Nuclear charge radii and E0 transitions in the IBM

S. Zerguine & A. Bouldjedri, University of Batna, Algeria
S. Heinze, University of Cologne, Germany
P. Van Isacker, GANIL, France

Motivation

Charge radii and E0 transitions

Application to rare-earth nuclei

Beauty in Physics, Cocoyoc, May 2012

Motivation

Origin of E0 transitions in nuclei:

Mixing of coexisting configurations with different shapes (Heyde & Wood);

Between β-vibrational states in the geometric collective model (Reiner).

In a geometric framework E0 strength should rise in the transition from spherical to deformed ⇒ Link with phase transitions in nuclei (von Brentano *et al.*).

Simultaneous treatment of charge radii and E0 transitions.

Electric monopole (E0) transitions

The probability for an E0 transition to occur is given by $P=\Omega\rho^2$ with Ω and ρ^2 electronic and nuclear factors.

The nuclear factor is the matrix element

$$\rho = \sum_{k \in \text{protons}}^{Z} \left\langle \mathrm{f} \left| \left(\frac{r_k}{R}\right)^2 - \sigma \left(\frac{r_k}{R}\right)^4 + \cdots \right| \mathrm{i} \right\rangle \quad \left(R = r_0 A^{1/3}, r_0 = 1.2\ \mathrm{fm}\right)$$

Higher-order terms are usually not considered, $\sigma=0$, (*cfr.* Church & Weneser) and hence contact is made with the charge radius.

E.L. Church & J. Weneser, Phys. Rev. **103** (1956) 1035

E0 and charge radius operators

Definition of a ‘charge radius operator’:

$$\langle \mathrm{s}|\hat{T}(r^2)|\mathrm{s}\rangle \equiv \langle r^2\rangle_{\mathrm{s}} = \frac{1}{Z}\sum_{k\in\mathrm{protons}}^{Z}\langle \mathrm{s}|r_k^2|\mathrm{s}\rangle \Rightarrow \hat{T}(r^2) = \frac{1}{Z}\sum_{k\in\mathrm{protons}}^{Z} r_k^2$$

Definition of an ‘E0 transition operator’ (for $\sigma=0$):

$$\rho \equiv \frac{\langle \mathrm{f}|\hat{T}(\mathrm{E0})|\mathrm{i}\rangle}{eR^2} \Rightarrow \hat{T}(\mathrm{E0}) = e\sum_{k\in\mathrm{protons}}^{Z} r_k^2$$

Hence we find the following (standard) relation:

$$\hat{T}(\mathrm{E0}) = eZ\,\hat{T}(r^2)$$

Effective charges

Addition of neutrons produces a change in the charge radius ⇒ need for effective charges.

Generalized operators:

$$\langle r^2 \rangle_{\mathrm{s}} = \frac{1}{e_{\mathrm{n}} N + e_{\mathrm{p}} Z} \sum_{k=1}^{A} \langle \mathrm{s} | e_k r_k^2 | \mathrm{s} \rangle \Rightarrow \hat{T}(r^2) = \frac{1}{e_{\mathrm{n}} N + e_{\mathrm{p}} Z} \sum_{\mathrm{k}=1}^{A} e_k r_k^2$$

$$\hat{T}(\mathrm{E0}) = \sum_{k=1}^{A} e_k r_k^2$$

Generalized (non-standard) relation:

$$\hat{T}(\mathrm{E0}) = \left(e_{\mathrm{n}} N + e_{\mathrm{p}} Z\right) \hat{T}(r^2)$$

E0 transitions in nuclear models

- **Nuclear shell model**: E0 transitions between states in a single oscillator shell vanish.
- **Geometric collective model**: Strong E0 transitions occur between β- and ground-state band.
- **Interacting boson model** (intermediate between shell model and collective model): The IBM can be used to test the relation between radii and E0 transitions.

Application to rare-earth nuclei

Application to even-even nuclei with Z=58-74.

Procedure:

Fix IBM hamiltonian parameters from spectra with special care to the spherical-to-deformed transitional region.

Determine α and η from measured isotope and isomer shifts.

Calculate ρ^2 (depends on η only).

S. Zerguine *et al.*, Phys. Rev. Lett. **101** (2008) 022502

S. Zerguine *et al.*, Phys. Rev. C **85** (2012) 034331

Energy spectra

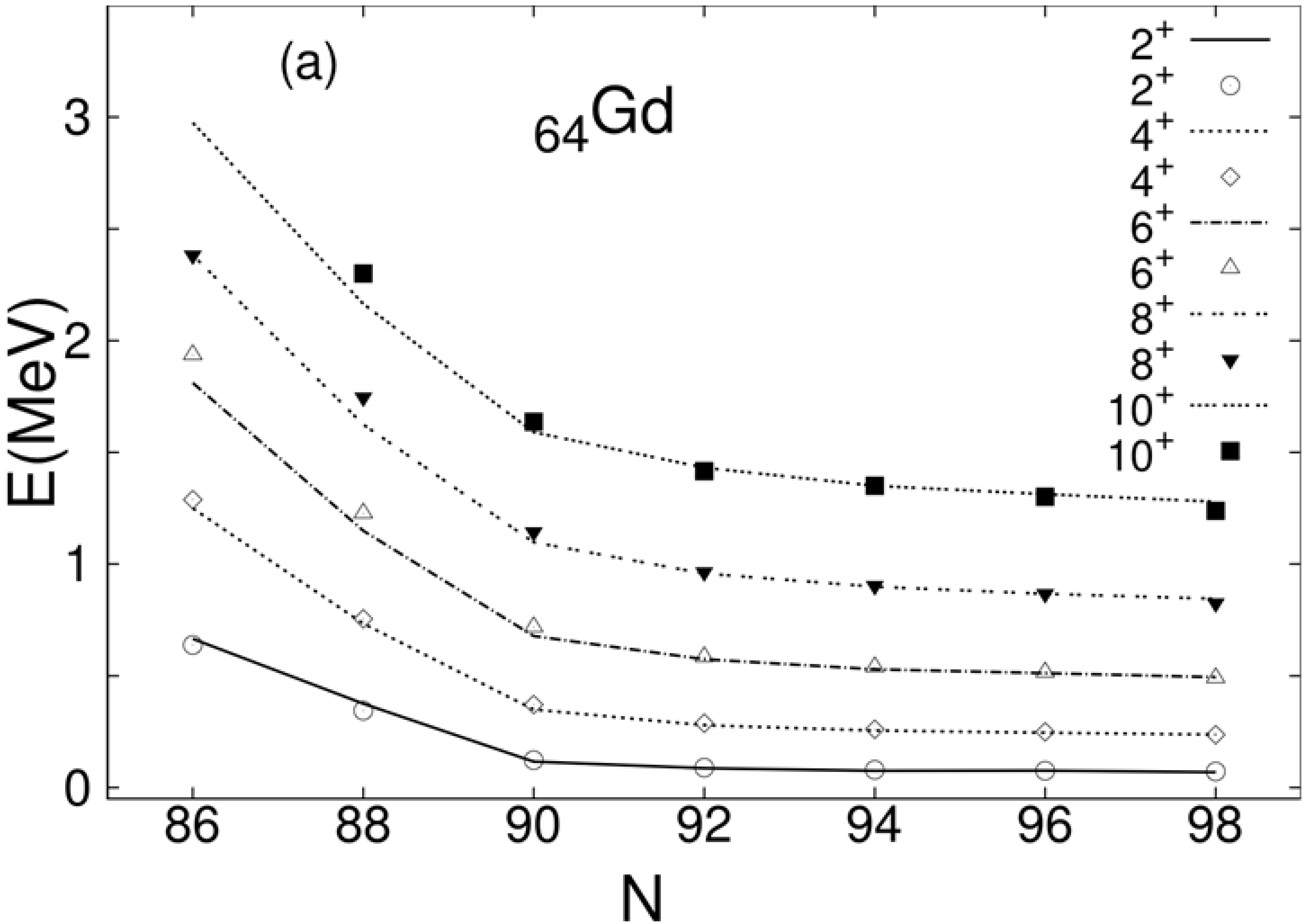
The standard (1+2)-body IBM hamiltonian:

$$\hat{H} = \varepsilon \hat{n}_d + a_0 \hat{P}_+ \hat{P}_- + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4$$

Constant parameters for a given isotopic chain except for the quadrupole strength:

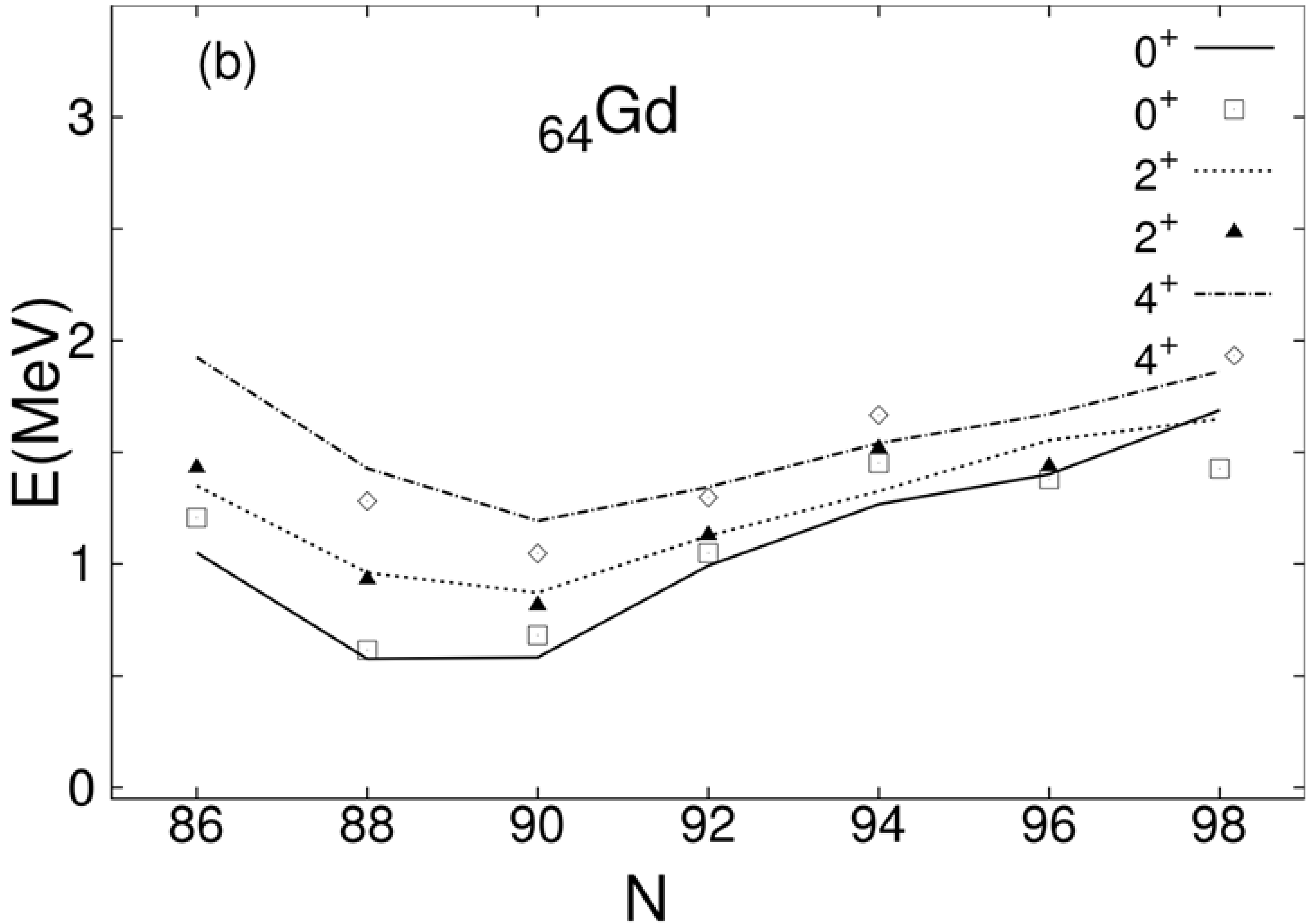
$$a_2 = a_2' + \frac{N_\nu N_\pi}{N_\nu + N_\pi} a_2''$$

Example: gadolinium isotopes

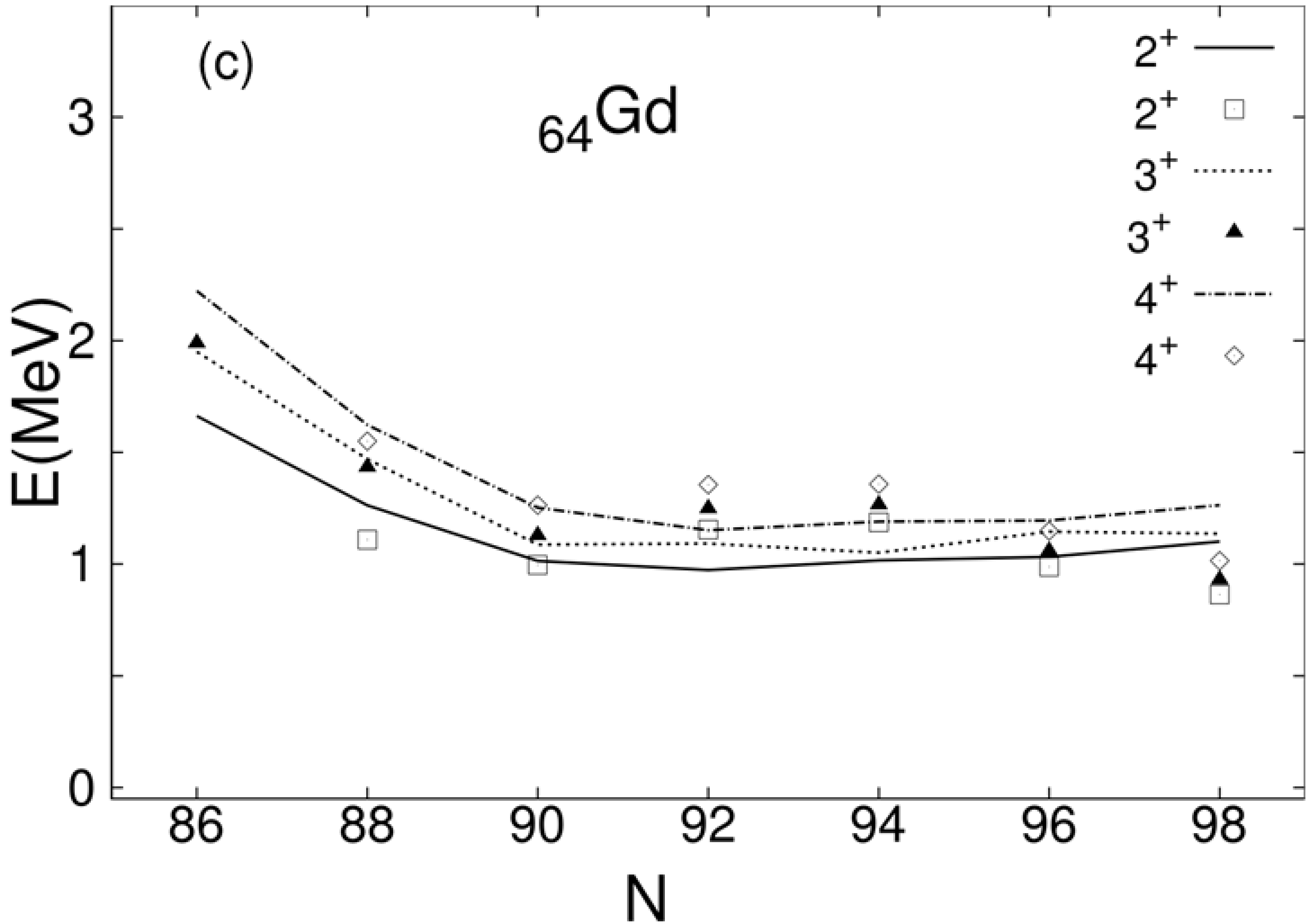


Beauty in Physics, Cocoyoc, May 2012

Example: gadolinium isotopes



Example: gadolinium isotopes



Charge radii

The charge radius operator in IBM:

$$\hat{T}(r^2) = \langle r^2 \rangle_{\text{core}} + \alpha N_b + \eta \frac{\hat{n}_d}{N_b}$$

Standard parametrization (cfr. Iachello and Arima):

$$\hat{T}(r^2) = \langle r^2 \rangle_{\text{core}} + \alpha' N_b + \eta' \hat{n}_d$$

F. Iachello and A. Arima, The Interacting Boson Model

Isotope shifts

Isotopes shifts depend on the parameters α and η:

$$\Delta\left\langle r^2\right\rangle \equiv \left\langle r^2\right\rangle_{0_1^+}^{(A+2)} - \left\langle r^2\right\rangle_{0_1^+}^{(A)} = |\alpha| + \eta\left(\left\langle \frac{\hat{n}_d}{N_b}\right\rangle_{0_1^+}^{(A+2)} - \left\langle \frac{\hat{n}_d}{N_b}\right\rangle_{0_1^+}^{(A)}\right)$$

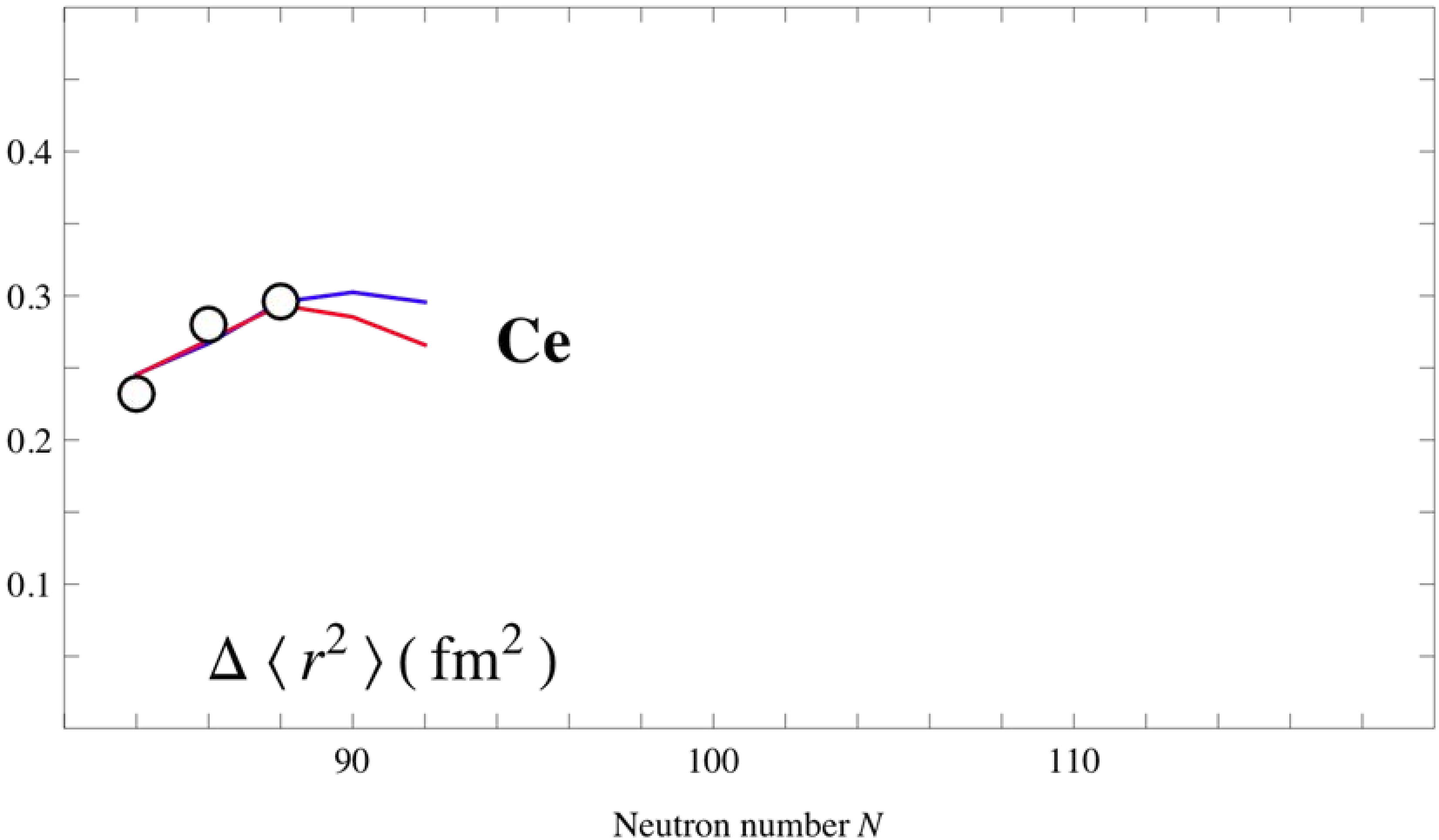
α *(linear slope) varies between 0.10 and 0.25 fm²;*

η *(deformation dependence) equals 0.5 fm² (constant for all nuclei).*

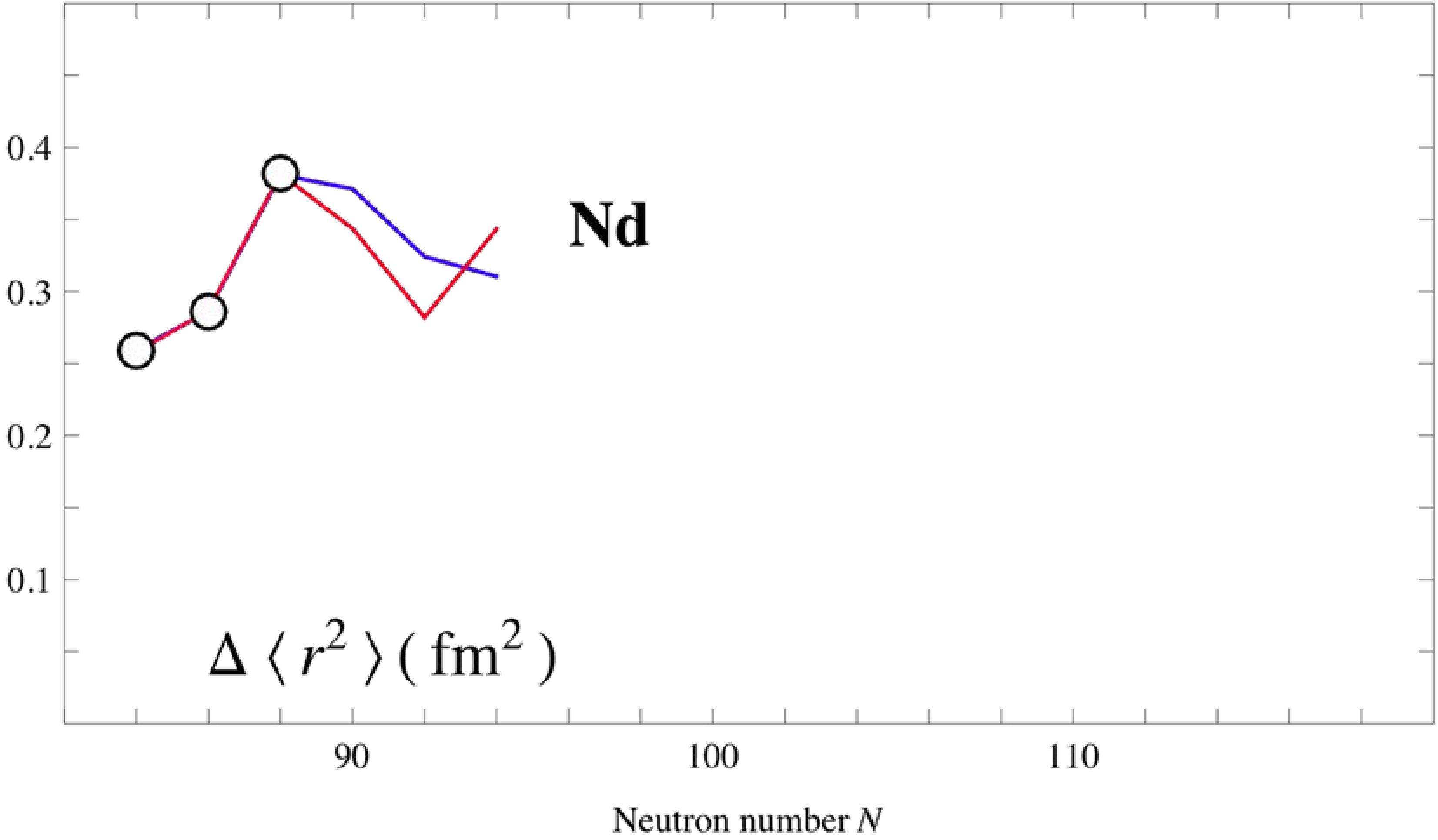
Standard parametrization:

$$\Delta\left\langle r^2\right\rangle \equiv \left\langle r^2\right\rangle_{0_1^+}^{(A+2)} - \left\langle r^2\right\rangle_{0_1^+}^{(A)} = |\alpha'| + \eta'\left(\left\langle \hat{n}_d\right\rangle_{0_1^+}^{(A+2)} - \left\langle \hat{n}_d\right\rangle_{0_1^+}^{(A)}\right)$$

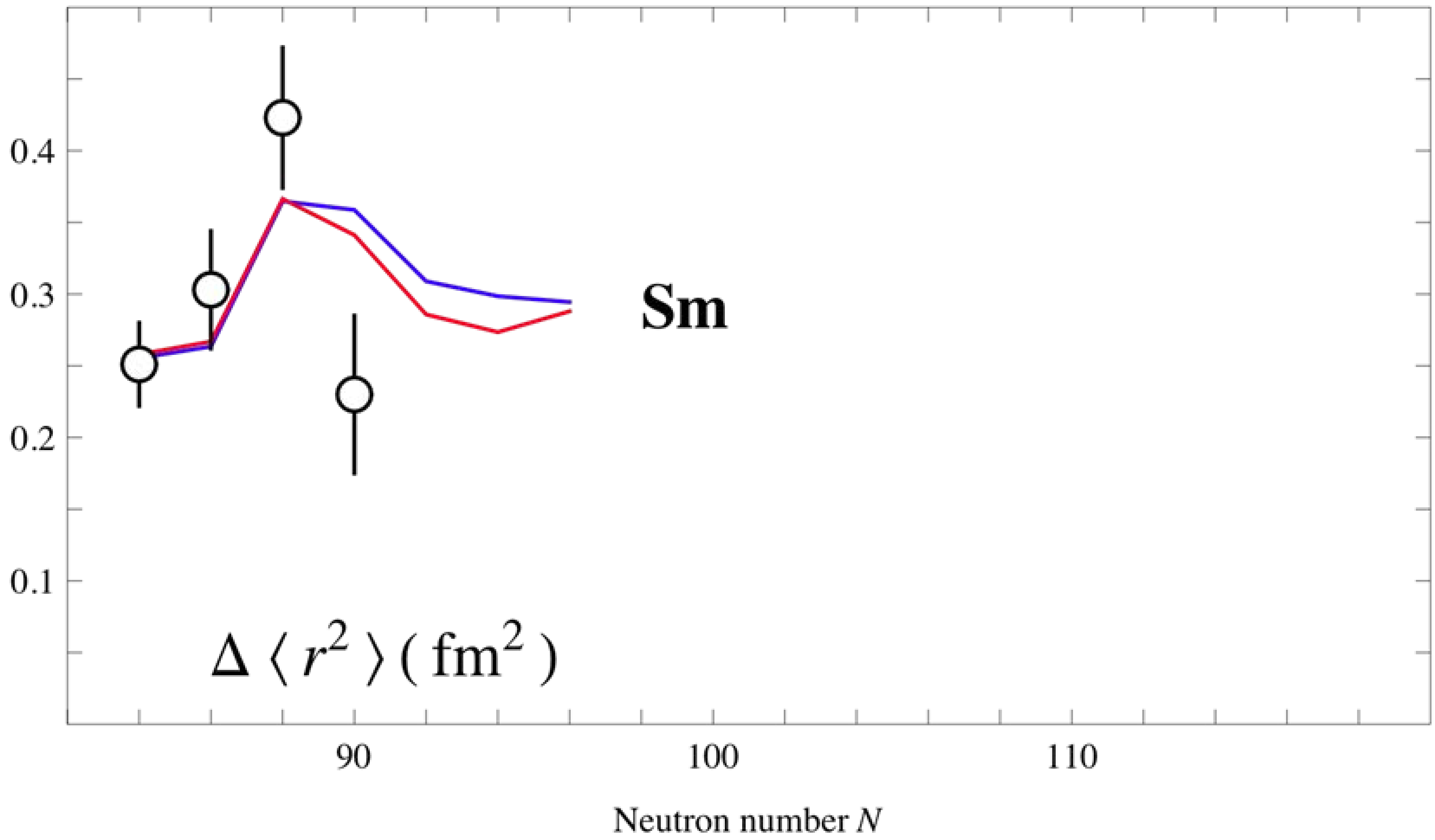
Isotope shifts in cerium



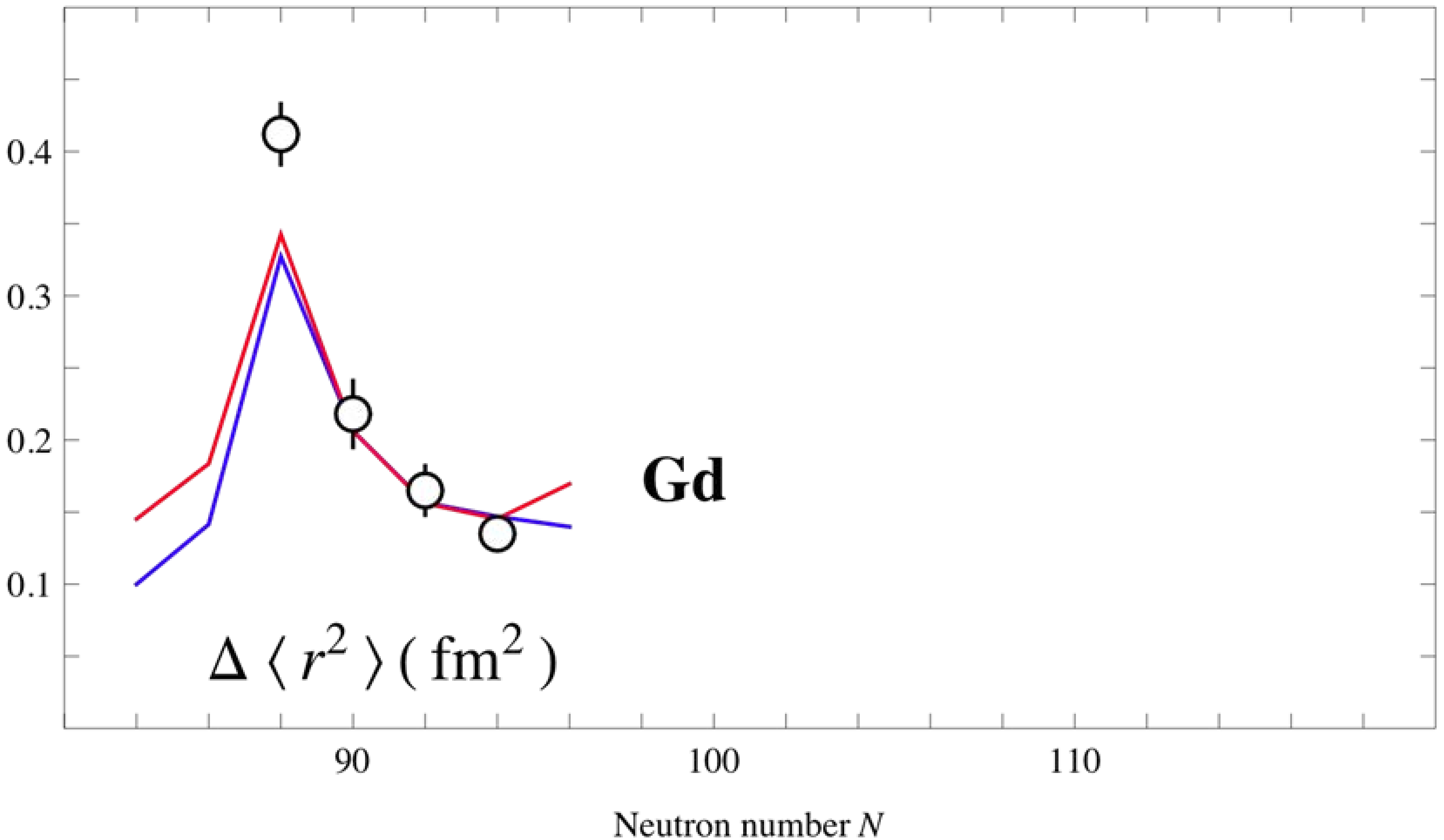
Isotope shifts in neodymium



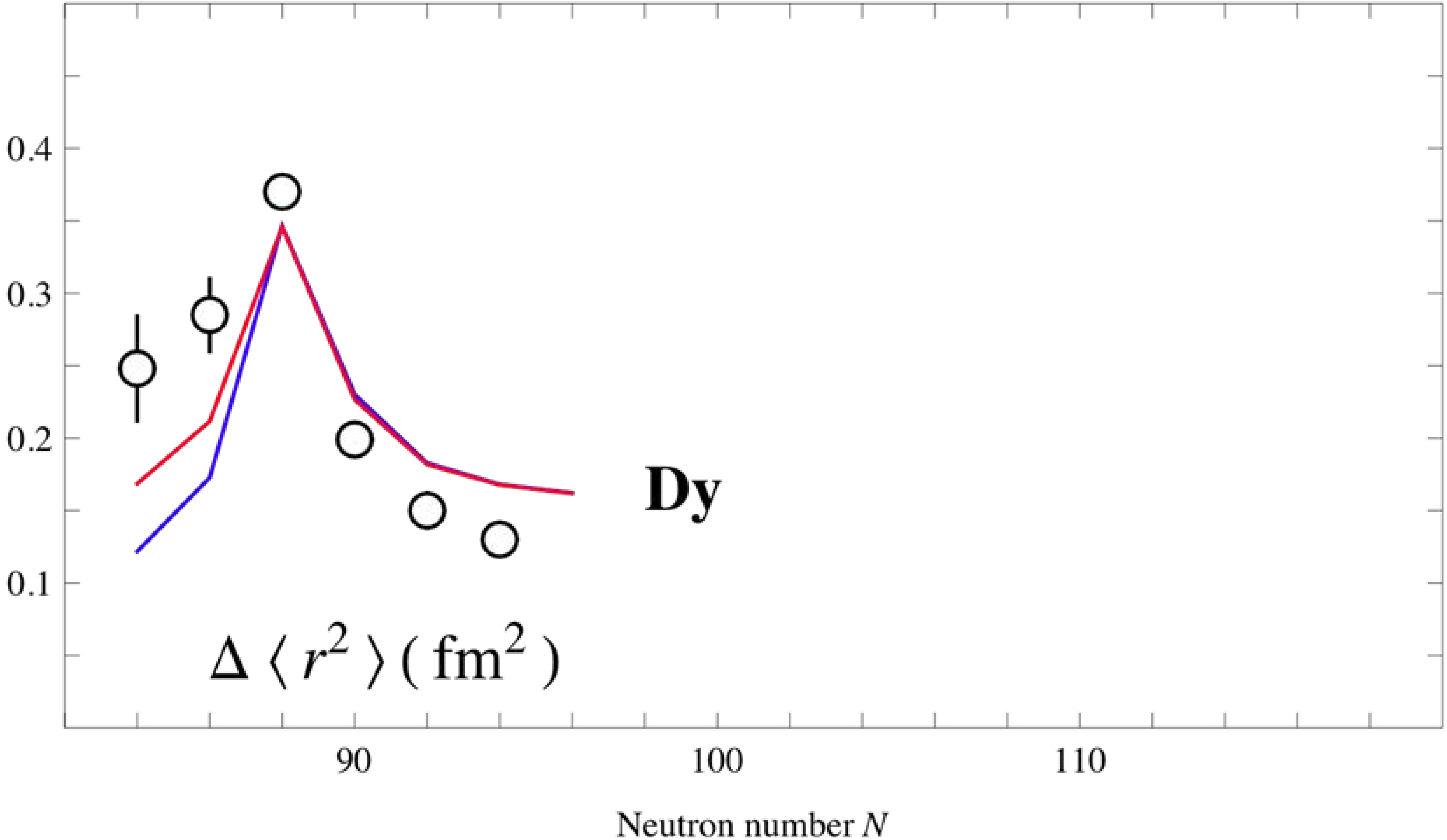
Isotope shifts in samarium



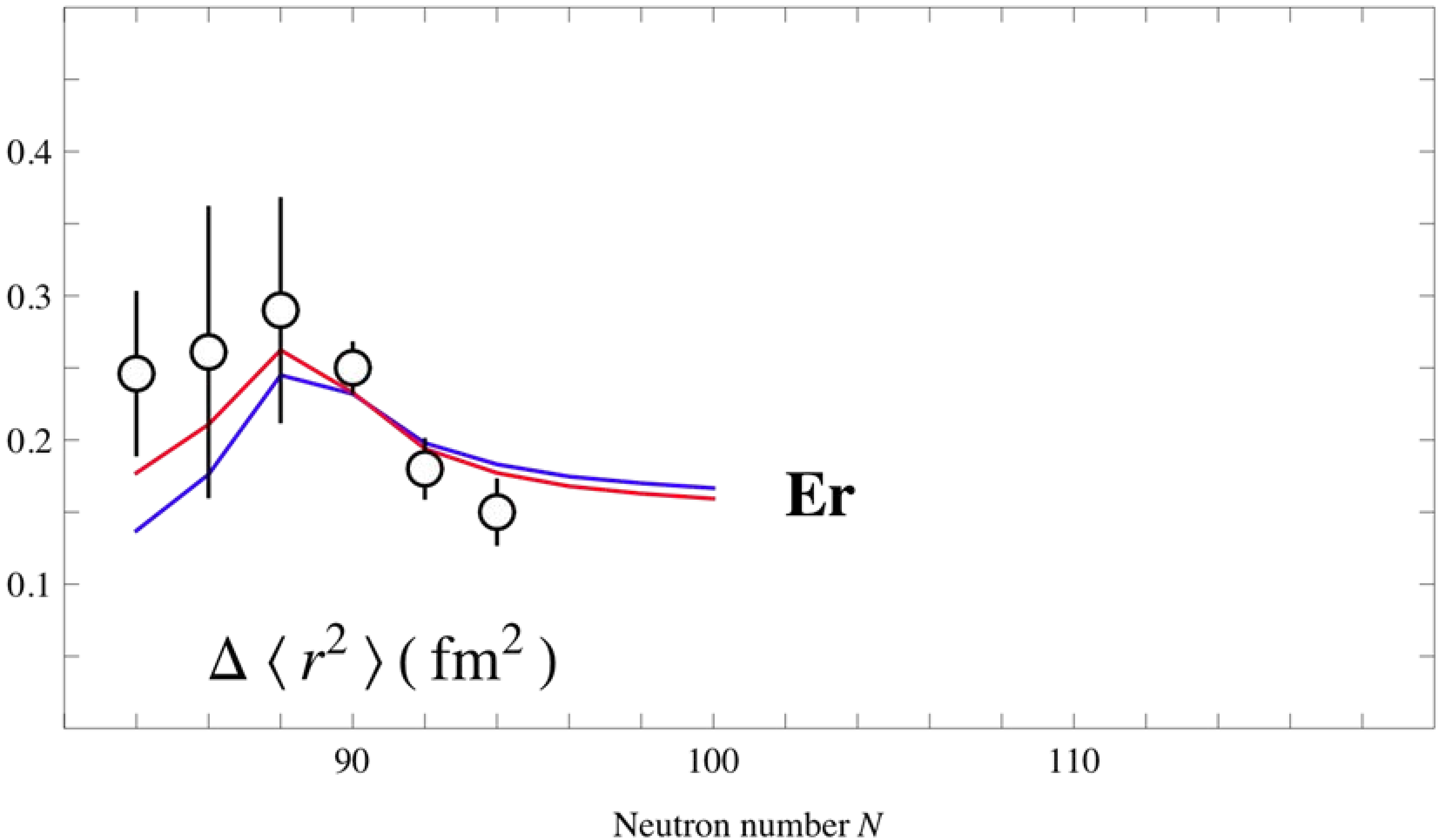
Isotope shifts in gadolinium



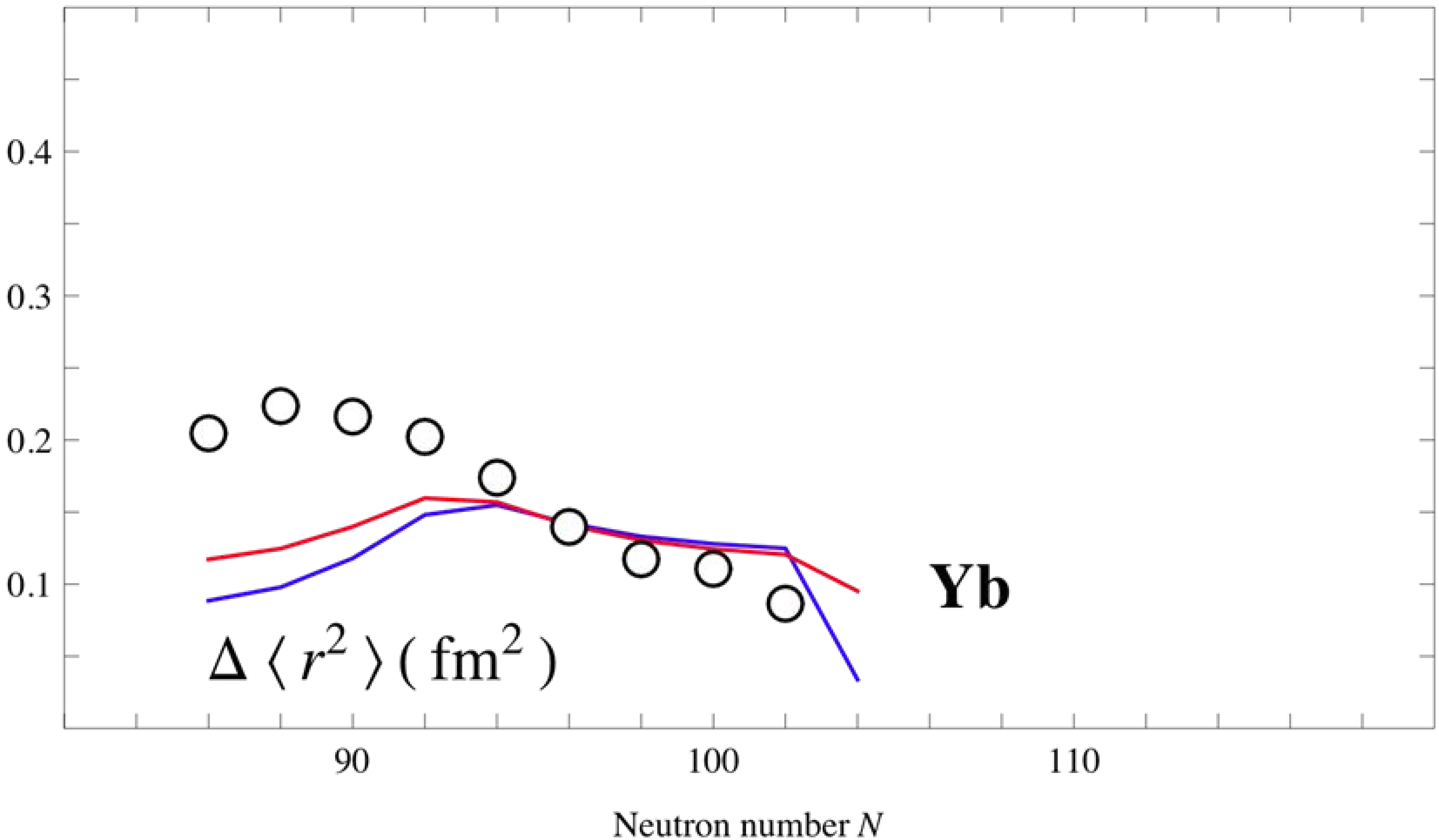
Isotope shifts in dysprosium



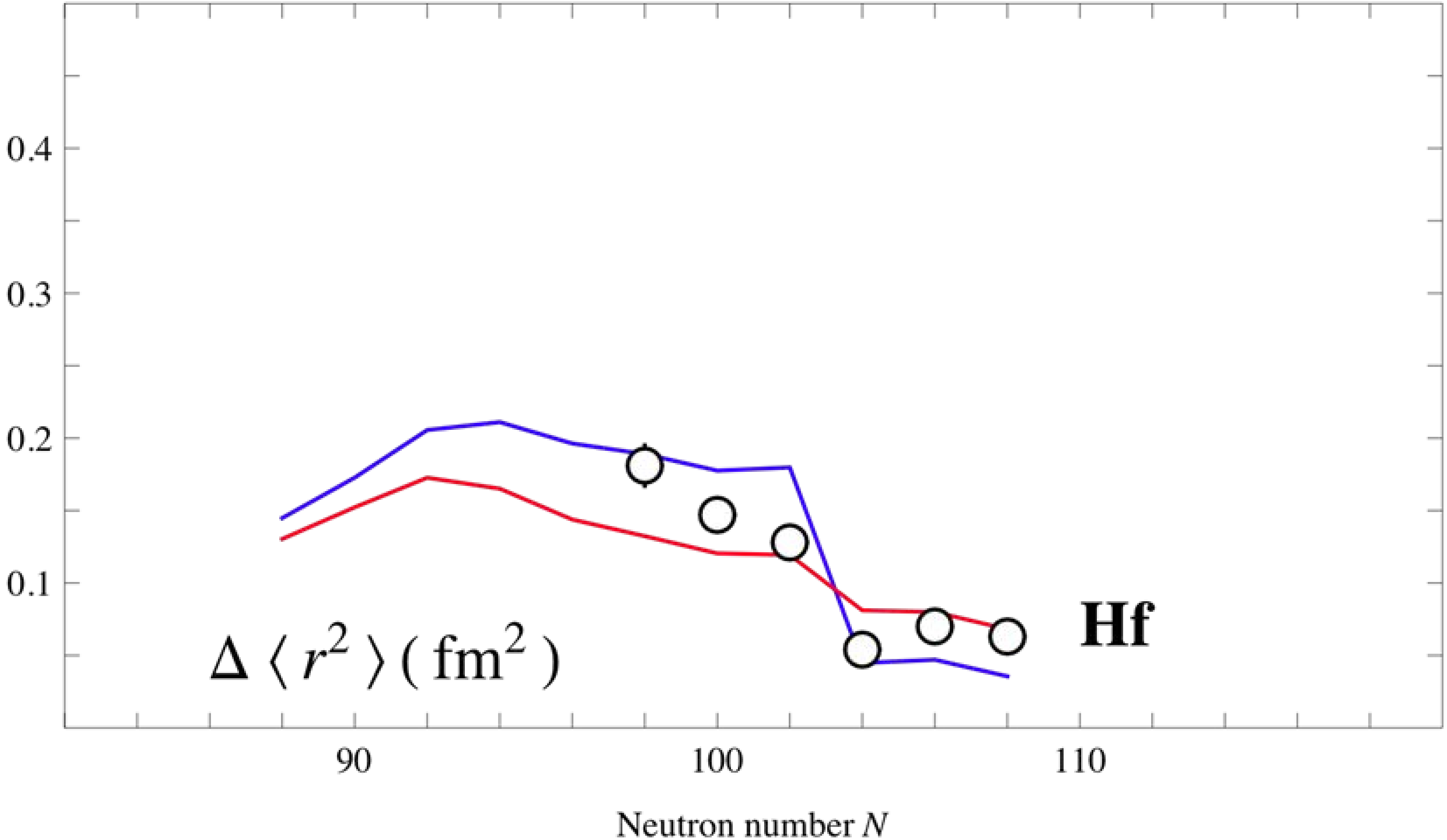
Isotope shifts in erbium



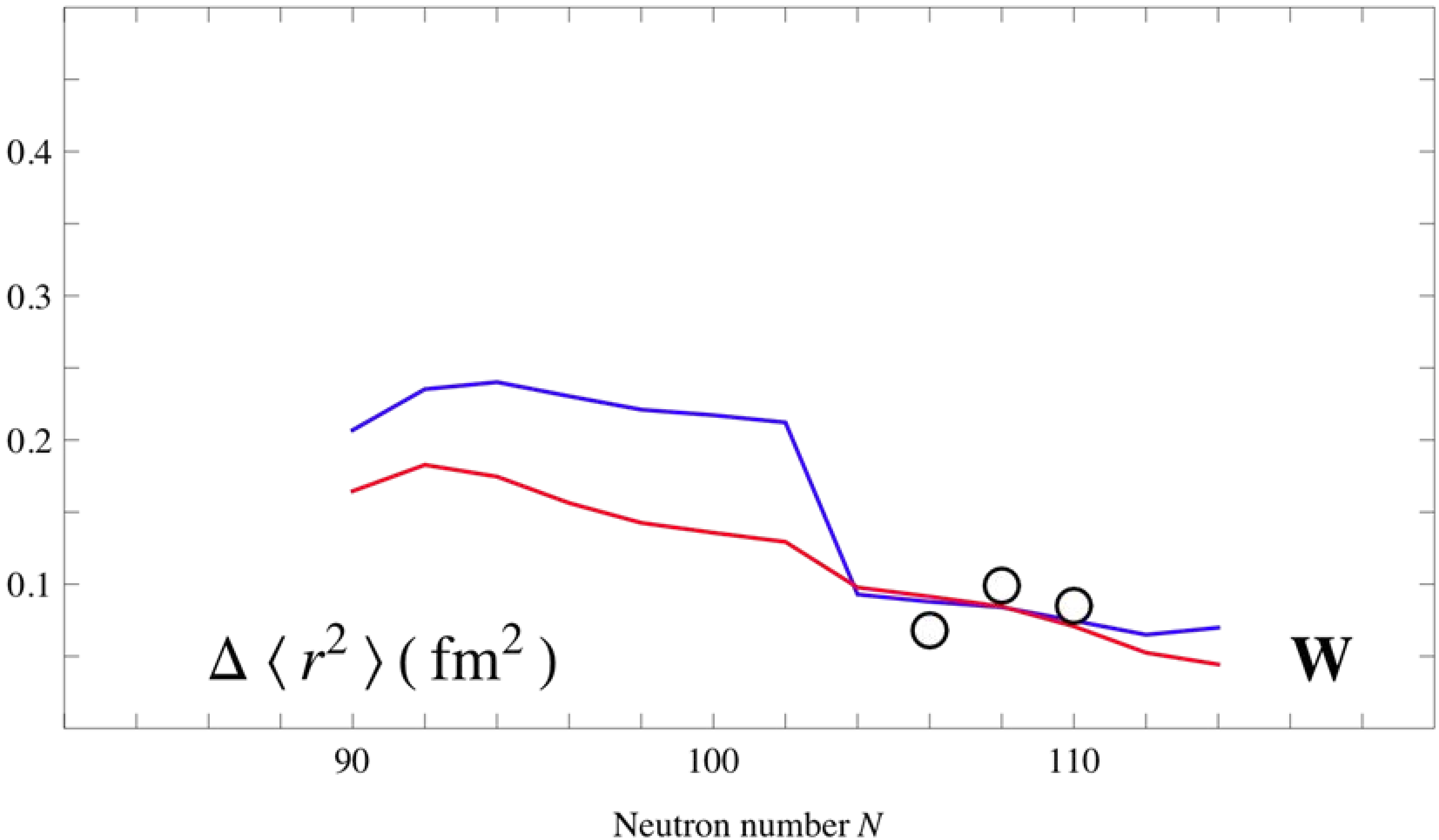
Isotope shifts in ytterbium



Isotope shifts in hafnium



Isotope shifts in tungsten



Isomer shifts

Isotopes shifts depend on the parameter η:

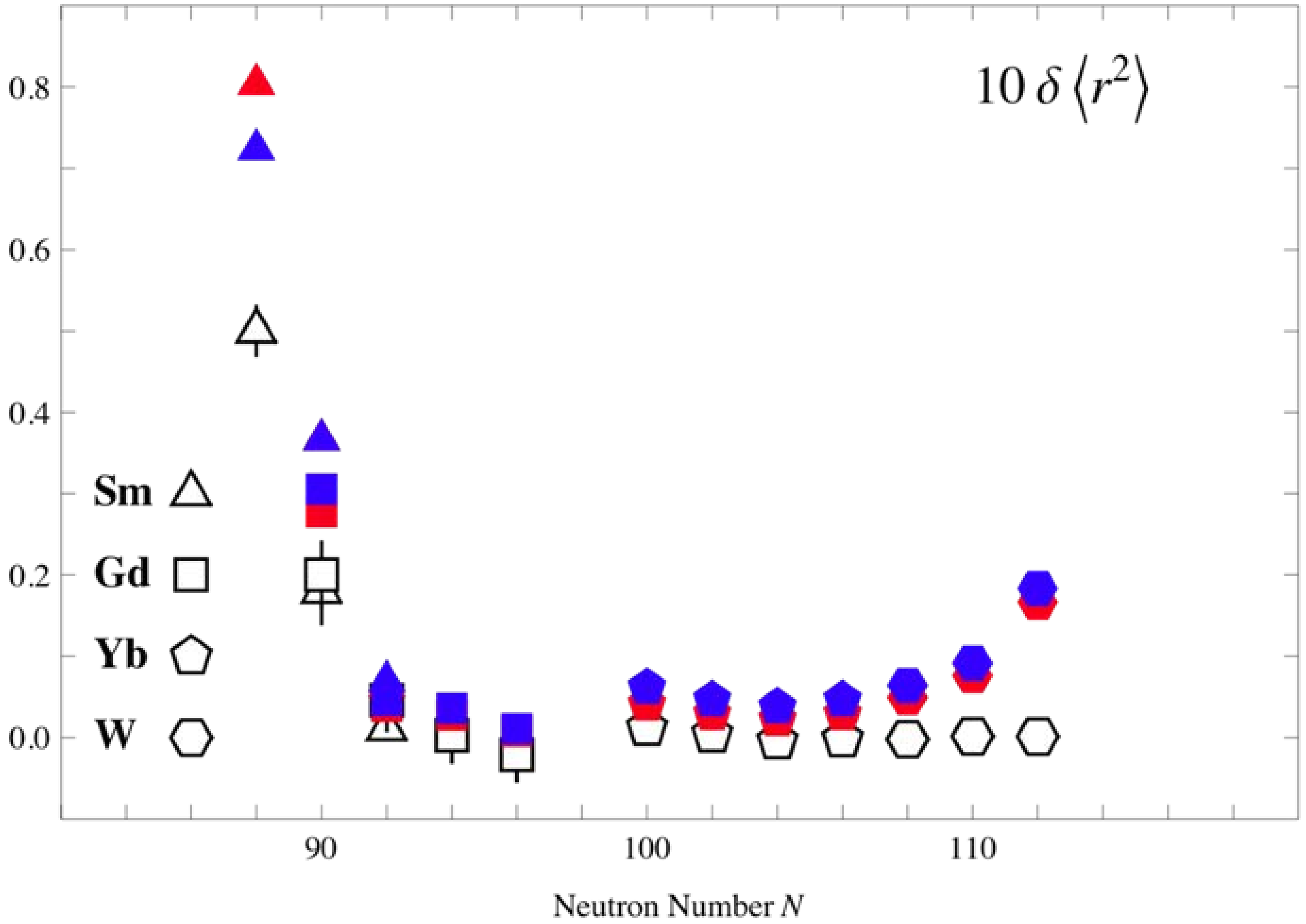
$$\delta\langle r^2\rangle \equiv \langle r^2\rangle_{2_1^+}^{(A)} - \langle r^2\rangle_{0_1^+}^{(A)} = \eta\left(\left\langle\frac{\hat{n}_d}{N_b}\right\rangle_{2_1^+}^{(A)} - \left\langle\frac{\hat{n}_d}{N_b}\right\rangle_{0_1^+}^{(A)}\right)$$

η (deformation dependence) equals 0.5 fm^2 (constant for all nuclei).

Standard parametrization:

$$\delta\langle r^2\rangle \equiv \langle r^2\rangle_{2_1^+}^{(A)} - \langle r^2\rangle_{0_1^+}^{(A)} = \eta\left(\langle\hat{n}_d\rangle_{2_1^+}^{(A)} - \langle\hat{n}_d\rangle_{0_1^+}^{(A)}\right)$$

Isomer shifts



E0 transitions and ρ^2 values

We apply the general relation between E0 and charge-radius operators.

The E0 operator in the IBM is therefore

$$\hat{T}(r^2) = \langle r^2 \rangle_{\text{core}} + \alpha N_b + \eta \frac{\hat{n}_d}{N_b} \Rightarrow \hat{T}(\text{E0}) = \eta \frac{e_{\text{n}} N + e_{\text{p}} Z}{N_b} \hat{n}_d$$

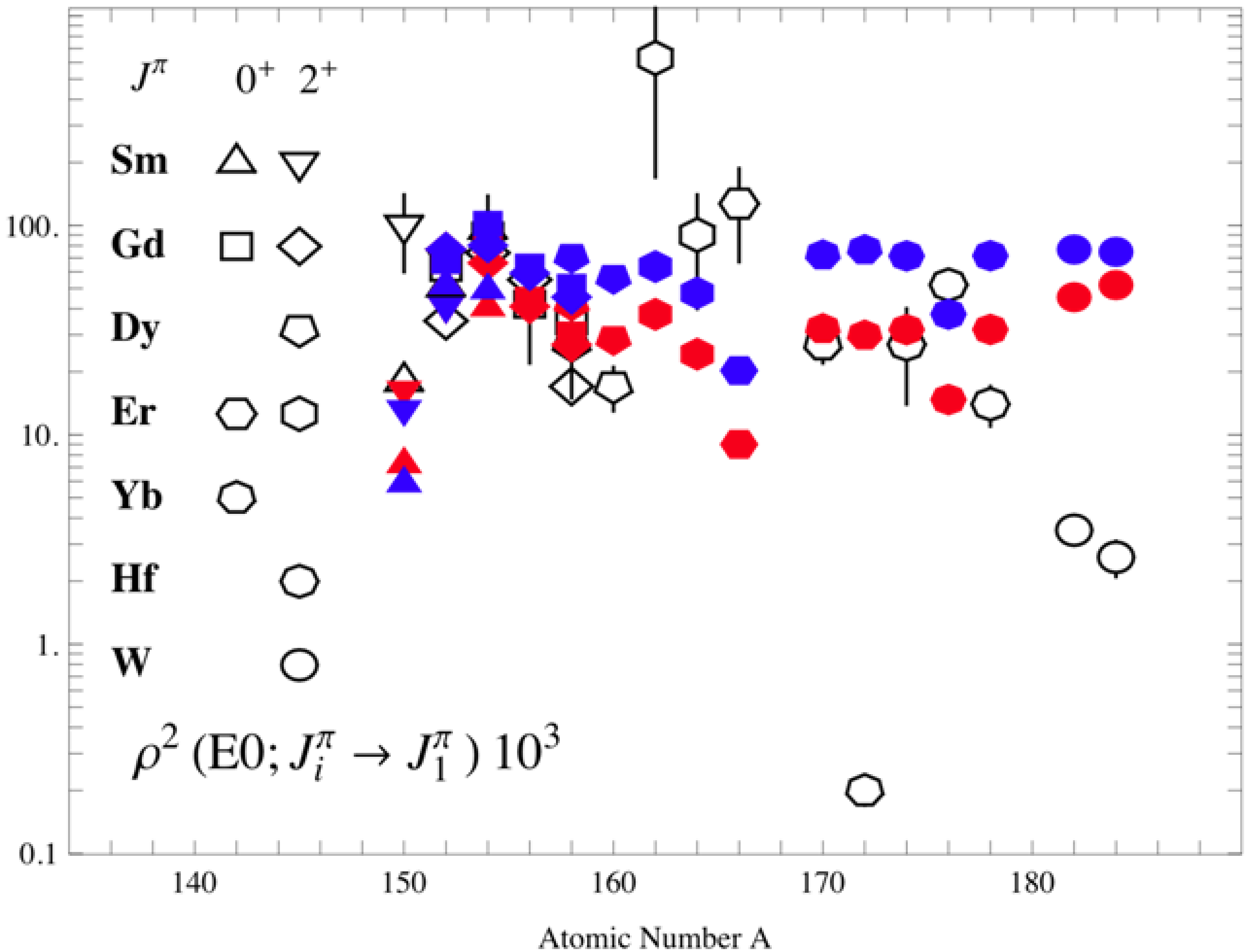
In standard parametrization

$$\hat{T}(r^2) = \langle r^2 \rangle_{\text{core}} + \alpha' N_b + \eta' \hat{n}_d \Rightarrow \hat{T}(\text{E0}) = \eta' \left(e_{\text{n}} N + e_{\text{p}} Z \right) \hat{n}_d$$

The ρ^2 value is defined as

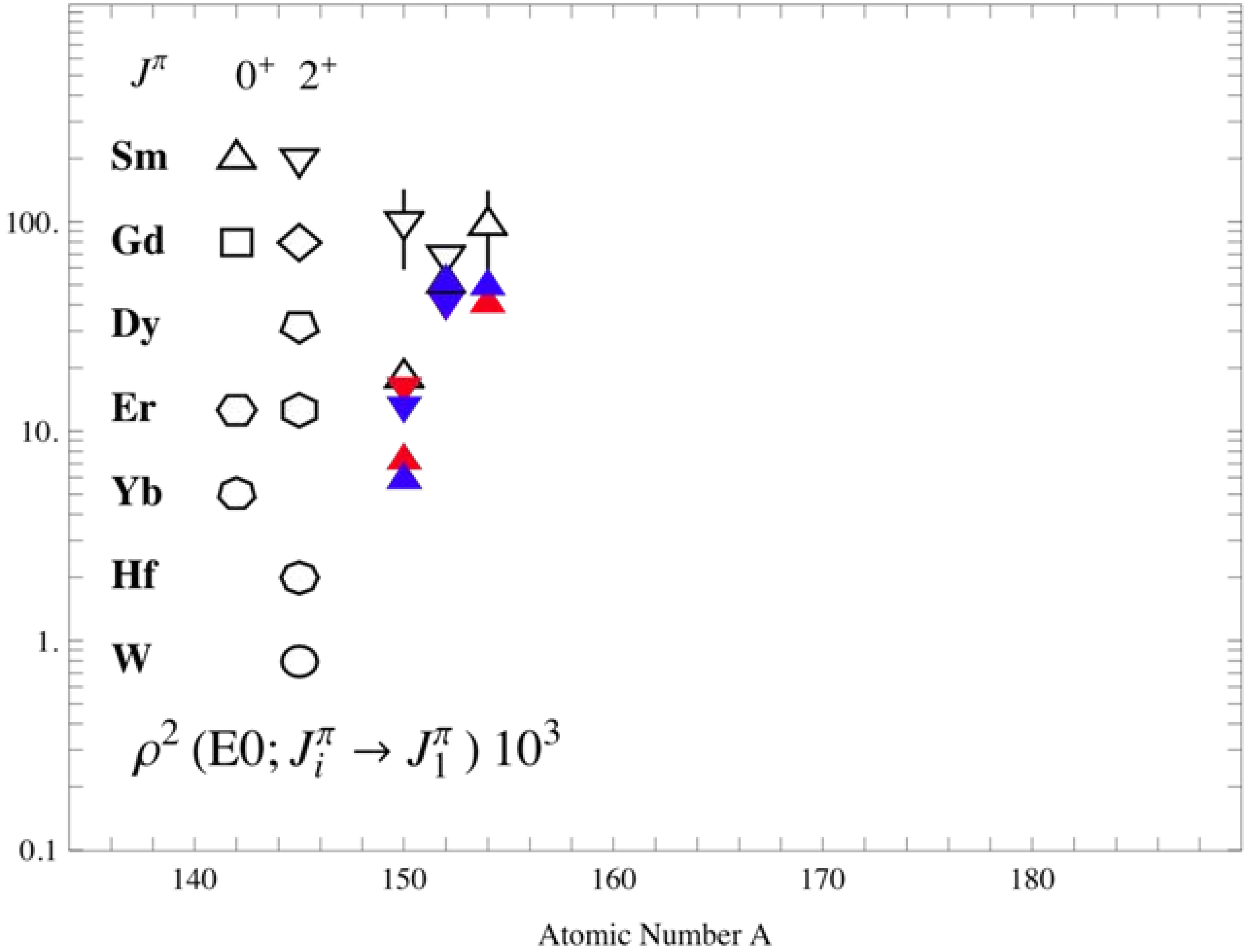
$$\rho^2 \equiv \frac{\langle \text{f} | \hat{T}(\text{E0}) | \text{i} \rangle^2}{e^2 R^4}$$

ρ^2 values

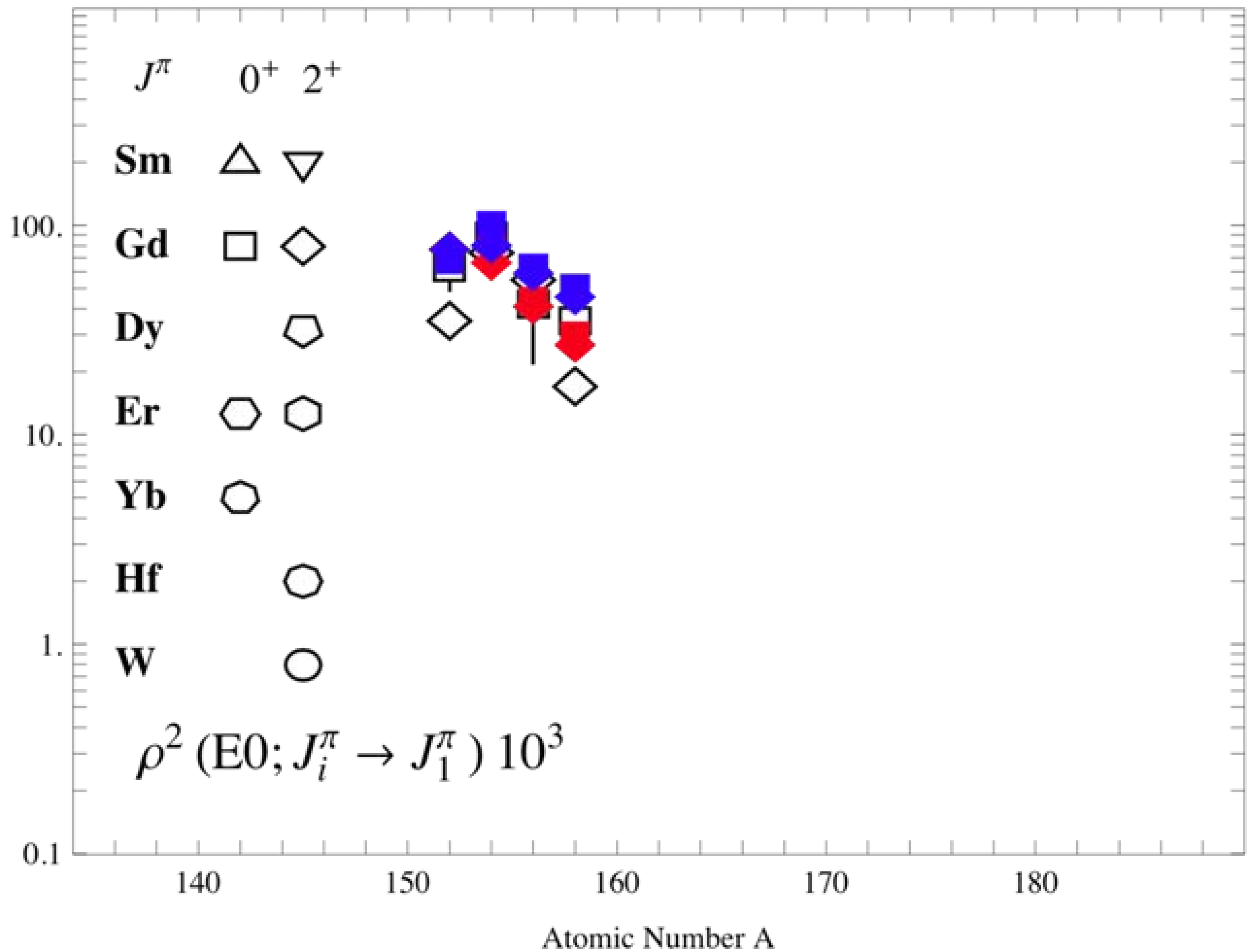


J^π 0⁺ 2⁺
Sm
Gd
Dy
Er
Yb
Hf
W
ρ^2 (E0; $J_i^\pi \to J_1^\pi$) 10^3
100.
10.
1.
0.1
140
150
160
170
180
Atomic Number A

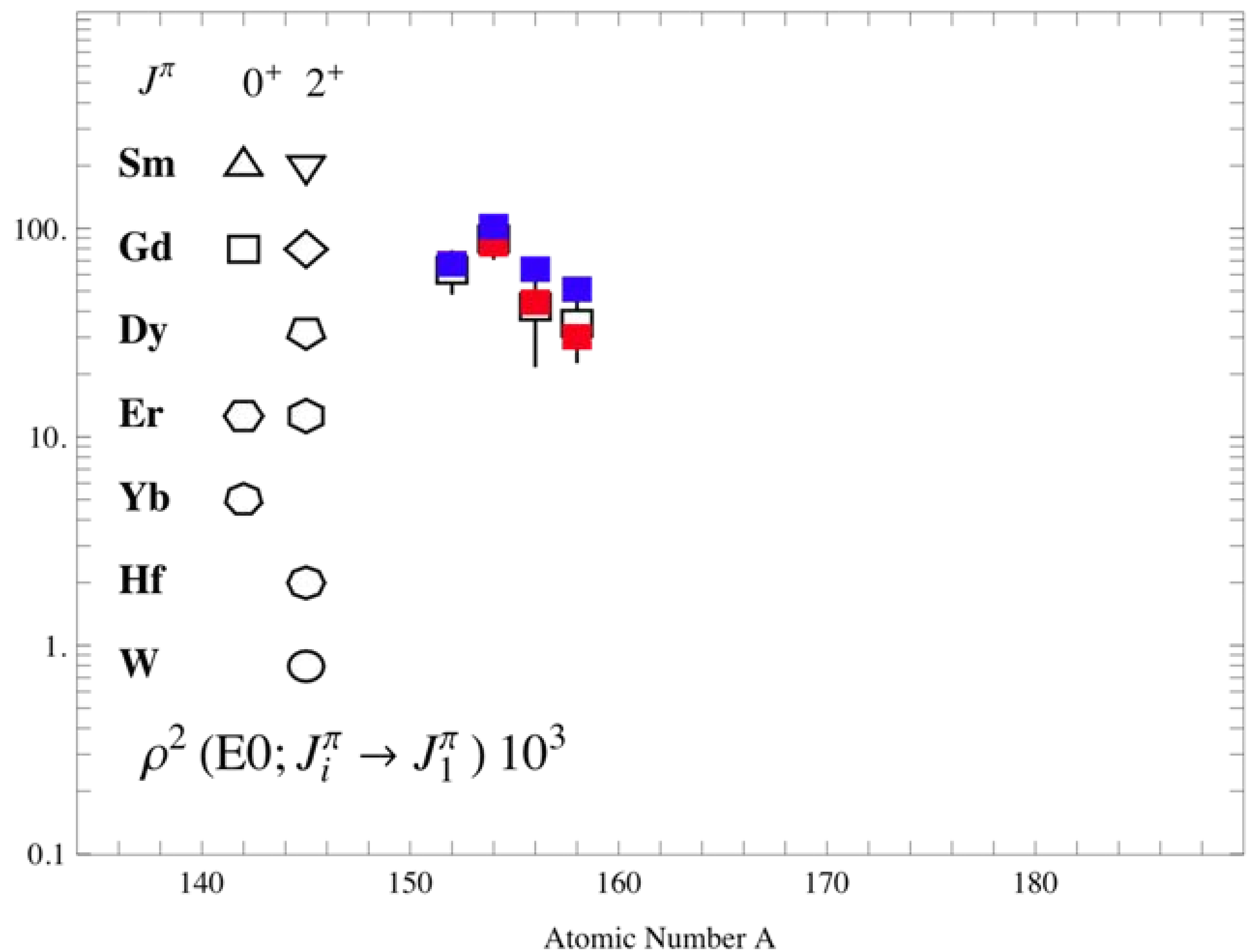
ρ^2 values in samarium



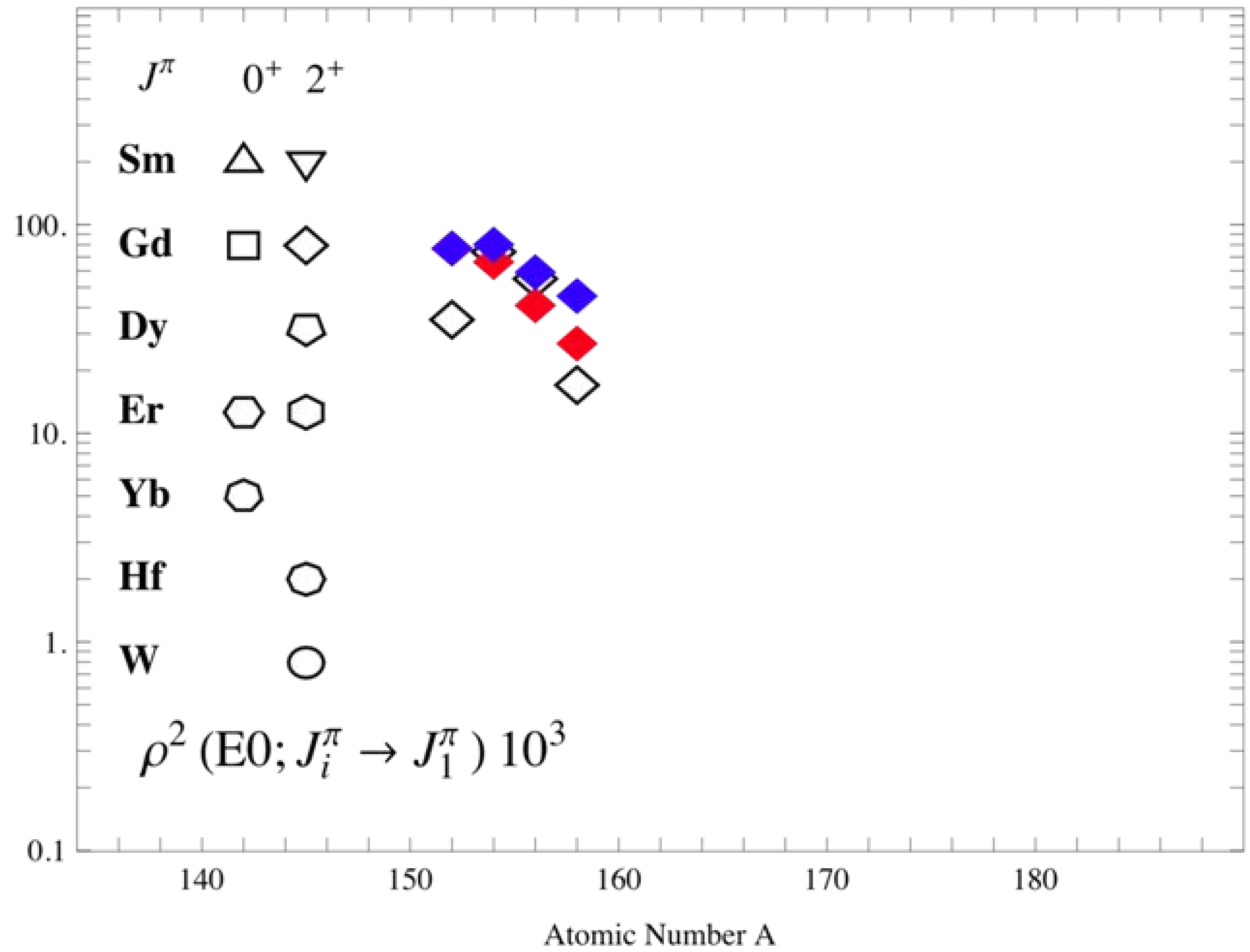
ρ^2 values in gadolinium



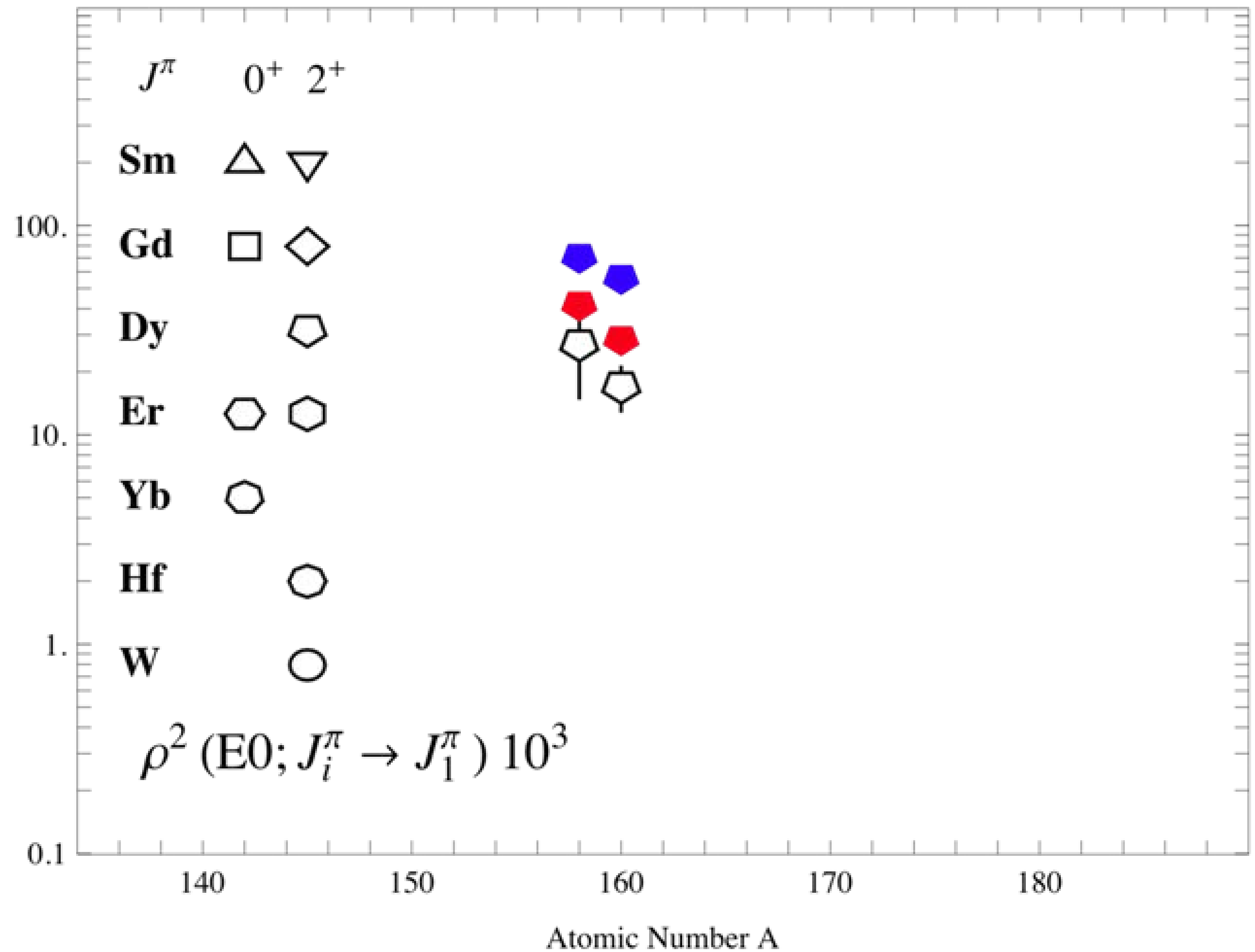
ρ^2 values in gadolinium



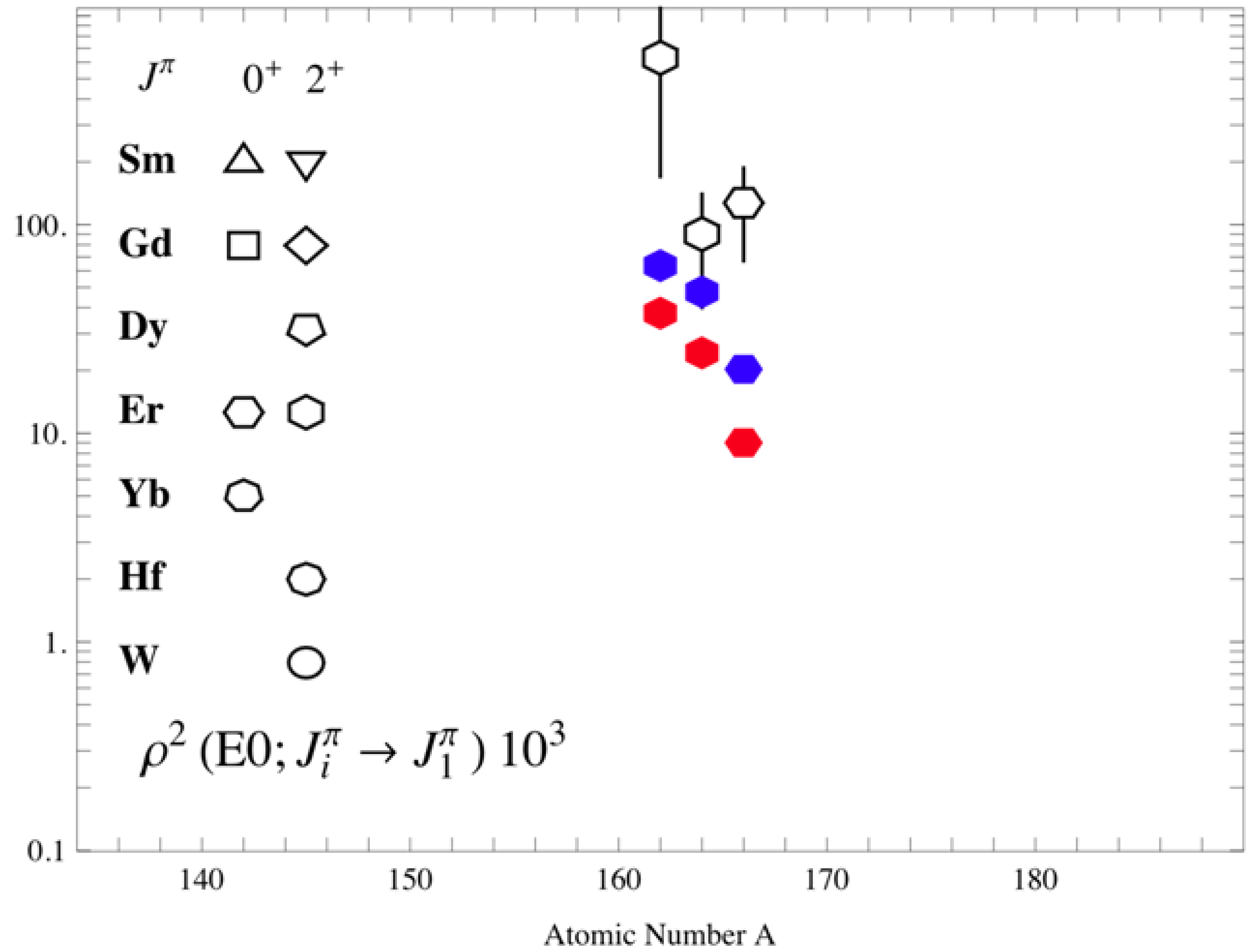
ρ^2 values in gadolinium



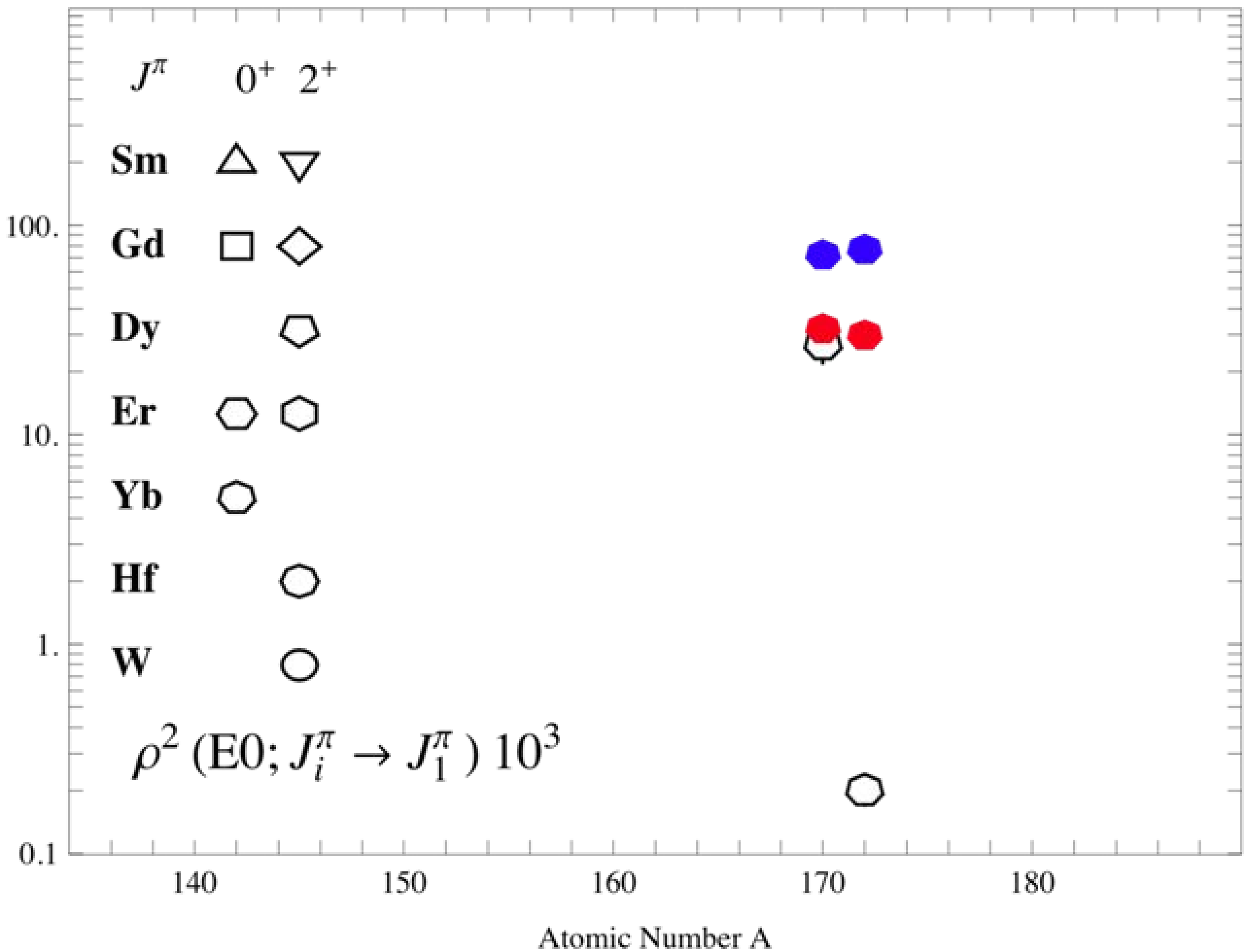
ρ^2 values in dysprosium



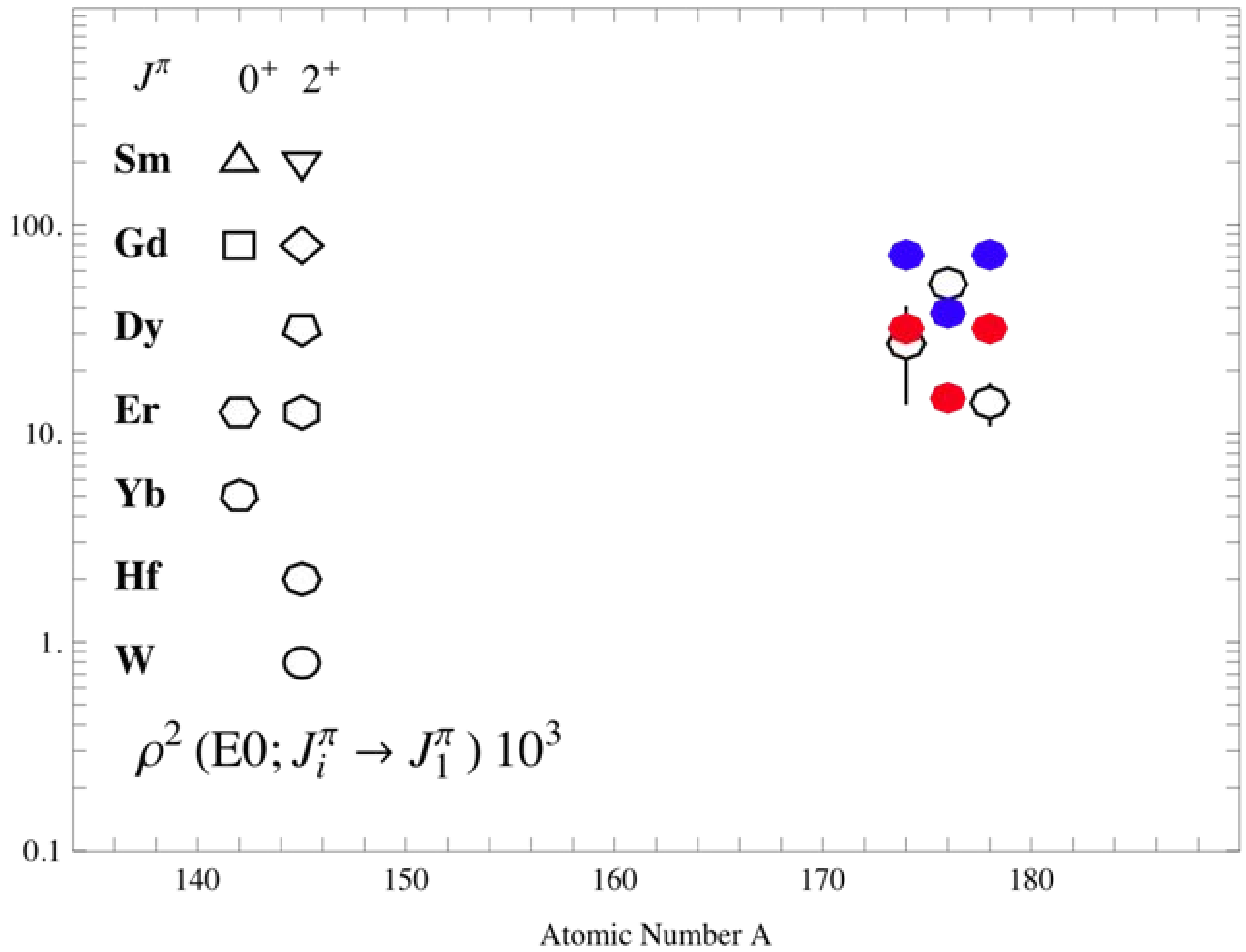
ρ^2 values in erbium



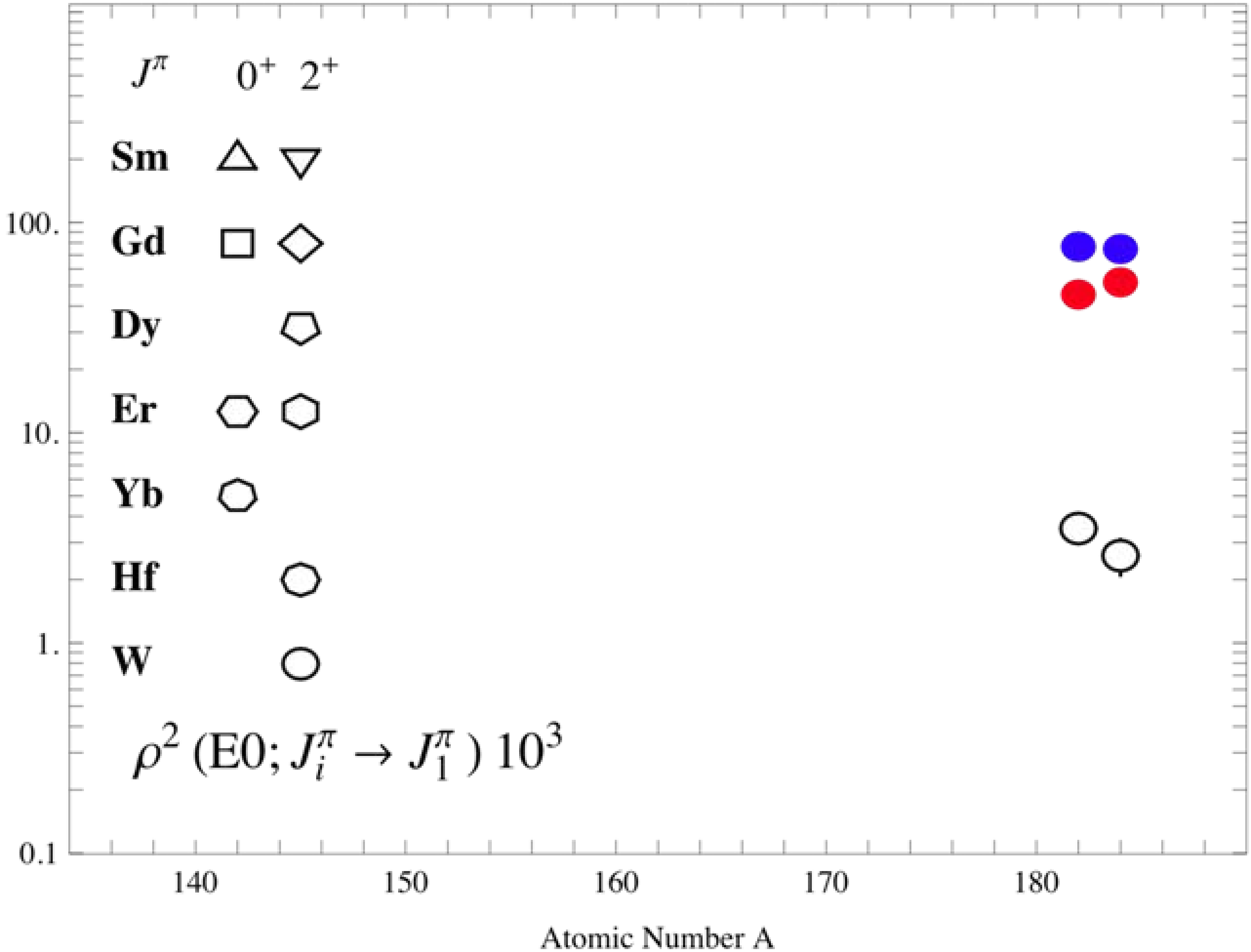
ρ^2 values in ytterbium



ρ^2 values in hafnium



ρ^2 values in tungsten



Conclusions

- Consistent treatment of charge radii and E0 transitions assuming the same effective charges.
- Yet another application of the IBM to explain nuclear data in a simple and comprehensive fashion.
- Thanks to Franco and Akito for this model that continues to give interesting new results to this day.

Estimate of parameters

The `average' increase of the charge radius with particle number:

$$\left\langle r^2 \right\rangle_{\mathrm{av}} \approx \frac{3}{5} r_0^2 A^{2/3} \Rightarrow |\alpha| \approx \frac{4}{5} r_0^2 A^{-1/3} \sim 0.2\ \mathrm{fm}^2$$

The increase of the charge radius due to deformation:

$$\left\langle r^2 \right\rangle_{\mathrm{def}} \approx \frac{3}{4\pi} \beta^2 r_0^2 A^{2/3} \Rightarrow \eta \approx \frac{4}{3} \left(1 + \bar{\beta}^2\right) r_0^2 N_b^2 A^{-4/3}$$

$$\sim 0.25 - 0.75\ \mathrm{fm}^2$$

Effective charges from radii

Estimate with harmonic-oscillator wave functions:

$$\langle r^2 \rangle_{\mathrm{s}} = \frac{1}{e_{\mathrm{n}}N + e_{\mathrm{p}}Z} \sum_{k=1}^{A} \langle \mathrm{s} | e_k r_k^2 | \mathrm{s} \rangle$$

$$= \frac{3^{4/3}}{4} \frac{b^2}{e_{\mathrm{n}}N + e_{\mathrm{p}}Z} \left(e_{\mathrm{n}} N^{4/3} + e_{\mathrm{p}} Z^{4/3} \right)$$

$$= \frac{3\sqrt[3]{2}}{5} r_0^2 \frac{A^{1/3} \left(e_{\mathrm{n}} N^{4/3} + e_{\mathrm{p}} Z^{4/3} \right)}{e_{\mathrm{n}}N + e_{\mathrm{p}}Z}$$

Fit for rare-earth nuclei (*Z*=58 to 74) gives: r_0=1.24 fm, e_{n}=0.50*e* and e_{p}=*e*.

Influence of g boson

Spherical-to-deformed transitional hamiltonian in sdg-IBM-1:

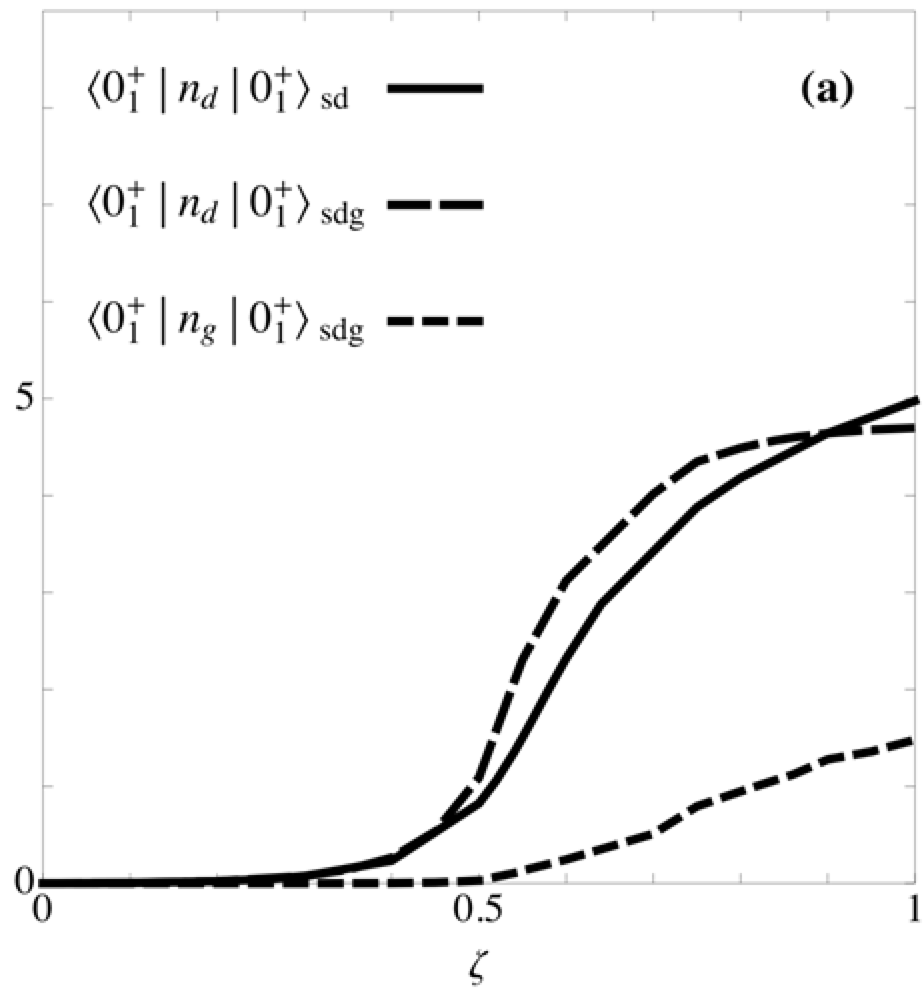
$$\hat{H} = c\left[(1-\varsigma)\left(\hat{n}_d + \lambda \hat{n}_g\right) - \frac{\varsigma}{4N_b} Q\cdot Q\right]$$

$$Q_\mu = \left[s^+ \times \tilde{d} + d^+ \times \tilde{s}\right]^{(2)}_\mu - \frac{11}{14}\left[d^+ \times \tilde{d}\right]^{(2)}_\mu$$

$$+ \frac{9}{7}\left[d^+ \times \tilde{g} + g^+ \times \tilde{d}\right]^{(2)}_\mu - \frac{3}{14}\left[g^+ \times \tilde{g}\right]^{(2)}_\mu$$

Take λ=1.5 and let ζ vary from 0 (spherical) to 1 (deformed).

Effect of *g* boson on radii



Effect of *g* boson on E0s