Excited state quantum phase transition and chaos in the Dicke model

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Beauty in Physics: Theory and Experiment

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Introduction

- **Quantum phase transitions** (QPT's) describe the change in the ground state wave function of a many particle system due to quantum fluctuations.
- An **excited quantum phase transition** (ESQPT) is similar to a QPT but affecting to excited states.
- We study the relationship between ESQPT and chaos in the Dicke model.

The Models: Algebraic structure

- We consider a single-mode bosonic field (HW(1)) interacting with an algebraic subsystem:
  - SU(2) $\longrightarrow$ HW(1)$\otimes$SU(2).
- Operators for HW(1): $b^\dagger$, b.
- Operators for SU(2): $J_\pm = J_1 \pm iJ_2$, $J_0 = J_3$.
- Commutation relation of SU(2) algebra:

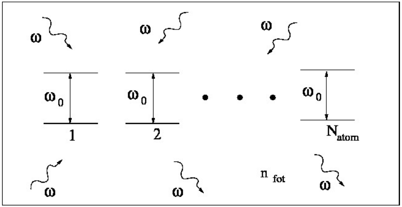
$$[J_0, J_\pm] = \pm J_\pm, \; [J_+, J_-] = 2J_0$$

Dicke and Jaynes-Cumming models

- Its generators can be constructed from fermionic operators:

$$J_+ = \sum_{i=1}^{2j} a^\dagger_{\uparrow i} a_{\downarrow i}, \quad J_- = \sum_{i=1}^{2j} a^\dagger_{\downarrow i} a_{\uparrow i}, \quad J_0 = \frac{1}{2} \sum_{i=1}^{2j} \left(a^\dagger_{\uparrow i} a_{\uparrow i} - a^\dagger_{\downarrow i} a_{\downarrow i} \right)$$

- The Hamiltonian reads:



$$\begin{aligned} \mathrm{H}_2 &= \omega_0 J_0 + \omega b^\dagger b \\ &+ \frac{\lambda}{\sqrt{M_3}} \left(b J_+ + b^\dagger J_- \right) \\ \mathrm{H}_3 &= \omega_0 J_0 + \omega b^\dagger b \\ &+ \frac{\lambda}{\sqrt{M_2}} \left((b + b^\dagger)(J_- + J_+) \right) \end{aligned}$$

The Models: Jaynes-Cummings and Dicke model

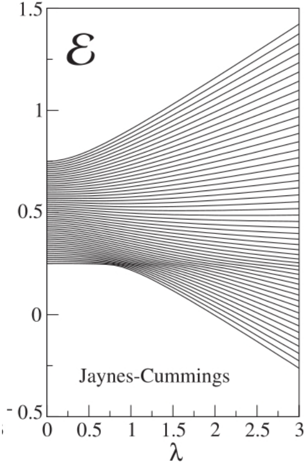
- $\mathrm{H}_2 \rightarrow$ Tavis(Jaynes)-Cummings Model.
  - It conserves the quantity:

$$M_2 = 2(N_b + \mathrm{J}_0 + j)$$

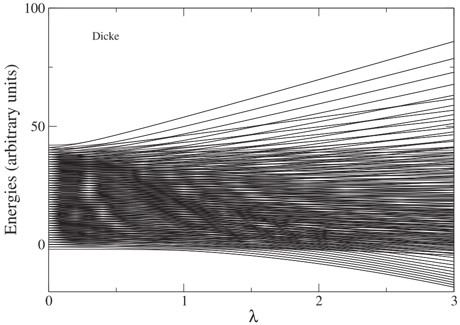
- $\mathrm{H}_3 \rightarrow$ Dicke Model.
  - This model violates the conservation of M_2, but it still conserves the parity:

$$\Pi = (-1)^{M_2/2}$$

  - M_3 is defined as $M_3 = 4j$.



ℰ
Jaynes-Cummings
1.5
1
0.5
0
- 0.5
0
0.5
1
1.5
2
2.5
3
λ



Dicke
Energies (arbitrary units)
100
50
0
0
1
2
3
λ

The Models: Classical limits

Coherent states

$$|\Psi\rangle = |\zeta\rangle \otimes |\xi\rangle,$$
$$|\zeta\rangle \propto e^{\zeta b^\dagger}|0\rangle,$$
$$|\xi\rangle \propto e^{\xi \hat{J}_+}|j,-j\rangle$$

Holstein-Primakoff transformation + position and momentum operators

$$\begin{aligned} J_+ &= c^\dagger\sqrt{2j - c^\dagger c}, \\ J_- &= \sqrt{2j - c^\dagger c}\, c, \\ J_0 &= c^\dagger c - j \end{aligned}$$

The Models: Classical limits

Coherent state for the HW(2) algebra

$$|\zeta,\xi\rangle \propto e^{\zeta b^\dagger + \xi c^\dagger}|0\rangle$$

Semiclassical aproximation

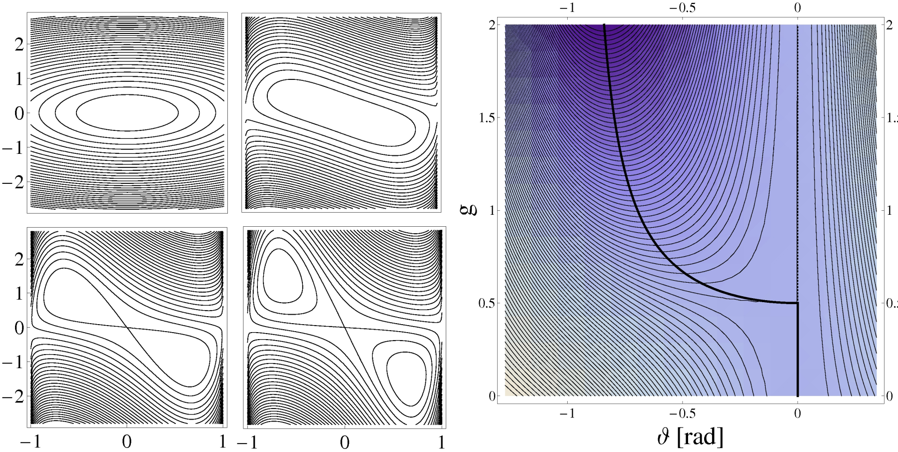
$$\frac{c}{\sqrt{M_n}} = \frac{\hat{x}+i\hat{p}}{\sqrt{2}}$$
$$\frac{b}{\sqrt{M_n}} = \frac{\hat{y}+i\hat{q}}{\sqrt{2}}$$
$$.[\hat{x},\hat{p}] = [\hat{y},\hat{q}] = \frac{i}{M_n}$$

The Models: Classical limits

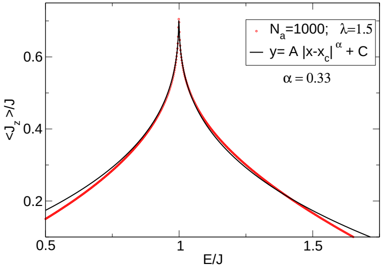
- QPT Dicke model is well known: $\lambda_{QPT} = \sqrt{\frac{\omega\omega_0}{2}}$. (C. Emary and T. Brandes, Phys. Rev. E 67, 066203).
- QPT for Jaynes-Cummings: $\lambda_{QPT} = \frac{|\omega_0 - \omega|}{\sqrt{2}}$.

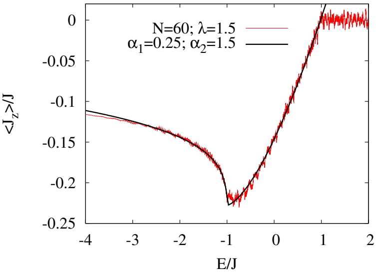
$$
\begin{aligned}
H_n &= H_{n0} + \lambda H'_n \\
H_{10} &= \frac{-R_1\omega_0}{2} + \frac{\omega_0}{2}(p^2 + x^2) + \frac{\omega}{2}(q^2 + y^2) \\
H'_1 &= \sqrt{2R_1 + (p^2 + x^2)}(xy + pq)/\sqrt{2}
\end{aligned}
$$

The Models: Classical limits



Order parameters

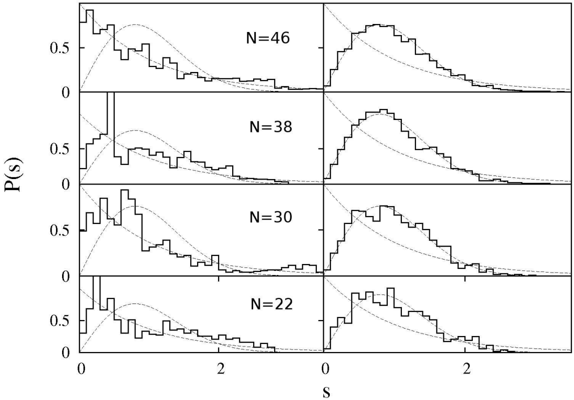




$$\langle \hat{J}_z \rangle = \langle \hat{J}_z \rangle_c + A\,|E - E_c|^{\alpha}$$

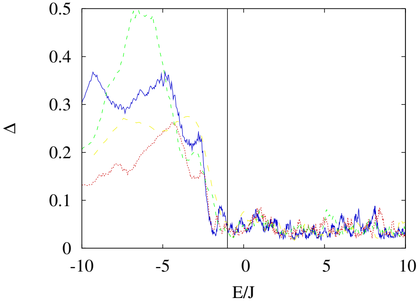
ESQPT and onset of Chaos

Regular regime → Poisson: $P(s) = e^{-s}$
Chaotic regime → Wigner: $P(s) = \frac{\pi}{2} s e^{-\pi s^2/4}$



$E < E_c$ $E > E_c$

ESQPT and onset of Chaos



$$F(s) = \int_0^s dx P(x)$$

$$\Delta = \frac{\sum_i (F^w(s_i) - F(s_i))^2}{\sum_i (F^w(s_i) - F^p(s_i))^2}$$

Conclusions

- We have studied the Dicke and Jaynnes-Cummings model.
- We have demostrated the existence of an ESQPT in two models describing the collective matter-light interaction.
- We have calculated the value for λ_{QPT} using a semiclassical aproximation for the Jaynnes-Cummings model.
- For the Jaynes-Cummings model, the ESQPT leads to a neat nonanalyticity of the order parameter $< J_z >$ at the critical energy E_c.

Conclusions

- The Dicke model exhibits a similar type of ESQPT than the Jaynes-Cummings model, but with signatures blurred by the onset of chaotic behaviour in the spectrum.
- Our numerical calculations show that a crossover from the regime with no level repulsion to the one with the Wigner level statistics takes place precisely around the critical energy. These results are compatible with the hypothesis that the abrupt emergence of level repulsion is caused by the precursors of the ESQPTs.
- We anticipate the existence of a similar qualitative behaviour in other non-integrable systems with ESQPTs.