



**Pairing correlations and two-particle
transfer reactions**
Interplay of reaction mechanism and
structure properties

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How to use dynamics to study pairing correlations?

The main road is clearly provided by the study of those processes where a pair of particles is involved, e.g. transferred from/to another nucleus (two-particle transfer) or ejected onto the continuum (two-particle break-up or two-particle knock-out). Clearly the probabilities for such processes must be influenced by the particle-particle correlations, but the quantitative connection is not obvious.

Coulomb excitation, inelastic to collective states and single-particle transfer reactions are to large extent well described by first-order one-step mechanism and this allows to extract easily spectroscopic information (for example in low-energy one-step Coulomb excitation the excitation probability is directly proportional to the $B(E\lambda)$ values). But **two-nucleon transfer reactions** are rather complicated processes

It is widely accepted that pairing correlations strongly effect (and enhance) two-particle transfer reactions. But the quantitative connection is not obvious. Will cross sections scale with the square of the two-particle transfer matrix elements? Or the radial dependence of the two-particle transition densities contain more information? And how this information enter into the reaction mechanism?

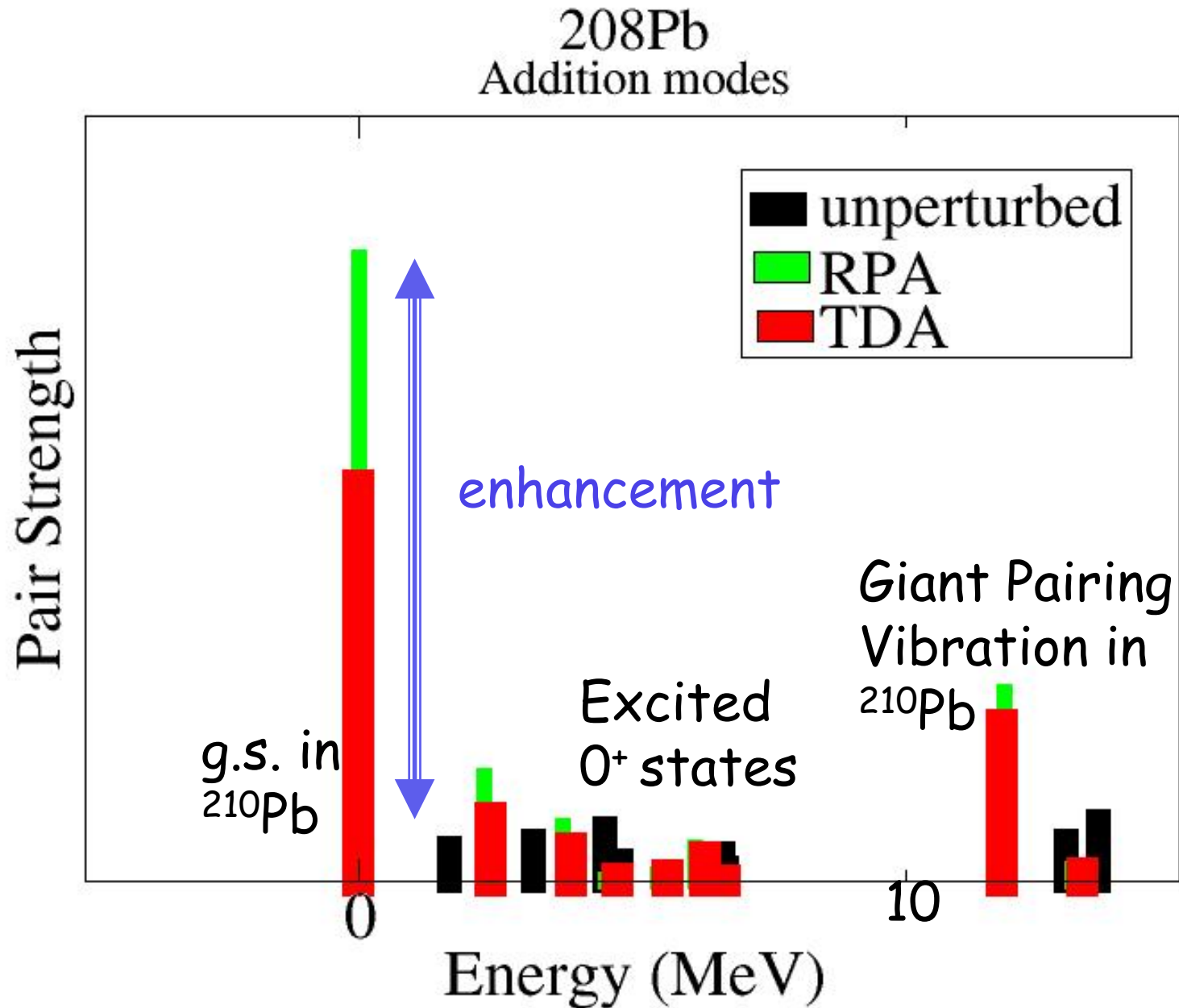
It is often assumed that the cross section for two-particle transfer just scale with T_0 , the square of the matrix element of the pair creation (or removal) operator

$$P^+ = \sum_j [a_j^+ a_j^+]_{00}$$

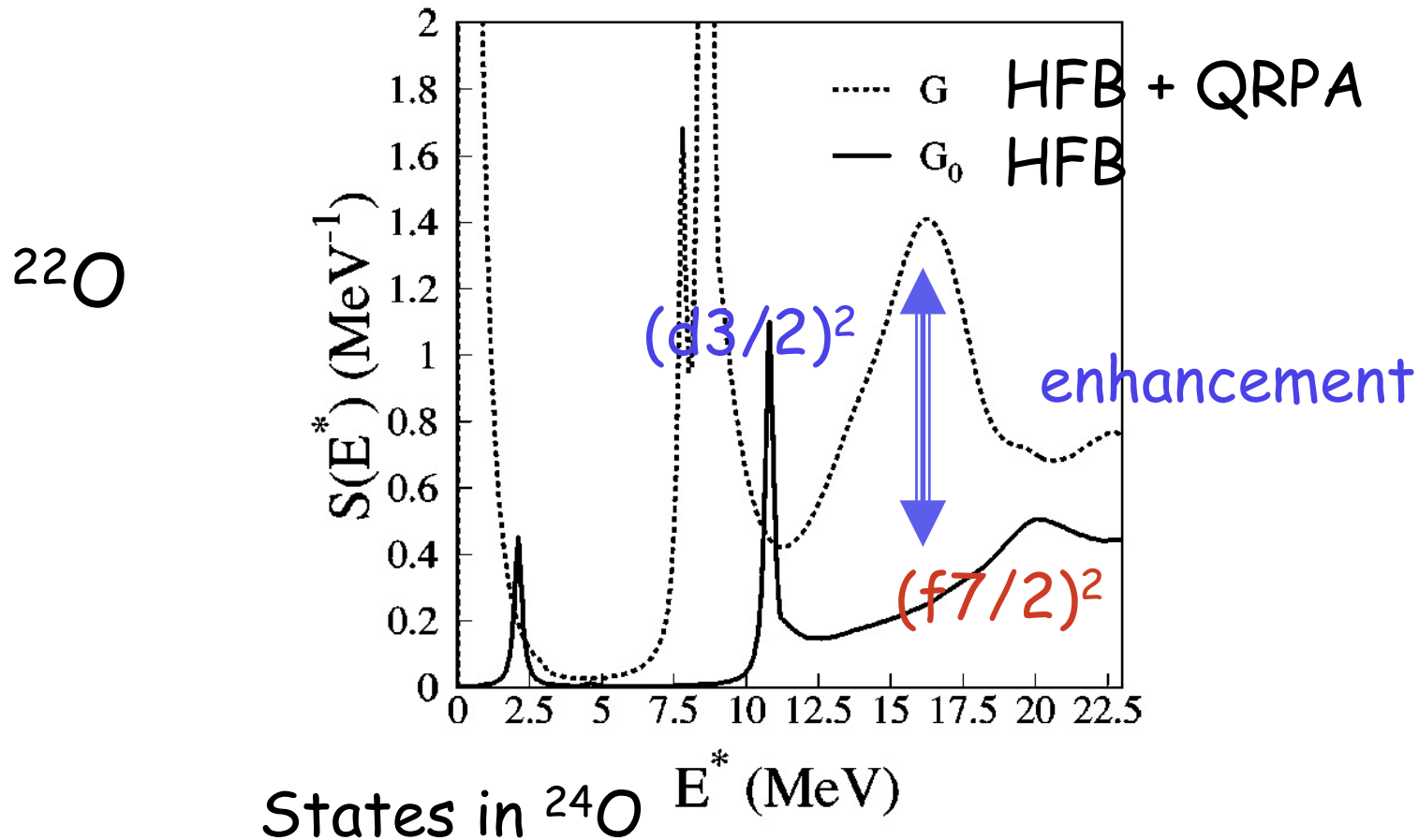
For this reason the traditional way to define and measure the collectivity of pairing modes is to compare with **single-particle** pair transition densities and matrix elements to define some "pairing" single-particle units and therefore "pairing" enhancement factors.

Obs: We discuss here monopole pairing modes, i.e. 0^+ states

Example



A system close to the drip line: pair strength function in ^{22}O



Khan, Sandulescu, Van Giai, Grasso

In models that do not explicitly use the fermion degrees of freedom the "form" of the two-particle addition operator has to be assumed. For example, within the IBM, one assumes as addition operator

$$s^+$$

or, from boson mapping procedure, one also includes higher-order terms

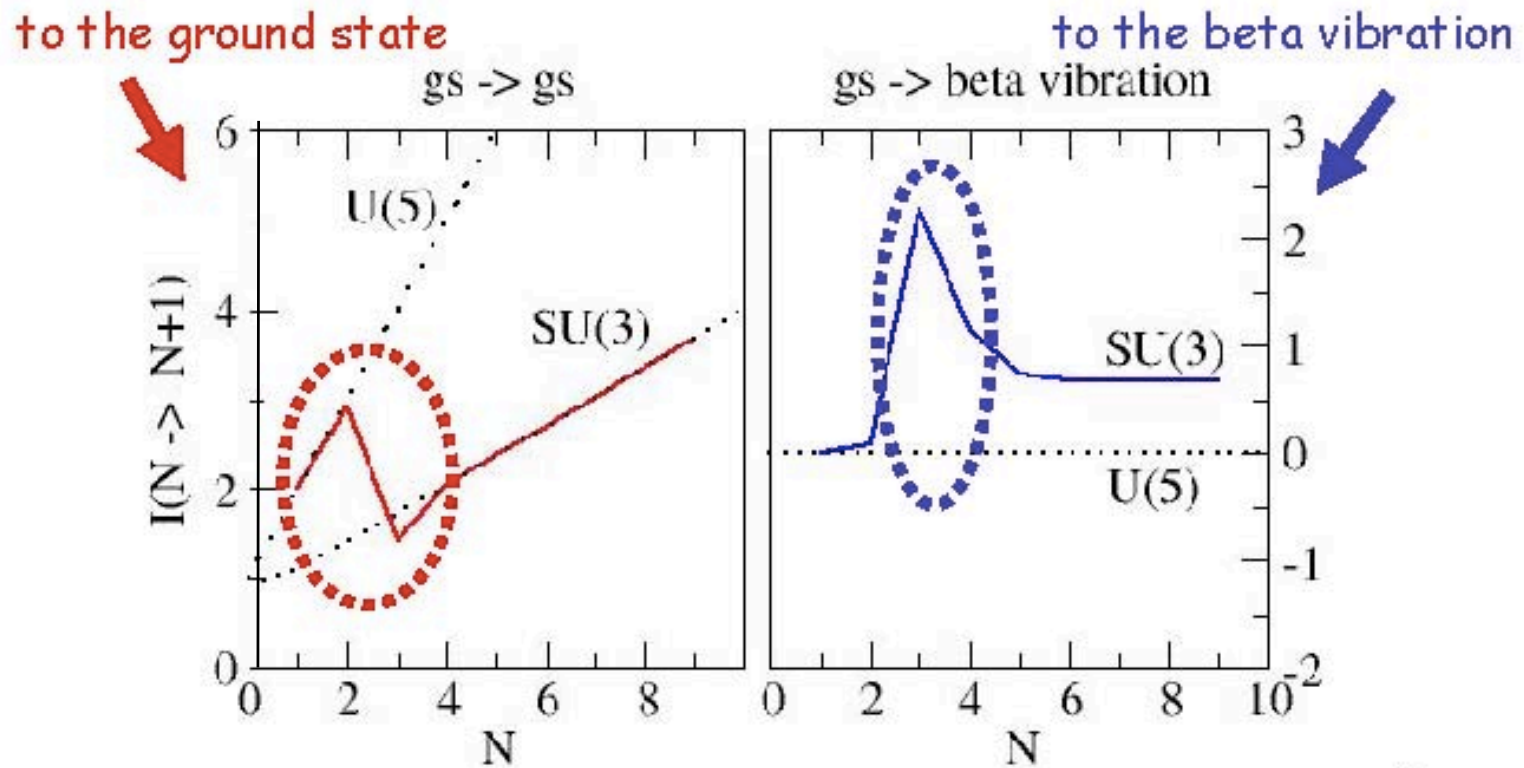
$$s^+ + \alpha s^+ s^+ s + \beta [d^+ d^+]_0 s + \gamma [d^+ s^+ d]_0 + \dots$$

OBS: The model provides relative intensities, not absolute values

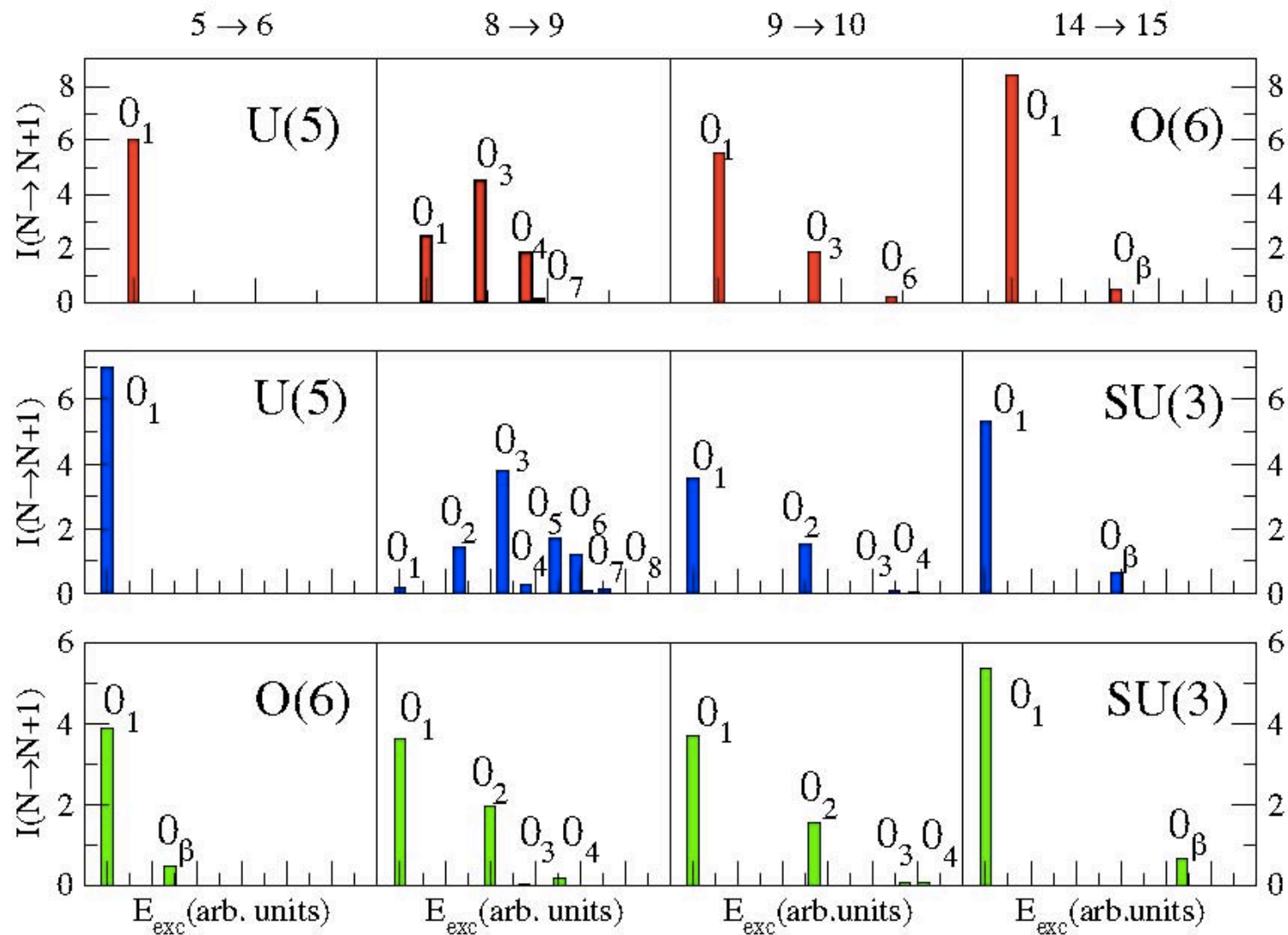
EXAMPLE: signature of the phase transitions

Example: $L=0$ pair transfer in a phase transition from spherical to axial deformation

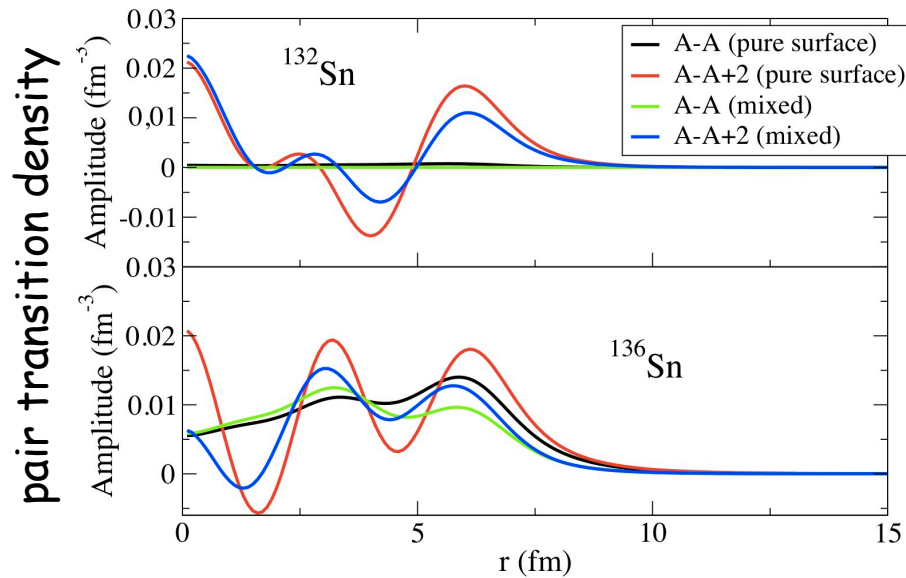
(from Fossion et al, PRC, 2007)



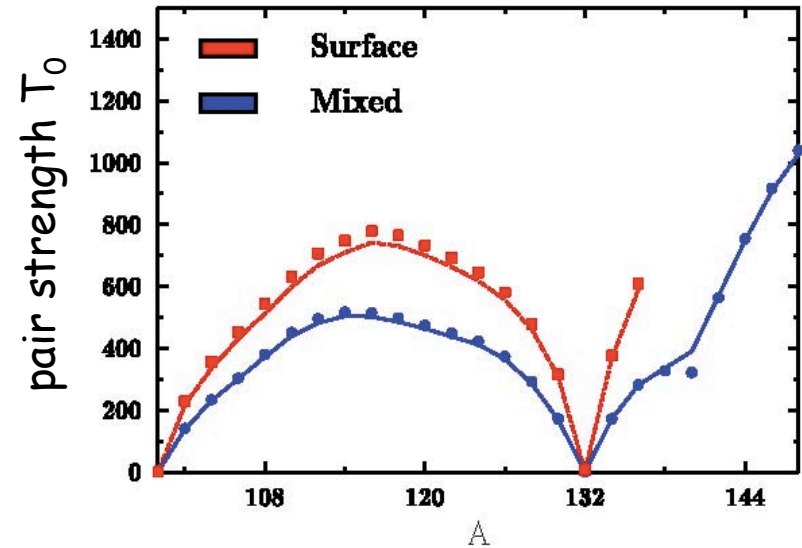
Obs: fragmentation of the strength in
 correspondence to phase transitions
 (from Fossion et al, PRC, 2007)



Example of structure investigation of pairing correlations: can we discriminate among different forms of pairing interactions? Example: can we distinguish pure surface pairing interaction from mixed (volume plus surface) used in HFB calculations?
 Grasso, Lacroix, AV



$A(gs) \rightarrow A+2 (gs)$



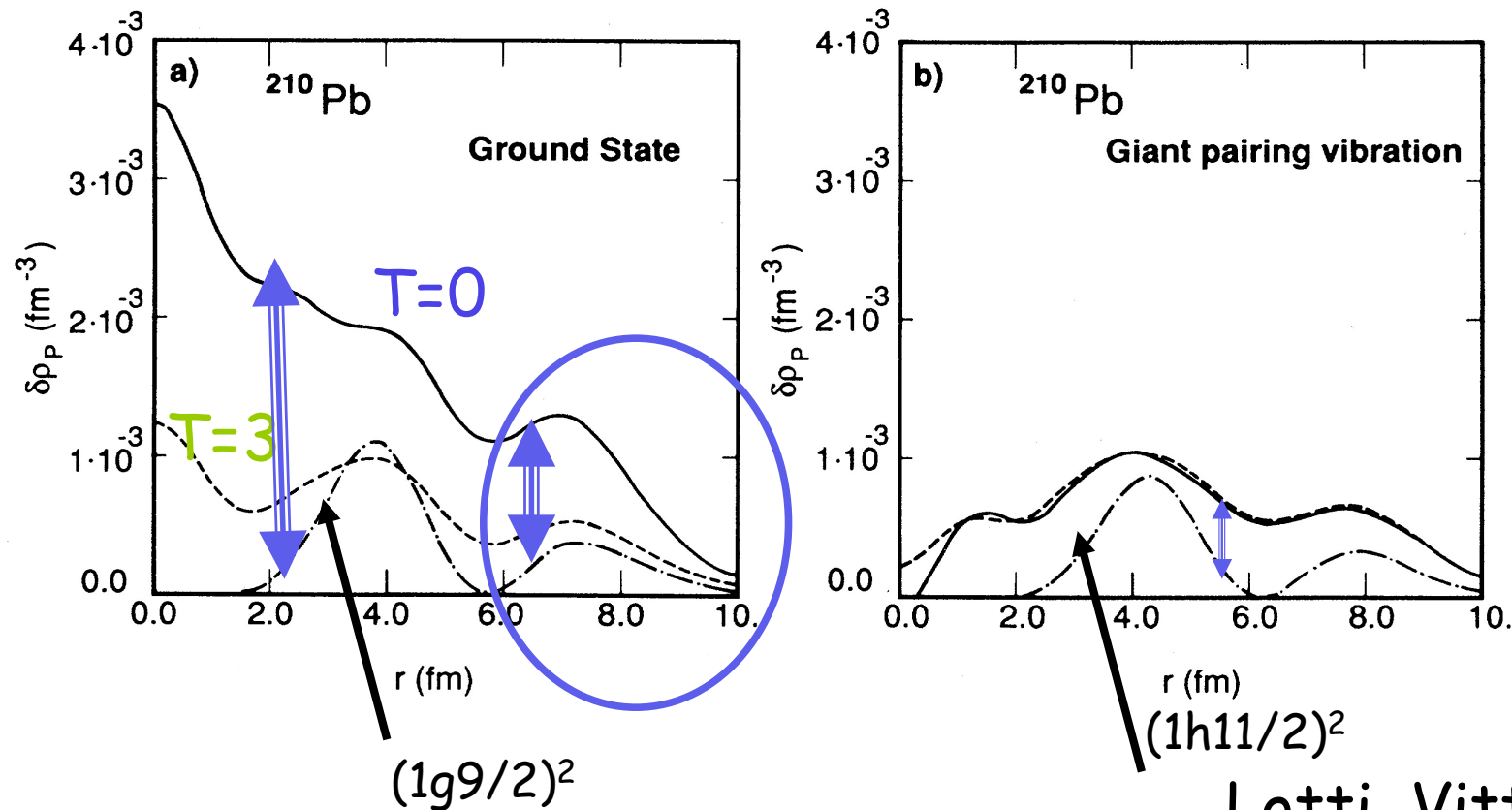
$A(gs) \rightarrow A+2 (gs)$

How the different behavior in the tail enter in the reaction mechanism? Is only the integrated value pair strength T_0 relevant?

But the two-particle transfer process is not sensitive to just the pair matrix element. We have to look at the radial dependence, which is relevant for the reaction mechanism associated with processes involving pairs of particles (pair transfer, pair break-up, pair knock-out, ...).

Comparison with pure single-particle configurations pair transition density

$$\rho^{\nu}_p(r,r) = \kappa^{\nu}(r\sigma) = \langle 0 | c(r\sigma) \bar{c}(r\sigma) | \nu \rangle$$



Lotti, Vitturi et al

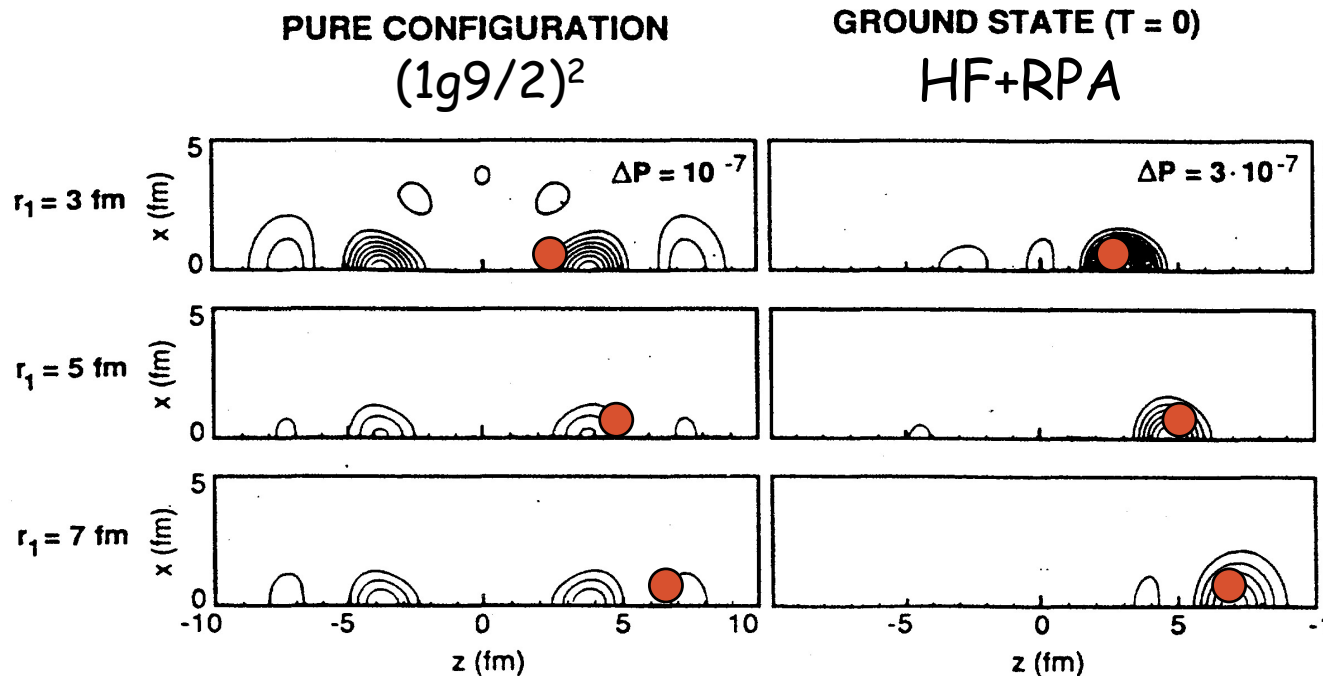
OBS: the enhancement is not the same for all values of r

particle-particle spatial correlations

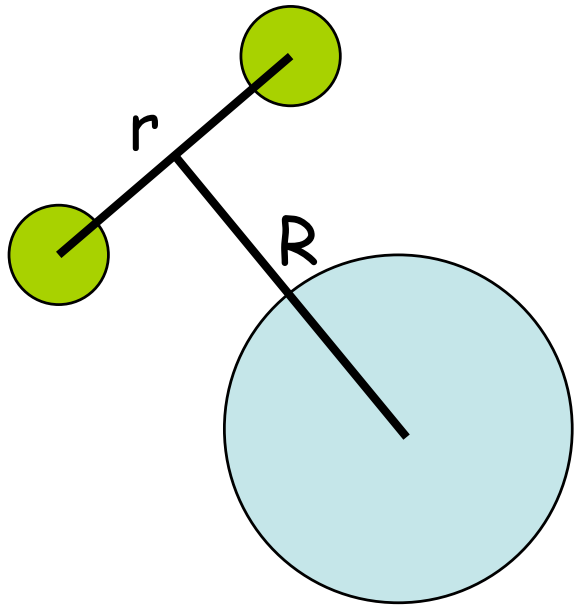
$|\Psi(r_1, r_2)|^2$ as a function of r_2 , for fixed r_1

Neutron **addition** mode: ground state of ^{210}Pb

● position of particle 1



OBS: mixing of configurations with opposite parity

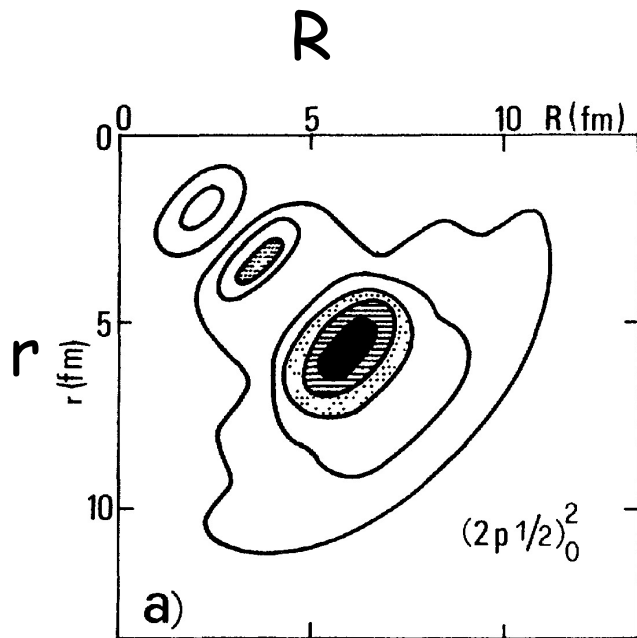


$$\delta\rho_p(R,r)$$

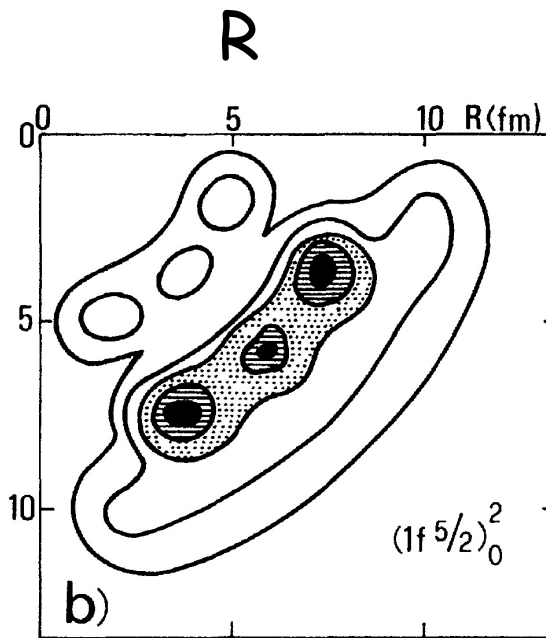
Catara et al

^{206}Pb

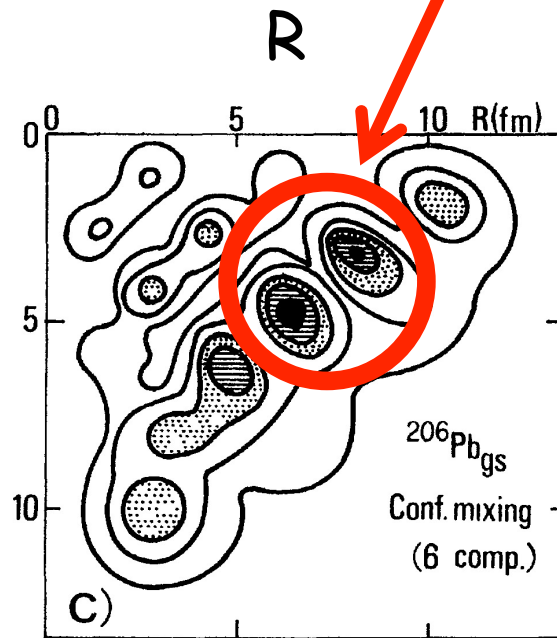
larger R , smaller r



$(3p_{1/2})^2$



$(2f_{5/2})^2$



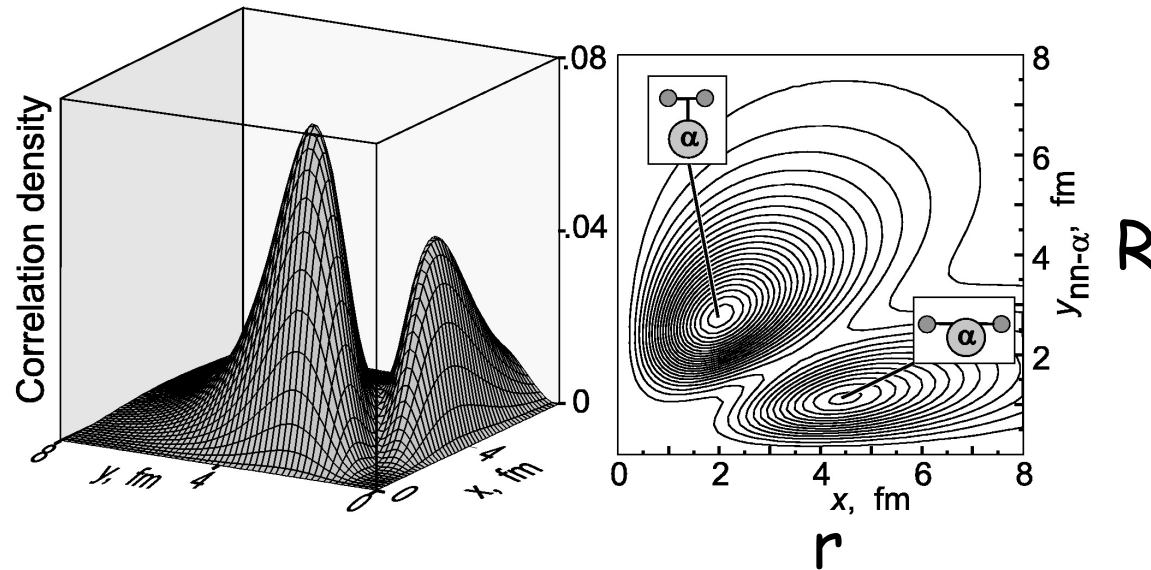
Correlated g.s. (RPA)

$^{206}\text{Pb}_{\text{gs}}$
Conf. mixing
(6 comp.)

Interesting problem:

how is changed the picture as we move closer or even beyond the drip lines?

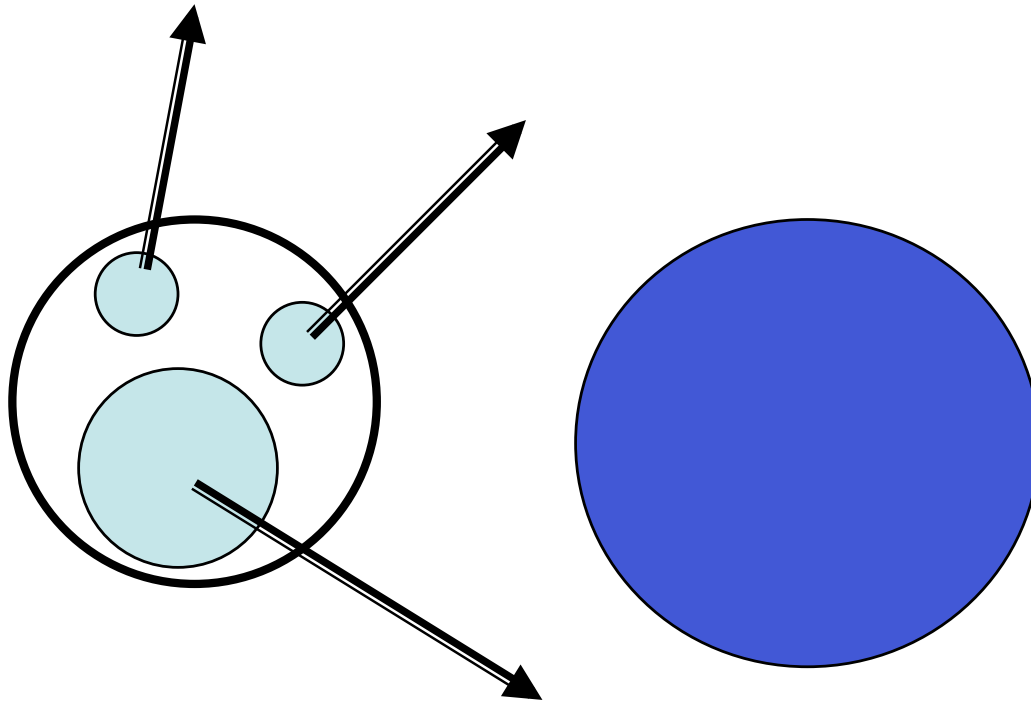
Example:
the case of
 ${}^6\text{He}$



Oganessian, Zagrebaev, Vaagen, 1999

As a first example of the dynamical effects of particle-particle correlations on reaction dynamics, let us consider the process of **two-particle break-up in Borromean nuclei**

The break-up process is induced by the action of the external one-body field (for example produced by the reaction partner in a heavy-ion reaction), but pairing correlations in the initial nucleus will influence the reaction mechanism enhancing two-particle break-up

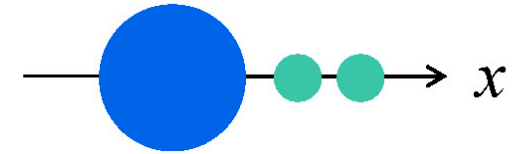


Break-up of a two-particle halo system is a rather complex 4-body process (at minimum).

To make it simpler let us consider as example an one-dimensional case

Hagino, Vitturi, Sagawa, Perez Bernal, 2011

One-dimensional three-body model

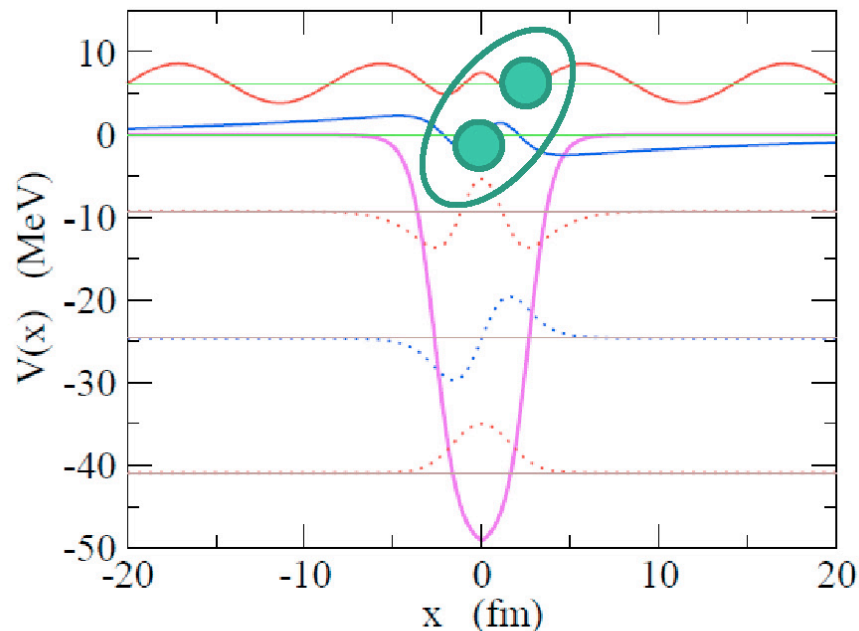


Two interacting neutrons in a one-dimensional potential well:

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx_1^2} + V(x_1) - \frac{\hbar^2}{2m} \frac{d^2}{dx_2^2} + V(x_2) + v_{nn}(x_1, x_2)$$

density-dependent contact interaction:

$$v_{nn}(x, x') = -g \left(1 - \frac{1}{1 + e^{(|x|-R)/a}} \right) \delta(x - x')$$



$$\Psi_{\text{gs}}(x_1, x_2) = \sum_{n \leq n'} \alpha_{nn'} \Psi_{nn'}(x_1, x_2)$$

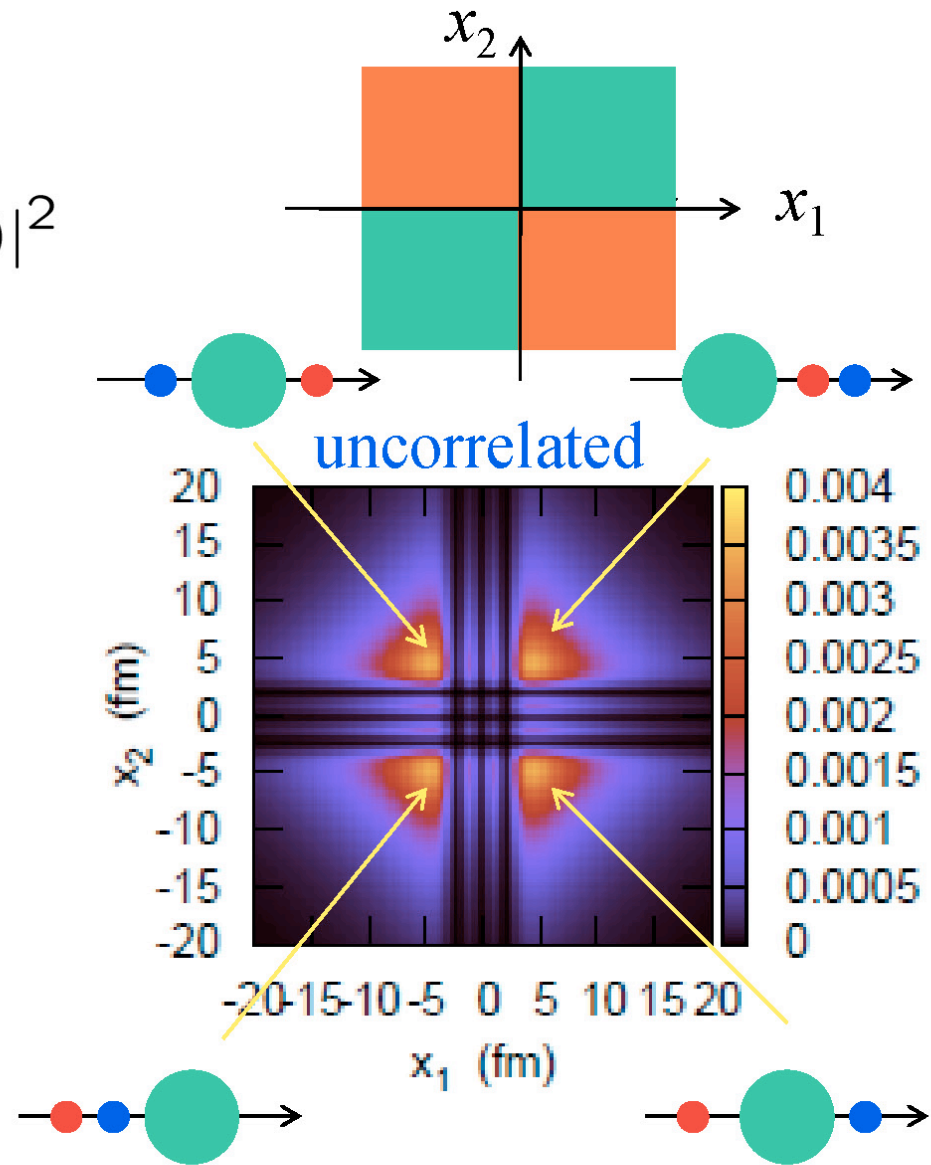
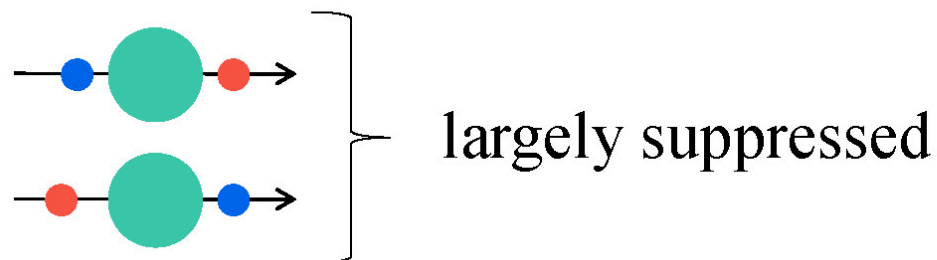
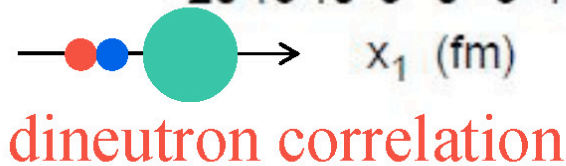
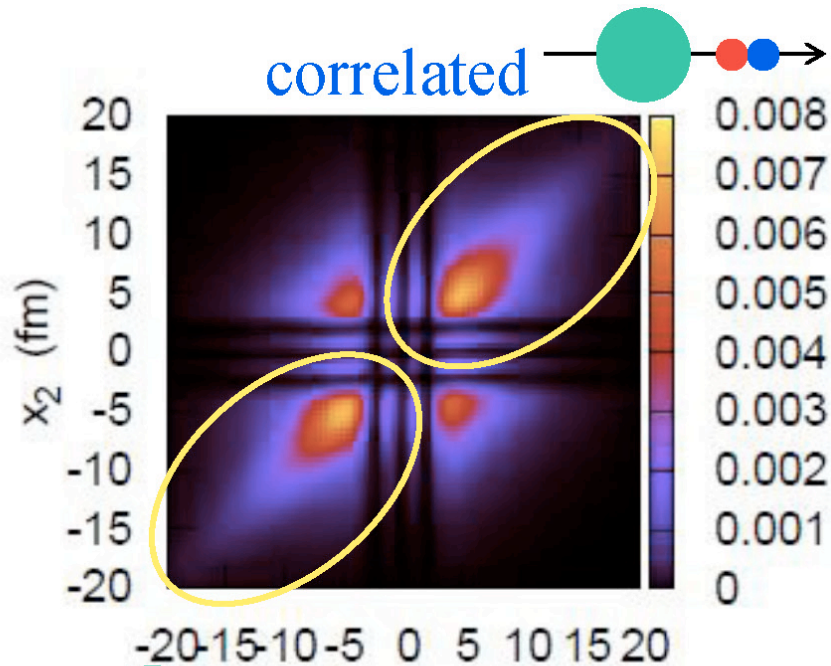
$$\Psi_{nn'}(x_1, x_2) \propto \mathcal{S}[\phi_n(x_1)\phi_{n'}(x_2)] \times |S = 0\rangle$$

• $S = 0$ state: symmetric for the spatial part of wf

• n, n' : the same parity

Ground state properties

two-particle density: $|\Psi_{\text{gs}}(x_1, x_2)|^2$

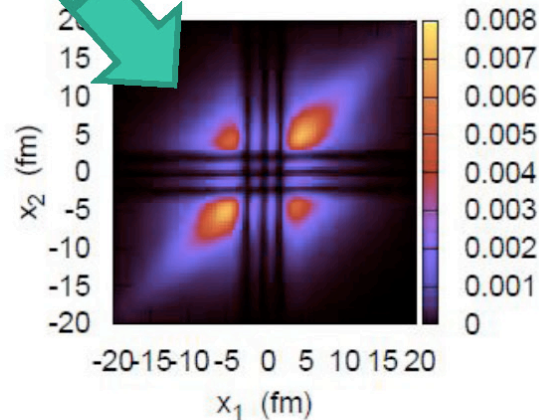


four symmetric peaks

Nuclear Breakup Process



(one-body) external field



$\Psi_{\text{gs}}(x_1, x_2)$

Time-dependent two-particle Schroedinger equation:

$$i\hbar \frac{\partial}{\partial t} \Psi(x_1, x_2, t) = [H + V_{\text{ext}}(x_1, x_2, t)] \Psi(x_1, x_2, t)$$

external one-body field

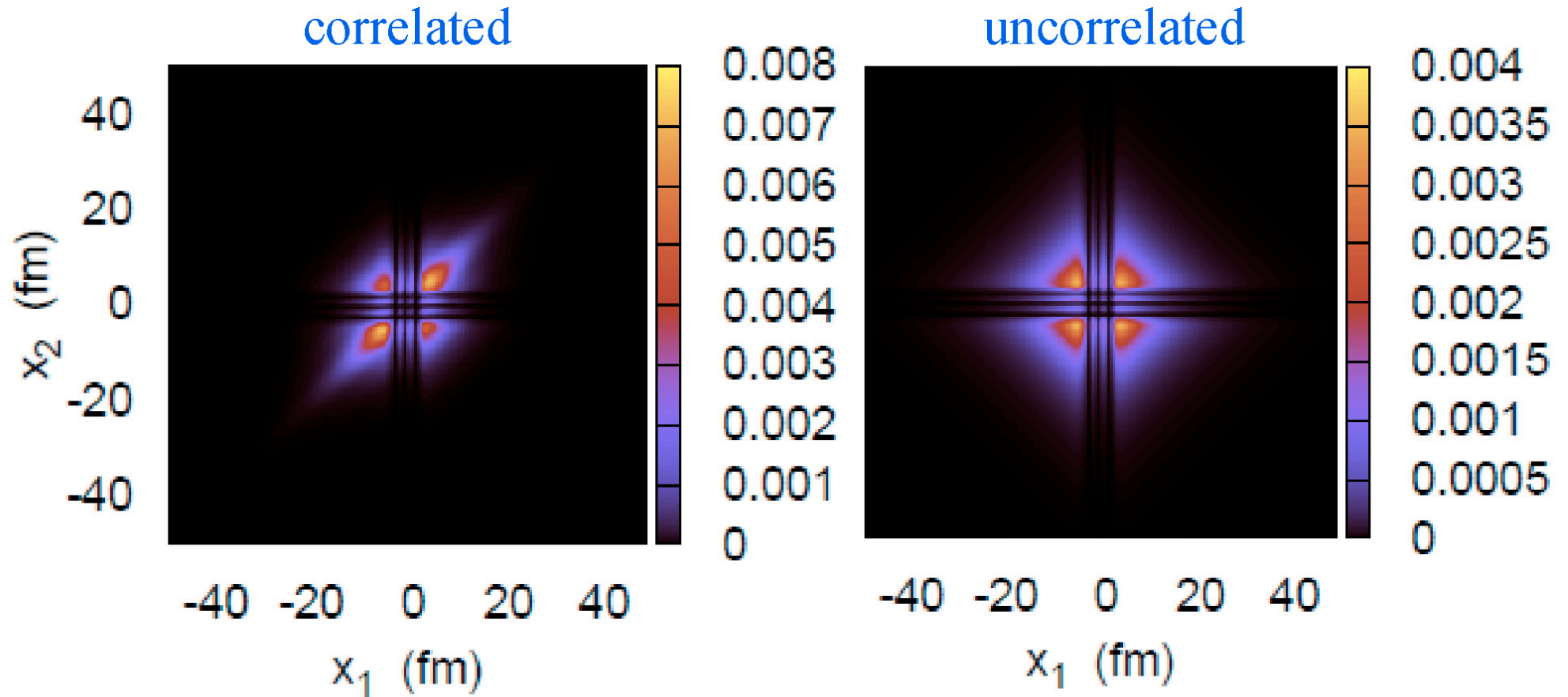
$$V_{\text{ext}}(x_1, x_2, t) = \sum_{i=1,2} V_c e^{-t^2/2\sigma_t^2} e^{-(x_i-x_0)^2/2\sigma_x^2}$$

$$V_c = 3 \text{ MeV}, \sigma_t = 2.1 \text{ hbar/MeV}, x_0 = 0$$

The perturbing interaction (that produces the break-up) is a **one-body field** (i.e. acting individually on each of the two particles). The enhanced two-particle break-up originates from the correlations in the two-particle wave function, and not from the use of a "cluster" interaction in the reaction mechanism

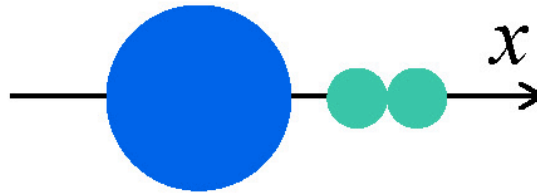
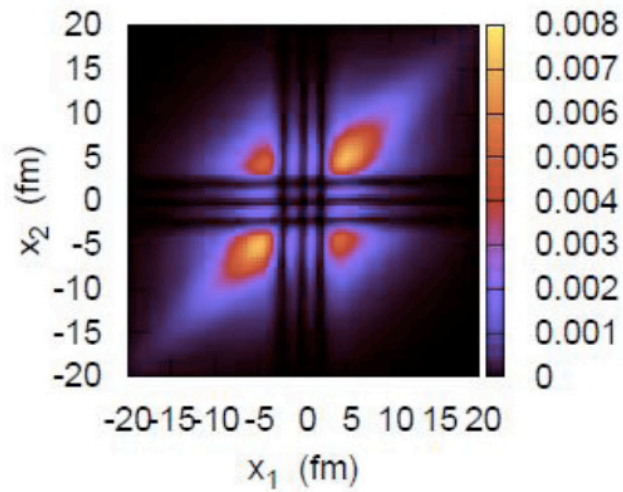
two-particle density at $t = t_{\text{ini}}$

Initial situation

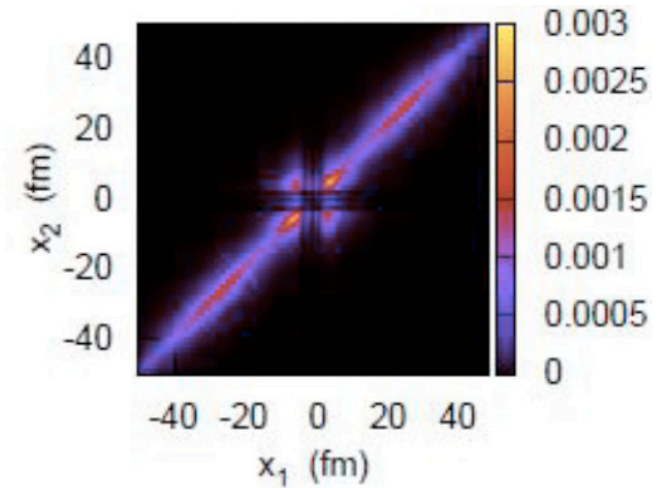


Time evolution

initial stage



final stage

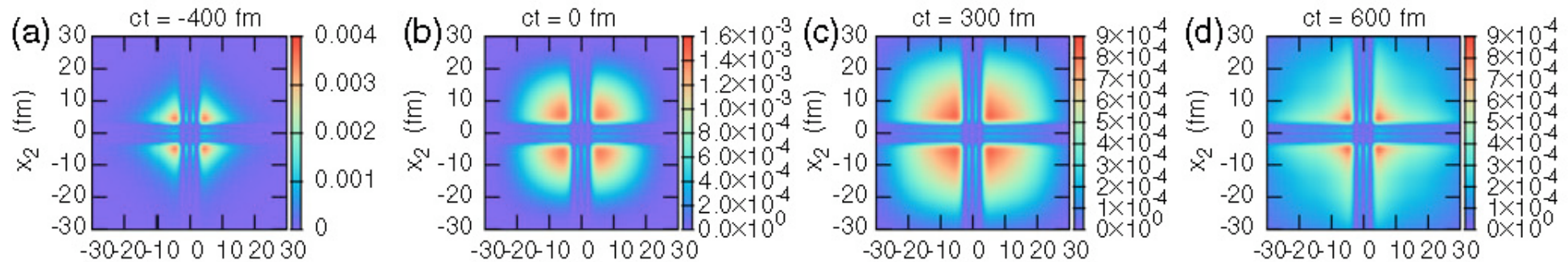


Particles are emitted in the
same direction
(di-neutron emission)

Time evolution (uncorrelated case)

initial stage

final stage

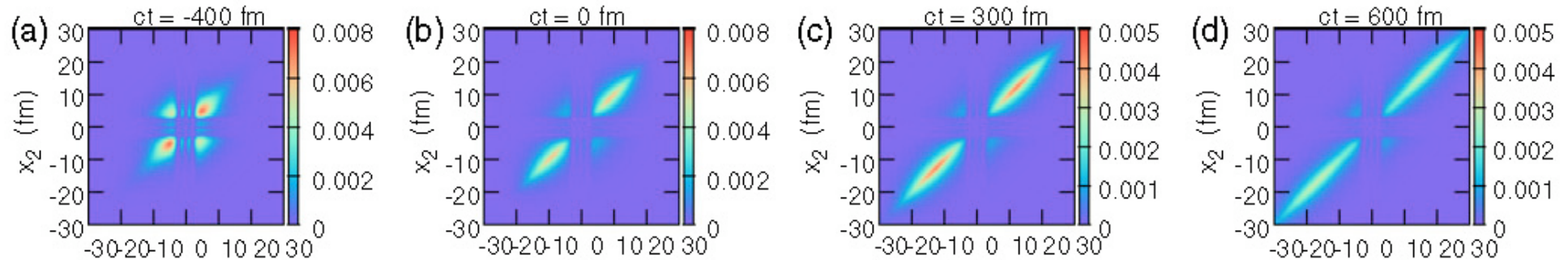


Obs: in absence of pair interaction there is no "angular" correlation between the two emitted particles

Time evolution (correlated case)

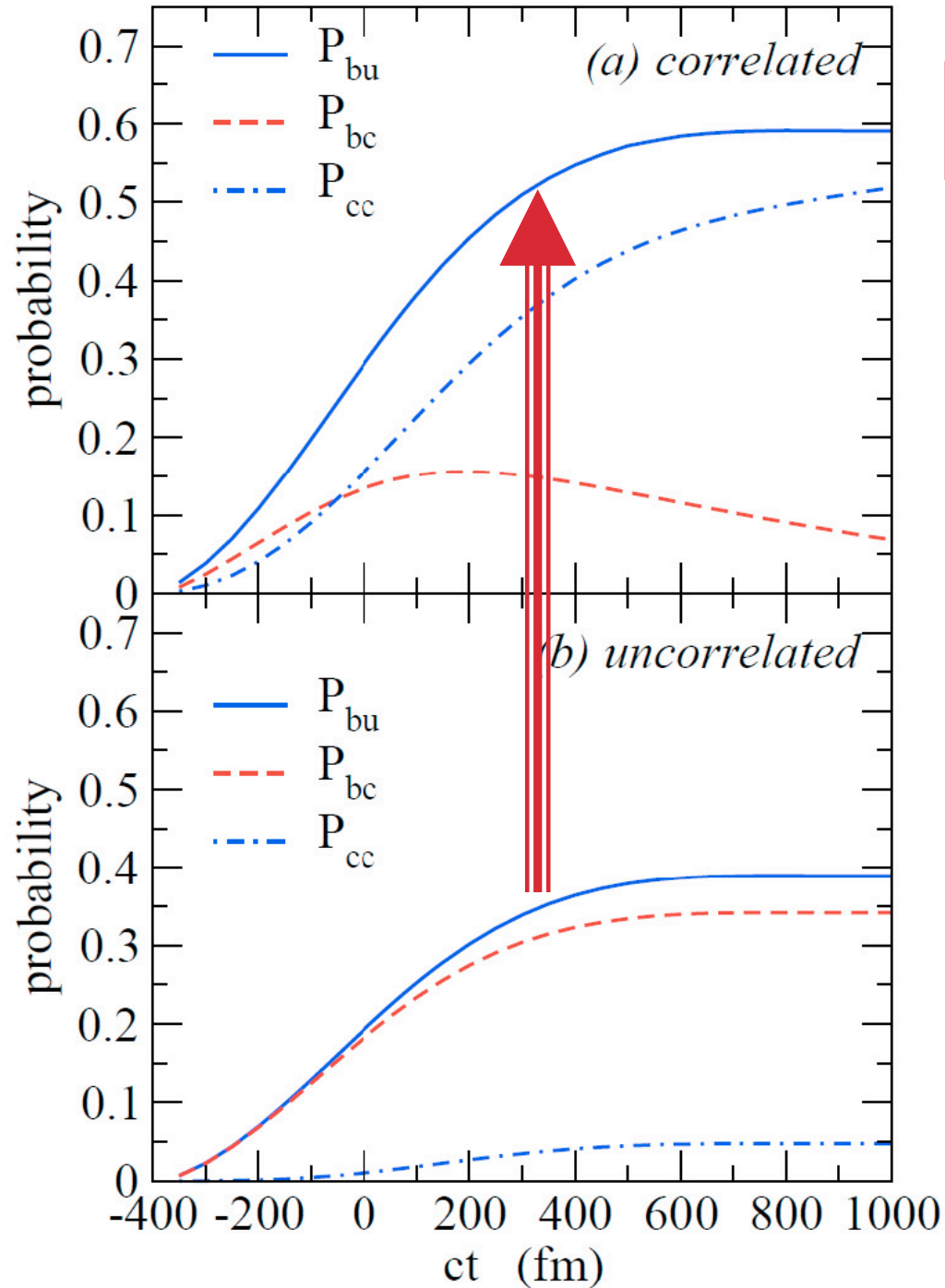
initial stage

final stage



Particles are emitted in the
same direction ($x_1 \sim x_2$)

Comparing one- and two-particle break-up



➤ Pairing: enhances the breakup

➤ Correlated: (cc) process
case dominant

➤ Uncorrelated: (bc) process
case dominant

P_{cc} : 2 neutron breakup

P_{bc} : 1 neutron breakup

But let us move to **two-particle transfer reactions** and the associated reaction mechanism (mainly for heavy-ion induced processes)

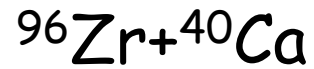
Large number of different approaches, ranging from macroscopic to semi-microscopic and to fully microscopic. They all try to reduce the actual complexity of the problem, which is a four-body scattering (the two cores plus the two transferred particles), to more tractable frameworks.

Aside from the precise description of the reaction mechanism (and therefore from the absolute values of the cross sections) one has to take into account **Q-value effects**.

In fact, keeping fixed any other parameter, the probability for populating a definite final channel depends on the **Q-value** of the reaction. The dependence (in first approximation a gaussian distribution centered in the optimum Q-value) is very strong in the case of heavy-ion induced reactions, weaker in the case of light ions.

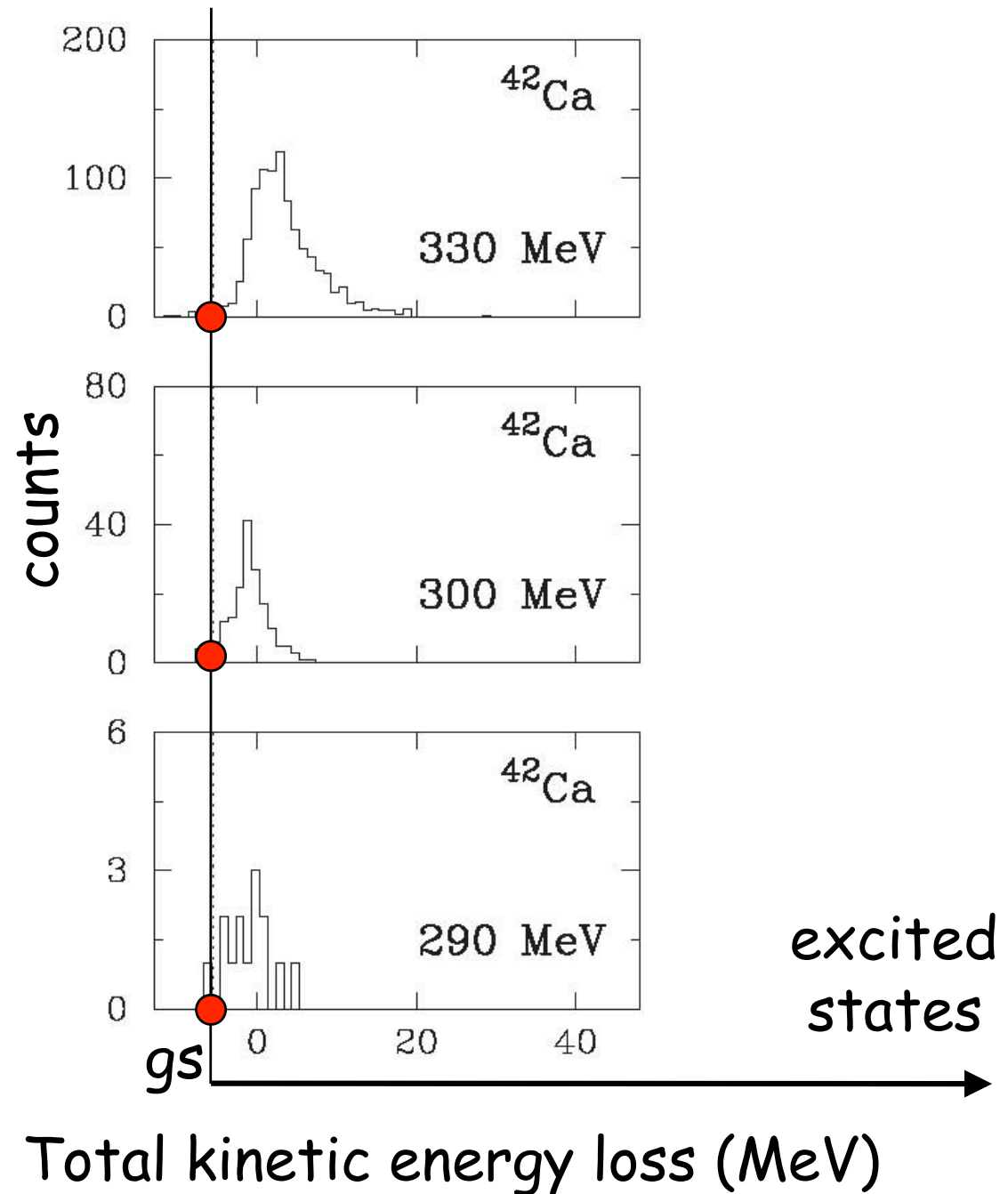
The optimum Q-value depends on the angular momentum transfer and on the charge of the transferred particles. In the specific case of $L=0$ two-neutron transfer, the **optimal Q-value is approximately zero**. But the actual Q-value for two-particle transfer to the ground states may be different from zero

Experimental evidence:
one example



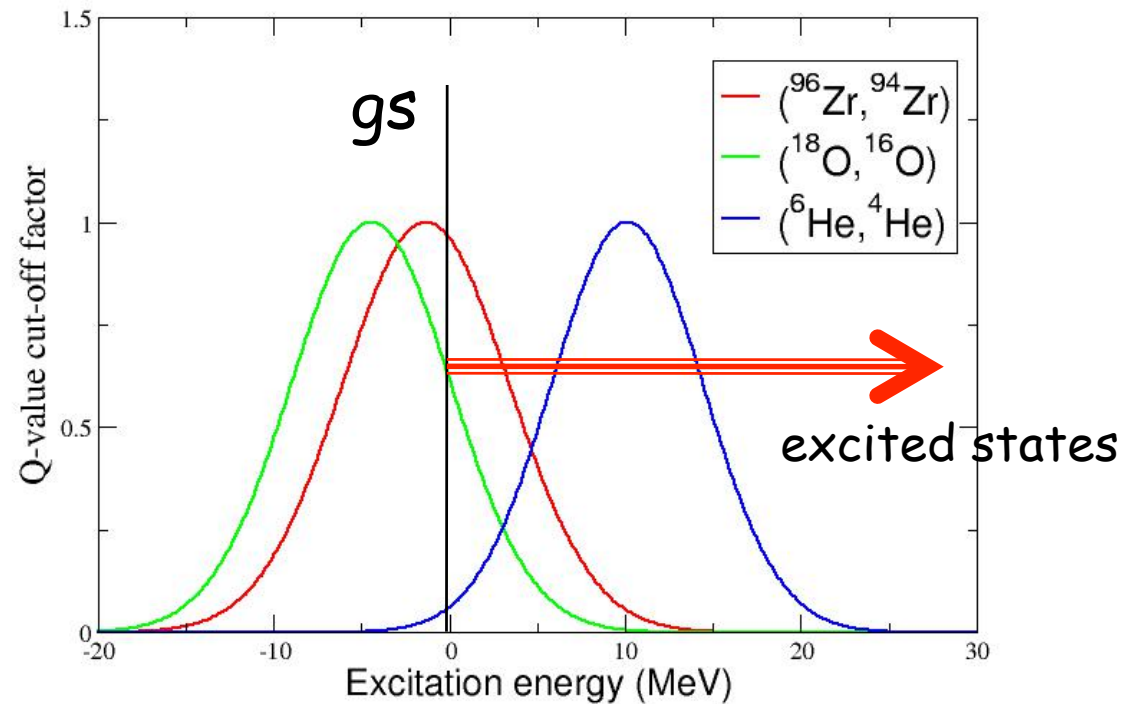
Selecting final
 ${}^{42}\text{Ca}$ mass partition

In this case the
population of the
"collective" ground
state is hindered
by the Q-value
window What
information on
pairing
correlations?



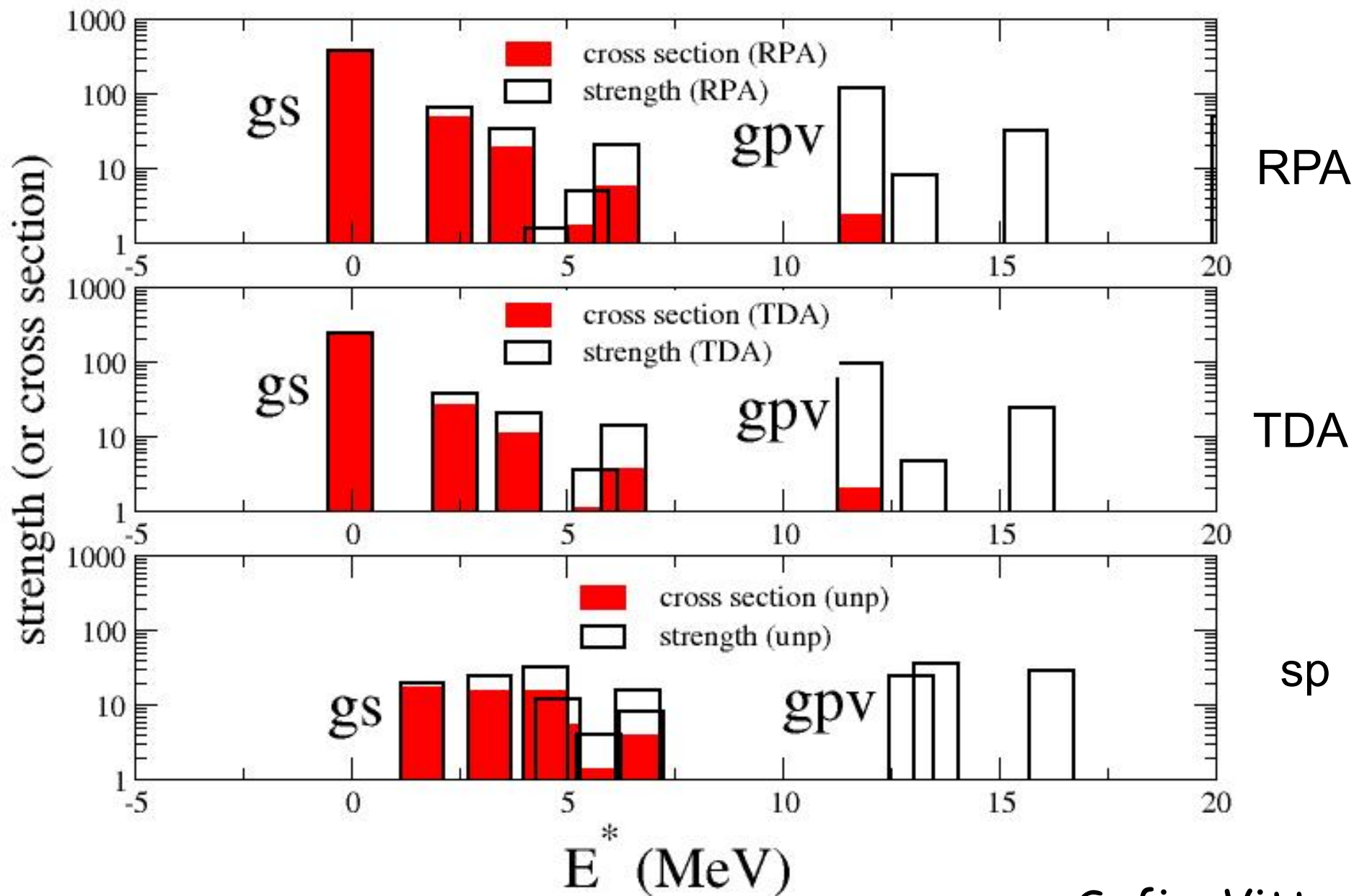
Playing with different combinations of projectile/target (having different Q_{gg} -value) one can favour different energy windows

Example: Target ^{208}Pb Final ^{210}Pb (at bombarding energy $E_{\text{cm}} = 1.2 E_{\text{barrier}}$)

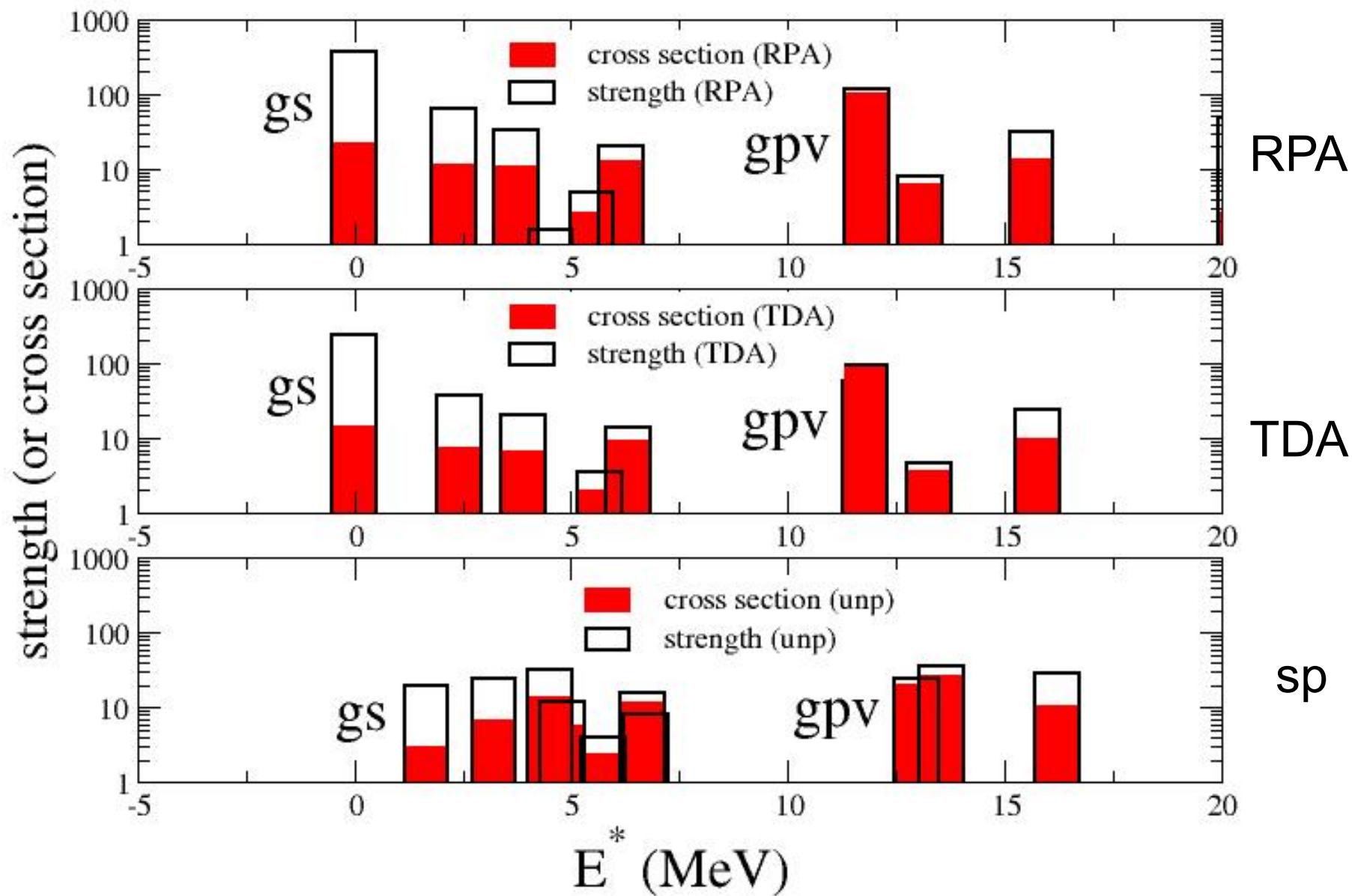


The pairing strength (as a function of the excitation energy) is therefore modulated by the Q-value cut-off to yield the final two-particle cross section

$^{208}\text{Pb}(^{18}\text{O}, ^{16}\text{O})^{210}\text{Pb} (0^+ \text{ states})$



$^{208}\text{Pb}(^6\text{He}, ^4\text{He})^{210}\text{Pb} (0^+ \text{ states})$



Another problem: multipole selection

“standard” pairing interaction is associated with $\lambda=0$ states and $\lambda=0$ two-particle creation (or removal) response *

To investigate pairing interaction one needs therefore to select the population of $\lambda=0$ states from the often overwhelming states with other multipolarities (trivial issue for theorists, not for experimentalists). Possible clue: shape of angular distributions

* note, however, the role played by “quadrupole” pairing in deformed nuclei

BASIC PROBLEM: The reaction mechanism (hopefully reasonably physically correct but on the same time sufficiently simple)

Large variety of models on the market (and this is not a good signal)

The fully microscopic approach is based on sequential two-step process (each step transfers one particle)

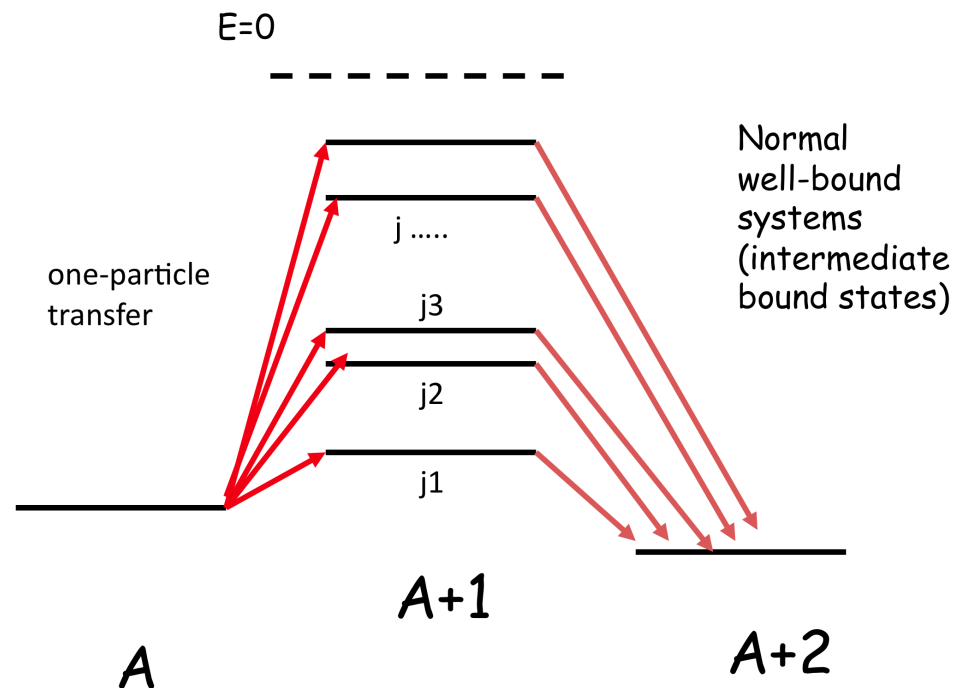
Microscopy: Pairing enhancement comes from the coherent interference of the different paths through the different intermediate states in $(a-1)$ and $(A+1)$ nuclei, due to the correlations in initial and final wave functions

Building blocks: single-particle formfactors and many-body wf's

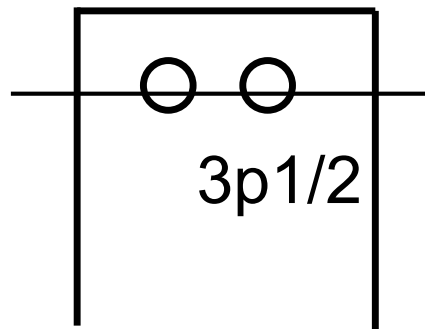
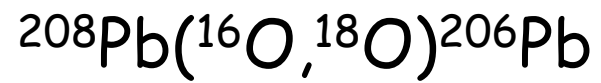
Problems: quantal calculations rather complex (taking into account full recoil), semiclassical more feasible (but approximate treatment of recoil)

All microscopy and nuclear structure information are contained in the two-particle transfer amplitudes (from correlated initial and final wave functions, so provided by structure models), which give the weight of each two-step path, and in the single particle transfer formfactors, which need single particle wavefunctions in target and projectile

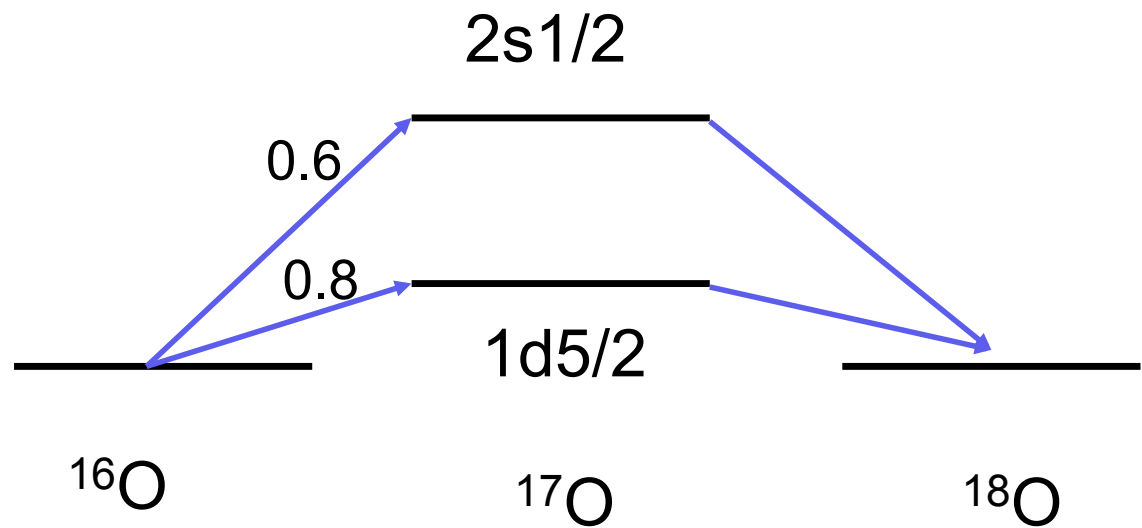
Obs: Basic idea: dominance of mean field, which provides the framework for defining the single-particle content of the correlated wave functions



Example

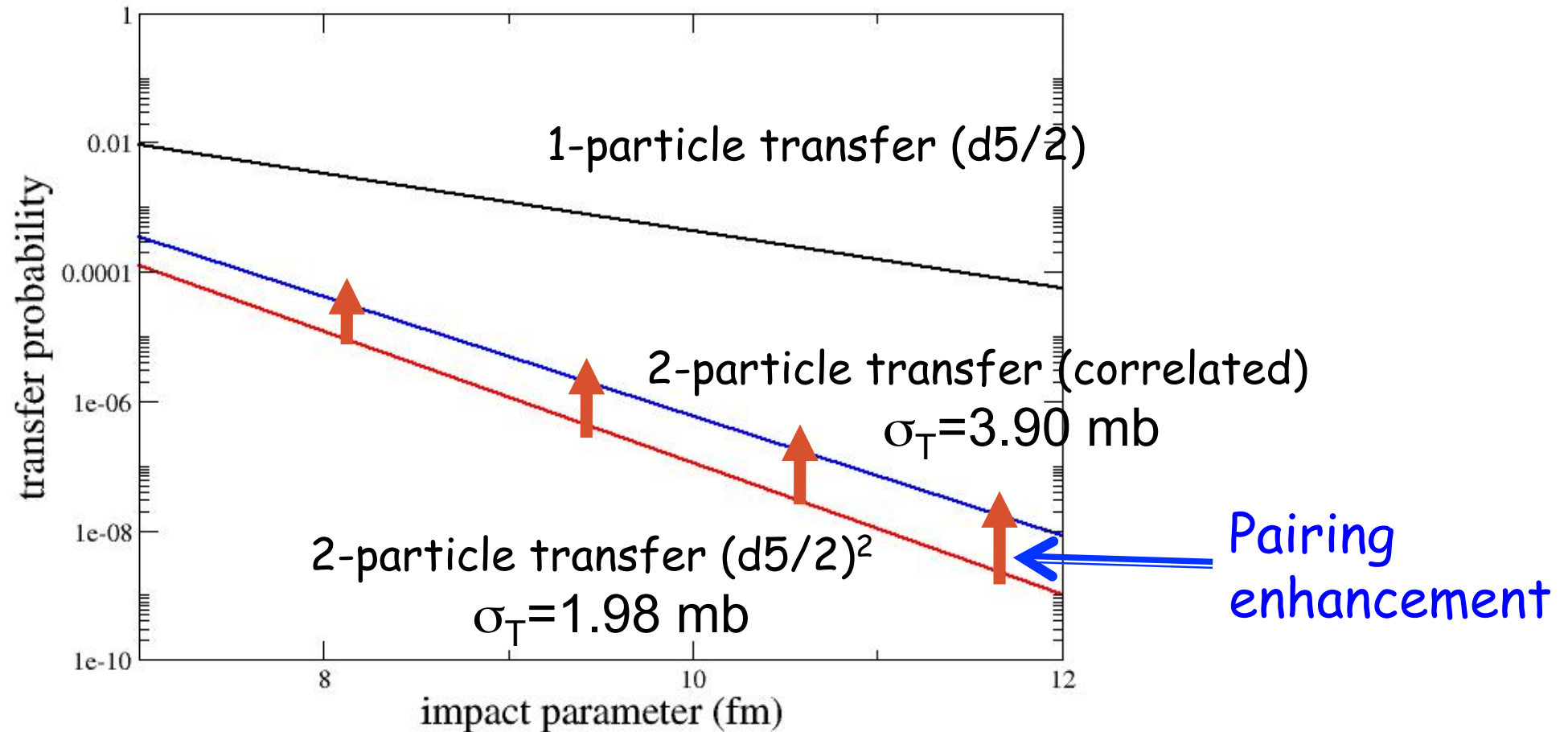
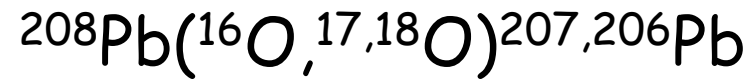


^{208}Pb



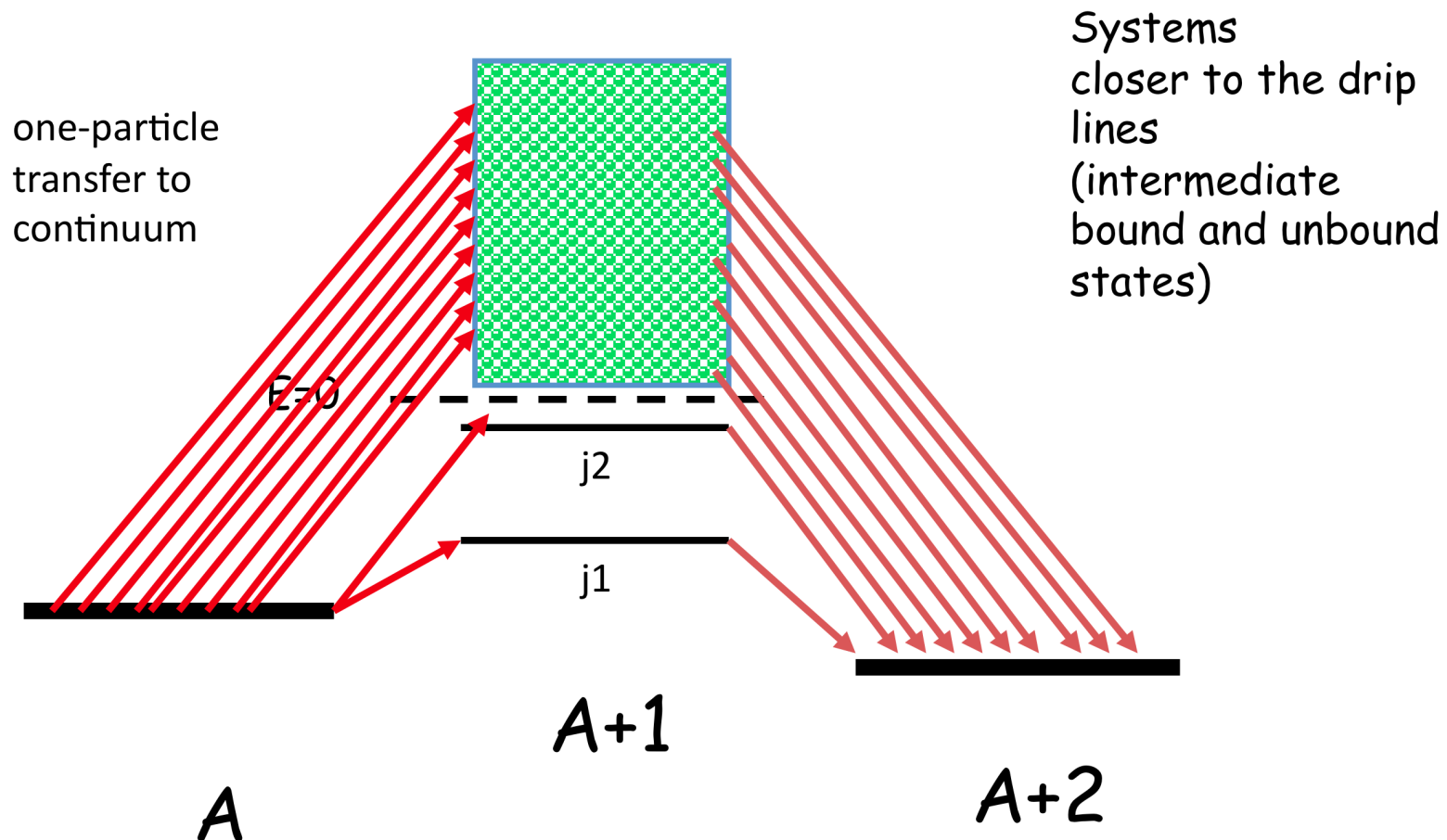
$0.8 (1d_{5/2})^2 + 0.6(2s_{1/2})^2$

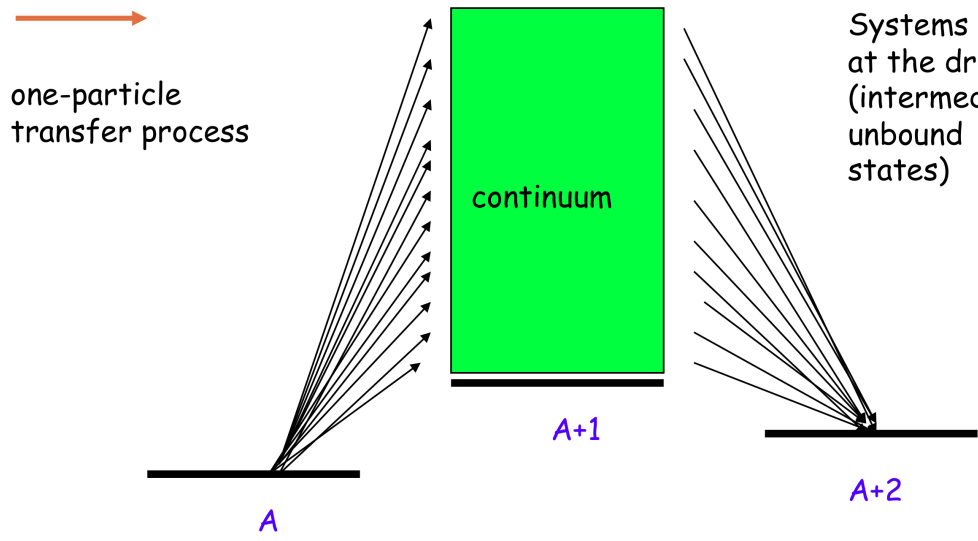
Example of calculation



Obs: to get cross sections one needs optical potentials

But moving from the stability line, how is the picture changing? Pairing effects will be enhanced by the diluted density situation?

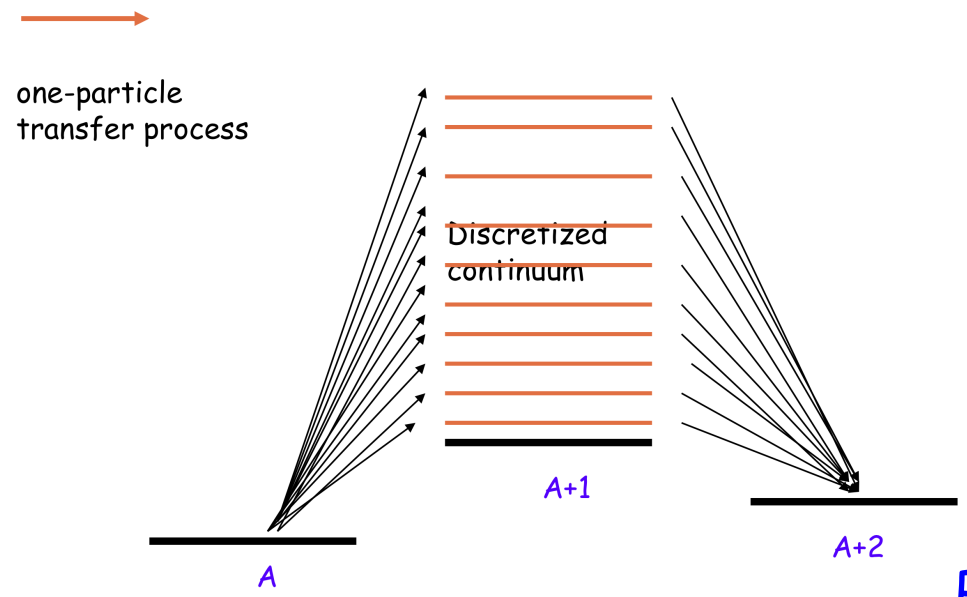




Systems at the drip lines (intermediate unbound states)

Finally reaching the dip lines

Two-particle transfer will proceed mainly by constructive interference of successive transfers through the (unbound) continuum intermediate states



The integration over the continuum intermediate states can become feasible by **continuum discretization**: but how many paths should we include? Thousands or few, for example only the resonant (Gamow) states?

But continuum is a Pandora vase

For **weakly-bound systems at the drip lines** it is **mandatory** to include in the models the positive energy part of the spectrum. If one wants to still use the same machinery used with bound states, the most popular approach is the **discretization of the continuum**. But the discretization **MUST** go in parallel in a consistent way both in the structure and in reaction parts.

All discretization procedures are equivalent as long as a full complete basis is used. In practice all procedures contain a number of parameters and criteria, that make not all procedures equally applicable in practical calculations. Computational constraints may in fact become a severe problem.

As possibilities we can consider

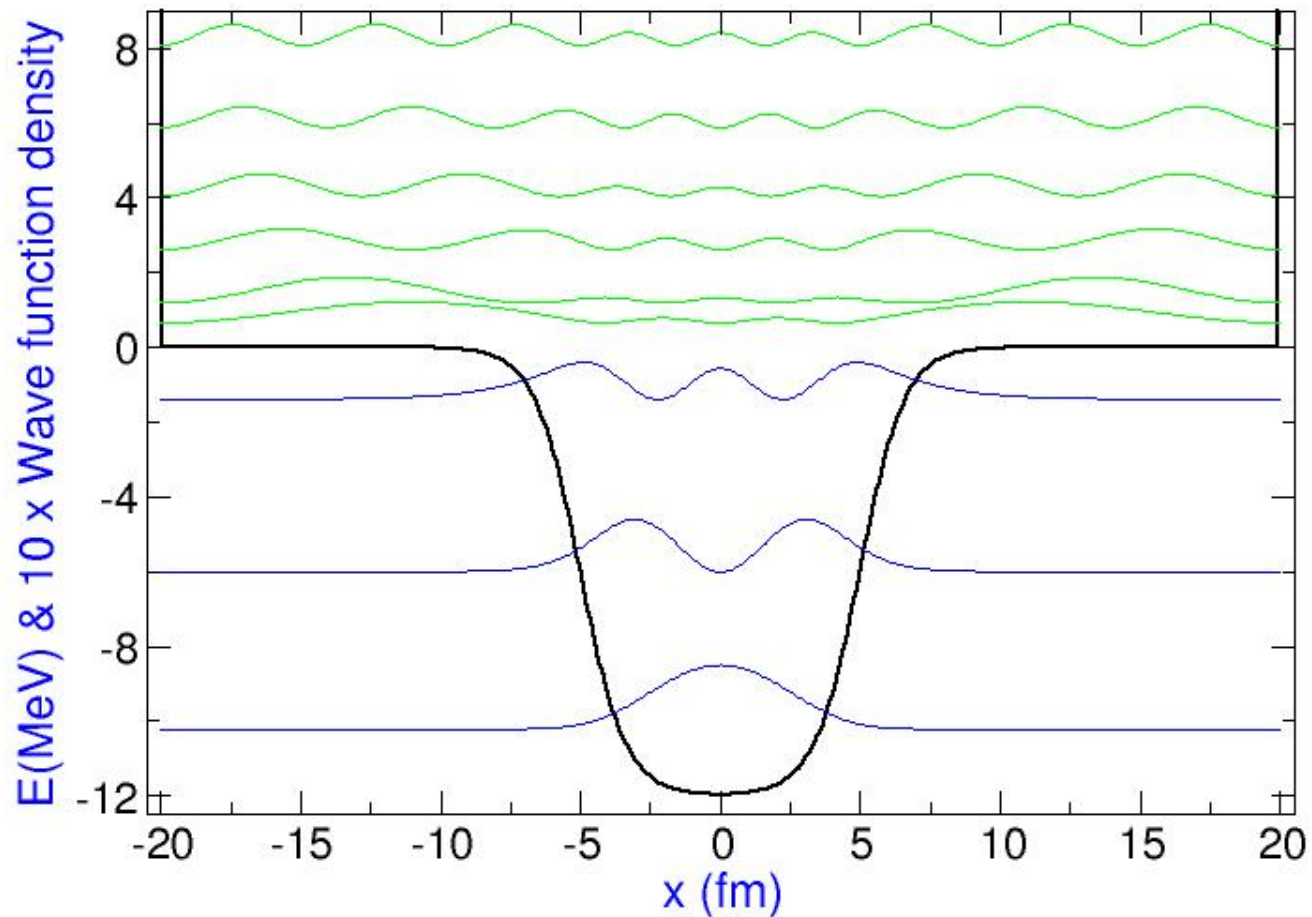
- diagonalization in a basis given by **HO** wave functions
- impose boundary conditions in a BOX
- the case of **discretized wave functions with scattering boundary conditions (CDCC)**
- Gamow states (complex energies)

Checking discretization methods in systems
with one particle in the continuum (where
the "exact" treatment of continuum is
feasible)

Diagonalization in a box

WS single-particle states obtained imposing boundary conditions at a box ($R=20$ fm)

Woods Saxon in a Box



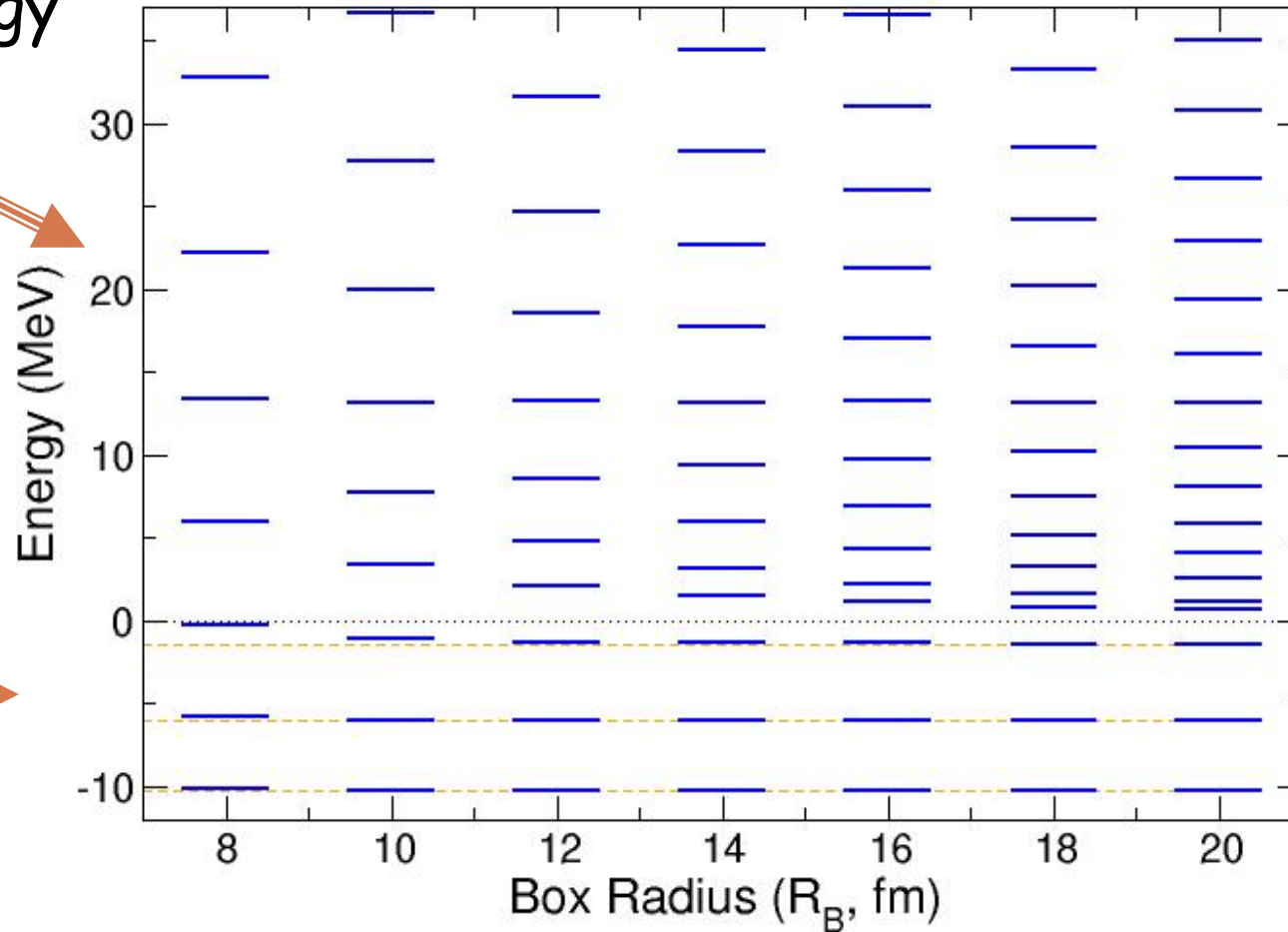
Single-particle energies

Woods-Saxon in the Box

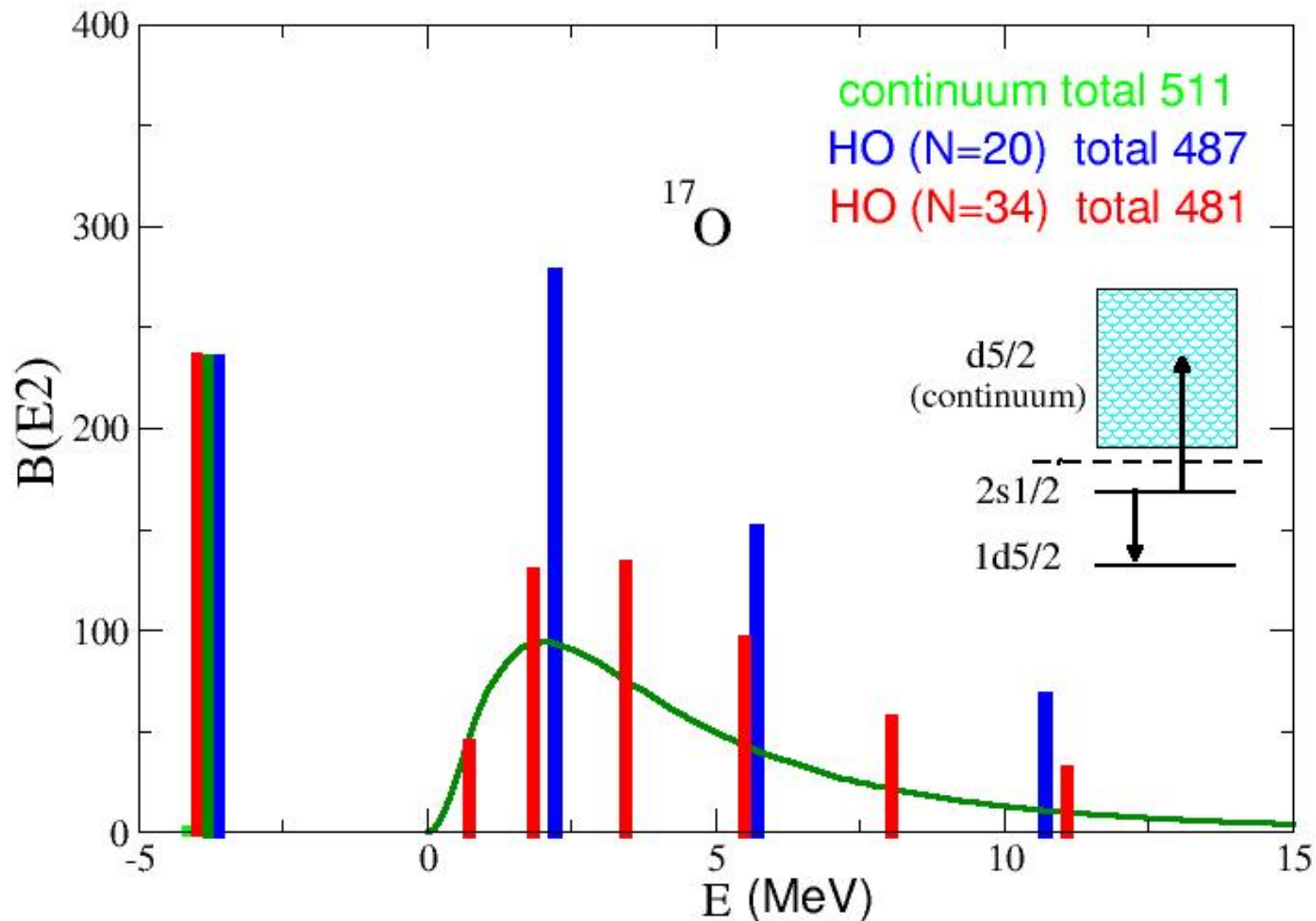
positive energy states



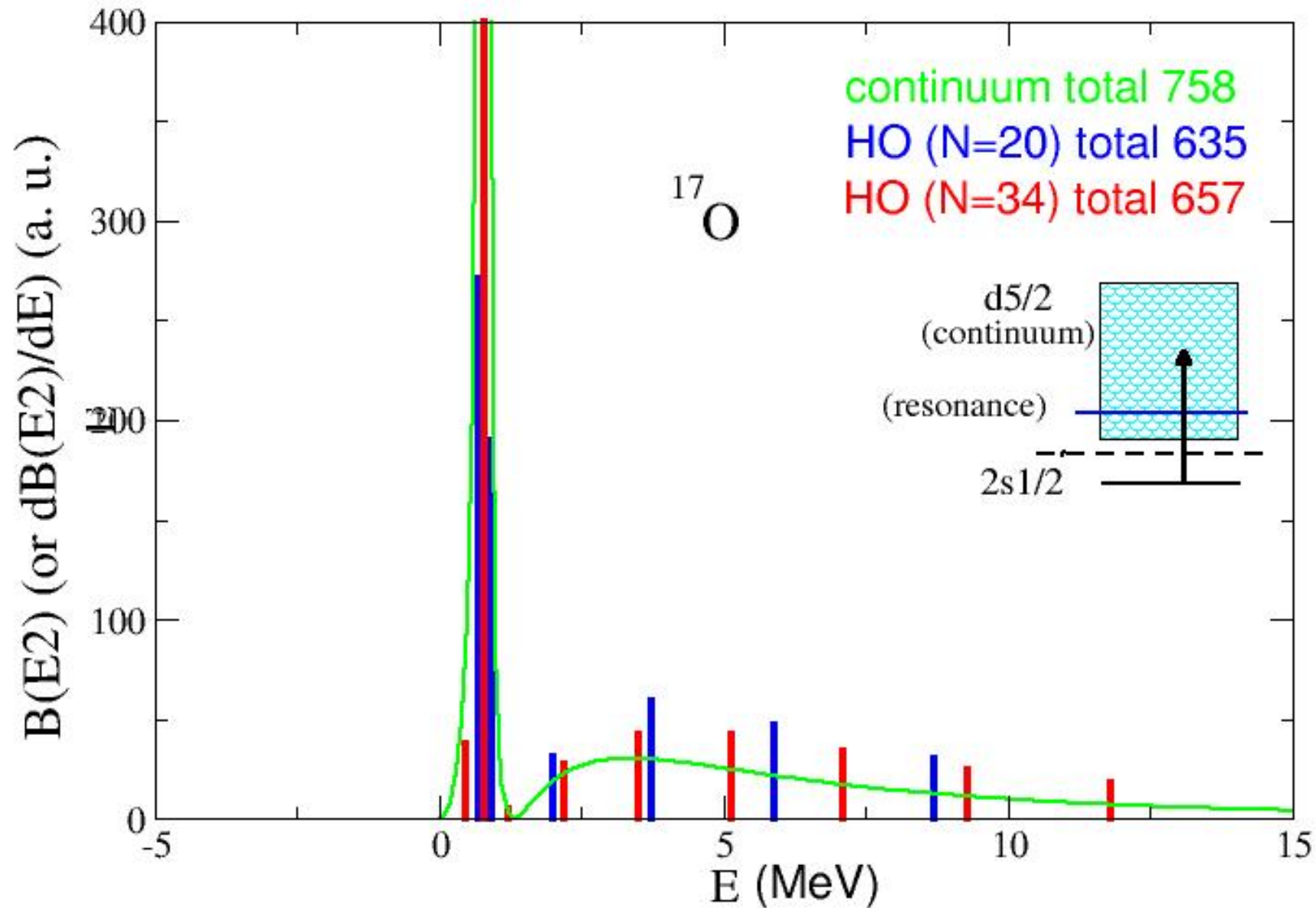
bound states



Case of non-resonant continuum
 (Woods-Saxon single-particle potential in a HO basis)
 Quadrupole transitions from excited $2s_{1/2}$ state in ^{17}O

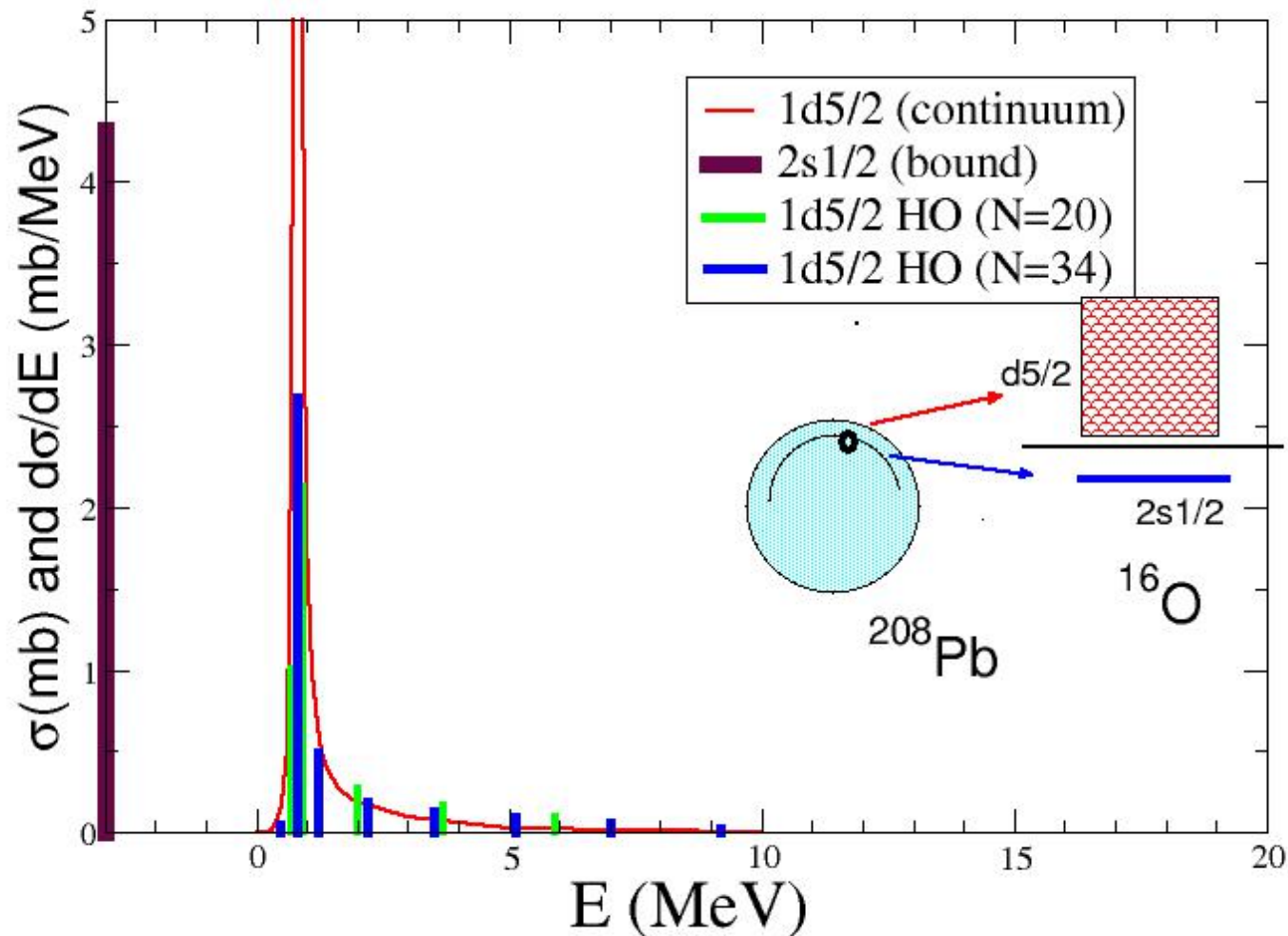


Case of resonant + non-resonant continuum



One-particle transfer (in DWBA)

Case of resonant + non-resonant continuum



Moving from the case of just one particle in the continuum to cases with more particles in the continuum (F. Perez Bernal and A. Vitturi, 2010,2011)

Simple test case

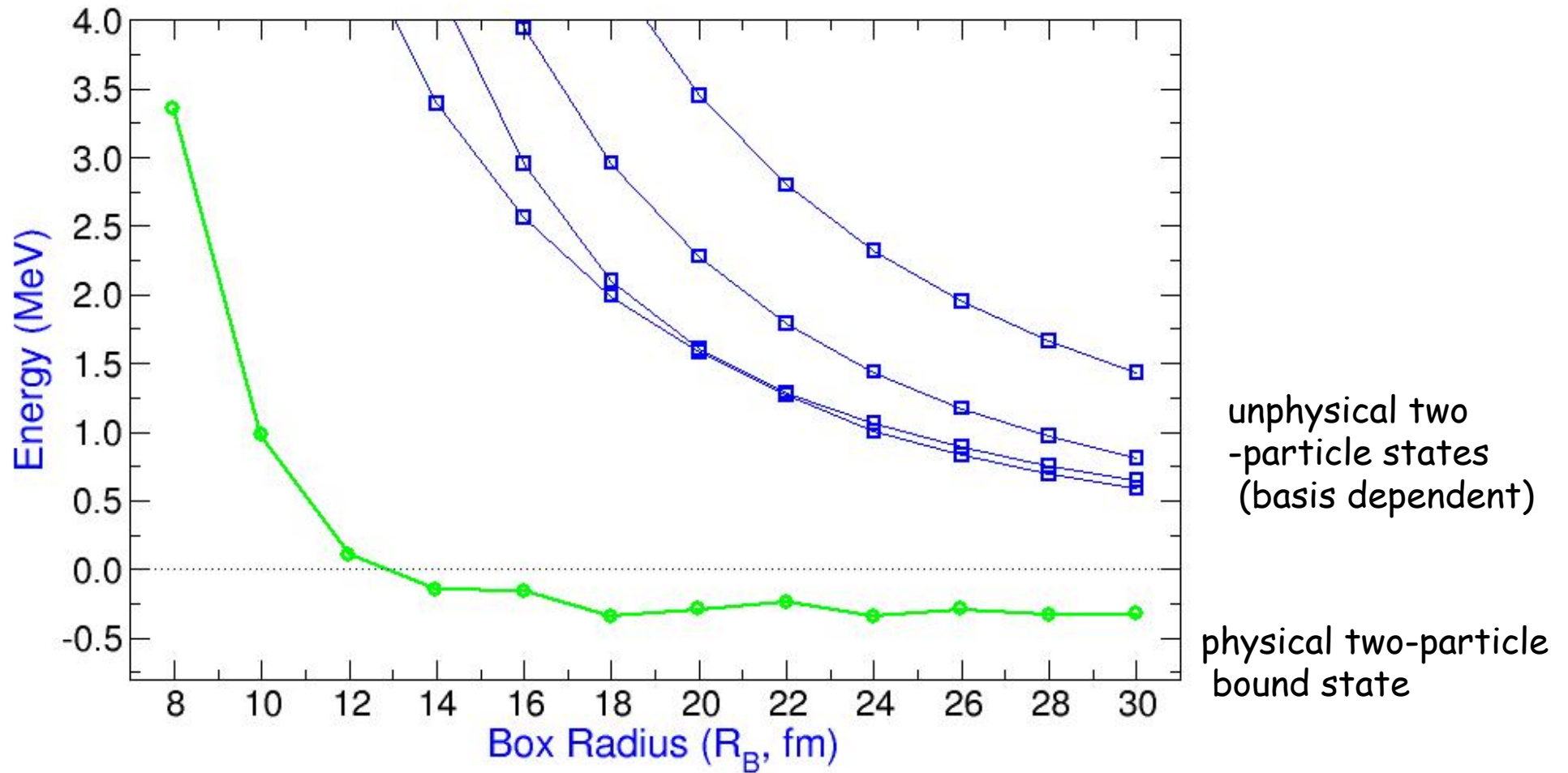
Two valence particles, moving in a one-dimensional Woods-Saxon potential V_0 , interacting via a residual density-dependent short-range attractive interaction.

Modelling a drip-line system, one can choose the Fermi surface in such a way that there are no available bound states, and the two unperturbed particles must be in the continuum. The residual interaction

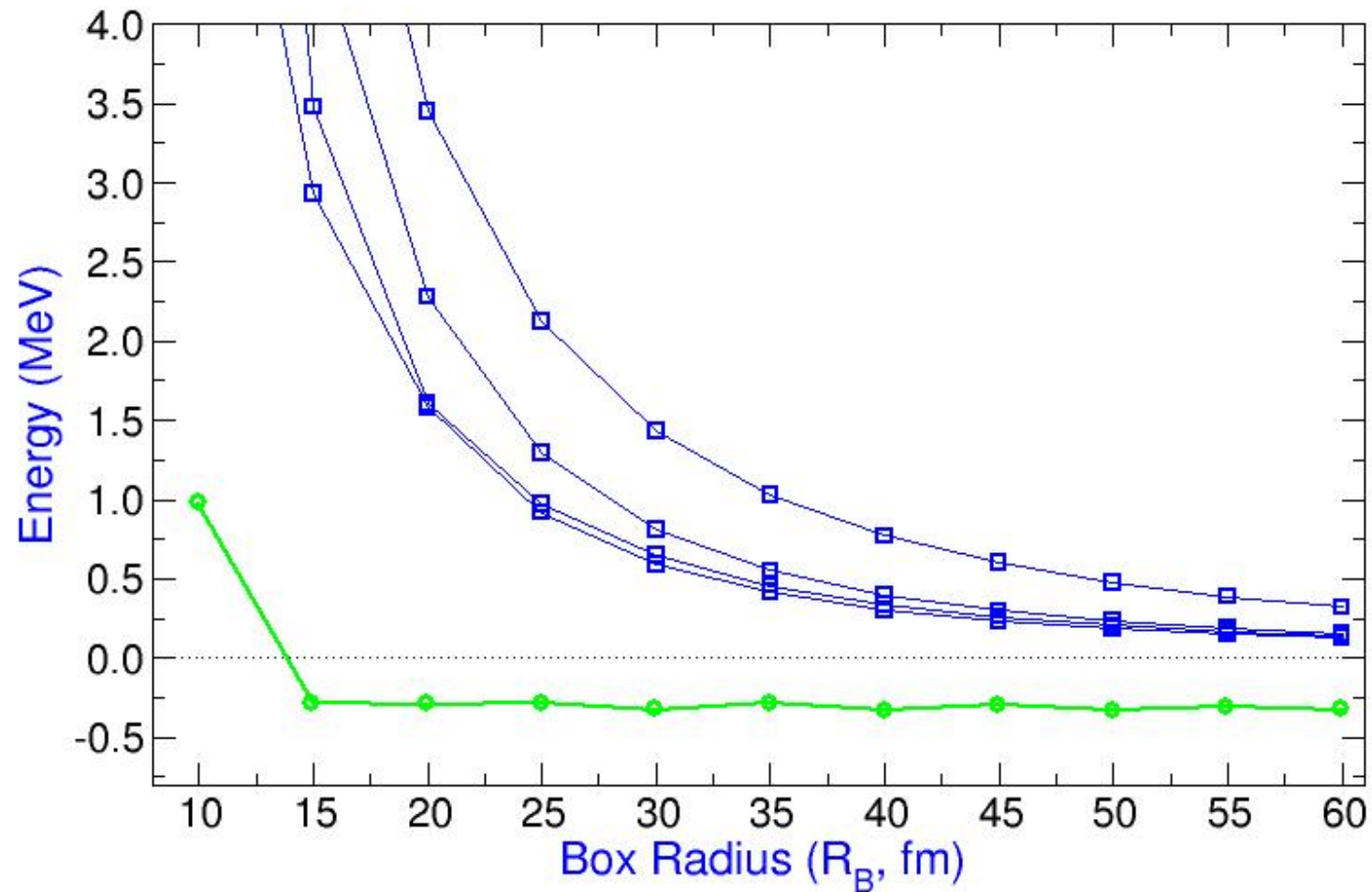
$$V(x_1, x_2) = V_0 \delta(x_1 - x_2) \rho((x_1 + x_2)/2) / \rho_0$$

can be chosen in such a way that the final correlated wave function is however bound. Such a system is normally called "Borromean"

Correlated energy of the two-particle system (as a function of the box radius)



The value of the binding energy is converging (with some oscillations) to the final value

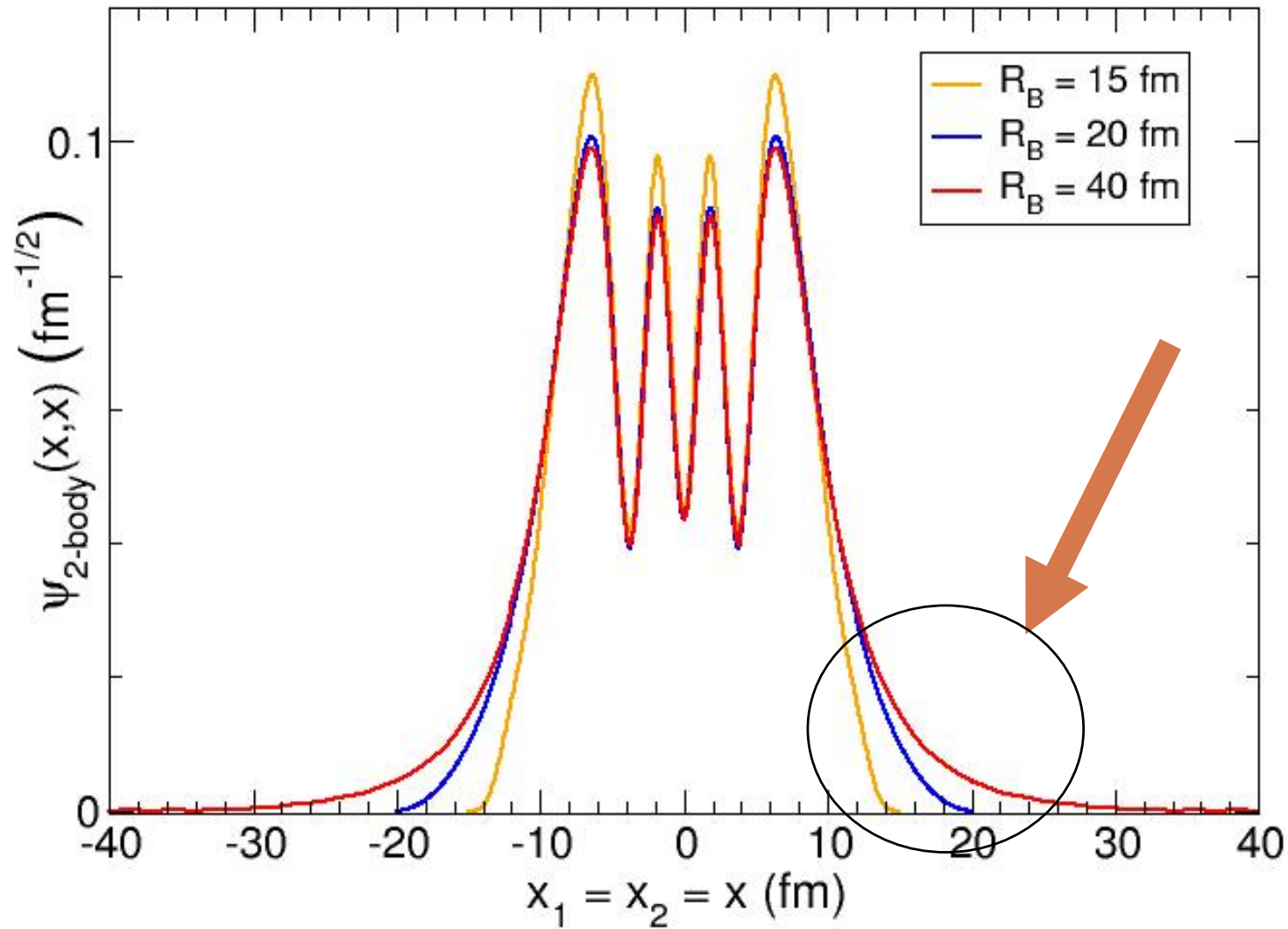


Energy already practically correct with a box of 15 fm, but what about the wave function? In particular, how does it behave in the tail?

Radial dependence

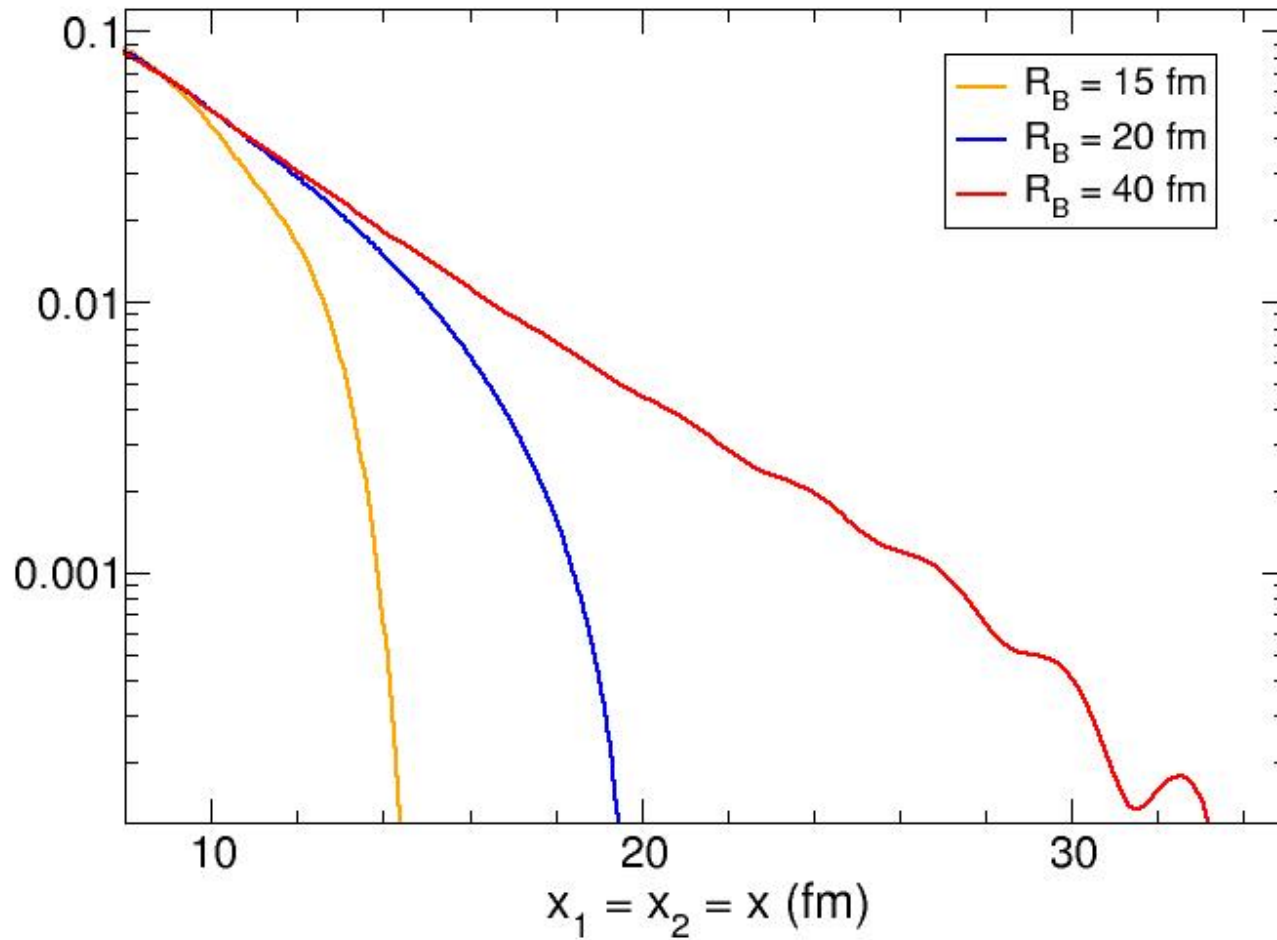
$$\delta\rho(x,x)$$

Woods-Saxon in a BOX



Logaritmik scale

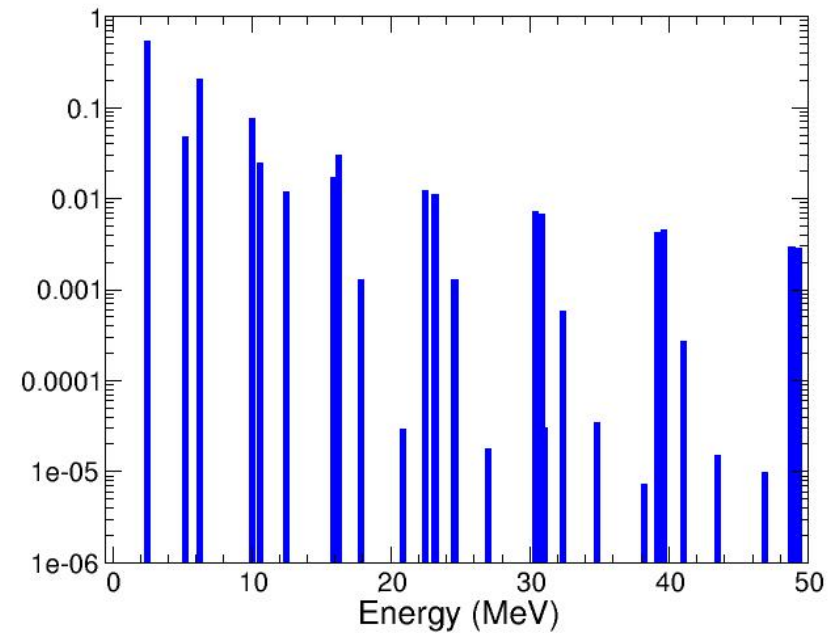
Woods-Saxon in a BOX



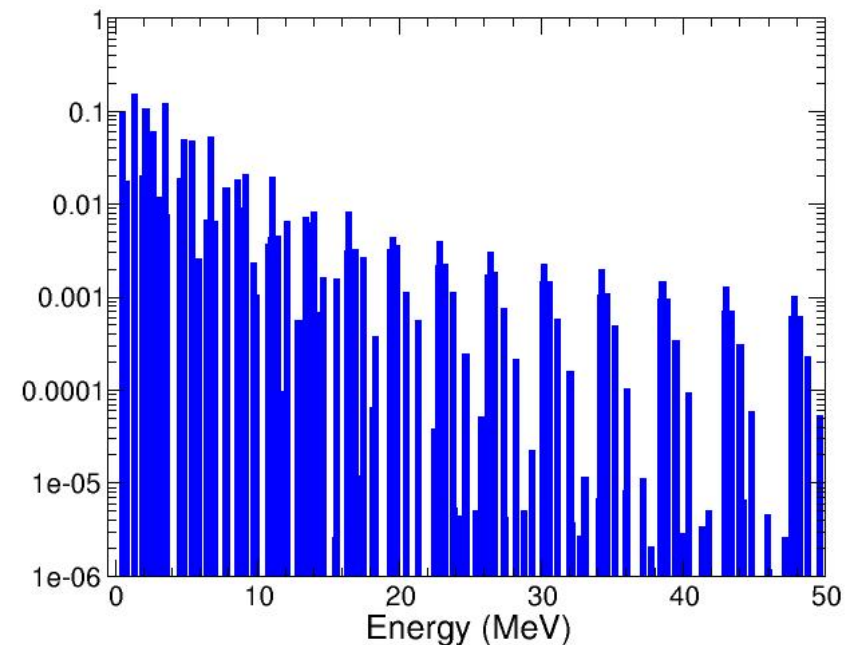
Correlated
two-particle
wave-function
expanded over
discretized
two-particle
positive energy
states

OBS Enormous
number of
components

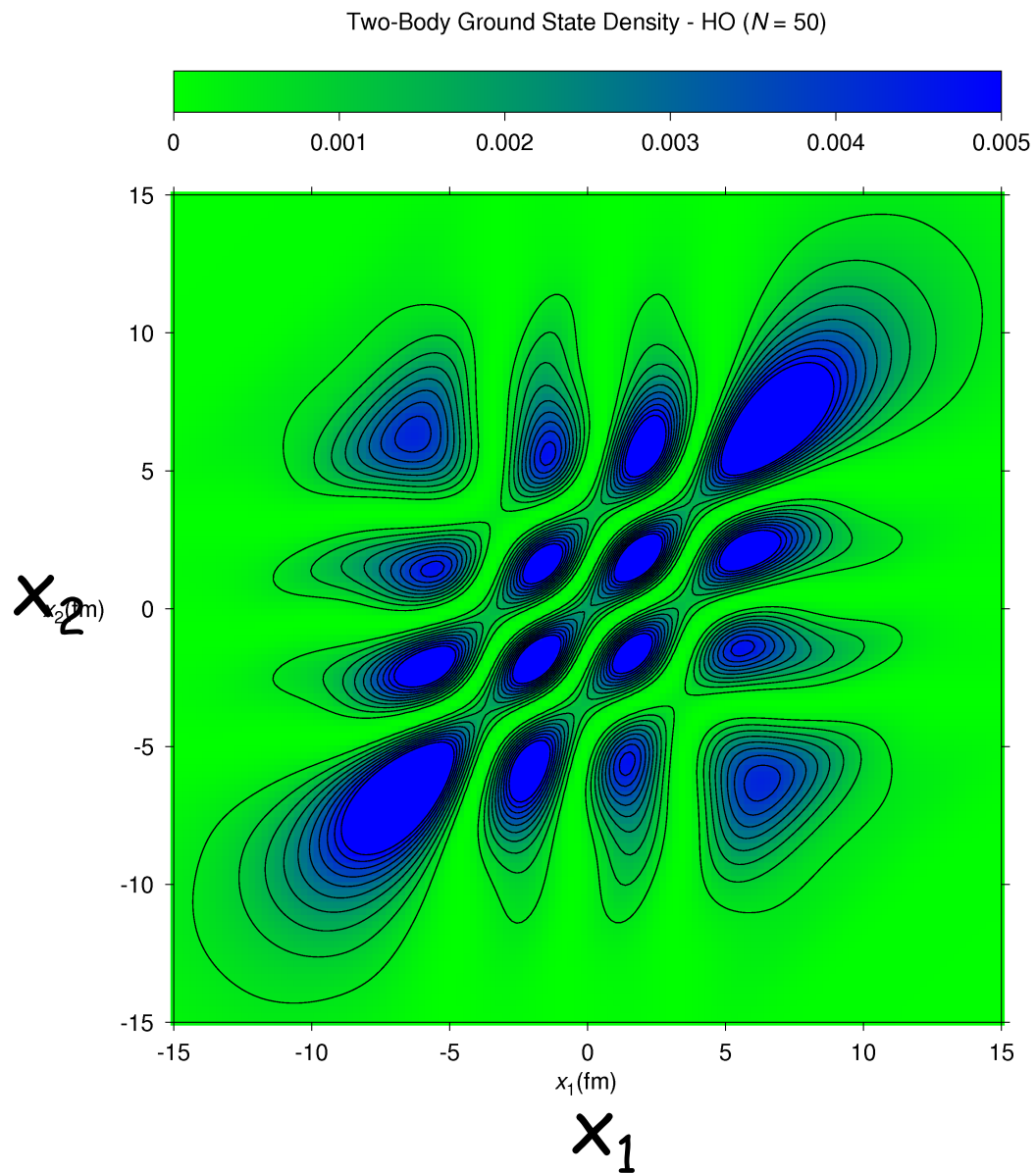
$R=15\text{fm}$



$R=40\text{fm}$

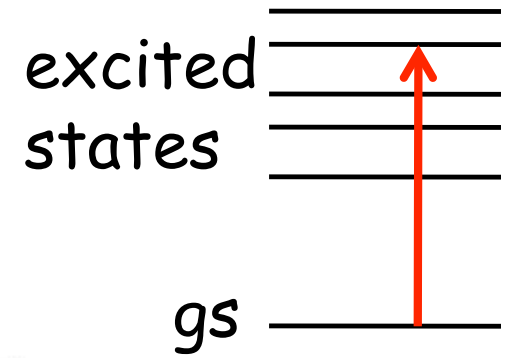


Space correlation: two-particle density as a function of x_1 and x_2

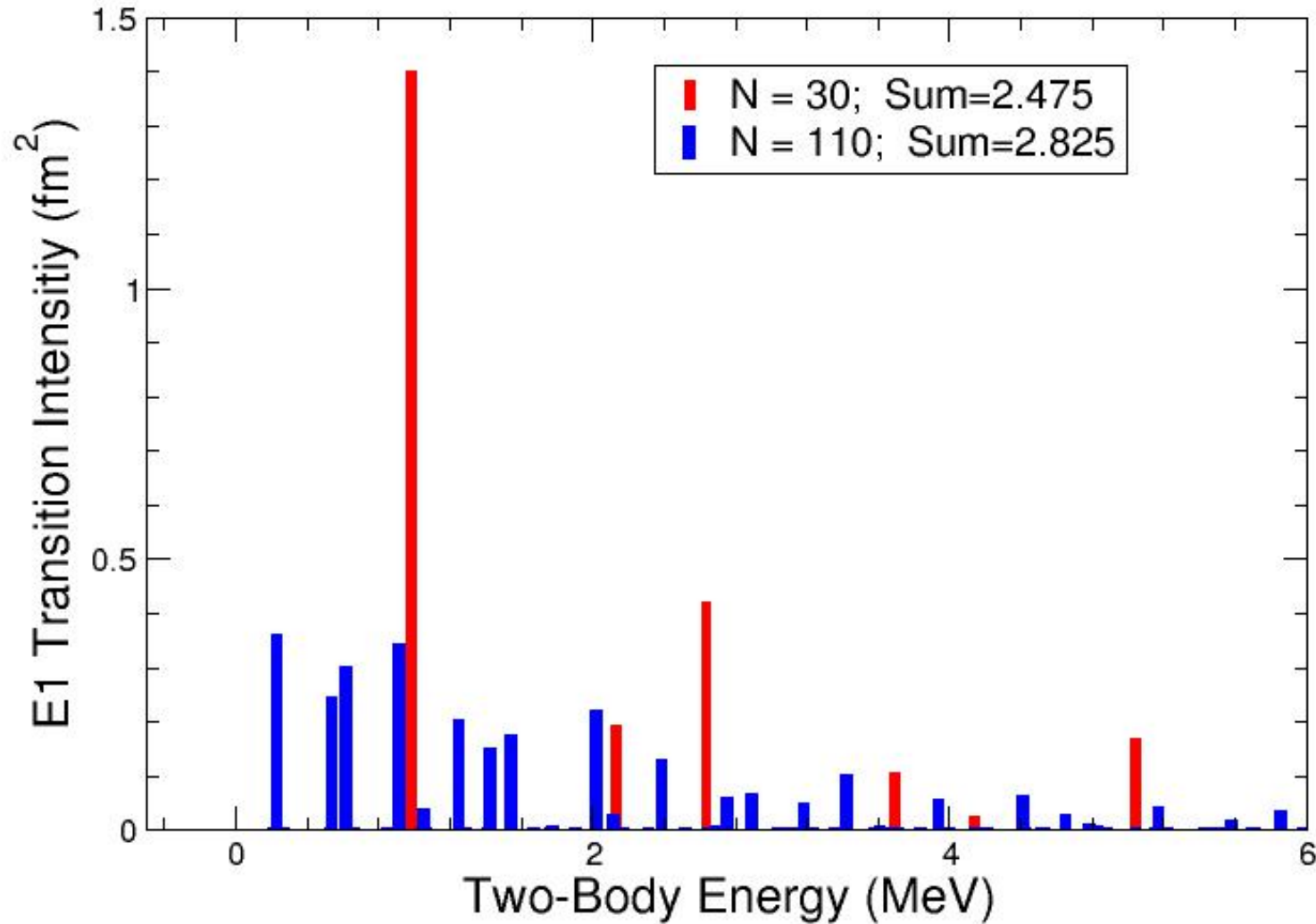


Let us consider now the response of our system to different particle-hole and particle-particle operators as a function of the dimension of the box

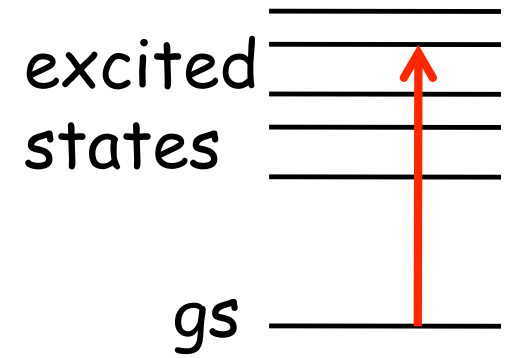
Response to "x" particle-hole operator



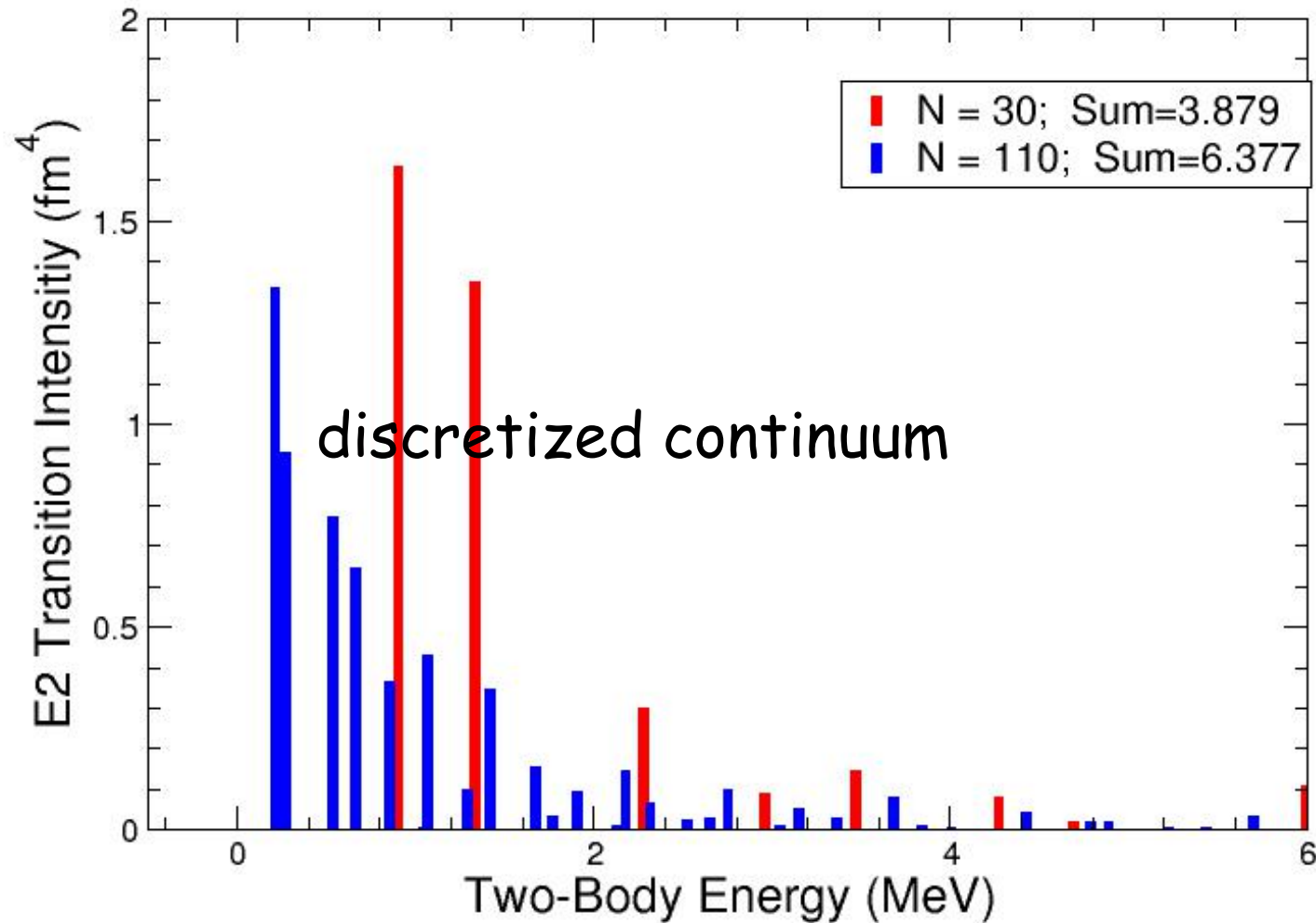
2-particle system



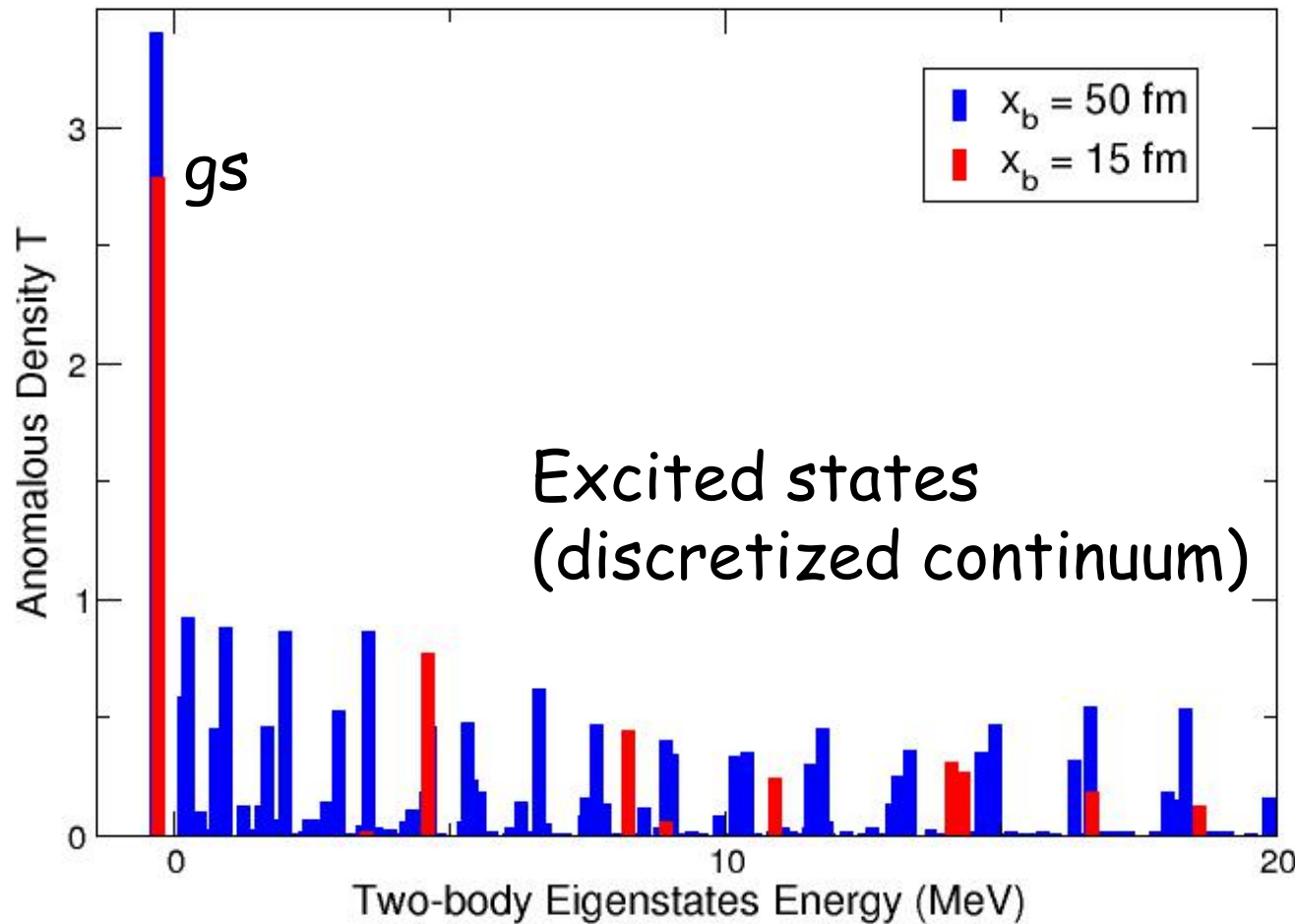
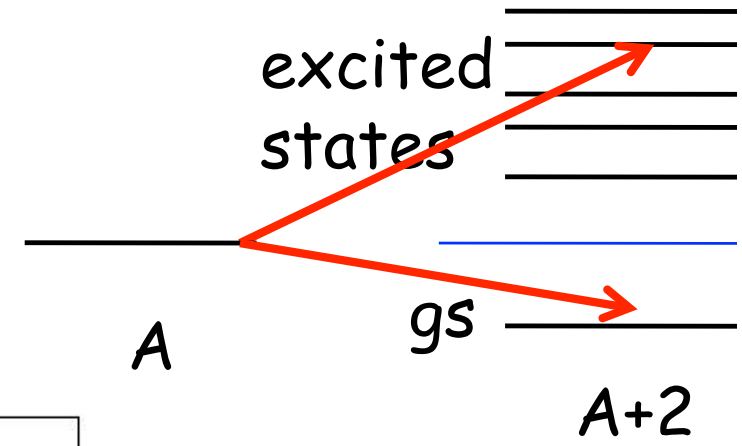
Response to "x²" operator



2-particle system

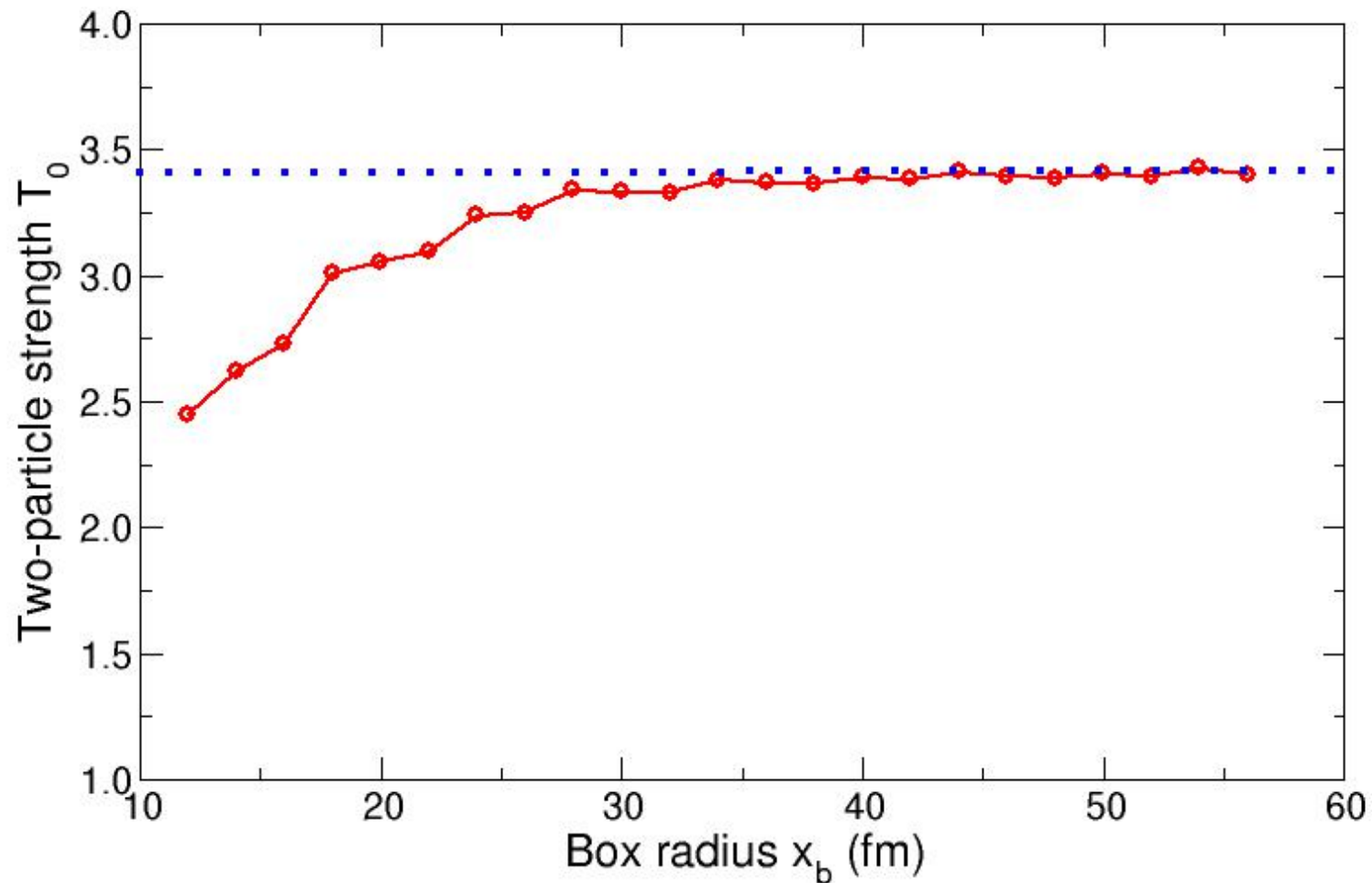


Response to "a+a+" operator



OBS: No convergence for the sum rule (it will diverge as r_{bax} will tend to infinity)

Pairing matrix element to the ground state
(two-particle creation matrix element)
as a function of the box radius



Similar results have been obtained by the diagonalization in a harmonic oscillator basis (with similar results, now as a function of the number of HO shells). Correct behavior on the tail and convergence of two-particle matrix element require very large number of shells (even more than 100)

Conclusions:

There is a non-trivial connection between pairing correlations and reactions involving two-particle dynamics (two-particle transfer, two-particle break-up, two-particle knock-out).

The connection is even more complicated in the case of weakly-bound systems due to the role of continuum states, which are not easily incorporated in standard reaction frameworks.