PHASE TRANSITIONS AND SPONTANEOUSLY BROKEN SYMMETRIES

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Summary

The concepts of spontaneously broken symmetries and phase transitions are widespread in physics. In addition to the examples chosen here, there are many other applications of spontaneously broken symmetries, e.g. superconductivity, superfluidity and Bose-Einstein condensation in condensed matter physics, and mean-field descriptions of correlated many-body systems as used extensively in nuclear physics, just to mention a few.

Spontaneously broken symmetries occur whenever the fundamental equations of a physical system possess a symmetry that is not displayed by the ground state. The phenomenon of spontaneous symmetry breaking (SSB) is closely related to the occurrence of phase transitions between two phases with different symmetry properties. SSB shows a remnant of a deeper symmetry of the physical laws that are hidden from sight, but which in principle can be restored by increasing the energy (or temperature) of the system.

1 Introduction

The concept of symmetry is a powerful tool that has inspired many generations of scientists in their search for beauty, harmony, order and regularity in Nature and in the understanding of the fundamental laws of physics. Nevertheless, at times it has also led physicists astray, trying to find higher, or more fundamental, symmetries before the time is ripe, i.e. without the availability of sufficient empirical information on the phenomena one seeks to understand.

At present, symmetries form one of the cornerstones of physics: its basic laws are based upon symmetry. The deep connection between three fundamental concepts in physics, symmetries, conservation laws and variational principles, was established in the beginning of the 20th century by the German scientist Emmy Noether (1882-1935). Nowadays it is known as the Noether theorem which states that

For every continuous symmetry of the laws in physics, there exists a corresponding conservation law. For every conservation law, there exists a continuous symmetry.

As a consequence, all conservation laws reflect fundamental symmetries of Nature. In classical physics one has the familiar examples of the conservation of angular momentum, momentum and energy which are related to the space-time symmetries of rotations and translations in space and time, respectively.

However, not only symmetries themselves, but also their breaking has provided deep insights into the nature of physical laws. The importance of symmetry breaking was realized as early as 1894 by Pierre Curie (1859-1906): the asymmetry of effects must be found in their causes and asymmetry is what creates a phenomenon. He also discussed some important aspects of what later became known as spontaneous symmetry breaking: the phenomena generally do not exhibit the symmetries of the laws that govern them. Symmetry breaking can occur in different ways. It can be broken explicitly as for example parity violation by the weak interaction, or spontaneously as in ferromagnetism, electroweak theory, superconductivity, superfluidity and Bose-Einstein condensation. In the case of spontaneously broken symmetries, the fundamental equations of a system possess a symmetry that is not displayed by the ground state.

The first time that the concept of spontaneous symmetry breaking came to be perceived as a general principle was when Yoichiro Nambu (1921) introduced this mechanism into particle physics using an analogy with superconductivity (the name was coined a few years later in a paper by Baker and Glashow). SSB is a general mechanism which plays an important role in many different branches of physics.

The phenomenon of SSB is most easily understood by examining a few examples. First, consider a ball which is located on top of a hill. The ball is in a completely symmetric state. However, this configuration is not stable: any small disturbance may cause the ball to roll down the hill in one direction or another. At the moment the ball starts to roll
down, the initial symmetry is broken, because the direction in which the ball rolls down has been chosen arbitrarily from all possible directions.

Another example is provided by Abdus Salam’s analogy of a dinner party at which the guests are seated around a circular table, and a napkin is placed between each pair of neighbors. The table setting is symmetrical until someone takes a serviette from his left or his right side. After that, the symmetry is broken, and the other guests can no longer choose between left or right napkins. The left-right symmetry is spontaneously broken.

This chapter briefly reviews the concept of spontaneously broken symmetries and phase transitions by examining the cases of ferromagnetism, global and local gauge symmetries and the electroweak unification.

2 Ferromagnetism

One of the best-known examples in physics of spontaneously broken symmetries is that of a ferromagnet. It was discovered by Pierre Curie that beyond a certain temperature ferromagnetic materials lose their magnetic properties, i.e. the ability to have a net magnetization in the absence of an external magnetic field. This so-called critical temperature $T_c$ (or the Curie point) is different for each material. The atoms in a ferromagnet interact through a spin-spin interaction

$$H = -\sum_{ij} \alpha_{ij} \vec{S}_i \cdot \vec{S}_j,$$

which is invariant under rotations. For temperatures below the critical value $T < T_c$, the magnetic moments of the atoms are partially aligned within magnetic domains in ferromagnetic materials. As the temperature is increased, this alignment is destroyed by thermal fluctuations, until the net magnetization vanishes at $T = T_c$. For temperatures above the critical point $T > T_c$ the ground state of the system is symmetric, i.e. the atoms are randomly oriented. There is no preferred direction in space. However, for temperatures below the critical value $T < T_c$, the rotational symmetry is broken since now the ground state consists of spins which are aligned within a certain domain. The orientation of the magnetization is random. Each of the possible directions is equally likely to occur, but nevertheless only one of them is chosen at random. The rotational symmetry of the ferromagnet is manifest for $T > T_c$, but is broken or hidden by the arbitrary selection of a particular (nonsymmetrical) ground state for $T < T_c$ (see Figure 1). According to a famous image by Coleman a little man living inside such a ferromagnet would have a hard time detecting the rotational invariance of the laws of nature. The ferromagnet provides an example of spontaneous symmetry breaking: the system itself has rotational symmetry, but the ground state is not invariant under that symmetry.

The vanishing of magnetization at the critical temperature $T = T_c$ is a second order phase transition in which the magnetization (order parameter) changes discontinuously
as a function of the temperature (control parameter). At the critical temperature, the magnetic susceptibility becomes infinite. In general, the spontaneous breaking of any continuous symmetry is marked by a nonzero order parameter.

Any situation in physics in which the ground state (i.e., the state of minimum energy) of a system has less symmetry than the system itself, exhibits the phenomenon of spontaneous symmetry breaking. For example, the state of minimum energy for an iron magnet is that in which the atomic spins are all aligned in the same direction, giving rise to a net macroscopic magnetic field. By selecting a particular direction in space, the magnetic field has broken the rotational symmetry of the system. However, if the energy of the system is raised, the symmetry may be restored, e.g., the application of heat to an iron magnet destroys the magnetic field and restores rotational symmetry.

3 Continuous Global Symmetry: Nambu-Goldstone Bosons

A simple example from mechanics can be used to demonstrate the essential features of a potential with exhibits spontaneous symmetry breaking. Consider a particle moving in the $\phi_1$-$\phi_2$ plane in a Mexican hat potential

$$V(\phi_1, \phi_2) = -\frac{1}{2} \mu^2 (\phi_1^2 + \phi_2^2) + \frac{1}{4} \lambda^2 (\phi_1^2 + \phi_2^2)^2,$$
with $\mu^2 > 0$ and $\lambda^2 > 0$. This potential only depends on the radius $\sqrt{\phi_1^2 + \phi_2^2}$, and is invariant with respect to a rotation in the $\phi_1$-$\phi_2$ plane

\[
\left( \begin{array}{c} \phi'_1 \\ \phi'_2 \end{array} \right) = \left( \begin{array}{cc} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{array} \right) \left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right).
\]

The potential $V$ has a maximum in the origin $\phi_1 = \phi_2 = 0$ and a set of degenerate (and equivalent) minima that lie along a circle of radius

\[
\phi_1^2 + \phi_2^2 = \frac{\mu^2}{\lambda^2}.
\]

As a consequence of the symmetry of the potential, the minima are related to one another by rotation in the $\phi_1$-$\phi_2$ plane (see Figure 2).

In scalar field theory one can have a similar situation, i.e. that the symmetry of the Lagrangian is not shared by the ground state solution. In this case, $\phi_1$ and $\phi_2$ become two scalar fields and the ground state associated with the minimum of the potential is called a vacuum expectation value $\langle \phi_k \rangle$. As an example, consider a complex $\phi^4$ theory with the Lagrangian

\[
\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi) (\partial^\mu \phi^*) - V(\phi, \phi^*),
\]

where the complex scalar field is given by $\phi = \phi_1 + i\phi_2$ and the potential has the same form as before

\[
V(\phi, \phi^*) = -\frac{1}{2} \mu^2 \phi \phi^* + \frac{1}{4} \lambda^2 (\phi \phi^*)^2.
\]

The Lagrangian is invariant under the global gauge transformation

\[
\phi'(x) = e^{i\Lambda} \phi(x).
\]

Due to the symmetry of the Lagrangian any one of the minima $\phi \phi^* = \mu^2/\lambda^2$ can be chosen as the vacuum state, however, whichever one is chosen, the global gauge symmetry of the Lagrangian is spontaneously broken. Let us choose a particular vacuum state

\[
\langle \phi_1 \rangle = \frac{\mu}{\lambda}, \quad \langle \phi_2 \rangle = 0,
\]
and introduce the fields $\eta$ and $\xi$ as fluctuations about this vacuum state

$$\eta = \phi_1 - \langle \phi_2 \rangle = \phi_1 - \mu/\lambda, \quad \xi = \phi_2 - \langle \phi_2 \rangle = \phi_2.$$  

This procedure leads to a massive field $\eta$ with $m_\eta = \mu\sqrt{2}$ and a massless field $\xi$ with $m_\xi = 0$. The occurrence of a massless field is a general phenomenon and is due to the Goldstone theorem which states that

The spontaneous breaking of a continuous global symmetry is always accompanied by the appearance of a massless scalar particle called a Nambu-Goldstone boson.

In the example of the Mexican hat potential, the $\eta$ field corresponds to excitations in the radial direction which require moving away from the minimum of the potential. The nonvanishing restoring force in the radial direction is reflected in the mass of the $\eta$ field. The occurrence of the massless $\xi$ field is related to the fact that there is no resistance to excitations in the angular direction around the circle.

In conclusion, the spontaneous breaking of a global gauge symmetry is accompanied by the appearance of a massless Nambu-Goldstone boson. If a global symmetry is spontaneously broken, the ground state must be degenerate, because the Lagrangian has the symmetry, but the ground state does not. The different ground states are related by symmetry transformations. The vacuum expectation value $\langle \phi_k \rangle$ plays the role of the order parameter.

Massless Nambu-Goldstone bosons associated with spontaneously broken symmetries appear in various many-body physical systems, as for example spin waves in a ferromagnet and phonons in crystalline solids and liquid helium.

4 Continuous Local Symmetry: Higgs Bosons

The spontaneous breaking of a global gauge symmetry leads to the emergence of a massless Nambu-Goldstone bosons, but what happens, if one changes the gauge symmetry to a local one? Let us consider the same system, but for the case of invariance under the local gauge transformations

$$\phi'(x) = e^{iA(x)}\phi(x).$$  

The system can be made invariant under local gauge transformations by the usual procedure of introducing a massless gauge field $A_\mu$ and replacing the derivatives with covariant derivatives $D_\mu = \partial_\mu + ieA_\mu$ to obtain

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi)(D^\mu \phi^*) - V(\phi, \phi^*) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$
where the latter term denotes the kinetic energy term associated with the vector field $A_\mu$. Next one applies the same procedure as before, i.e. introducing the fields $\eta$ and $\xi$ as fluctuations about a particular vacuum state

$$\eta = \phi_1 - \mu/\lambda, \quad \xi = \phi_2.$$  

This procedure leads to a massive field $\eta$ with $m_\eta = \mu \sqrt{2}$, whereas the vector field $A_\mu$ now acquires a mass $m_A = e\mu/\lambda$, and the $\xi$ field can be transformed away by an appropriate choice of the gauge $\Lambda(x)$. A massless vector field $A_\mu$ carries two degrees of freedom corresponding to the transverse polarizations, whereas a massive vector field picks up a third degree of freedom from the longitudinal polarization. This extra degree of freedom comes from the Nambu-Goldstone boson which has disappeared from the theory. In other words, the gauge field absorbed the Nambu-Goldstone boson, thereby acquiring both a mass and a third polarization state. This is the so-called Brout-Englert-Higgs mechanism:

If a continuous global symmetry is changed to a local one, the Nambu-Goldstone boson disappears and the gauge bosons acquire a mass.

The $\phi(x)$ field is referred to as the Higgs field, where $\eta$ denotes the Higgs boson.

The case discussed above is that of a so-called Abelian Higgs model which is relevant to the study of metals that exhibit the phenomenon of superconductivity, i.e. the loss of electrical resistance at very low temperatures. The superconductor is invariant under local gauge symmetry, but the ground state consisting of a condensate of Cooper pairs of two electrons breaks the symmetry. In the superconducting phase the electrical current flows freely without experiencing the effect of electrical resistance. These currents effectively screen out the magnetic flux (Meissner effect). In other words, the photons have acquired an effective mass, which is a characteristic feature of the Higgs mechanism.

A popular account of the Higgs mechanism can be found on the home pages of the CERN Physics Tour (adapted from a prize-winning essay by David J. Miller).

To understand the Higgs mechanism, imagine that a room full of physicists chattering quietly is like space filled with the Higgs field. A well-known scientist walks in, creating a disturbance as he moves along the room and attracting a cluster of admirers with each step. This increases his resistance to movement, in other words, he acquires mass, just like a particle moving through the Higgs field. If a rumor crosses the room, it creates the same kind of clustering, but this time among the scientists themselves. In this analogy, these clusters are the Higgs particles.

5 Electroweak Unification

The Higgs mechanism is usually associated with the Weinberg-Salam model in which the spontaneous breaking of the $SU(2) \otimes U(1)$ gauge symmetry forms a crucial ingredient
in the unification of the electromagnetic and weak interactions. The Higgs mechanism gives the mass to the vector bosons $Z$ and $W^\pm$, the force carriers of the weak interaction. Although at present the electromagnetic and weak interactions appear very different, the former long range (massless photon) and the latter short range (massive intermediate vector bosons), at some time in the early Universe with temperatures in excess of 100 GeV, the electromagnetic and weak forces were unified into a single electroweak interaction.

The large difference in mass between the photons and the $Z$ and $W^\pm$ vector bosons is attributed to the spontaneous symmetry breaking as the hot Universe cooled. The electroweak symmetry can be restored by increasing the temperature (or going back in time to the early Universe). Notice here the parallel between a physicist in the present low-temperature Universe and Coleman’s little man living inside the ferromagnet: it took a lot of imagination of the physicists to detect the symmetry of the laws of nature!

The idea of the unification of all fundamental interactions can be taken even further: it has been speculated that at even higher temperatures (or earlier times in the Universe) the electroweak and the strong interactions were unified in the so-called grand unification. At still higher temperatures, gravity may be added to obtain a unification of all four fundamental interactions that we know nowadays, the gravitational, strong, weak and electromagnetic forces as manifestations of a single unified interaction (see Figure 3).

If the Standard Model of particle physics were perfectly symmetric, none of the particles would have any mass. The fact that most fundamental particles do have masses is a consequence of the Higgs mechanism, a combination of local gauge invariance and spontaneous symmetry breaking by which the particles acquire their masses through interactions with the all-pervading Higgs field. In the early Universe, the Higgs field looked the same in all directions, but this symmetry was spontaneously broken shortly after the Big Bang, in much the same way the perfect symmetry of a pencil standing on its tip is spontaneously broken when the pencil falls over and defines a direction in space. One of the biggest challenges of particle physics is the discovery of the Higgs boson associated with the Higgs field. So far it has escaped detection in the laboratory, but the search for the Higgs boson is one of the main objectives of the Large Hadron Collider at CERN, which is scheduled to come into operation in 2007. Its discovery is of the utmost importance to the Standard Model since the Higgs boson is essential in understanding the origin of mass.

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Figure 3: Chronology of the Universe, taken from the HyperPhysics home page.

**Glossary**

**Electroweak theory**: Unified theory of the electromagnetic and weak interactions.

**Ferromagnet**: Material that can exhibit spontaneous magnetization, *i.e.* without the presence of an external magnetic field.

**Global symmetry**: Symmetry transformation which is independent on space-time points.

**Higgs boson**: Massive particle predicted by the Standard Model to explain the origin of mass, but which has not been observed yet experimentally.

**Local symmetry**: Symmetry transformation which depends on space-time points.

**Nambu-Goldstone boson**: Massless boson which arises when a continuous global symmetry is broken spontaneously.

**Phase transition**: Transformation of a physical system from one phase to another (*e.g.* solid,
liquid, gas) which is characterized by a sudden change in one or more physical observables.

**Spontaneously broken symmetry**: Symmetry of the fundamental equations of a physical system which is not exhibited by the ground state.

**Superconductivity**: The loss of electrical resistance at very low temperatures.

**Symmetry**: Invariance of a physical system under a certain transformation.

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http://hyperphysics.phy-astr.gsu.edu/hbase/astro/unify.html [HyperPhysics home page on Big Bang expansion and the fundamental forces].