurity is Helium.

Fuzzy systems have been applied to different control tasks for ITER or other systems. For instance, fuzzy logic was used to control the poloidal field AC/DC converter in ITER [4] or to estimate the electron cyclotron heating power deposition [5]. Also fuzzy logic was used to control the plasma vertical position in the STOR-M tokamak [6]. Another use of fuzzy login in the fusion context has been to predict disruptions on JET [7]. On the other hand the burning rate control in ITER has been addressed in many works [1–3,8–15] with different control

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systems such as nonlinear Lyapunov analysis [11,15], adaptive control [10] or neural networks [1,2]. However, to our knowledge, a fuzzy logic controller has not been used to control the fusion burning rate of a reactor. Therefore, the aim of the present study is to develop such a control system in order to evaluate its performance and compare it with a neural network control system.

In order to simulate the behavior of an ITER-like fusion reactor a code has been written that simulates the behavior of a "burning plasma" inside the reactor with a zero-dimensional model, based on a set of coupled nonlinear differential equations that describe the balance of D-T (deuterium–tritium) fuel density, alpha particle density and thermal energy. The reactor model was used previously in references [1,2,8,9].

Moreover a fuzzy control algorithm was developed that allows to manage the control variables, which are the D-T fuel feed, alpha particle source and auxiliary heating power, based on the information of electron density, fraction of alpha particles and temperature, obtained from the ITER-like reactor simulator, according to the parameters described by International Atomic Energy Agency in reference [16]. The intention of using a Helium source as an actuator to control the system is to have an extra degree of freedom that does not contaminate the plasma. Since it should not modify the energy input substantially, because that is done with the auxiliary power, the alpha particles are injected with the thermal energy, i.e. 12 keV. The other reason for choosing this set of control actuators is to compare with other work that uses a neural network control system using these control variables [2].

Finally, random perturbations were introduced to the energy confinement time(τ_E) of the reactor through a random function with Gaussian distribution, to simulate uncertainties in the measurements that feedback the controller, with the purpose of testing the robustness of the system. The good response under the perturbations indicates that the reactor-control system has a suitable behavior under conditions of inherent errors as well as in the ideal time dependent operation.

The paper is organized as follows. In Section 2 the model for the fusion reactor used in the control system is described. The time behavior towards relaxation is shown, pointing out that the resulting relaxation times are quite large and thus they have to be reduced with the control system. In Section 3 the fuzzy logic controller is presented, giving first a brief introduction to the ideas and methods of fuzzy logic. Next, in Section 4 the results of the coupled controlled system are presented, showing that the performance is quite good in the sense that the relaxation times are substantially reduced for all of the possible initial perturbations. Finally, in Section 5 the conclusions are presented. Since fuzzy inference systems are not familiar to many readers we present an introductory explanation with an example in Appendix.

2. Fusion reactor plasma model

The plasma in the thermonuclear reactor is described by means of a set of differential equations in zero dimensions. They represent the evolution of the state variables characterizing the system which for a Deuterium-Tritium burning plasma are, the density of D-T ions, the density of alpha particles and the thermal energy, as in [2]. The plasma is neutral and completely ionized, with a 50:50 D-T mixture, while the alpha particles resulting from the nuclear reactions are assumed to give their energy instantly (within the relevant time-scales) to the plasma which allows to consider the Helium ashes as thermalized with the same temperature of the main ions and electrons. A small fraction of impurities can be allowed which is included through the value of the effective charge Z_{eff}. It is assumed that the impurities are beryllium, nickel and tungsten but this information is unimportant for the simulation, we just use an average impurity with $Z_I = 14.7$. In the model it is also assumed that the only energy loss by radiation is bremsstrahlung.

The zero-dimensional equations can be obtained from the fluid equations by averaging over the whole volume of the reactor as in [17]. In that case the transport losses across the magnetic field are represented by the confinement times for energy, alpha particles and D-T (τ_E, τ_α, τ_P). Here we assume that $\tau_P = 3\tau_E$ and $\tau_P = 5.5\tau_E$, while τ_E is given by the ITER-98 scaling:

$$\tau_E = 0.031 I^{0.95} R^{1.92} B_0^{0.25} n^{0.35} A_i^{0.42} \epsilon^{0.08} \kappa^{0.63} P^{-0.67}, \tag{1}$$

where I is the plasma current in MA, R the major radius in meters, B_0 the toroidal magnetic field in T, n the electron density in $10^{19}\,\mathrm{m}^{-3}$, A_i the atomic mass of the main ions, P the injected power in MW, ϵ the inverse aspect ratio and κ the elongation. With the previous assumptions this system is governed by the following coupled nonlinear equations, representing D-T particle balance, Helium ash balance and thermal energy density balance, respectively [2]

$$\frac{d}{\mathrm{dt}} n_{\mathrm{DT}} = S_f - 2\left(\frac{n_{\mathrm{DT}}}{2}\right)^2 \langle\sigma v\rangle - \frac{n_{\mathrm{DT}}}{\tau_P} \tag{2}$$

$$\frac{d}{\mathrm{dt}} n_\alpha = S_\alpha + \left(\frac{n_{\mathrm{DT}}}{2}\right)^2 \langle\sigma v\rangle - \frac{n_\alpha}{\tau_\alpha} \tag{3}$$

$$\frac{d}{\mathrm{dt}}\left[\frac{3}{2}(n_e + n_{\mathrm{DT}} + n_\alpha + n_I)T\right] = P_{\mathrm{aux}} + Q_\alpha \left(\frac{n_{\mathrm{DT}}}{2}\right)^2 \langle\sigma v\rangle + \eta j^2 - A_B Z_{\mathrm{eff}} n_e^2 T^{1/2} - \frac{3}{2}(n_e + n_{\mathrm{DT}} + n_\alpha + n_I)\frac{T}{\tau_E} \tag{4}$$

Here $\langle\sigma v\rangle$ is the fusion reaction rate, $Q_\alpha = 3.4$ MeV the birth energy of the alphas, $\langle\sigma v\rangle$ the fusion reaction rate given by [18] and the η and A_B terms are the ohmic heating [19] and bremsstrahlung radiation [20]. The neutrality condition including impurities with density n_I is enforced: $n_e = n_{DT} + 2n_\alpha + Z_I n_I$. The equations are solved for the electron density n_e, the fraction of alphas $f_\alpha = n_\alpha/n_e$, and the temperature T following a Runge-Kutta method of fourth order. The explicit form of the equations that are actually solved can be found in Ref. [2]. Making the left hand sides equal to zero gives the parameters for steady state operation; this is the nominal state in which the reactor is expected to operate. We set the quasi-ignition condition $P_{aux} = 0$ and $S_\alpha = 0$ for this state which gives for the nominal operation, for ITER parameters, the values $n_e = n_0 = 10^{20}\,\mathrm{m}^{-3}$, $T = T_0 = 12$ keV, $f_\alpha \equiv n_\alpha/n_e = f_0 = 0.09$ and a fuel source $S_f = S_0 = 7.464 \times 10^{18}\,\mathrm{m}^{-3}$. The other ITER parameters used in this study that are always kept constant are, plasma current $I = 15$ MA, toroidal magnetic field $B = 5.3$ T, major radius $R = 6.2$ m, inverse aspect ratio $\epsilon = 2/6.2 = 0.323$, elongation $\kappa = 1.86$, isotopic number $A_i = 2.5$ and plasma volume $V_{core} = 831\,\mathrm{m}^3$; the main ion density is $n_i = n_{DT} = n_e - 2n_\alpha - Z_I n_I$, with $n_I = 7 \times 10^{17}\,\mathrm{m}^{-3}$ and $Z_I = 14.7$ fixed.

When the system departs from the equilibrium, the variables used to drive it back to the nominal state are the D-T source (S_f), the alpha particle source (S_α) and the injected auxiliary heating power (P_{aux}). These are normalized according to [2],

$$\hat{S}_f \equiv S_f/n_0, \quad \hat{S}_\alpha \equiv S_\alpha/f_0 n_0, \quad \hat{P}_{\mathrm{aux}} \equiv 2P_{\mathrm{aux}}/3n_0 T_0, \tag{5}$$

while the state variables are normalized to their nominal values, using the notation,

$$z_1 \equiv n_e/n_0, \quad z_2 \equiv f_\alpha/f_0, \quad z_3 \equiv T/T_0. \tag{6}$$

In case there is no control system, keeping the variables (5) to their nominal values, the solution of Eqs. (2)–(4) for given initial conditions for the variables (6) provides the time evolution of the reactor towards the nominal state. This is presented in Fig. 1 for a representative case. It is seen that the relaxation times required to return all z_i variables to one are of the order of 80 s.

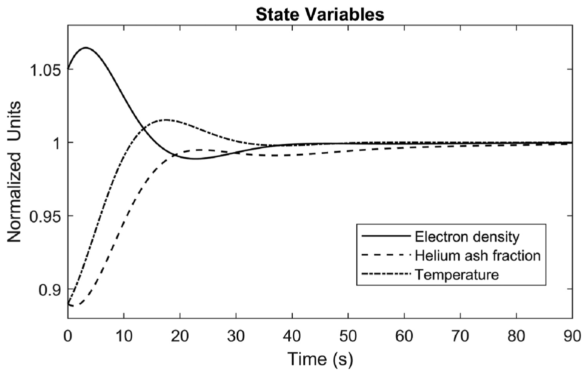

Fig. 1. Evolution of the state variables for an initial perturbation given by $z_1 = 1.05$, $z_2 = 0.89$, $z_3 = 0.89$, maintaining the control variables at their nominal values. Response time is about 80 s.

3. The control system

In the standard logic, it is possible to infer a conclusion out of a couple of statements based on the truthfulness of them. Furthermore they can only be true or false as well the conclusion. The concept can be extended to allow for statements to be only partially true assigning to them a certain degree of truth. This expands the possibilities of applicability of logical inference to situations where the variables defining a system (the statements) cannot be known with certainty so that they cannot be assigned sharp true or false values. Those cases are studied by fuzzy logic, which deals with variables that do not have sharp values of truth but can have different attributes simultaneously with different weights each. The relationships between the variables, given by some inference rules, then enable to reach certain decisions. This set of rules can thus be the basis for a control system. The convenience of using such a control system depends on the particular task it has to perform. In the present work a fuzzy logic controller is used to provide the steady state operation in a tokamak, in order to determine how suitable it is for this task.

The design of a control system usually involves the application of rigorous mathematical methods that determine the precise actions taken to provide the desired operation. On the other hand, the aim of fuzzy control systems (FCS) is to develop a framework for systems that are characterized loosely and as such it is more appropriate for solving real-world problems containing implicit errors. It is based on logical inferences, decision-making processes and linguistic variables, as means to formulate the initial problem. The denomination does not mean that fuzzy logic should be an inexact theory; vagueness and imprecision could be equally well described with the theory of probability. Actually it is very important to notice that the mathematical basis of fuzzy logic is as solid as those of other control theories.

As mentioned above, fuzzy logic works under the concept of logical variables in a non-binary way, which means that each variable has some degree of "trueness". The value of the variables is given by a set of membership functions, having different shapes according to the known behavior of the system. The values of variables can belong to different sets that have some attribute. For instance, if the variable is temperature, the sets can have the attributes warm, hot, very hot, cool, cold or very cold. The state of the variable could involve not only one set but a combination of them, according to the knowledge one has of the state. Thus, in a wider, sense we could introduce the fuzzy logic as a kind of blurring of the borders between two or more sets. This allows to incorporate common language sentences, such as, in the previous example, "a state with temperature that is 50% hot, 25% warm and 25% very hot". For this, one uses the so called linguistic variables [22]. These concepts are explained in more detail in Appendix that describes the ideas of a Fuzzy Inference System using a simple example.

There are two main types of fuzzy inference systems (FIS), developed by Mamdani and Sugeno. They are especially appropriate for capturing expert knowledge to design the inference rules, using a more intuitive language. The two methods differ in the way the crisp output is generated from fuzzy input. Essentially, the process followed to work with both FIS is composed by three stages: (1) fuzzification, in which the sharp variables are converted to fuzzy variables according to a set of membership functions that are defined to represent the properties of the system; (2) application of inference rules which determine the behavior of the system, in order to get the corresponding response; and (3) determination of the output variables which depends on the FIS type. The Mamdani type uses the defuzzification technique while the Sugeno type uses a weighted average of the consequences of the rules based on given functions. The defuzzification is the process to convert the fuzzy inference results into crisp values. The output of the FIS allows to take a definite action in order to maintain the proper state of the system. These systems are suitable for dealing with problems that involve the solution of real-world problems, formulated with words, for which the output signal returned is a real-world set [21]. Both FIS types have advantages and disadvantages. Mamdani method is more adequate for problems formulated with words (human input), but it is computationally more expensive, while Sugeno type is well suited for mathematical analysis and is thus more efficient.

In this work we use the Mamdani method since the reactor model is highly nonlinear which is more difficult to handle with the Sugeno method because it uses linear functions for the output. The three different stages of the proposed FIS are described below (an example is also given in Appendix).

3.1. Fuzzification of input variables

The input and output variables must be fuzzified giving a range of gradual membership to the different possible values, usually between zero and one. Each of the possible states for a variable is represented by a "membership function". It is important to notice that membership functions can take different shapes such as triangular, sigmoidal, Gaussian, etc., depending on the expected properties of the variable particular state. Both input and output variables are associated to membership functions; they are related through a set of inference rules that determine the possible values for the output. Subsequently the output variables are defuzzificated by an algorithm that transforms from fuzzy states to sharp values.

The input signals in a control system are generally provided by sensors or some other experimental measurement in the form of a number or a set of numbers. The incoming information to the system is well defined as a single point on the set of values for the input, it is a sharp signal. On the other hand, a fuzzy system has to work with fuzzy variables, so the signals are transformed to fuzzy signals by means of a set of membership functions that are blended, with a specific "weight" or level of truthfulness for each one. To translate from sharp signal to fuzzy signal, first of all the membership functions of each variable are carefully defined, both in range and shape including all real possibilities according to the upper and lower limits of the system. Then, according to the value of the signal a degree of membership is assigned for each function.

This procedure was followed for the input variables which are the electron density, the alpha particle fraction and the temperature, as well as for the output variables, namely, the D-T fuel source, the alpha particle source and the auxiliary heating power (with the respective normalizations of Eq. (5)). For each one, a set of membership functions is chosen. Fig. 2 shows the shape and the bounds selected in this FCS for the membership functions of the variables. The first plot contains the input variables (the membership functions for the three variables have the same shape and limits) for which the range of values is centered at one, extending from zero to two, according to the assigned normalization. The second plot represents the output variables, taken between −1 and 1 as limits; the functions have all the same shape but differ

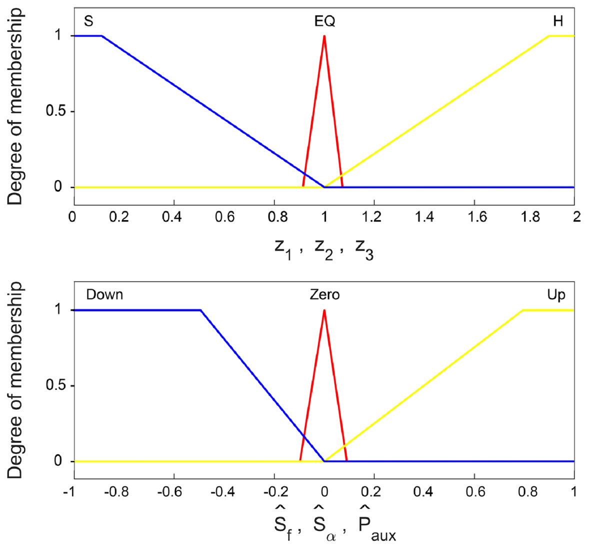

Fig. 2. Membership functions for state variables which include three states (top panel) and for control variables which have three states centered on zero (bottom panel).

from those for the input variables. Notice that the values of the control variables obtained from these functions could be positive or negative, although the actual values of $\hat{S}_f$, $\hat{S}_\alpha$ and $\hat{P}_{\mathrm{aux}}$ cannot be negative since they represent sources. This condition will be imposed later on. The chosen form of the membership functions is based on having a simple functional form that also contains the expected behavior for the variables. For the central functions corresponding to EQuilibrium, they are centered at zero since one expects to have no variation when the variable is already in equilibrium and the width is narrow in order to have small departures from there. For the others there is a gradual increase in the degree of membership until a reasonable value is reached for being considered fully characterized by that value. There is not a unique way to achieve this but the good performance of the FIS validates the choice.

Using these membership functions we can explain how a variable is fuzzified in a FIS. In Fig. 2 the variable is divided in and defined by the three states EQ, S and H, meaning that the variable is in EQuilibrium, or has a Small or High value, respectively. Each one is represented by a membership function defined between 0 and 1. On the horizontal axis one locates the sharp value of the variable. When, for instance, the input value is 1.05, the membership functions EQ and H will be launched as 30% EQ and 5% H, triggering corresponding actions on each space. Although it is still just one single variable, it is now defined by two different co-existing functions, with different degrees of truthfulness; furthermore, each one of these respectively activates the related actions on output variables with different values and shapes through the "inference rules". The values chosen in our FIS for the input variables are S, EQ and H, while for the output variables they are represented by the membership functions: DOWN, ZERO or UP, meaning that the control variable is decreased DOWN, stays at zero or is raised UP. Notice that in actuality the reduction cannot be so as to produce negative values since particle sources and auxiliary heating power can only add mass and energy to the system. But this restriction will be imposed outside the FIS.

3.2. Inference rules

Usually the inference rules of the system are obtained by the experience and knowledge of an expert. They determine the concrete actions to be taken based on "logic" statements, equivalent to the traditional syllogisms. Here, they were deduced following the observed trends of the system, as obtained by following the system response to the variation of each one of the input and output variables in Eqs. (2)–(4) that govern the deterministic evolution of the system.

The inference rules are the intermediate step of Mamdani FIS (whenever an initial fuzzification and a final defuzzification of the signals is applied). It is the step where fuzzy input variables are related to fuzzy output variables, whose relationship emerges with a meaning that incorporates the dynamics of the system. It allows the self-regulation process, indirect measurements or even forecasting depending on the problem one is dealing with. It is important to have a reliable set of rules since this is the most important part of the control system.

According to model equations (2)–(4) for the system evolution of the zero-dimensional reactor, a solution can be found for different initial conditions, thus providing a way to determine the response of the system to given initial perturbations from the equilibrium state. When all the variables have their nominal values, which for the normalized state variables (z_1, z_2, z_3) is one and $S_f = S_0$, $S_\alpha = P_{aux} = 0$, the system stays in the same state. Any variation from this initial condition produces a response in time as displayed, for example, in Fig. 1. By modifying the vales of the control variables one can determine how the state variables respond. Doing this for all possible types of initial perturbations in the variables, we can obtain a set of relations based on the observed trends, for the normalized variables, given in the following table. For instance, in the solution for the system evolution, when S_α was reduced from its current value, it was found that the density was reduced ($dn_e/dt < 0$) as well as the alpha fraction ($df_\alpha/dt < 0$) while the temperature was raised ($dT/dt > 0$); then a rule for decreasing the variable z_1 when it is high, or increasing z_3 when it is small has to be to decrease S_α (rule VI) and also if z_2 is high it is reduced by reducing S_α (rule IX). The combination of all possible variations produces the full set of rules and this substitutes the information from an expert.

I.	If (z_3 is H) then ($\hat{S}_f$ is Up)($\hat{S}_\alpha$ is zero)($\hat{P}_{\mathrm{aux}}$ is Down)
II.	If (z_1 is S) and (z_2 is S) then ($\hat{S}_\alpha$ is Up)
III.	If (z_2 is H) then ($\hat{S}_f$ is Up)
IV.	If (z_1 is EQ) or (z_2 is EQ) then ($\hat{P}_{\mathrm{aux}}$ is zero)
V.	If (z_1 is EQ) and (z_3 is H) then ($\hat{P}_{\mathrm{aux}}$ is Down)
VI.	If (z_1 is H) or (z_3 is S) then ($\hat{S}_f$ is Down)($\hat{S}_\alpha$ is Down)($\hat{P}_{\mathrm{aux}}$ is zero)
VII.	If (z_2 is S) then ($\hat{S}_f$ is Down)($\hat{S}_\alpha$ is zero)($\hat{P}_{\mathrm{aux}}$ is zero)
VIII.	If (z_2 is EQ) and (z_3 is H) then ($\hat{P}_{\mathrm{aux}}$ is Down)
IX.	If (z_2 is H) then ($\hat{S}_\alpha$ is Down)($\hat{P}_{\mathrm{aux}}$ is Up)
X.	If (z_2 is H) or (z_3 is H) then ($\hat{S}_f$ is Up)
XI.	If (z_1 is S) then ($\hat{S}_f$ is Up)($\hat{S}_\alpha$ is Up)($\hat{P}_{\mathrm{aux}}$ is Up)
XII.	If (z_1 is EQ) and (z_2 is EQ) and (z_3 is EQ) then ($\hat{S}_f$ is zero)($\hat{S}_\alpha$ is zero)($\hat{P}_{\mathrm{aux}}$ is zero)

Notice that the last rule is an obvious solution of the system and is the goal for the system once it has reached the complete equilibrium: it will not move from that state at the subsequent times. It means that when the system achieves the nominal state for z_1, z_2 and z_3, the D-T source, alpha particle source and auxiliary heating power should stay constant, to avoid disturbing the system.

3.3. Defuzzification of output variables

The result of the process of applying the inference rules is a fuzzy output set. However, every control task should imply the existence of a crisp value at the fuzzy controller output. Defuzzification is the procedure to extract a crisp output value from fuzzy outputs. There are different methods to achieve defuzzification. However, a crisp output is frequently achieved using the center of area (COA) principle, which is based on the following expression and is the one adopted in this work,

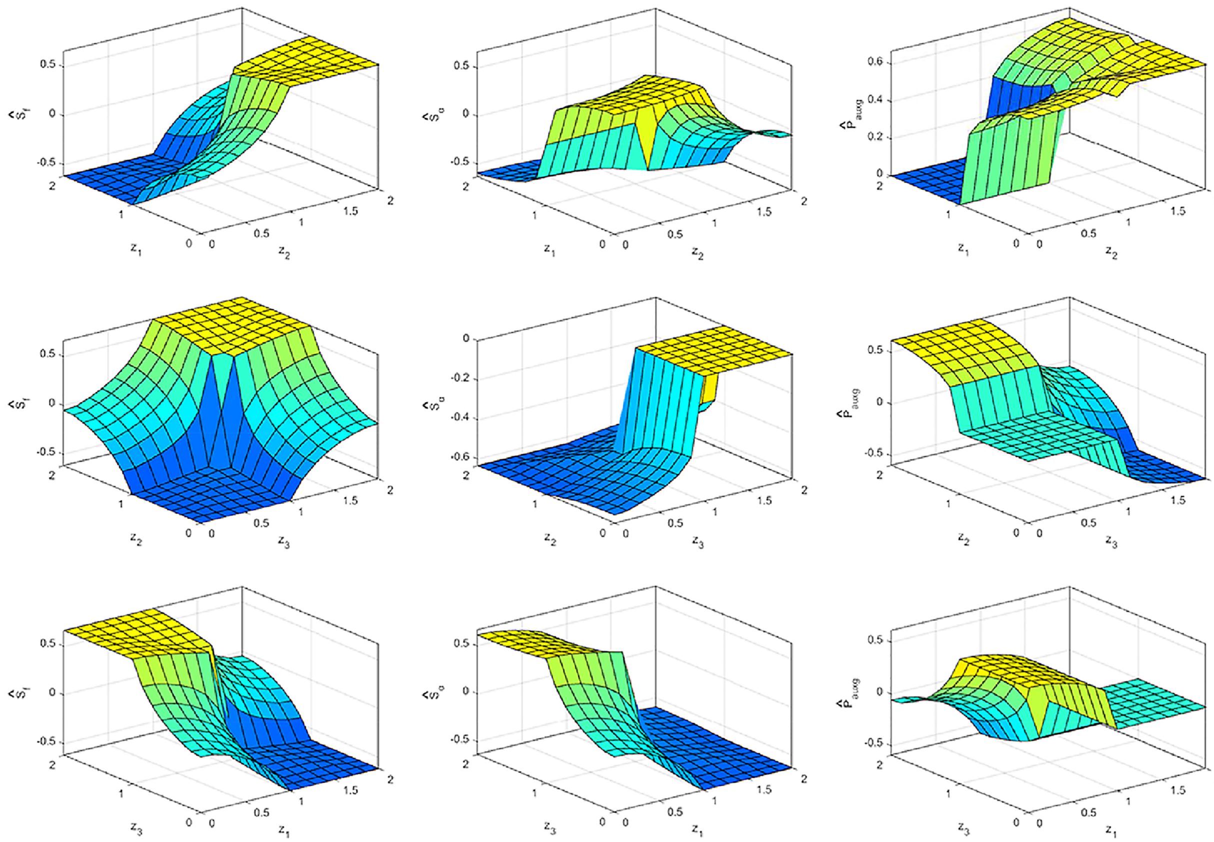

Fig. 3. Surface plots for the fuzzy inference system giving the normalized output variables as function of a pair of the normalized input variables ($z_1 \equiv n_e/n_0$, $z_2 \equiv f_\alpha/f_0$, $z_3 \equiv T/T_0$) when the third variables is fixed to one.

$$u_{FC}(\chi_k, y_k) = \frac{\Sigma_i u_i \cdot \mu_u(\chi_k, y_k, u_i)}{\Sigma_i \mu_u(\chi_k, y_k, u_i)} \quad (7)$$

where $u_{FC}(\chi_k, y_k)$ represents the crisp value of the fuzzy controller output, $u_i \in U$ is a discrete element of an output fuzzy set, and $\mu_u(\chi_k, y_k, u_i)$ is its membership function" [23].

In summary, defuzzification mainly consists of determining a precise return value obtained from a set of fuzzy output values. The results of the complete FIS can be visualized in surface plots that represent the value of each of the output variables as function of the input variables. Fig. 3 shows the surface plots for the three output variables as functions of two of the input variables maintaining the third constant (equal to one). This gives the crisp values of the output variables after the three steps of the FIS have been applied, for given values of the input variables z_1, z_2, z_3.

4. Performance of the control system

The implementation of the FIS in our control system is done in such a way that the instantaneous values of the control variables S_f, S_α and P_{aux} are updated by the FIS output at every time step according to

$$\hat{S}_f = S_0/n_0 + \hat{S}_{f,\mathrm{FIS}}, \quad \hat{S}_\alpha = \hat{S}_{\alpha,\mathrm{FIS}}, \quad \hat{P}_{\mathrm{aux}} = \hat{P}_{\mathrm{aux,FIS}}$$

where the sub-labels FIS mean the output from the fuzzy system.

The control system described above is coupled to the actual physical system which in this work is simulated by the model presented in Section 2. As mentioned before, the idea is to control the operation of a fusion reactor to stay at a nearly steady state. The goal is to maintain the system operating at the nominal levels, so that any departure from the nominal operation state should be restored in as short a time as possible. In our simulations the state variables coming out of the reactor model are input to the FCS which gives the control variables as output and these are used as input to the reactor model. In this way the new state variables are obtained, which should be closer to the nominal operation state. The process is iterated until the nominal state is reached.

The behavior of the reactor is critically determined by the measure of transport losses which in our model is given by the energy confinement time for which we used the ITER scaling, given in Eq. (1). We will show that continuous variations of τ_E can lead to unsteady operation of the reactor. In applying the FIS to the fusion reactor it is necessary to take into account that the control mechanisms consist of particle and energy sources and therefore their values cannot be negative, since that would imply extracting particles or energy. We assume this cannot be done since it would need sophisticated physical processes (in case it could be implemented). Therefore, if the result of the fuzzy control system is negative it is set to zero.

$$\begin{aligned} &\text{If } \hat{S}_f < 0 \text{ then } \hat{S}_f = 0 \\ &\text{If } \hat{S}_\alpha < 0 \text{ then } \hat{S}_\alpha = 0 \\ &\text{If } \hat{P}_{\mathrm{aux}} < 0 \text{ then } \hat{P}_{\mathrm{aux}} = 0 \end{aligned} \quad (8)$$

The performance of the FIS as a part of the control mechanism is tested in the following way: When all plasma variables have the values of the nominal operation state the control system does not take any action and the reactor stays in this state. Then, a perturbation is applied in the form of a variation of the initial value for the state variables z_1, z_2 and z_3. The response of the FCS is to adjust the control variables $\hat{S}_f$, $\hat{S}_\alpha$ and $\hat{P}_{\mathrm{aux}}$ to try to take this changes back to zero. This process is dynamical and takes place in a short time compared to the time it would take the system alone to return to its equilibrium state, as was the case of Fig. 1. We have taken the initial perturbations within the range ± 25% in our simulations since these are reasonable variations that could be expected during the actual operation. Many different initial values were

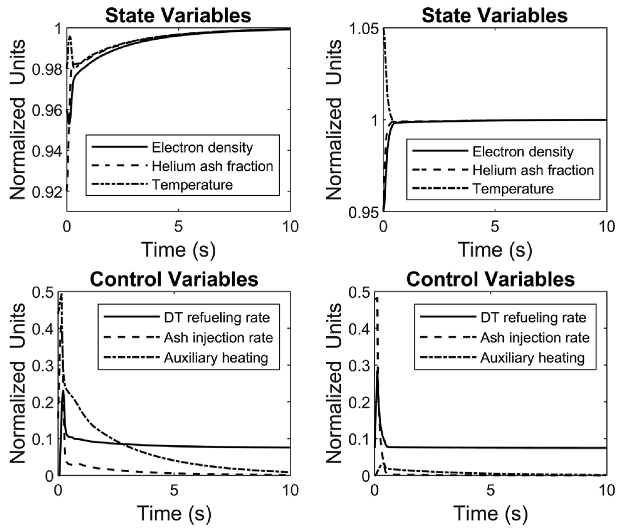

Fig. 4. Evolution of the system after an initial perturbation with two different initial conditions without any noise. Left-hand side: $z_1 = 0.96$, $z_2 = 0.92$, $z_3 = 0.98$ and right-hand side $z_1 = 0.95$, $z_2 = 0.95$, $z_3 = 1.05$.

tested within this range to compare the behavior and in all of them the control system works well.

In Fig. 4 one can see the behavior for two different initial perturbations when the reactor is controlled by the FIS. The upper plots show how the state variables return to the nominal values ($z_i = 1$) while the lower plots present how the control variables had to be adjusted to achieve the task. For the initial conditions of the right hand side diagrams there is a rapid damping and stabilization of the initial perturbation, taking about one second. Notice that the fuel source remains finite at $\hat{S}_f = 0.074$ after the nominal state has been reached. This is to be expected because in order to stay in the nominal state the equilibrium value $S_f = S_0$ in needed (recall that for this state $\hat{S}_f = S_0/n_0 = 0.07464$). On the other hand, the initial perturbation on the left hand side seems to be damped in two different steps, an initial fast one followed by a slower relaxation, which takes a longer time to reach an equilibrium state, of the order of 10 s. For this case the initial perturbations were of different sizes, as opposed to the right-hand-side case which had ± 0.05 for all variables, with the purpose of comparison.

From these illustrations it is clear that the response time of the controller will depend on the type of initial perturbation varying from less than one second to tens of seconds for the more difficult initial conditions. Some perturbations are more easy to neutralize than others, due to the limited effect the control actions have on the state variables. For instance, since there is no way to directly cool the reactor, an initial perturbation that rises the temperature is harder to overcome than one that reduces the temperature. After testing all possible combinations of initial perturbations to the state variables one can classify them according to the easiness of being canceled. This is presented in Table 1, where the range of relaxation times together with the average relaxation time is presented for all the possible cases. The finite range of τ_R is due to the fact that the decay time depends also on the amplitude of the perturbation and different amplitudes were tested in generating the table, i.e. for each entry of the table, many different sizes of initial perturbations were used that satisfy the corresponding condition. The criterion used to quantitatively determine the time of relaxation was that this is the time at which the three variables enter the band $z_i = 1 \pm 0.001$. It can be appreciated that the most difficult combinations to stabilize are those for which all the initial values are larger than one. This is because the control actions cannot simultaneously reduce the three state variables and therefore the system has first to thermalize (cool down) on its own, in order to lower the fusion power, which in turn reduces the temperature. This process takes longer times, which are comparable to the natural relaxation seen in Fig. 1.

Table 1
Relaxation time, τ_R, range and average for the different combinations of initial values for the perturbations of the state variables z_i.

Case	z_1	z_2	z_3	Average τ_R	Min–Max τ_R (s)
I.	> 1	> 1	> 1	41.1	34–48
II.	> 1	> 1	< 1	31.2	24–39
III.	> 1	< 1	> 1	32.8	23–40
IV.	> 1	< 1	< 1	13.5	3.5—28
V.	< 1	> 1	> 1	15.3	5.7–30
VI.	< 1	> 1	< 1	11.5	9–14
VII.	< 1	< 1	> 1	9.9	0.5–22
VIII.	< 1	< 1	< 1	11.8	9–18

5. Robustness test

The information provided to the controller comes from the measurements of the state variables which have always a degree of uncertainty. These in turn are used in the determination of τ_E which is used in the reactor model. Therefore, there could be a certain level of indetermination when the model is applied to a real setting. This could lead to a failure of the control system if it is not robust enough to these variations.

In order to simulate the noisy measurement environment or an uncertain scaling law of the energy confinement time τ_E (since this is empirical), we added some level of noise to τ_E. Each time step, the confinement time is multiplied by a random variable which has a Gaussian distribution function with a standard deviation of $0.04 \times \tau_{ELMy}$. Here τ_{ELMy} is a slightly different scaling law (given below) for the energy confinement time. The reason for choosing this one is to test the effect of modifying the confinement scaling on the system response, which is also a way of probing the robustness of the controller. It is worth mentioning that the value 0.04 for the deviations was selected because for larger values the FCS cannot return the state variables to their nominal values. For the larger fluctuations there is a relaxation but to values slightly different from $z_i = 1$.

This methodology to test for robustness was also used in a previous work that used an artificial neural network controller for a fusion reactor [2]. In a similar way, the mean value of the energy confinement time was taken to be the instantaneous value of τ_{ELMy}. Therefore, it is possible to compare the results and the behavior between this FIS and the artificial neural network as control systems. The expression for τ_{ELMy} is associated with the H-mode operation with ELMs, which is slightly degraded respect to the ELM-free operation [24]

$$\tau_{\mathrm{ELMy}} = 0.291\, I^{0.9}\, B^{0.2}\, P_{\mathrm{net}}^{-0.66}\, n^{0.4}\, R^{2.03}\, \epsilon^{0.19}\, k^{0.92}\, M^{0.2}. \tag{9}$$

Here $P_{net} = V_{core}(P_\alpha + P_{ohm} + P_{aux} - P_{rad})$ includes all power sources and sinks and V_{core} is the plasma volume.

The result of including the noise is presented in Fig. 5 for a given initial perturbation. It is interesting to notice the strong influence that small noisy fluctuations, introduced by the energy confinement time τ_E, have on the system evolution. The dotted lines represent the response of the system without a controller. This noise is enough to prevent the system from returning to the nominal state without the FCS, thus the plasma does not relax to the equilibrium values, no matter how much time is elapsed. On the other hand, the continuous lines show that the FCS allows the damping of all initial perturbations in the electron density $z_1 = n_e/n_0$, helium fraction $z_2 = f_\alpha/f_0$ and temperature $z_3 = T/T_0$, even after relatively large initial perturbations. This shows the control system is quite robust when it is subject to variations not contemplated in the original design.

Fig. 6 shows two other examples of relaxation with noise that reinforce the assertion about the robustness of the system. In the left-hand side of Fig. 6 one can compare the performance with noise with that

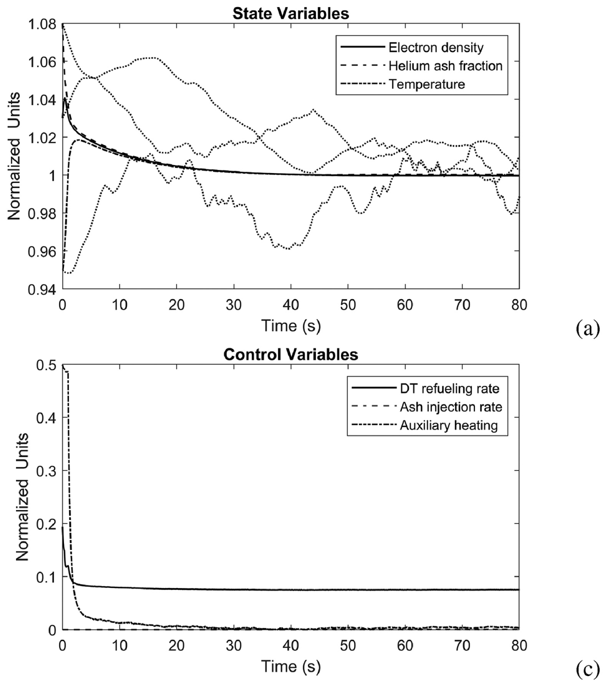

Fig. 5. (a) Response to an initial perturbation of the state variables given by $z_1 = 1.03$, $z_2 = 1.08$, $z_3 = 0.95$, with added noise without control system (dotted lines) and with the action of the FCS (continuous lines). (b) Random variation of the energy confinement time normalized to the nominal confinement time: τ_E/τ_{E0} and (c) evolution of the control variables, showing no fluctuations.

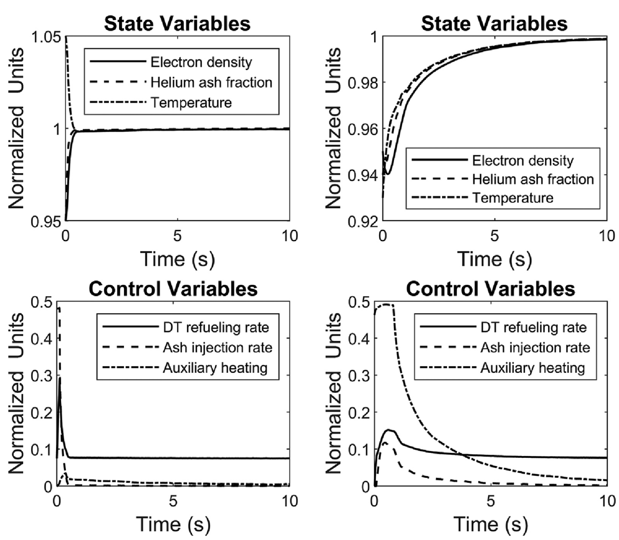

Fig. 6. Evolution of the system after an initial perturbation with two different initial conditions and a noise signal within τ_e. Left side: $z_1 = 0.95$, $z_2 = 0.95$, $z_3 = 1.05$ and right side: $z_1 = 0.95$, $z_2 = 0.94$, $z_3 = 0.93$. The energy confinement time varies with z_1 and P_{aux} according to $\tau_E = 1.77 z_1^{0.4}(1+f)/\hat{P}_{\rm net}^{0.66}$, where f is the random function and $\hat{P}_{\rm net} = (P_{\rm aux} + P_{\rm oh} - P_{\rm rad})/3n_0T_0$, thus it evolves in a way of the type shown in Fig. 5(b).

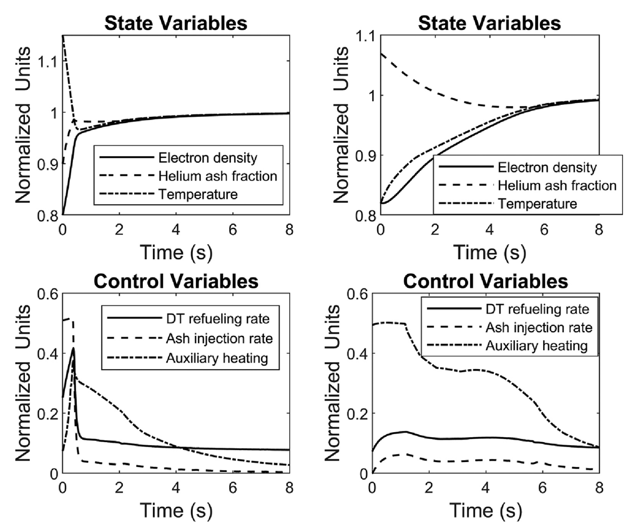

Fig. 7. Evolution of the system for two initial perturbations with different initial conditions corresponding to the examples of Ref. [2] (Fig. 7) including noisy fluctuations to τ_E. Left-hand side: $z_1 = 0.8$, $z_2 = 0.9$, $z_3 = 1.15$ and right-hand side: $z_1 = 0.82$, $z_2 = 1.07$, $z_3 = 0.82$.

with no noise, since it has exactly the same initial conditions as the right-hand side plot in Fig. 4. It can be appreciated that there is almost no difference for this fast-relaxing case. In general, this happens for all the different types of initial perturbations, and in some cases the relaxation time is even shorter when noise is included. For the case shown in the right-hand side, all initial perturbations are negative and the presence of noise does not give the fast initial evolution seen in Fig. 4left but the relaxed state is reached in about the same time. Apparently, the reason for a faster convergence in some cases is that the fluctuations may push the system in the right direction and once the states closer to equilibrium are reached the controller maintains them there and continues the evolution, arriving at the nominal state in shorter times.

In order to compare the results of the FIS with those obtained with an artificial neural network we show in Fig. 7 two cases simulated with the same conditions as those in Fig. 7 of [2]. For these cases the relaxation times are shorter for the fuzzy controller, being 5 s and 8 s, while for the neural network they were 8 s and 15 s, respectively. For the cases we tested, it turns out that the FIS is faster than the times obtained when a neural network controller is used, which is always of the order of around ten or more seconds [2]. In this respect, it is worth mentioning that in several studies with FIS it has been found that it performs better than other traditional control systems like PID [6] or PI [4].

6. Summary and conclusions

The stabilization of burn conditions for an ITER-like fusion reactor is properly controlled by a fuzzy logic system of the Mamdani type with a set of inference rules properly obtained from the evolution equations of the dynamical system. The reactor is simulated with a zero-dimensional model for the burning plasma in which the transport losses are incorporated through the energy confinement time given by the ITER-98 scaling. It is coupled to the fuzzy controller in a way that the electron density, alpha particle fraction and temperature in the plasma are fed into the controller as input, whose output (the fuel and alpha particle sources and the auxiliary heating power) then provides the actions to be taken to maintain the reactor operating in a quasi-steady state. The critical points in designing the FIS are the determination of the membership functions and of the inference rules. The former are chosen with the criterion of simplicity but also delimiting the ranges to well established limits, while the latter are obtained from analyzing the evolution response of the equations to all possible variations of the control variables. The coupled system provides a stable operation for initial perturbations within the range ± 25% about the nominal state. Furthermore, the system developed is also capable of balancing the variables when random disturbances are introduced in τ_E to simulate uncertainties in the system. Introduction of noisy fluctuations on τ_E maintains the good performance under the action of the control variables $\hat{S}_f$, $\hat{S}_\alpha$ and $\hat{P}_{\text{aux}}$. An interesting result is that some cases with noise converge a little faster than without noise because the fluctuations may take the system closer to equilibrium and the controller continues from there.

When comparing the FCS and an artificial neural network control system with the same initial conditions, similar responses are obtained in time evolution, although the FCS responds somewhat faster for some initial perturbations. This is in line with previous findings where a FIS works better than traditional control systems like PID [6] or PI [4]. It is important to recall that each method, both artificial neural network and FCS, follow different procedures to arrive at the nominal state, thus control variables are activated in different ways for each case. But, although they all achieve the goal of damping the initial perturbations, the FCS is quite easier to implement since it does not need to go over a long training process. In that sense we claim that it is better to use a fuzzy controller.

In conclusion, the work presented gives the following results:

1. The use of a FCS has shown to provide a good relaxation against initial perturbations in temperature, density of electrons and ions and alpha particles in reasonable times.
2. The efficiency of FCS is dependent on the initial relationship between variables, which means it is more efficient to control initial perturbations when they cool the plasma below the nominal state than when they heat it up over the nominal state, because there are not many possible cooling mechanisms. Sometimes, the convergence rate is slower because the control is not enough to take the system back to the nominal operation and then it evolves at the natural rate (the one with no control system). For some perturbations there is an initial fast evolution during which the three state variables get together and then they proceed at a slower pace all at once.
3. The FCS advantages are to have a short response time and easy operational implementation on the electronic systems of the experiments. Furthermore, data saved over long time operation allows to develop better updates to improve the system and its results.
4. When there are fluctuations on energy confinement time (τ_E) the FCS still returns the three state variables to their nominal states, showing the robustness of the system to small uncertainties.

It is clear that the validity of the results presented here is restricted to the range of applicability of the model used. Being a 0D model it is just a first approximation to the global behavior of the reactor. A more appropriate description would be to go to higher dimensions although this introduces important complications. Staying within our model, we should validate the results of our simulations with experimental results but this is not possible because there are none for ITER or use a computational database. Of course there are very sophisticated simulations of ITER we could use, but to consider them is not needed since it is enough for our purposes to be able to reproduce the nominal state of operation as attractor states. However, a natural next step is to use a 1D model.

Acknowledgments

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Appendix A

Fuzzy Inference Systems. A fuzzy logic inference system (FIS) is the basic unit of a fuzzy controller It based on the premises of fuzzy logic which is a computational paradigm that is based on how humans think. It describes the world in imprecise terms, in a way similar to how our brain takes in information (e.g. temperature is hot, speed is fast), and responds with precise actions. Fuzzy logic is in fact, a precise problem-solving methodology, able to simultaneously handle numerical data and linguistic knowledge. As opposed to traditional logic, a statement in fuzzy logic can assume any real value between 0 and 1, representing the degree to which an element belongs to a given set. A FIS has the capability of making decisions working with fuzzy variables. It uses the inference rules of the type “if … then”, along with connectors “or” or “and” for taking actions.

Although the FIS works with fuzzy variables, when it is used as a controller the input and output of variables have crisp values. Therefore at the entrance and exit of the FIS there has to be a unit to convert from crisp to fuzzy (fuzzification) and then from fuzzy to crisp (defuzzification). At the

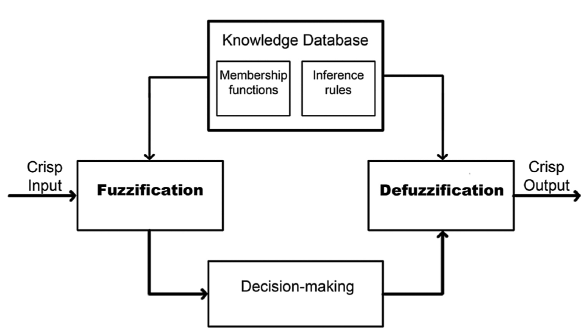

Fig. 8. Structure of a FIS.

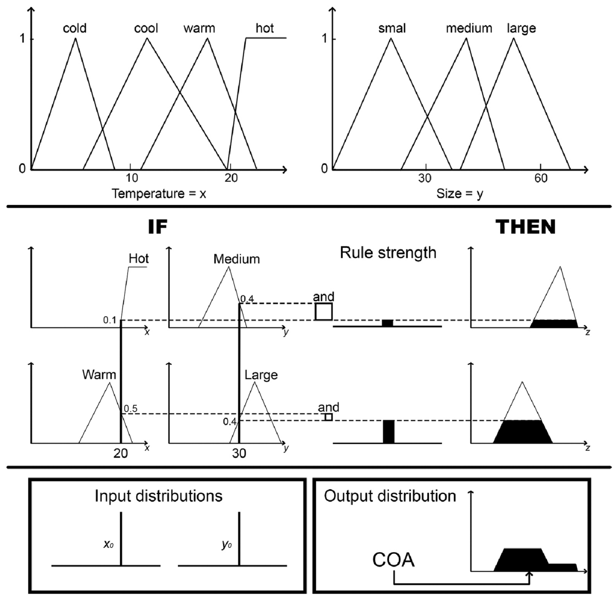

Fig. 9. Membership functions and inference flow for a Mamdani FIS.

core there is a unit for decision making which is the main part. This unit makes use of a set of inference rules and of a database which is expressed in terms of membership functions of fuzzy sets. The structure of a FIS is represented in Fig. 8. There are two methods that are usually used to carry out the task in a FIS: Mamdani and Sugeno. The Mamdani method is the one used here and is based on setting membership functions for the values of each of the input and output variables. In addition, the set of inference rules has to be established using some kind of expert information. Once this is established the inference process proceeds according to the following steps: (1) The crisp input is converted to fuzzy values using the input membership functions; (2) establish the rule strength by combining the fuzzified inputs according to the inference rules; (3) determine the consequent of each rule by combining the rule strength and the output membership function; (4) combine all the consequents to obtain the fuzzy output distribution which is then defuzzified to get the crisp output.

On the other hand, the Sugeno method is characterized by giving the output in terms of a mathematical function. It has the format: *IF x is A and y is B THEN z = f(x,y)*, where *A* and *B* are fuzzy sets in the input and *z* is the crisp output as given by the function *f*(*x*, *y*). Then, for this method the inputs are first made fuzzy with the membership functions and then the rules are applied obtaining at the same time the crisp output. In this case there are more adjustable parameters through the functions.

As an example of the Mamdani system let us consider a case consisting of bodies of different sizes having different temperatures, which have to be classified as coolants or heaters and the ability to handle them. The input fuzzy sets are SIZE with linguistic values {small, medium, large} and TEMPERATURE with linguistic values {hot, warm, cool, cold}. The output fuzzy sets are COOLANT with values {good, fair, bad} and MANEUVERABILITY with values {easy, hard}. Each set value has a membership function that defines how to assign the associated quality to given numerical values. One of the most common functions is the triangular which could be used to define the temperature ranges in the way shown in Fig. 9. Also, semi-trapezoidal functions can be used for the end values. For instance, for a temperature 20, there is 0.1 weight of cool, 0.5 weight of warm and 0.1 weight of hot, while for a size of 30 the weights are 0.4 for medium and 0.4 for small. Out of the whole set of rules (usually given as a look-up table with the two or more entries, that in our case are *size* and *temperature*) those that are satisfied are activated and produce values for the output, as shown also in Fig. 9. Two possibly activated rules presented here are: IF Temp is Hot AND Size is Medium THEN Coolant is Good and the other IF Temp is Warm AND Size is Large THEN Coolant is Fair. Finally, there is a defuzzification method, here the center of area (COA), to get a final value for the output *z* from the fuzzy distribution. The COA is a way of taking an average value based on the area under the distribution, but other ways of “averaging” are possible. A similar analysis should be done for other output variables like the maneuverability.

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