Interacting boson model from microscopic theory

Kosuke Nomura (Tokyo --> Köln)

Beauty in Physics @ Cocoyoc, May 2012

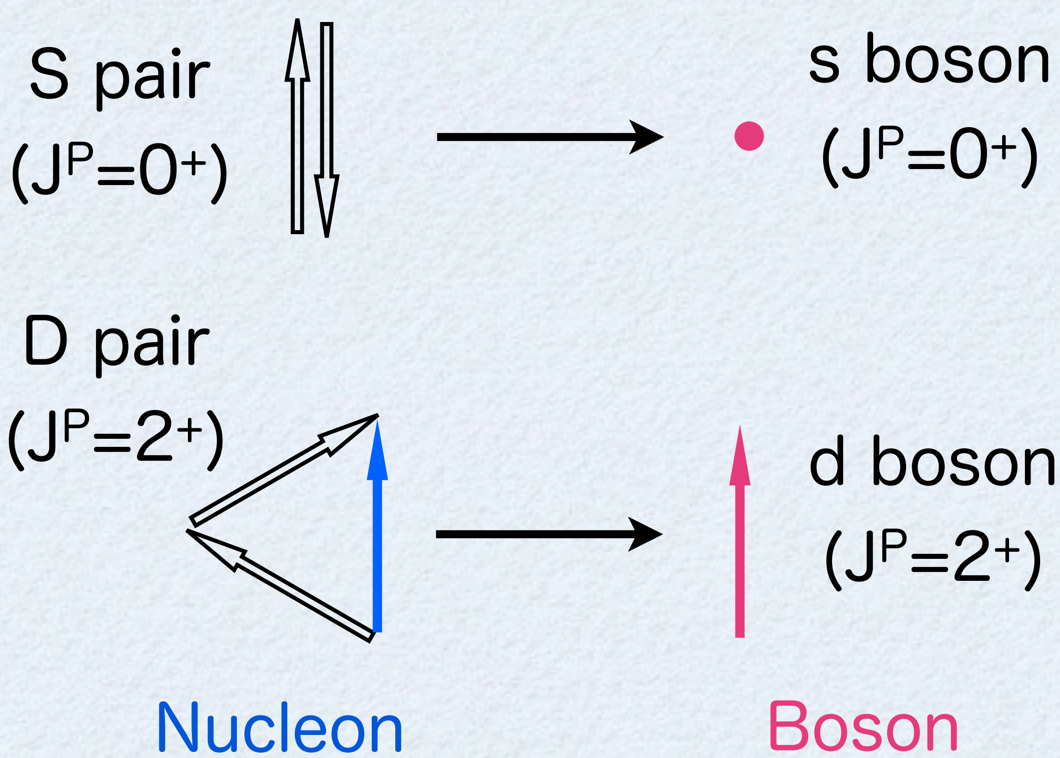
IBM and its “microscopic foundation”

- collective pairs of valence nucleons
- shell-model derivation for modest deformation

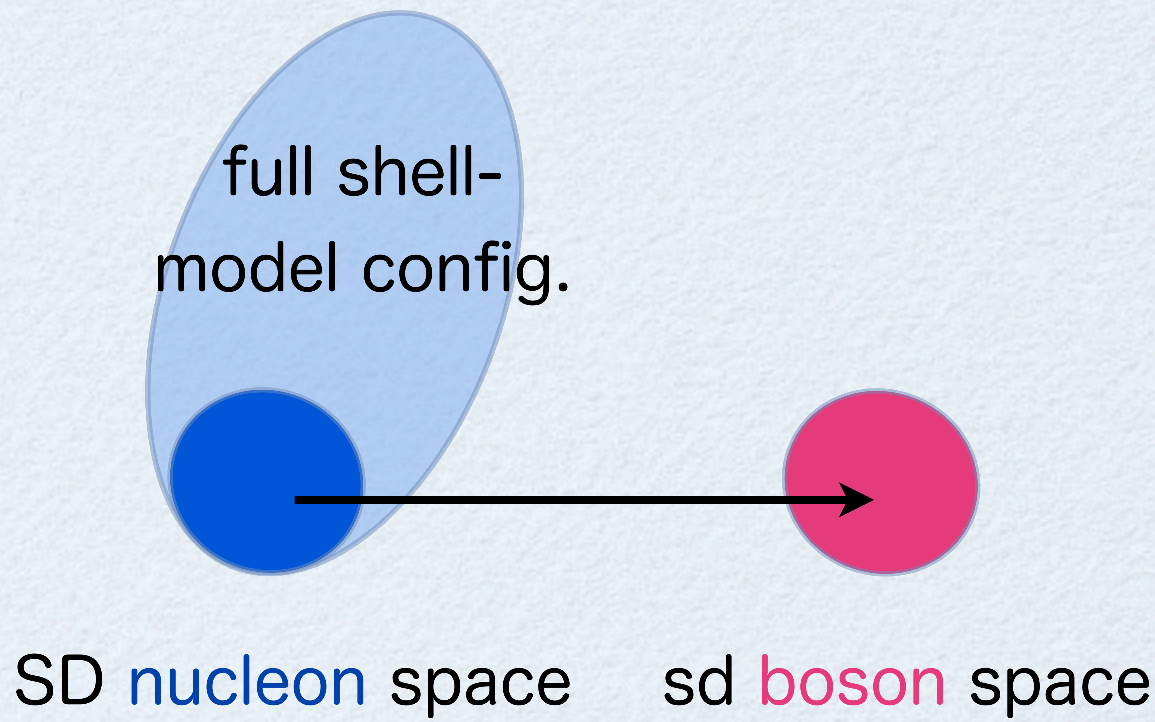
Refs:

- A. Arima & F. Iachello (1974)
- T. Otsuka, A. Arima & F. Iachello (1978)
- T. Mizusaki & T. Otsuka (1997)

Building blocks

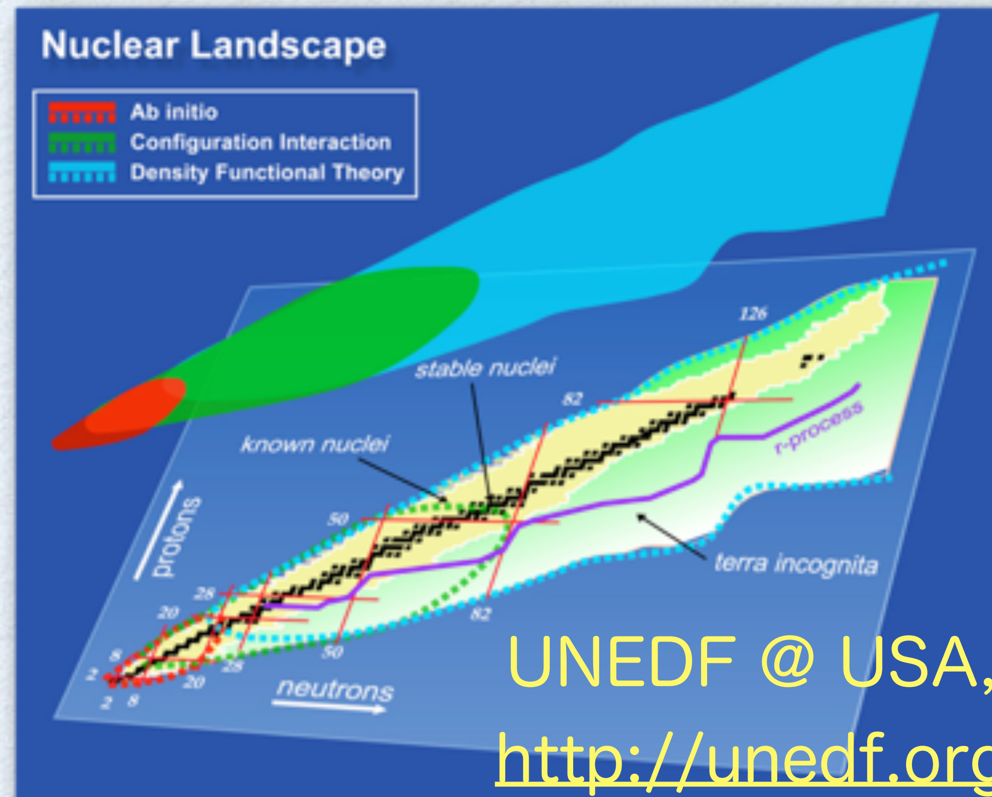


OAI mapping



Q. How to derive IBM for general cases ?

Energy density functional (EDF)



UNEDF @ USA,
http://unedf.org

- Mean-field model with EDFs: Skyrme, Gogny, RMF, etc. for nuclear properties. Universal.
    - Textbook: P. Ring & P. Schuck (1985)
    - Review: M. Bender et al. RMP (2003)

- Methods to derive spectra (with symmetry restoration and/or fluctuation of collective variables). Complicated.

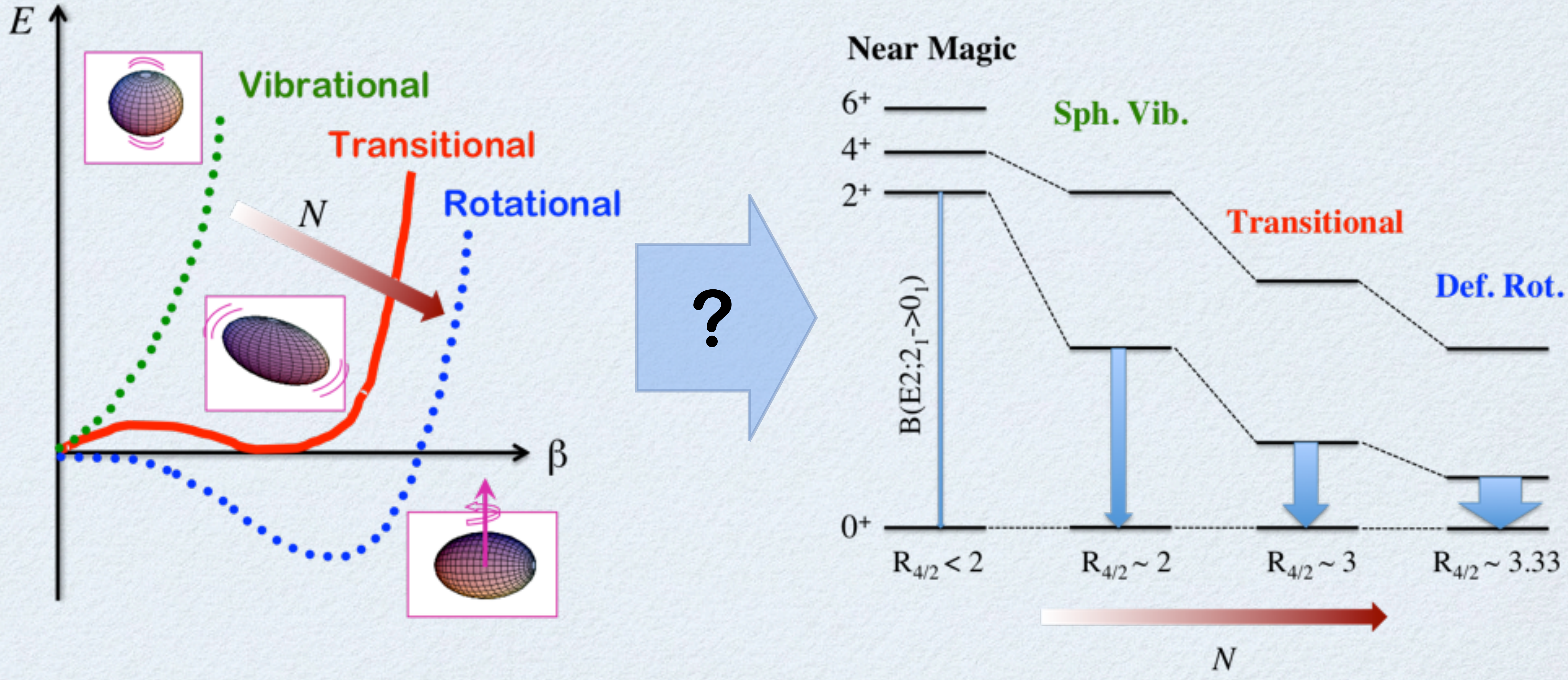
  much involved for well deformed and/or triaxial configs.

    - Skyrme-GCM: M. Bender & P.-H. Heenen PRC (2008)
    - Gogny-5DCH: J.-P. Delaroche et al. PRC (2010)
    - RMF-GCM: J. M. Yao et al. PRC (2011)

Q. Exploit the merit of EDF to formalize the IBM ?

Potential energy surface

- intuitive picture of geometry, deformation, QPT, ...
- For spectroscopy, it is also suitable to start with.
  (e.g., GCM, 5-dim. Collective Hamiltonian)
- then, can we construct IBM Hamiltonian in a similar way ?



Contents of the talk

1. Introduction
2. Basics, and application to axially-deformed case

    Derive IBM Hamiltonian from EDF, Sph.-Def. transition

3. Shape phenomena involving triaxiality

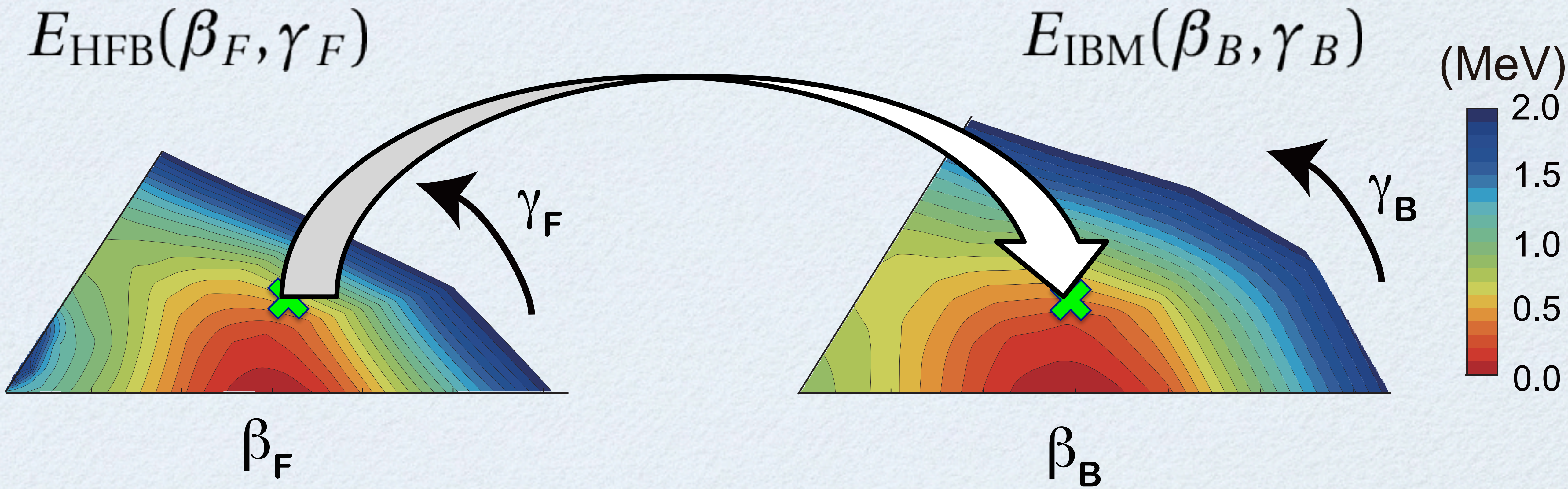
    Shape coexistence, prolate-oblate transition in A~190

4. Robust regularity in triaxially-shaped systems
5. Summary

Basics, and
axially-deformed nuclei

Refs. Phys. Rev. Lett. 101, 142501 (2008)
Phys. Rev. C 81, 044307 (2010)
Phys. Rev. C 83, 041302(R) (2011)

“Mapping” the energy surfaces



- Total energy from constrained self-consistent mean-field method (HF+BCS, HFB) with any type of EDF

- Total energy for a boson condensation (energy expectation value in the coherent state)

IBM parameters are obtained through this process.
Diagonalize boson Hamiltonian ⇒ Spectra & transition rates

Geometry in IBM

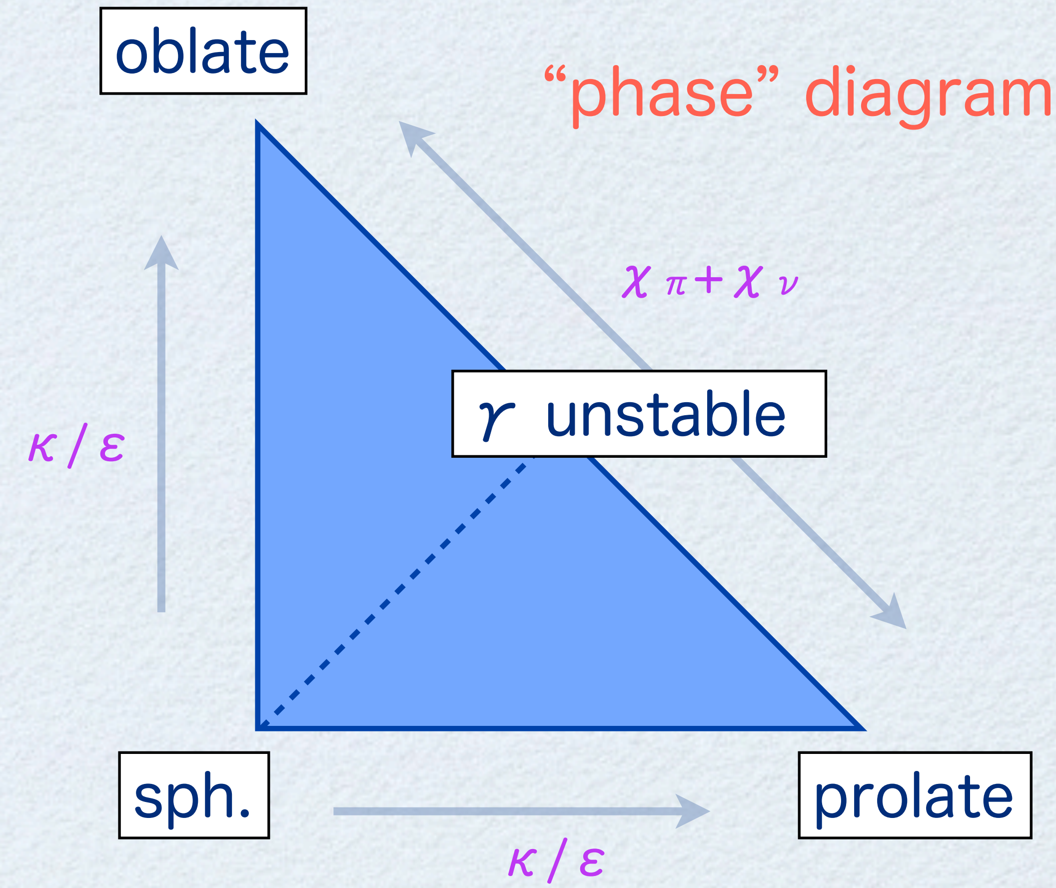
Ginocchio & Kirson, 1980

- Simplest Hamiltonian (up to two body)

$$\hat{H}_{\rm IBM} = \epsilon(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa\hat{Q}_\pi \cdot \hat{Q}_\nu$$

Sph. Driving — Def. Driving

$$\hat{n}_{d\rho} = d^\dagger_\rho \cdot \tilde{d}_\rho \qquad \hat{Q}_\rho = s^\dagger_\rho \tilde{d}_\rho + d^\dagger_\rho \tilde{s}_\rho + \chi_\rho [d^\dagger_\rho \tilde{d}_\rho]^{(2)}$$



- Coherent state

$$|\Phi\rangle = \prod_{\rho=\pi,\nu} \frac{1}{\sqrt{N_\rho!}} (\lambda^\dagger_\rho)^{N_\rho} |0\rangle \qquad \lambda^\dagger_\rho = \frac{1}{\sqrt{1+{\beta_\rho}^2}} \left[s^\dagger_\rho + d^\dagger_{\rho 0} \beta_\rho \cos\gamma_\rho + \frac{1}{\sqrt{2}} (d^\dagger_{\rho+2} + d^\dagger_{\rho-2}) \beta_\rho \sin\gamma_\rho \right]$$

- Energy surface

$$E(\beta_B, \gamma_B) = \langle \Phi | \hat{H}_{\rm IBM} | \Phi \rangle \qquad \begin{cases} \beta_\pi = \beta_\nu \equiv \beta_B (\propto \beta) \\ \gamma_\pi = \gamma_\nu \equiv \gamma_B (= \gamma) \end{cases}$$

This encompasses entire class of symmetries for intrinsic shape.

Wavelet transform

cf. G. Kaiser, “A Friendly Guide to Wavelets” (1994)

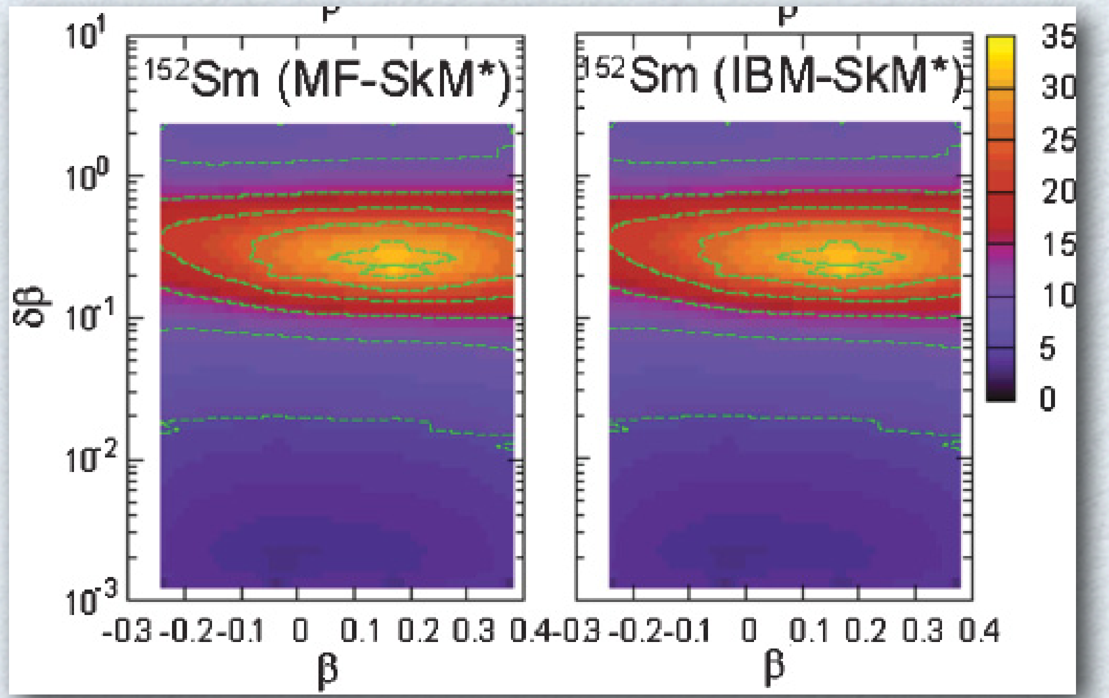
- extracts global features of PES: curvature, minimum, ...
- hence, eliminating any irrelevant local pattern.

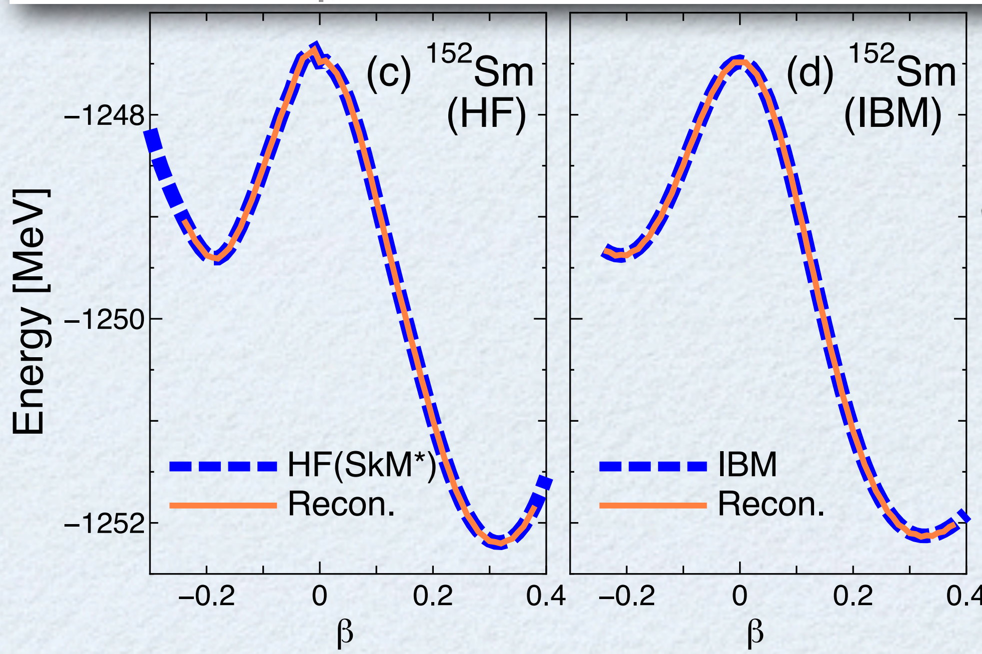
For axial symmetry,

$$\tilde{E}(\delta\beta,\beta)=\frac{1}{\sqrt{\delta\beta}}\int E(\beta')\Psi^{*}\left(\frac{\beta-\beta'}{\delta\beta}\right)d\beta'$$

Scale (frequency): $\delta\beta$; Position: β; Signal (Energy surface): $E(\beta')$; Basis (wavelet): Ψ^*

$|\tilde{E}(\delta\beta,\beta)|^2$

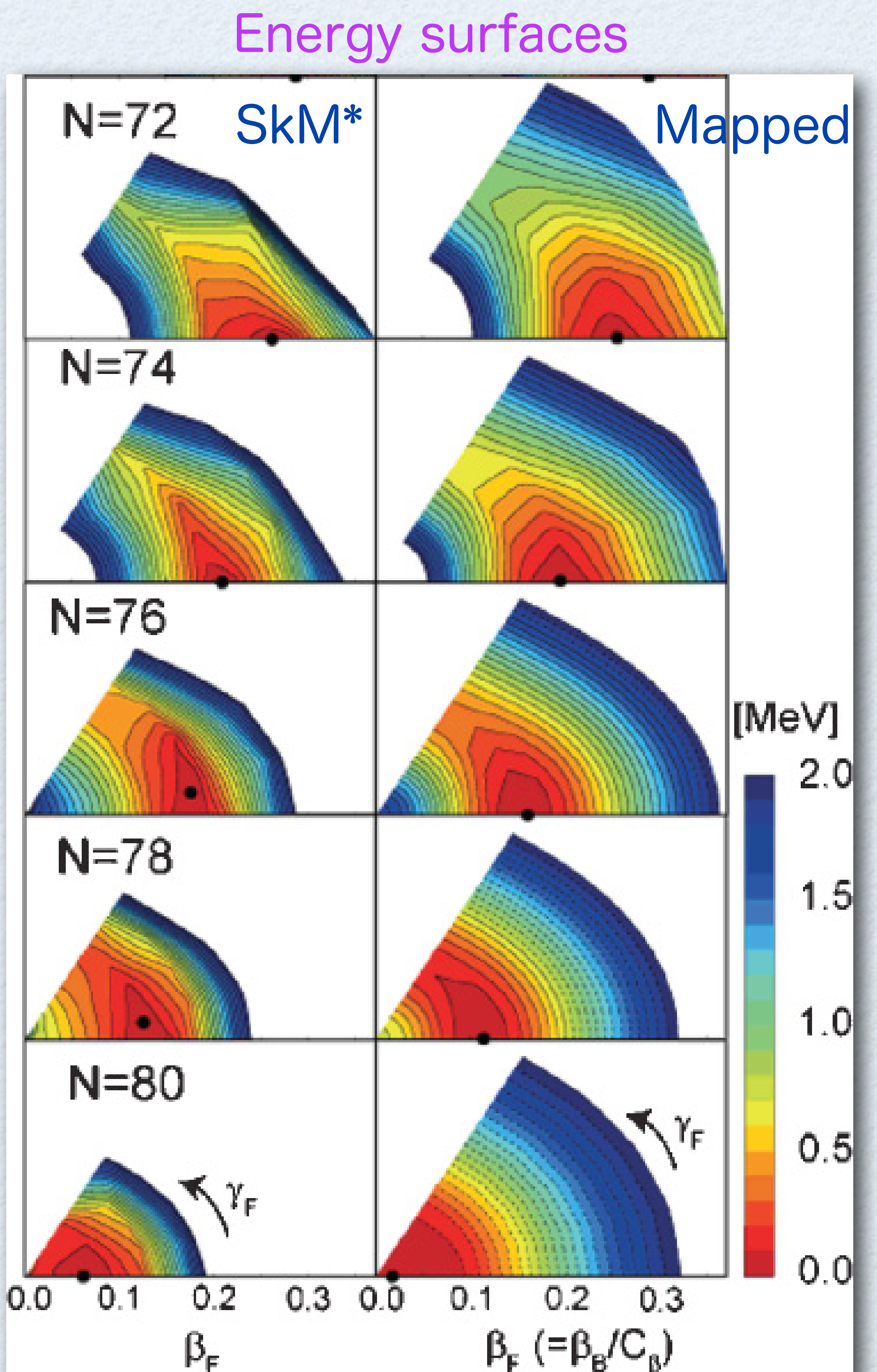


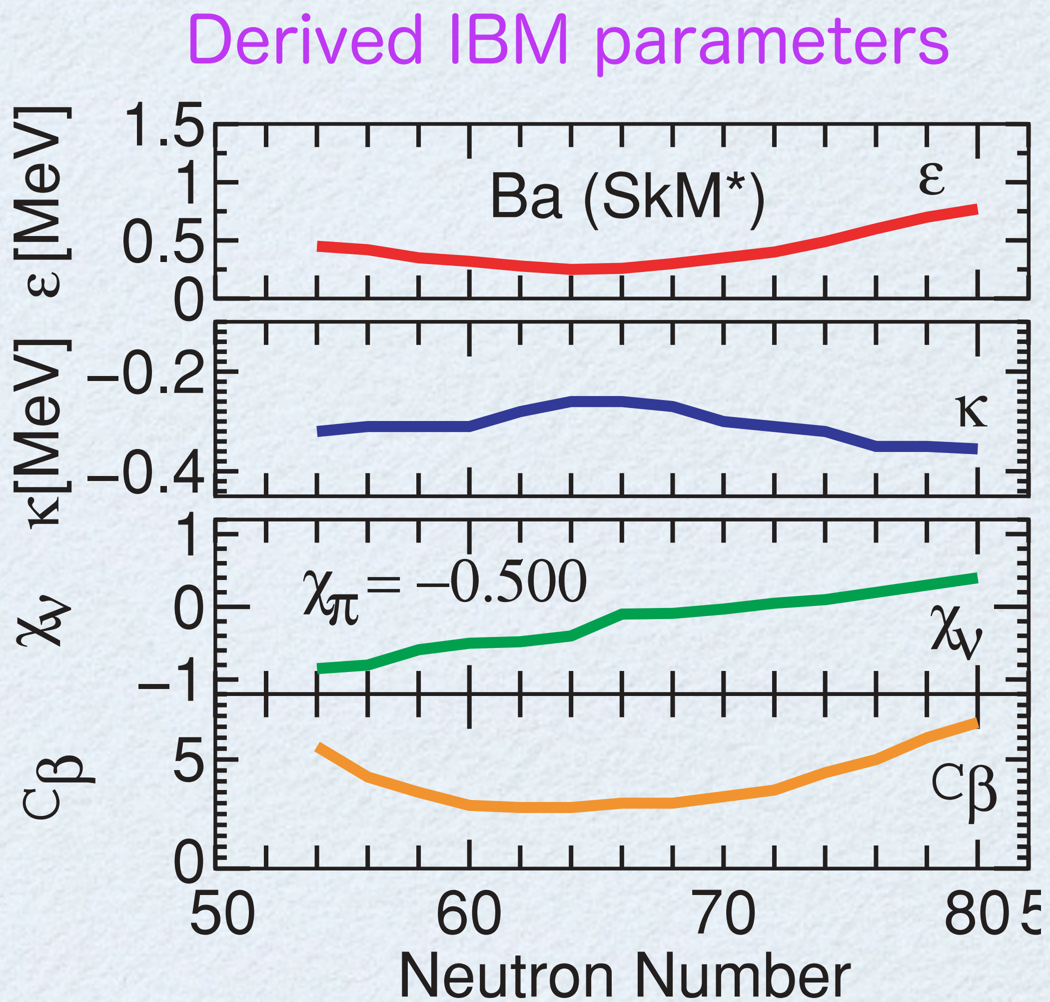


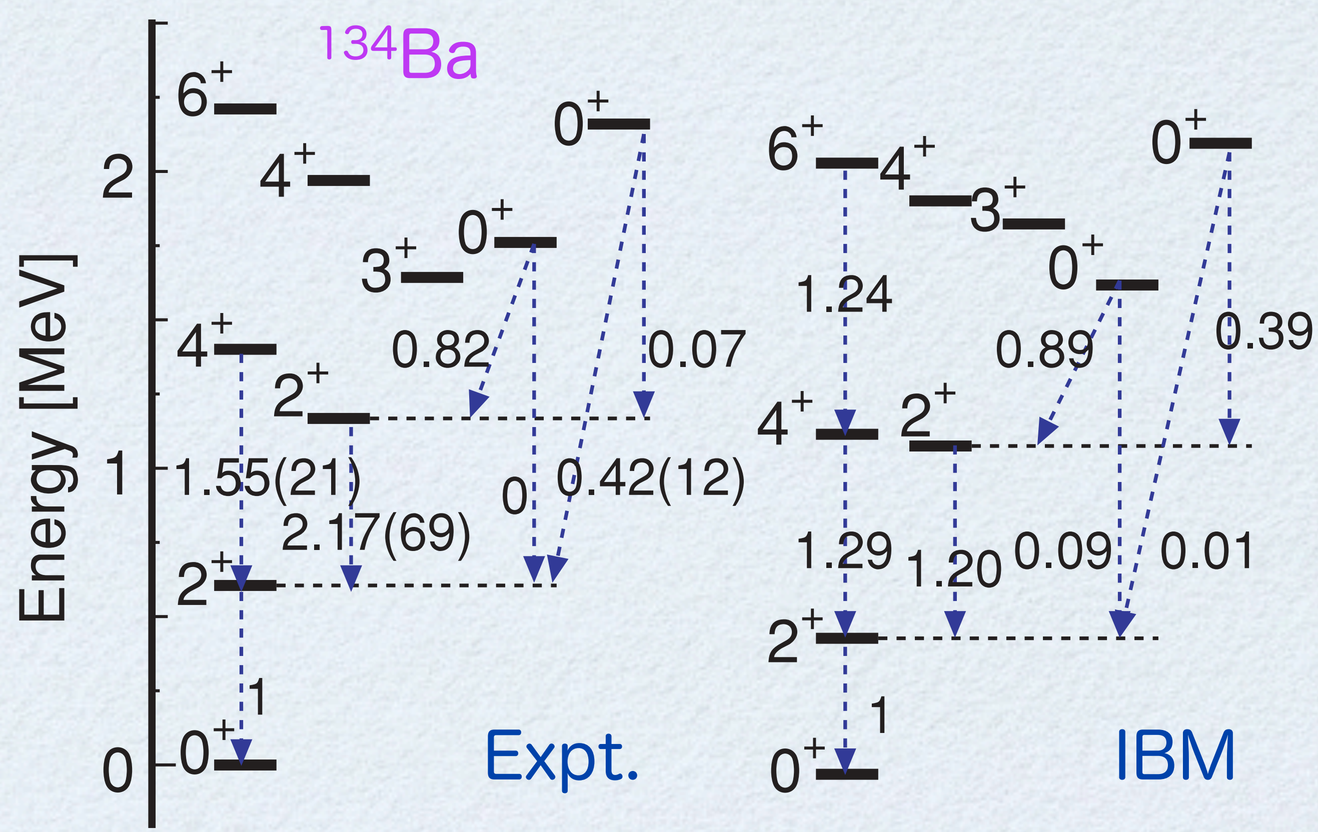
$E(\beta)$

IBM parameters are fixed by the fit of the wavelet transform of energy surface.

Example - Ba isotopes



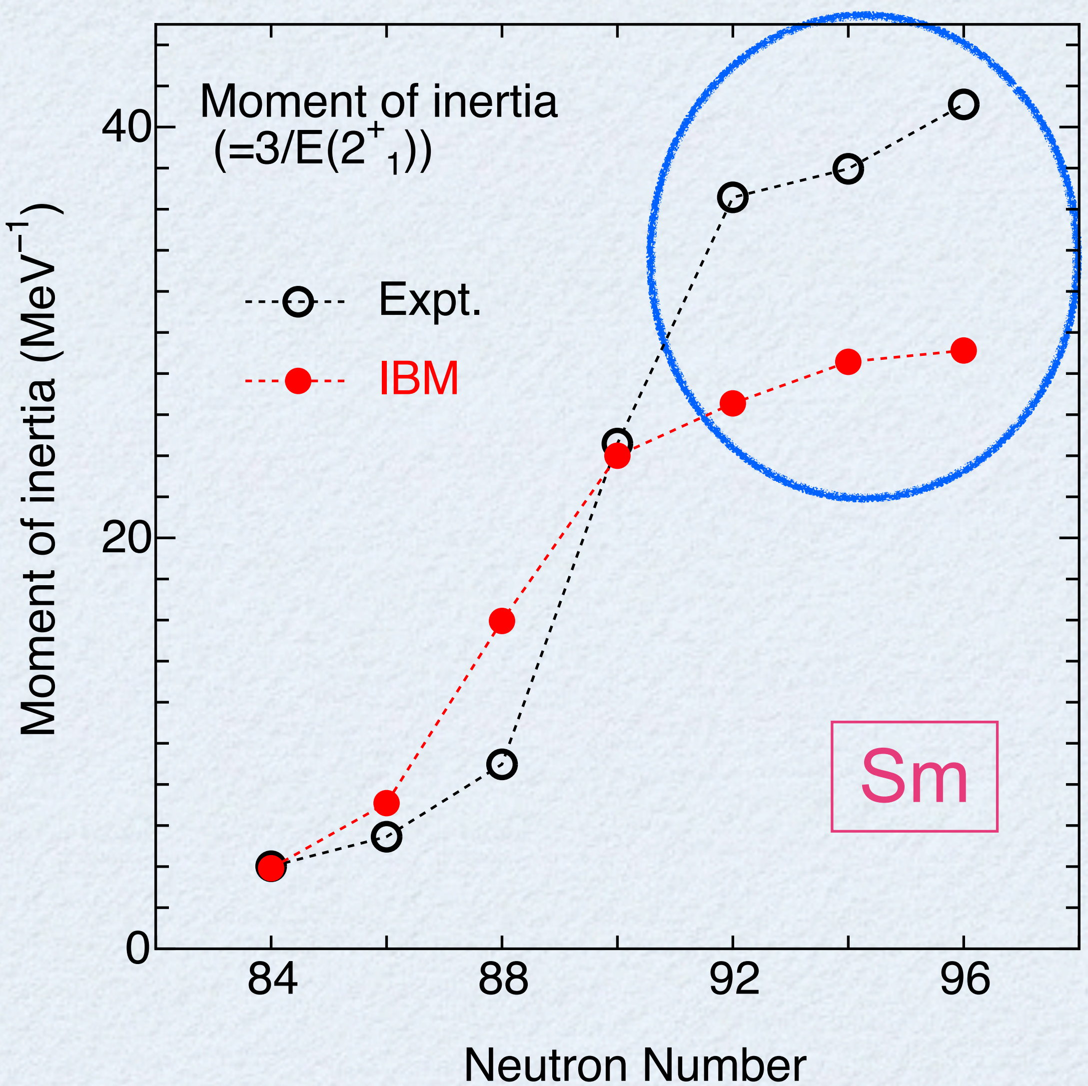


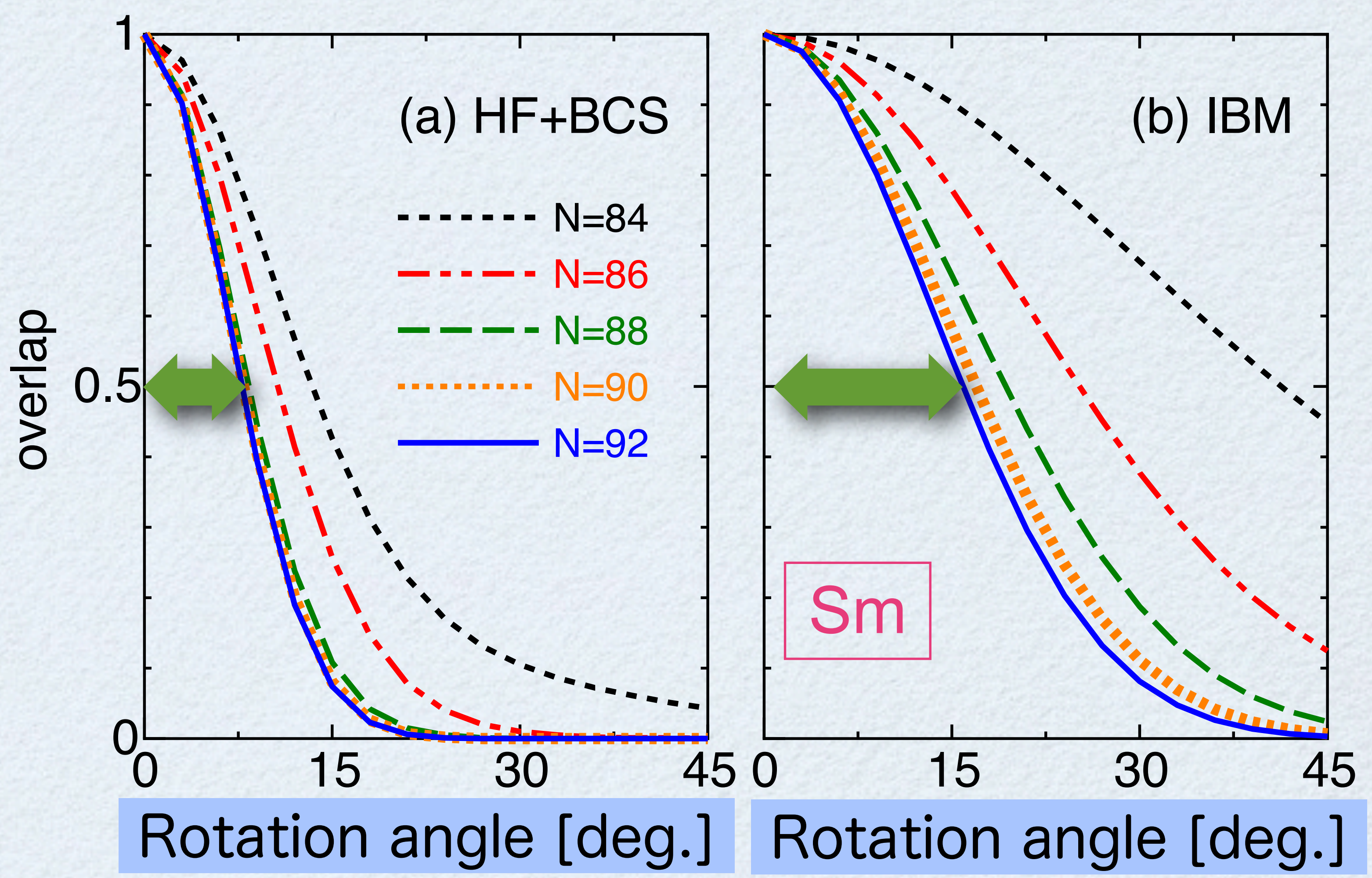


Problem with deformed rotor

- Moment of inertia is too underestimated in IBM when formulated microscopically. Why ?

- Boson's intrinsic wave function is different in its change with the rotation from the nucleon's.

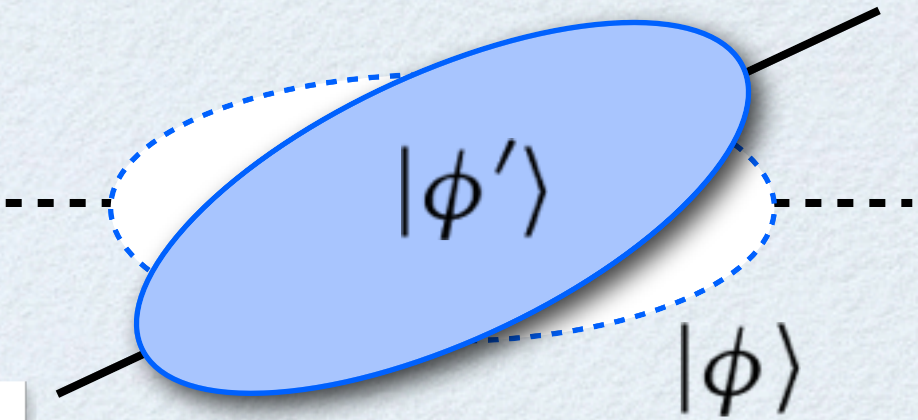




Rotational “response”

$$\delta\langle\phi_F|\hat{H}_F|\phi'_F\rangle \mapsto \delta\langle\phi_B|\hat{H}_B|\phi'_B\rangle$$

intr. state   Rotated intr. state



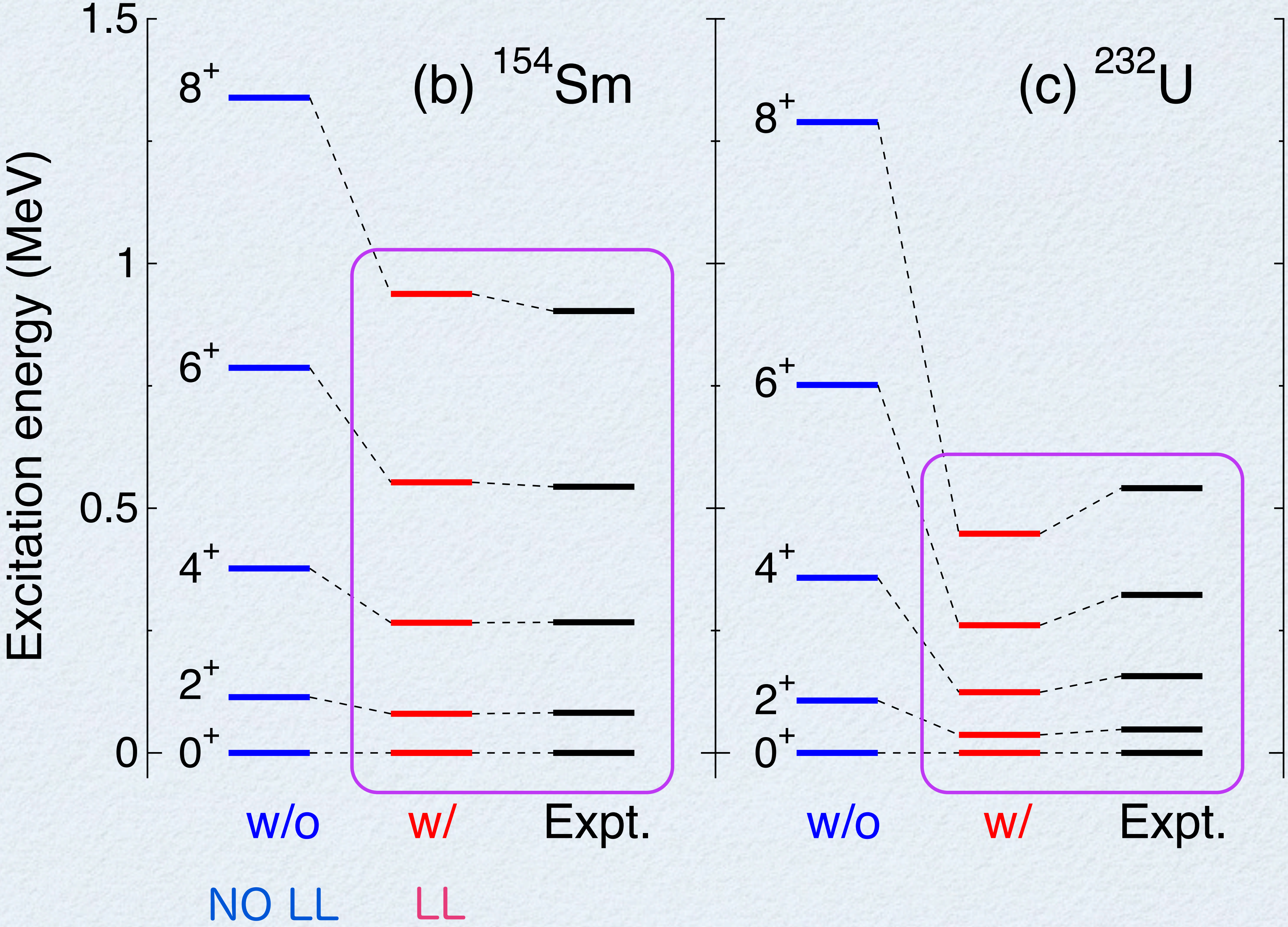
$$\hat{H}_B = \epsilon(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa\hat{Q}_\pi \cdot \hat{Q}_\nu + \alpha\hat{L}\cdot\hat{L}$$

This part does not change.   Rot. kinetic term

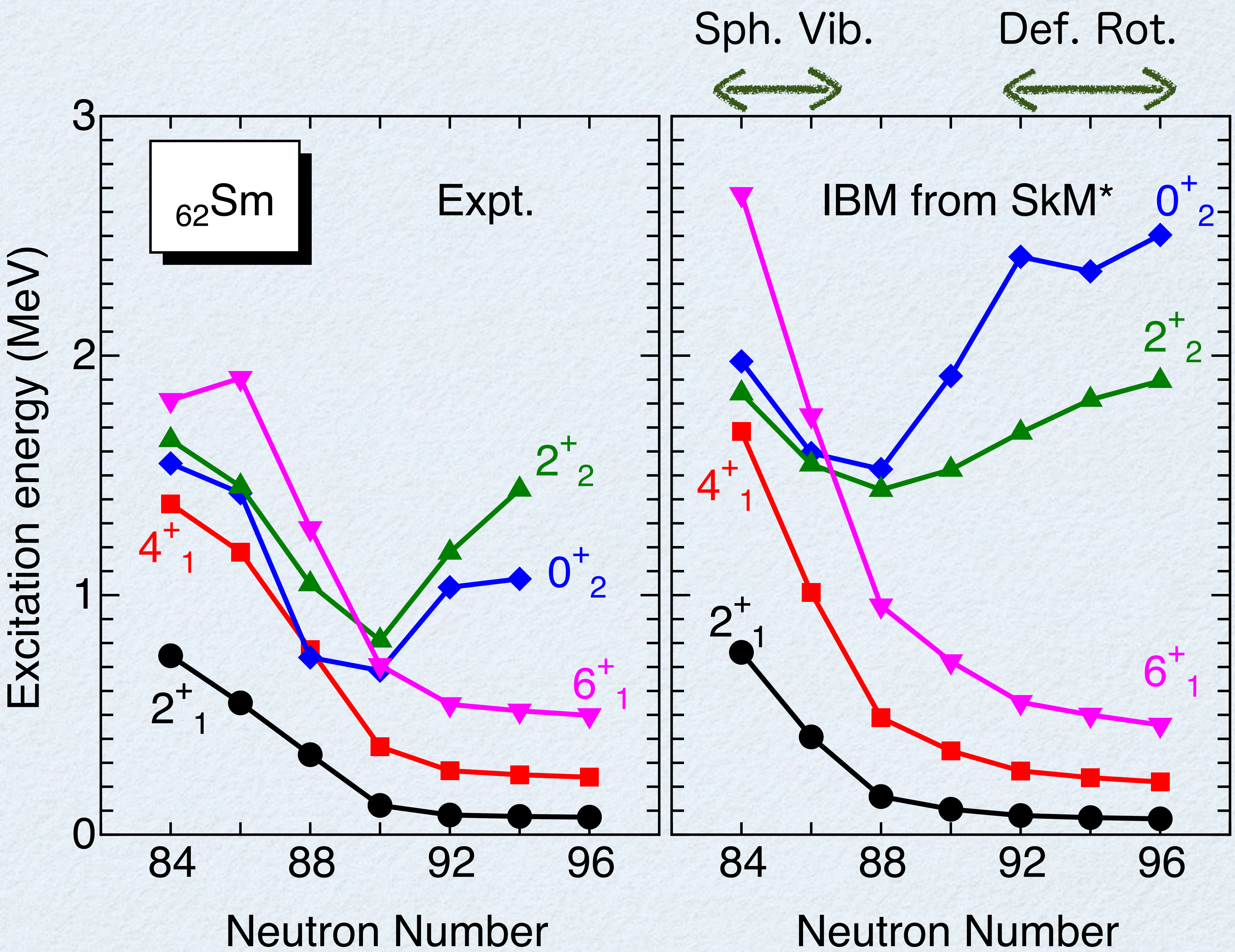
Principal idea:

- Rotational response of a fixed shape at equilibrium should be reproduced. Hence, PES is kept the same.
- To do this, LL term becomes necessary.

Impact on rotational bands

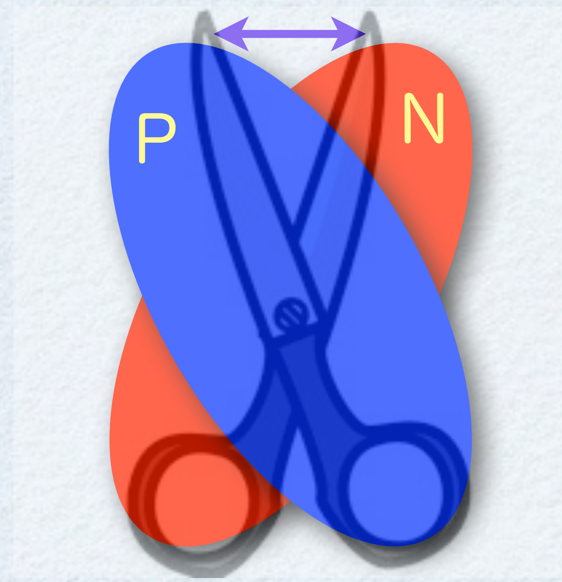


Spherical-deformed shape transition



Scissors mode

K.N., T. Otsuka et al., in preparation (2012)



- isovector collective excitation of valence shells
- observed in general two-fluid quantal systems:
  Trapped BEC, elliptically deformed quantum dots, ...
- strong $1^+ \to 0^+$ M1 transition:
  characteristic of proton-neutron IBM (IBM-2).

Fixing parameters of Majorana term:

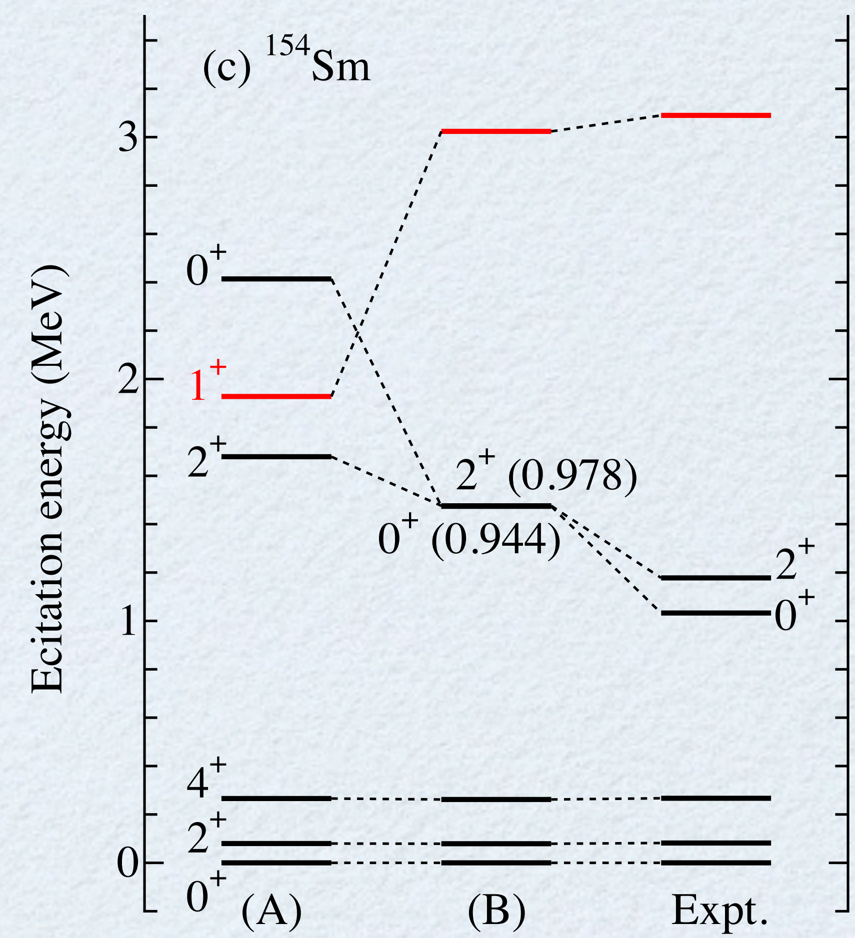
from isovector constraint on quad. mom.

$$\hat{M}_{\pi\nu} = \frac{1}{2}\xi_2[d_\pi^\dagger s_\nu^\dagger - s_\pi^\dagger d_\nu^\dagger]^{(2)} \cdot [\tilde{d}_\pi s_\nu - s_\pi \tilde{d}_\nu]^{(2)} + \sum_{k=1,3} \xi_k [d_\pi^\dagger d_\nu^\dagger]^{(k)} \cdot [\tilde{d}_\pi \tilde{d}_\nu]^{(k)}$$

from isovector rotational oscillation

With the Majorana term, 1^+ level (~3 MeV) and $B(M1;1^+ \to 0^+)$ of $2.7\,\mu_N^2$ (expt: $2.65\,\mu_N^2$) are reproduced for axially-deformed nucleus.

Scissors 1^+ level



w/o M   w/ M

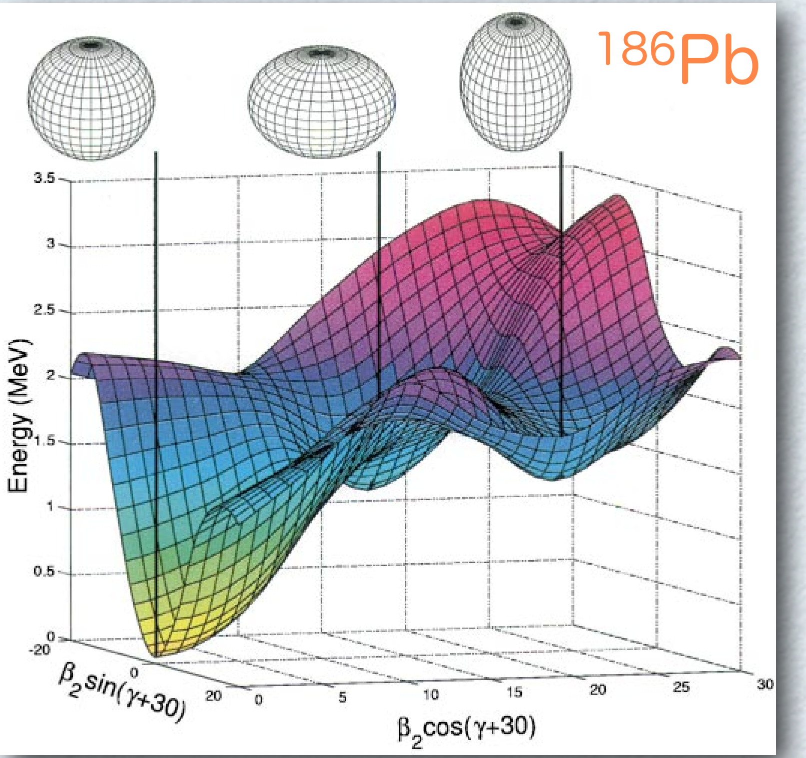
Shape phenomena involving triaxiality

Refs: Phys. Rev. C 83, 014309 (2011)
Phys. Rev. C 83, 054303 (2011)

Nuclear structure in A~190 region

Rich in nuclear shape phenomena:

Prolate-oblate shape transition, shape-coexistence, competing single-particle and collective dynamics, etc.



- Evidence for O(6) sym. (Casten & Cizewski, 1978)
- Shape coexistence (Review: Andreyev et al., 2005; Heyde & Wood, 2011)

Relevant theoretical works:

(beyond) mean field, phenomenological IBM, etc...

- Configuration mixing in IBM for Hg (Duval & Barrett, 1982)
- Nilsson-Strutinsky method for Pb-Hg (W. Nazarewicz, 1993)
- Skyrme+GCM for Pb (T. Duguet et al., 2003; M. Bender et al., 2004)
- Gogny+GCM for Pb (R. Rodríguez-Guzmán et al., 2004)

Shape coexistence

K.N., R. Rodriguez-Guzman et al., in preparation (2012)

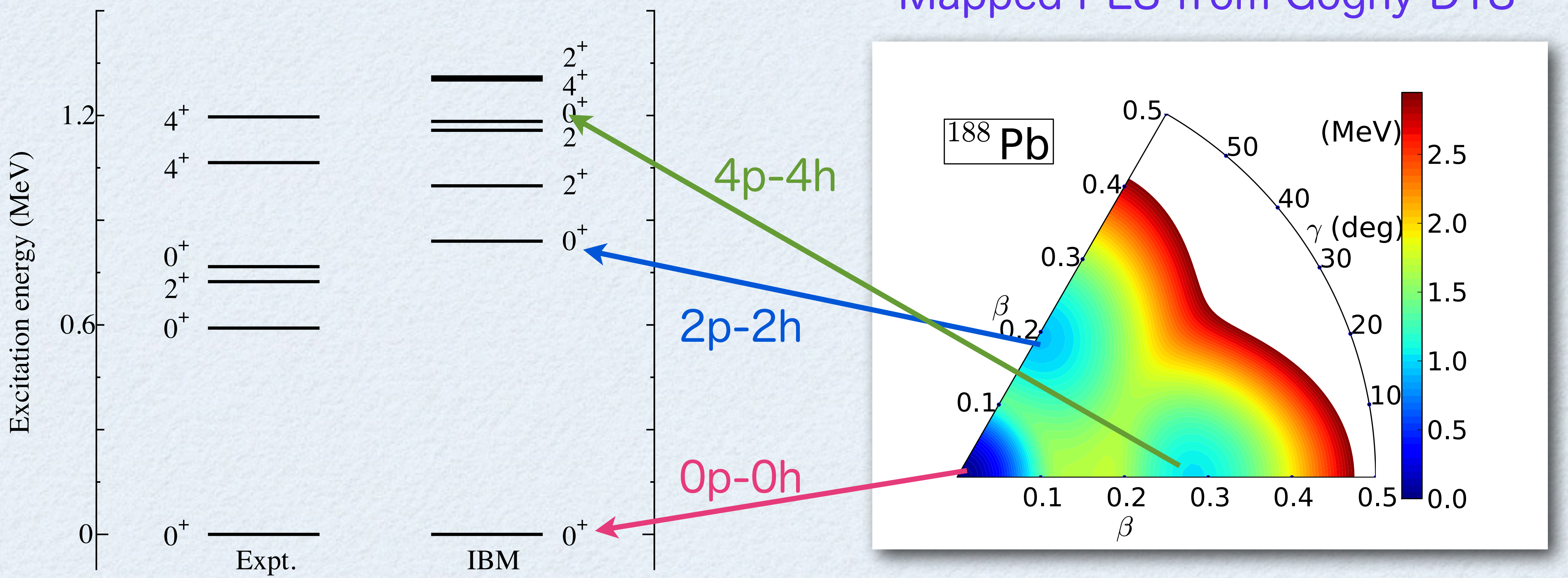
- Mix IBM Hamiltonian for cross-shell (0p-0h, 2p2h, ...) excitations, using Duval-Barrett's procedure (1982)

$$\hat{H} = \hat{H}_{0p-0h} + \hat{H}_{2p-2h} + \hat{H}_{4p-4h} + \hat{H}_{mix}$$

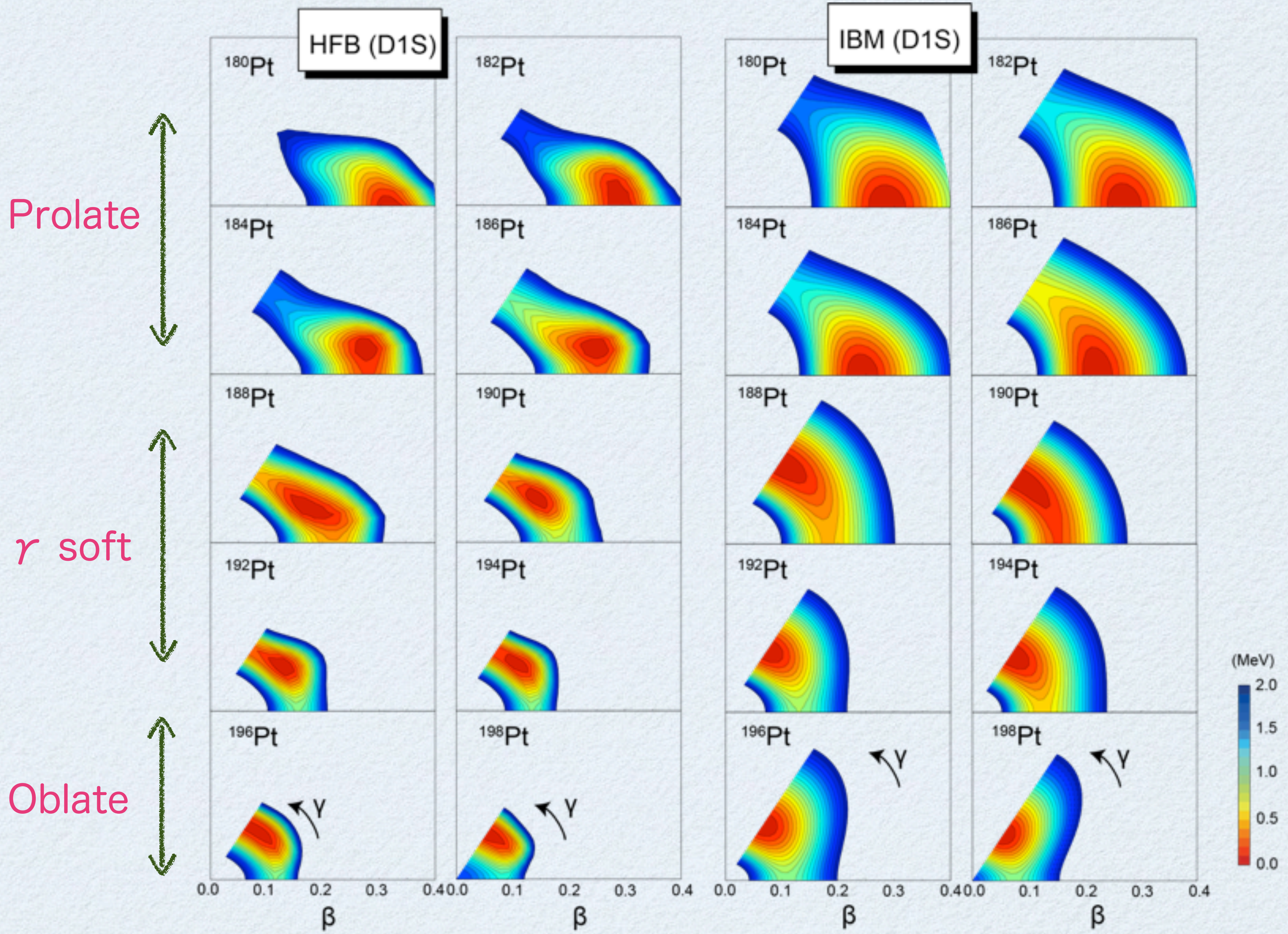
$$\hat{H}_i = \epsilon_i \hat{n}_d + \kappa_i \hat{Q}_\pi^{\chi_{\pi,i}} \cdot \hat{Q}_\nu^{\chi_{\nu,i}} + \Delta_i$$

$$\hat{H}_{mix} = \sum_{i=2p-2h,4p-4h} \alpha_i (s^\dagger \cdot s^\dagger + s \cdot s) + \beta_i (d^\dagger d^\dagger + \tilde{d} \cdot \tilde{d})$$

Mapped PES from Gogny D1S



Pt isotopes: ground-state shape (Gogny D1S)

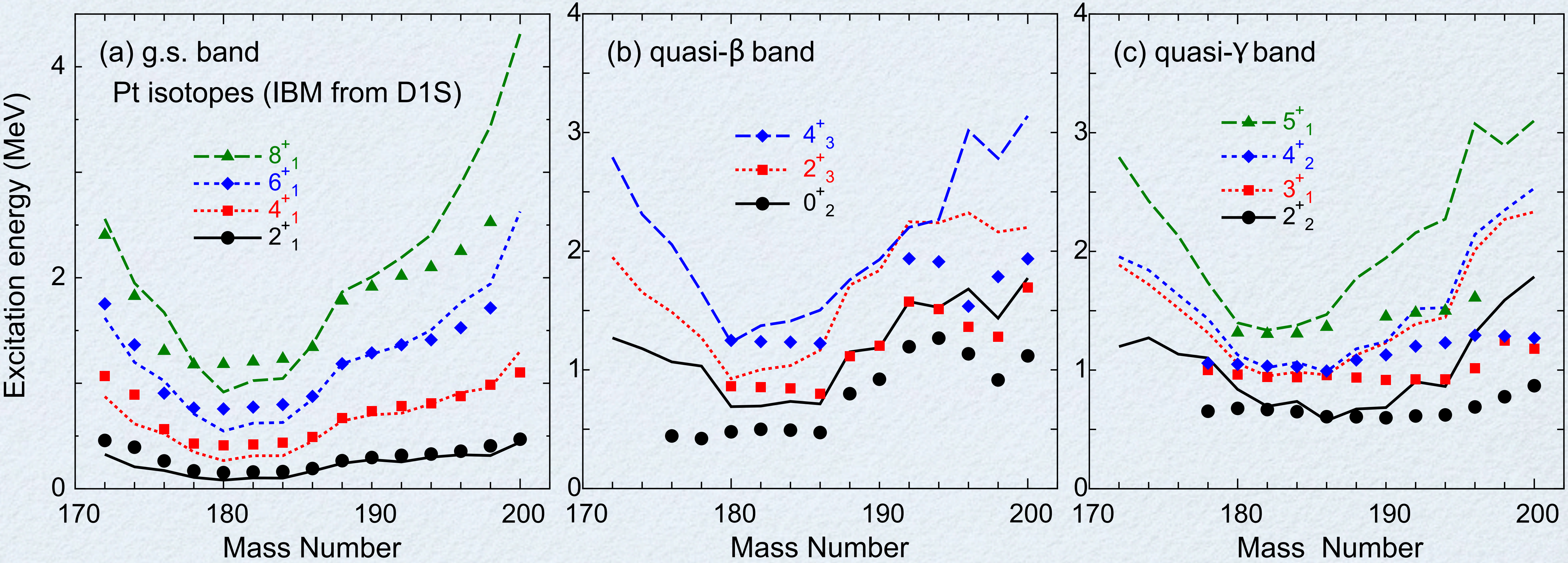


K.N., T. Otsuka, R. Rodríguez-Guzmán et al., PRC83, 014309 (2011)

Low-lying spectra

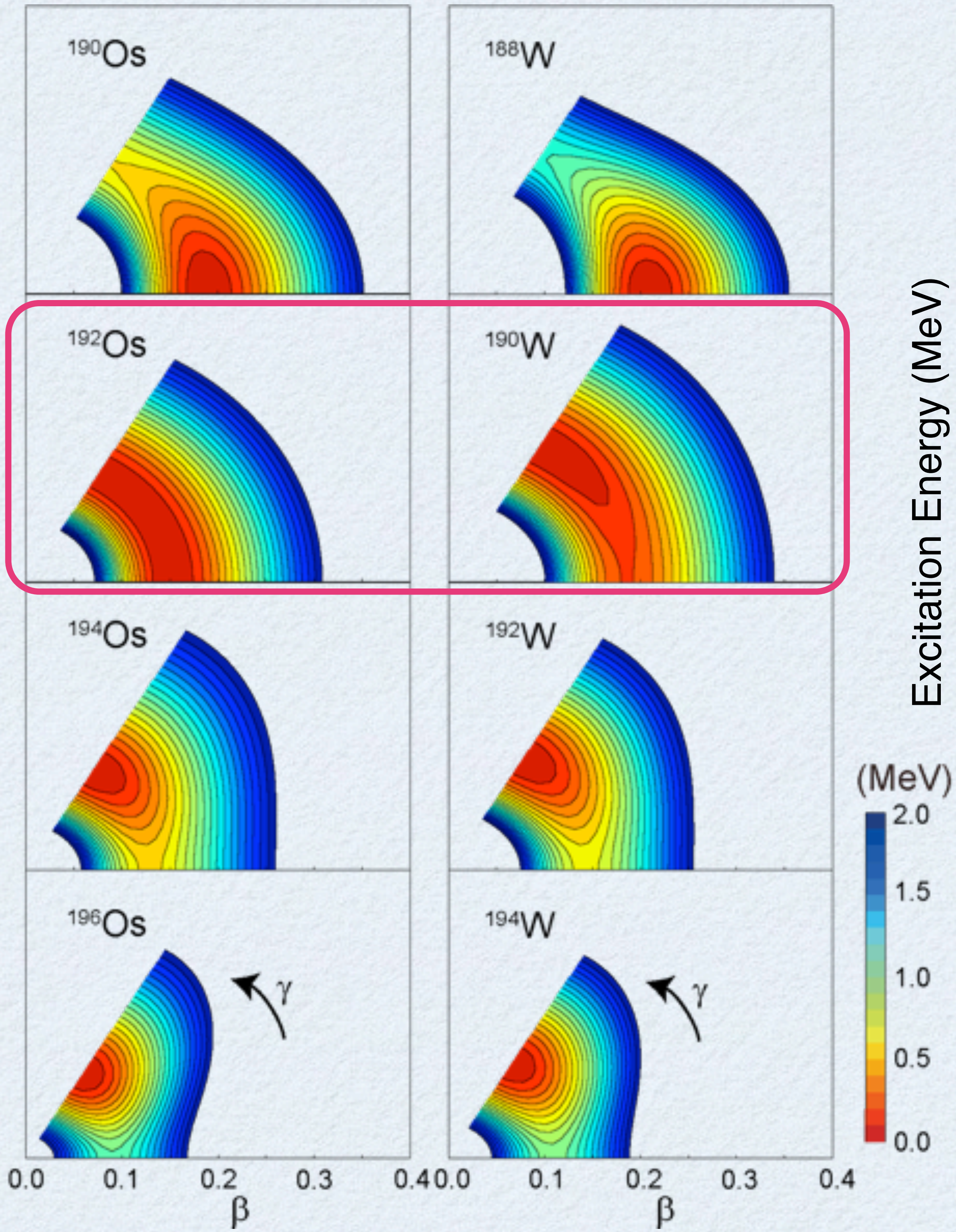
K.N., T. Otsuka, R. Rodríguez-Guzmán et al., PRC83, 014309 (2011)

- Consistent with experiment for g.s. band.
- NO need for config. mixing, as Gogny-D1S PES is concerned.
  But, experimental 0^+_2 energy is very low for A<180 (future work).
- Level pattern of quasi-γ band for A>190 (discussed later).

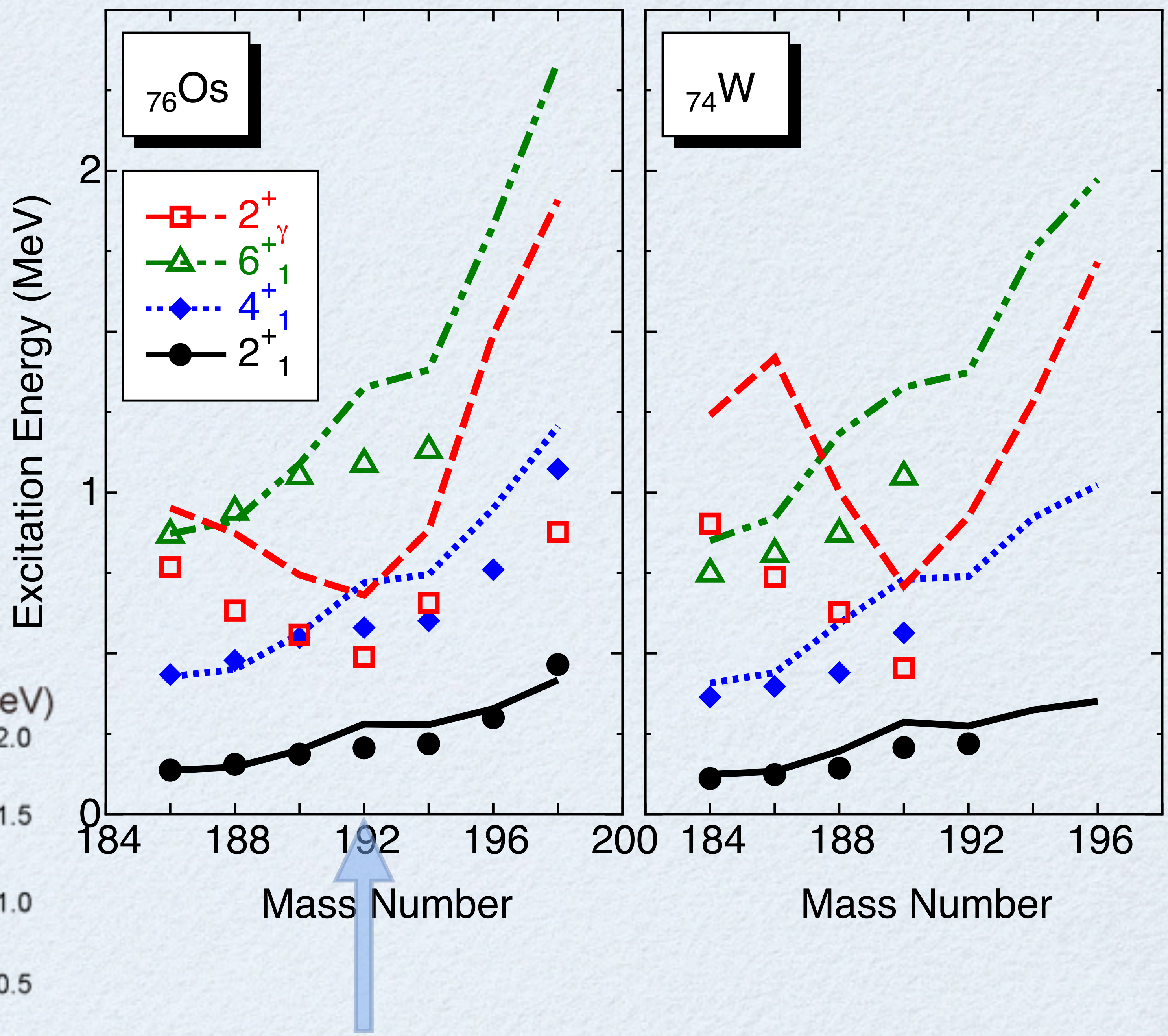


Exotic Os-W (from Gogny D1S)

Mapped IBM



K.N., T. Otsuka, R. Rodríguez-Guzmán et al., PRC83, 054303 (2011)



2_{γ}^{+} changes at N=116:
prolate-oblate transition

δV_{pn}: Empirical average p-n interaction

Double difference of BE(Z,N):

$$\delta V_{pn} = \frac{1}{4}[\{BE(Z,N) - BE(Z,N-2)\} - \{BE(Z-2,N) - BE(Z-2,N-2)\}]$$

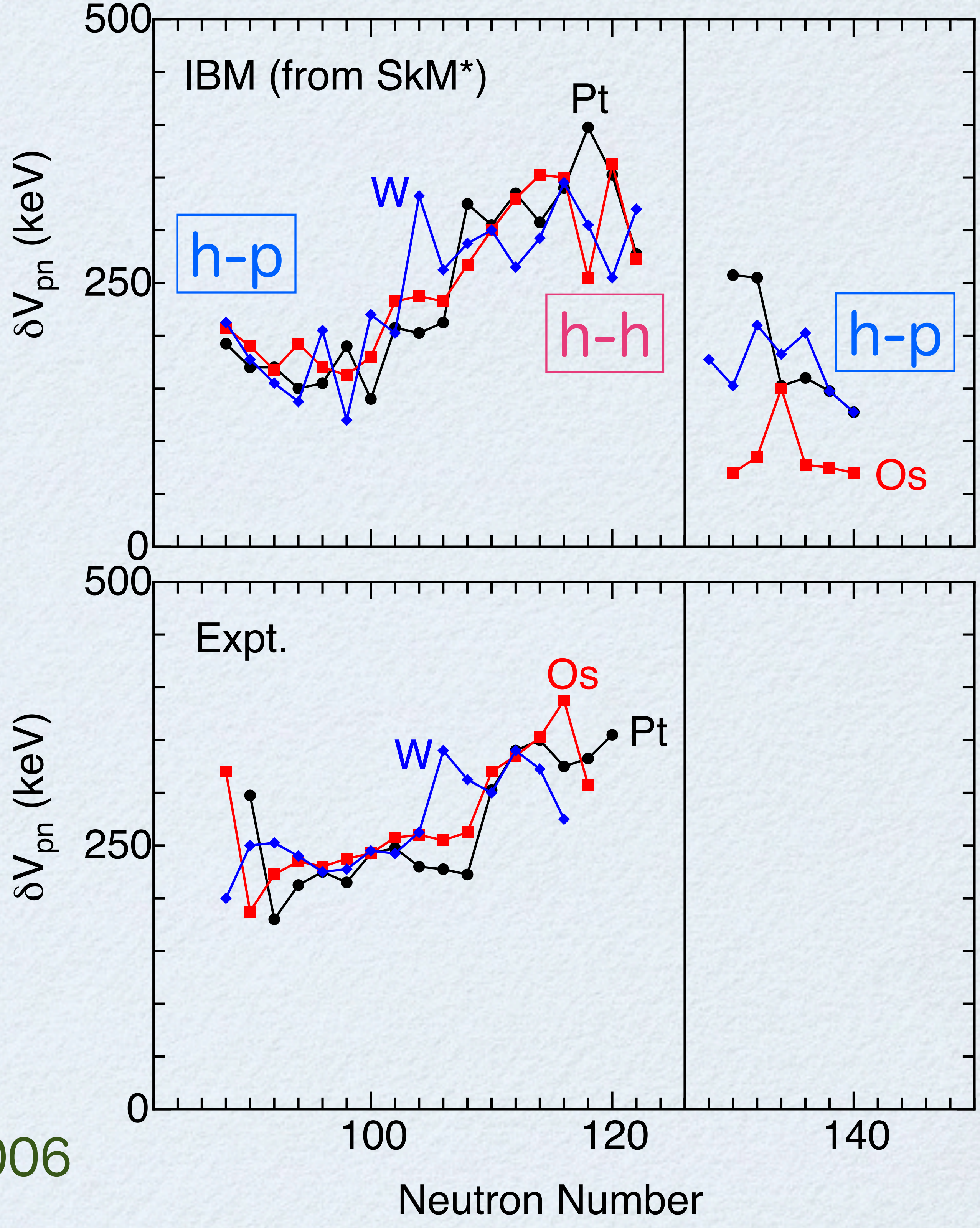
Collectivity, deformation, shell structure, ...

Larger δV_{pn} value for p-p and h-h than p-h and h-p configs.

This trend is predicted also for right-lower quadrant of ^{208}Pb.

- Federman & Pittel 1978
- J.-Y. Zhang et al. 1989
- Cakirli et al. 2005, Cakirli & Casten 2006

K.N., PhD thesis (U. Tokyo, 2012)



Robust regularity in non-axially symmetric nuclei

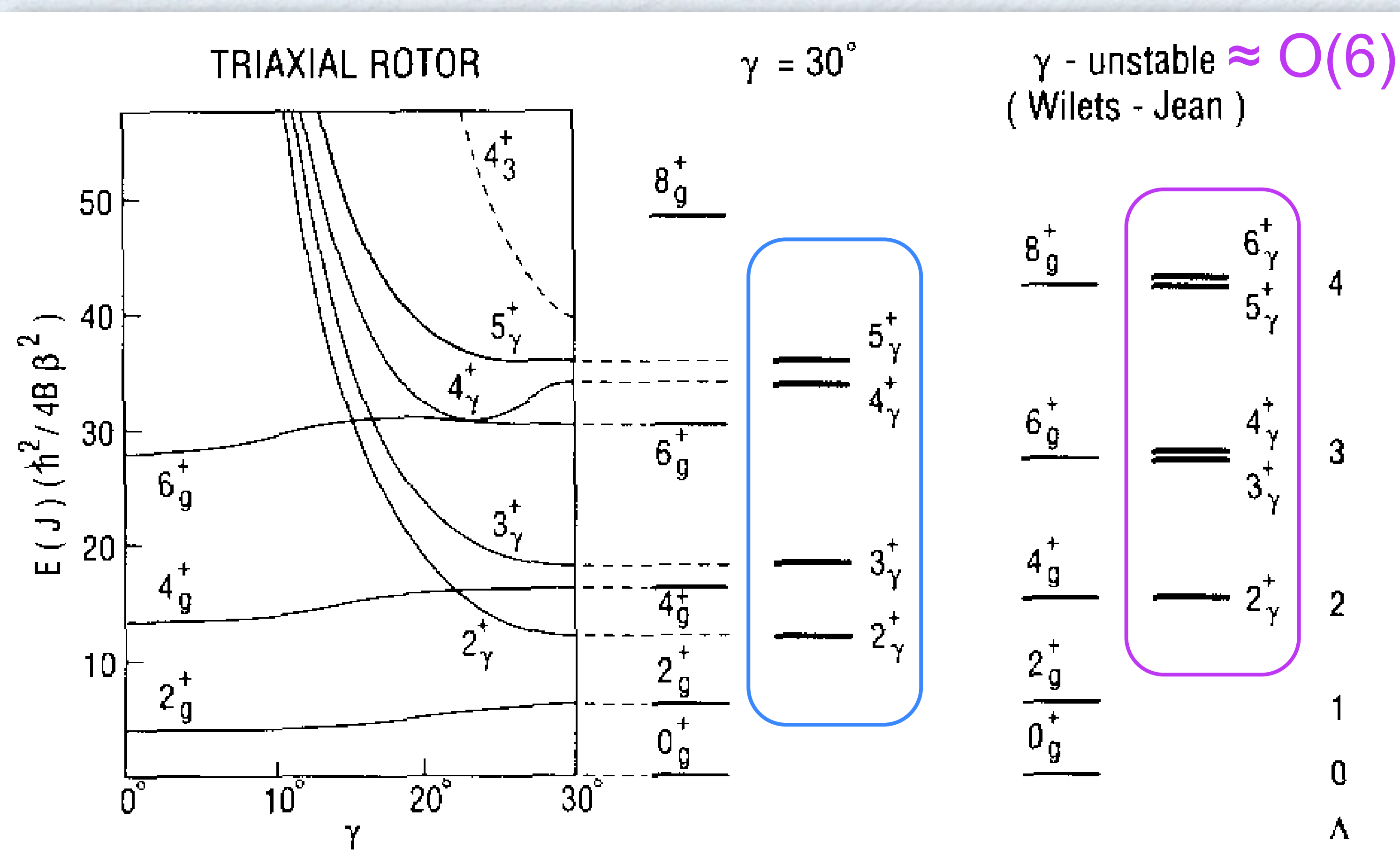
Ref. Phys. Rev. Lett. 108, 132501 (2012)

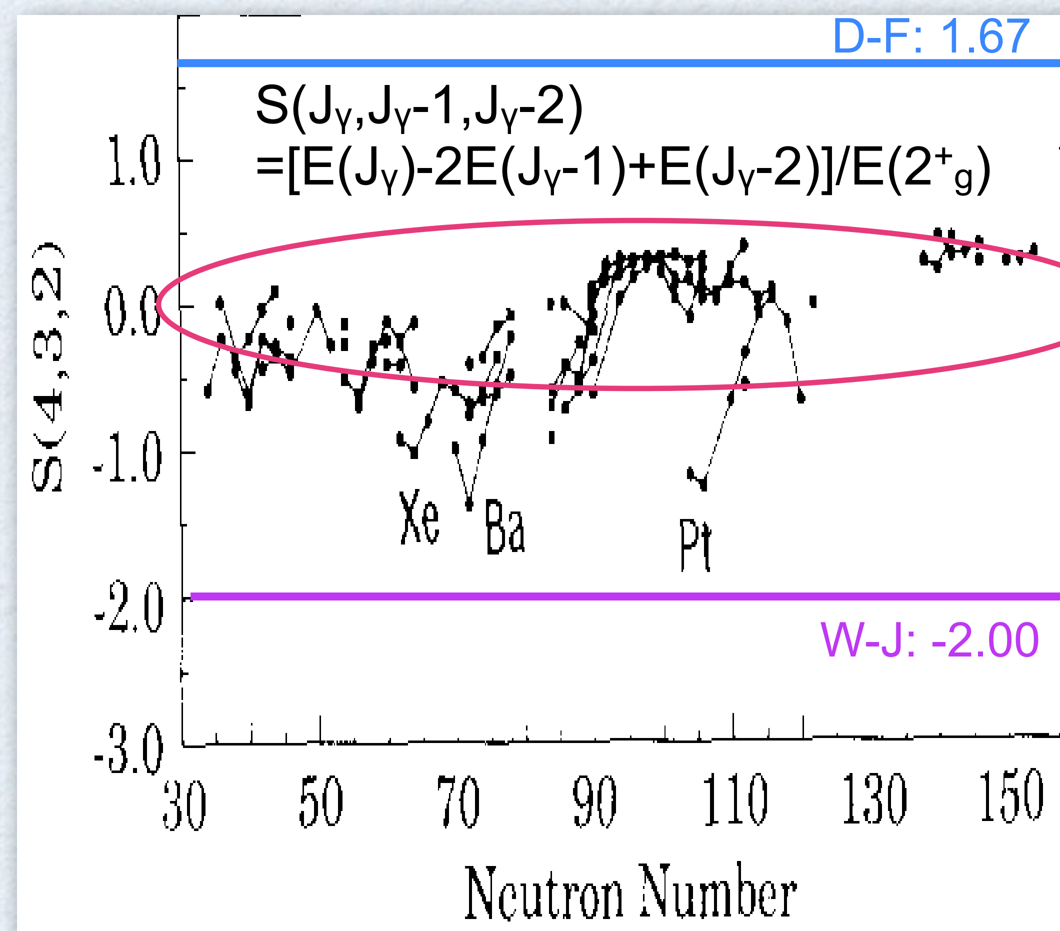
Is a triaxial nucleus γ rigid or unstable ?

Majority of observed triaxial nuclei are middle in between.

This regularity is not explained in major geometrical models.

- Rigid triaxial rotor model (Davydov & Filippov, 1958)
- γ-unstable rotor model (Wilets & Jean, 1956)
- Equivalence between W-J and O(6) in IBM (Ginocchio & Kirson, 1980)





Three-body term in IBM-2

- Hamiltonian $$\hat{H}_{\mathrm{IBM}} = \epsilon(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa \hat{Q}_\pi \cdot \hat{Q}_\nu + \hat{H}_{\mathrm{3B}}$$

- Three-body term $$\hat{H}_{3B} = \sum_{\rho' \neq \rho} \theta_\rho [d^\dagger_\rho d^\dagger_\rho d^\dagger_{\rho'}]^{(3)} \cdot [\tilde{d}_{\rho'} \tilde{d}_\rho \tilde{d}_\rho]^{(3)}$$

  - IBM-1: P. Van Isacker & J.-Q. Chen (1981); K. Heyde et al. (1984)

- Energy surface

  - Up to 2B terms: Minimum at $\gamma \sim \pi/3, 0$

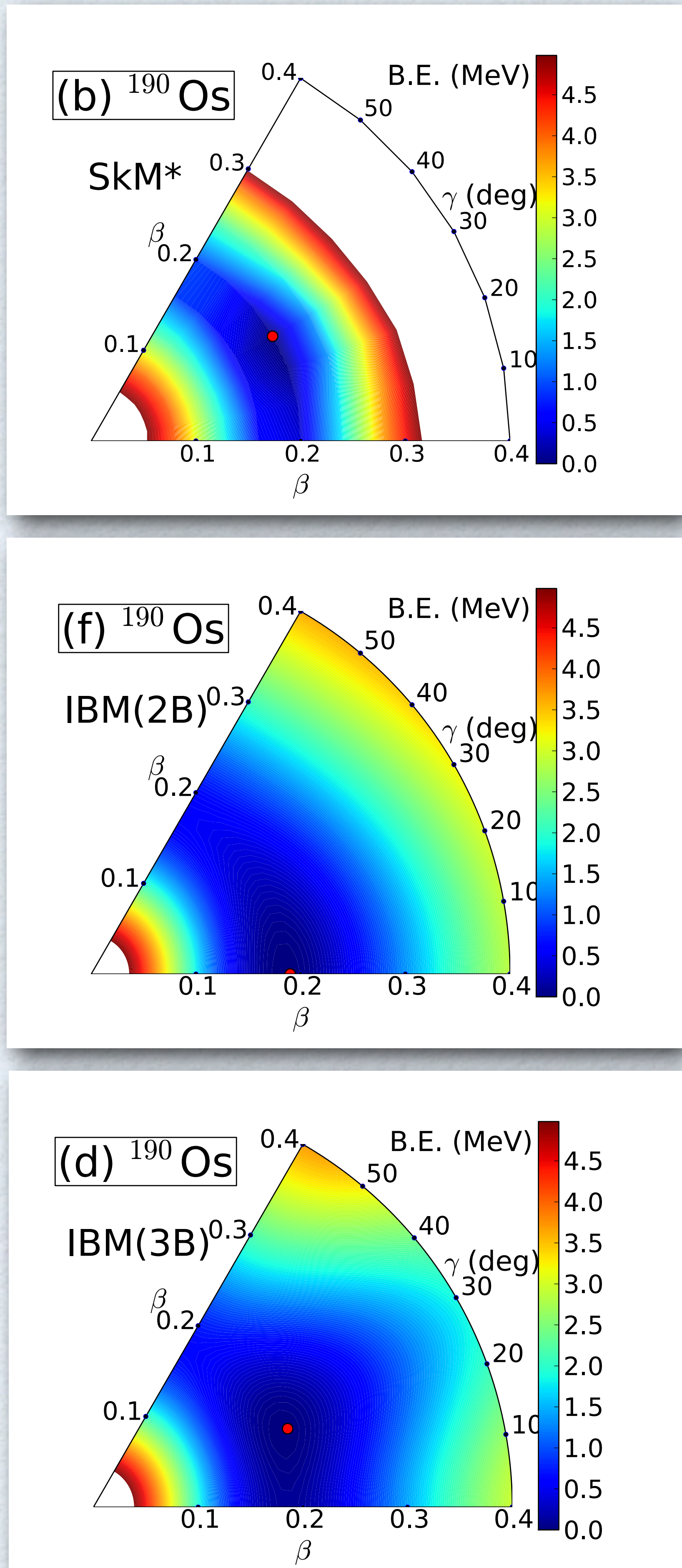
$$\langle \epsilon(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa \hat{Q}_\pi \cdot \hat{Q}_\nu \rangle = f_1(\beta) + f_2(\beta) \cos 3\gamma$$

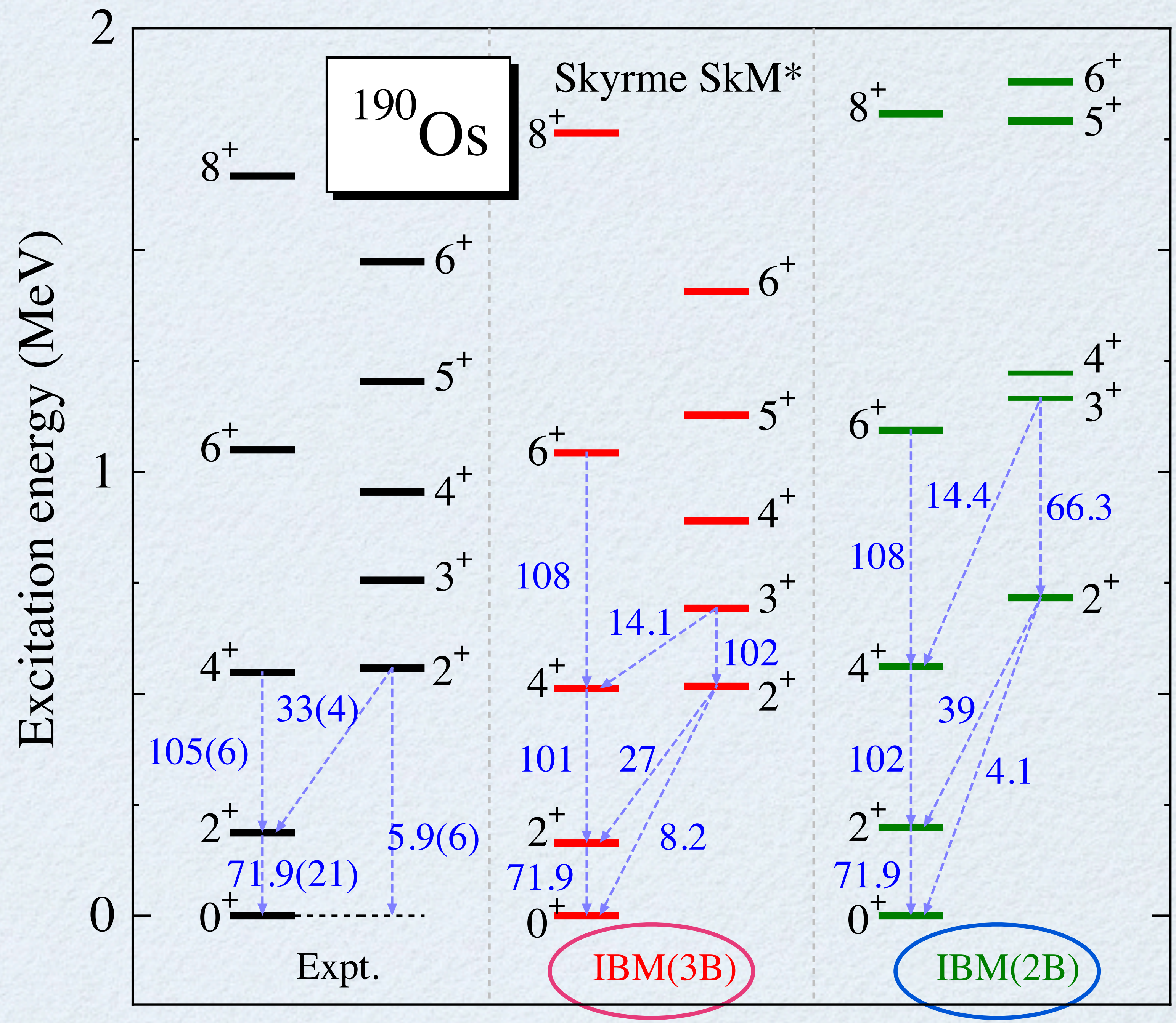
  - 3B term: Minimum at $\gamma \sim \pi/6$

$$\langle \hat{H}_{3B} \rangle = f_3(\beta) + f_4(\beta) \cos^2 3\gamma$$

^{190}Os (from Skyrme SkM*)

Energy surface

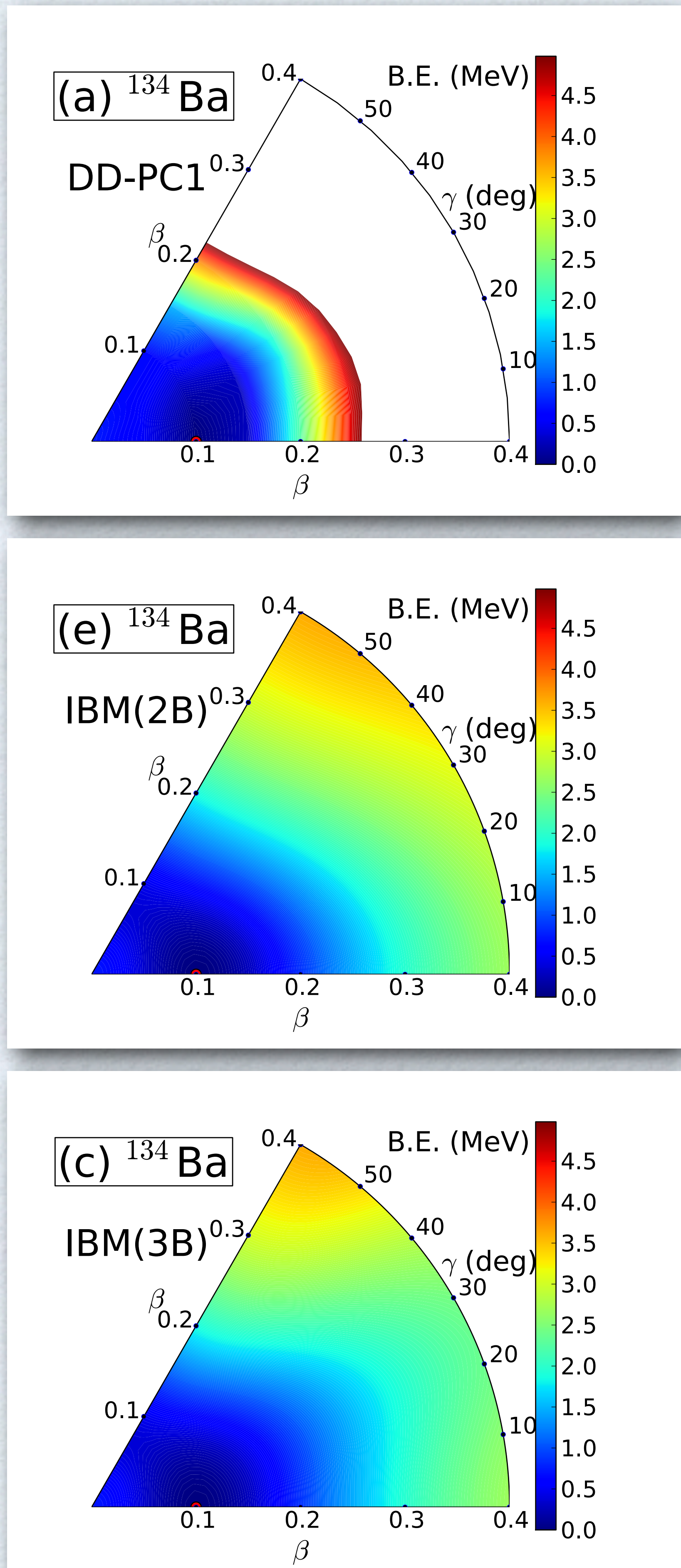




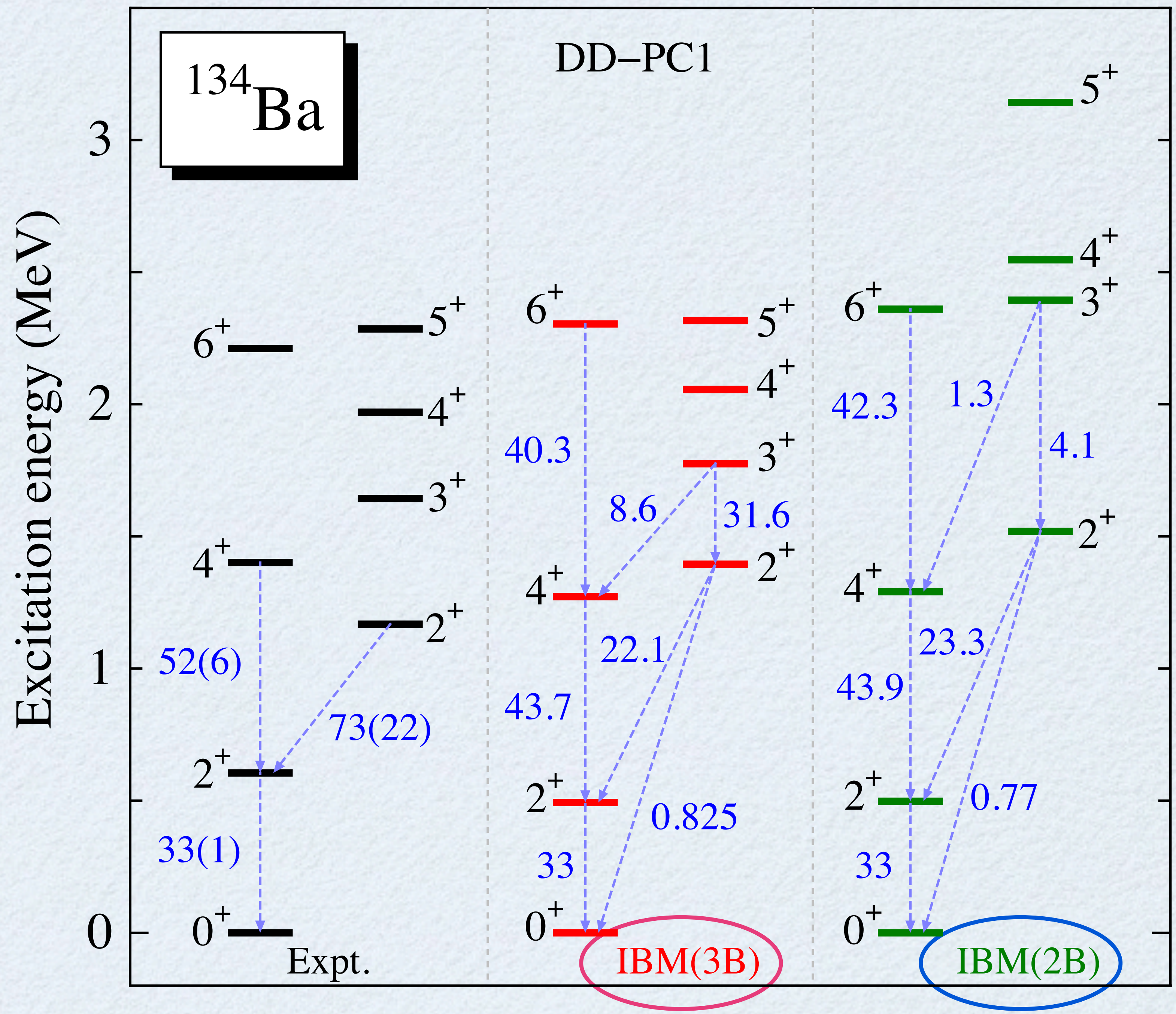
More γ rigid
with 3B

O(6) like
without 3B

Energy surface



^{134}Ba (from DD-PC1)

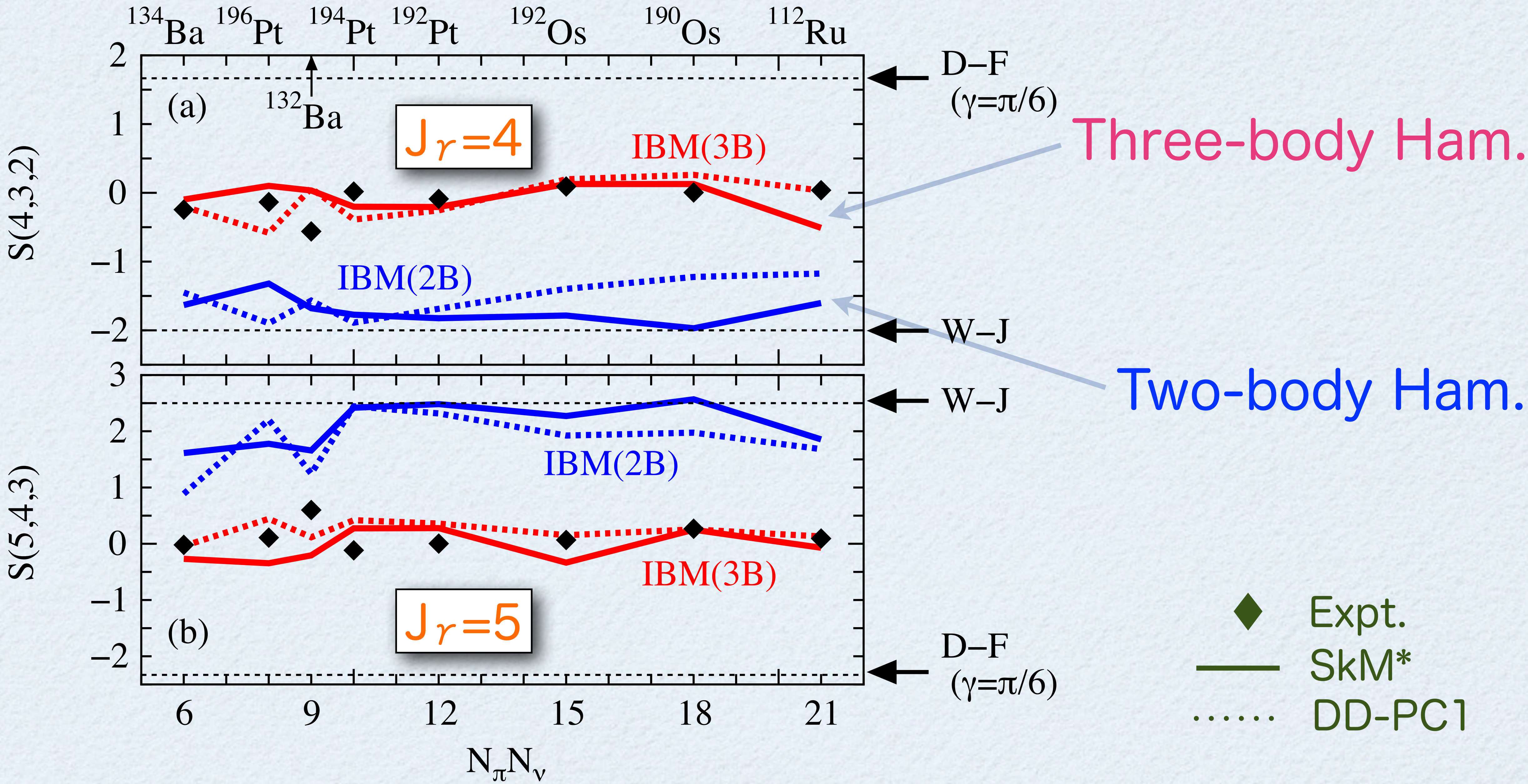


More γ rigid with 3B

O(6) like without 3B

Robustness

- Independently of EDFs, neither W-J nor D-F picture is realized in presumably all triaxial nuclei.
- In the IBM, this regularity never arises without the 3B term.



Summary

Bridge over the gap between IBM and nuclear DFT

- Gives spectra and transition rates with good J and N.
- Works out for general cases:
  - Main part ⇐ energy surface with varying deformation
  - LL part ⇐ rotational response of a fixed shape
  - 3B part ⇐ stable triaxial minimum
  - Config. mixing ⇐ more than one minimum

Work in progress

- Use more realistic interaction. Shell model will catch up.
- Application to other finite quantal system

Collaborators

T. Otsuka, N. Shimizu (Tokyo)
L. Guo (Beijing)

R. Rodríguez-Guzmán (Rice U., TX)
L. M. Robledo, P. Sarriguren (Madrid)

P. H. Regan, P. D. Stevenson, Zs. Podolyák (Surrey)

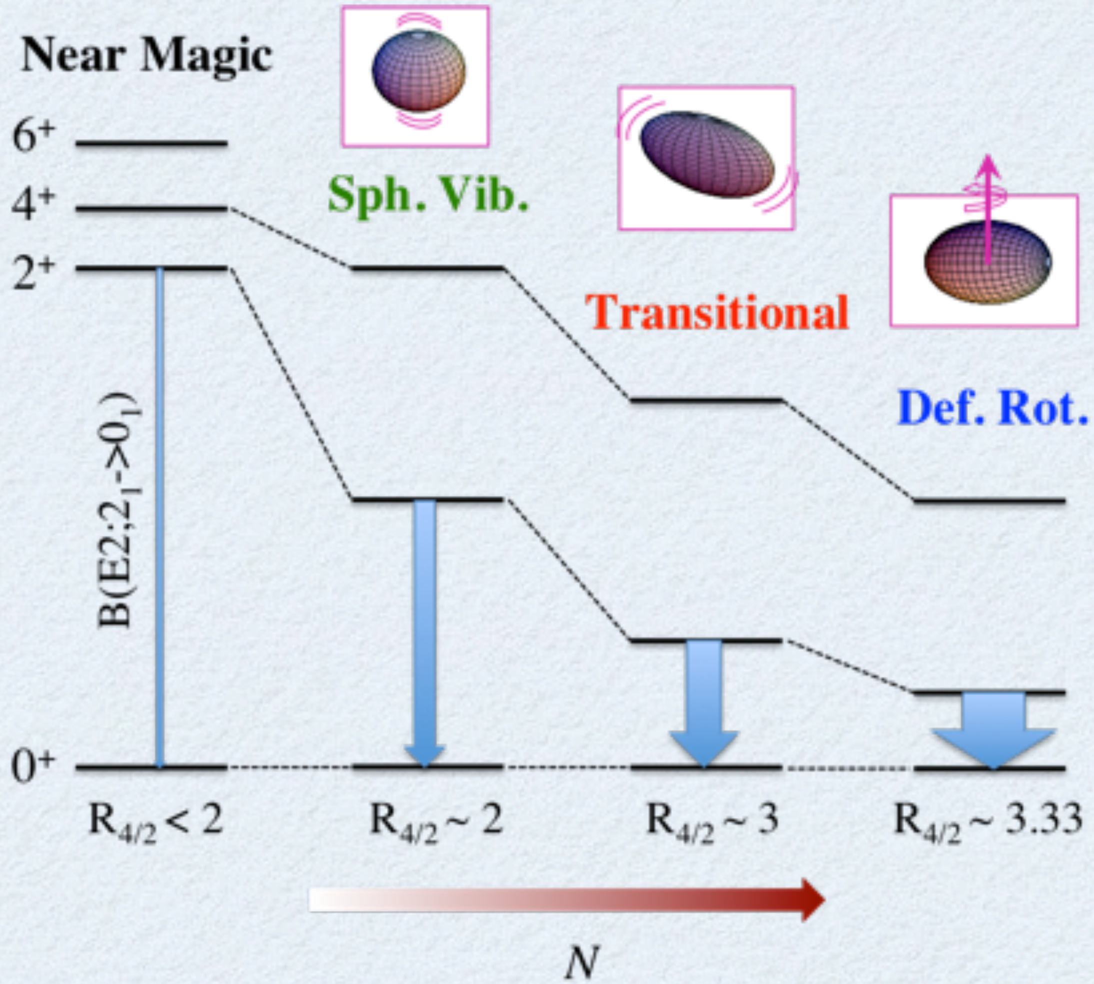
D. Vretenar, T. Nikšić (Zagreb)

Introduction

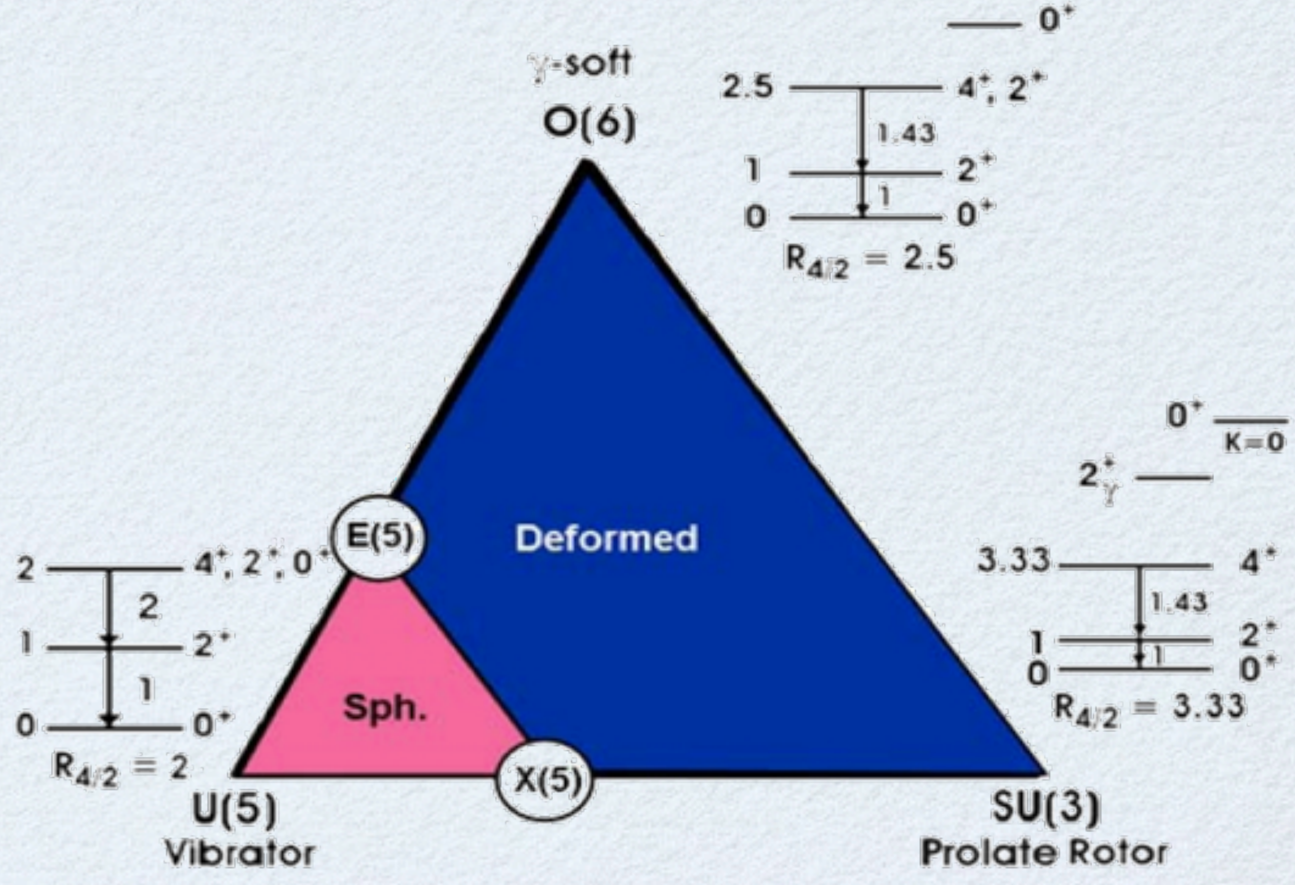
- Variety of nuclear shapes: onset of deformation, QPT, etc. These are governed by multi-nucleon dynamics

- Derived from nucleons ? Prediction ?

We shall work in the interacting boson model (IBM)



Quantum Phase Transition QPT



A piece of history

- Bohr & Mottelson (1980):

  “SD truncation is far from perfect to describe the intrinsic state of rotational nucleus”

  ⇒ Validity of IBM for strongly deformed nuclei ?

  - A. Bohr & B. R. Mottelson, Phys. Scr. 22, 468 (1980)

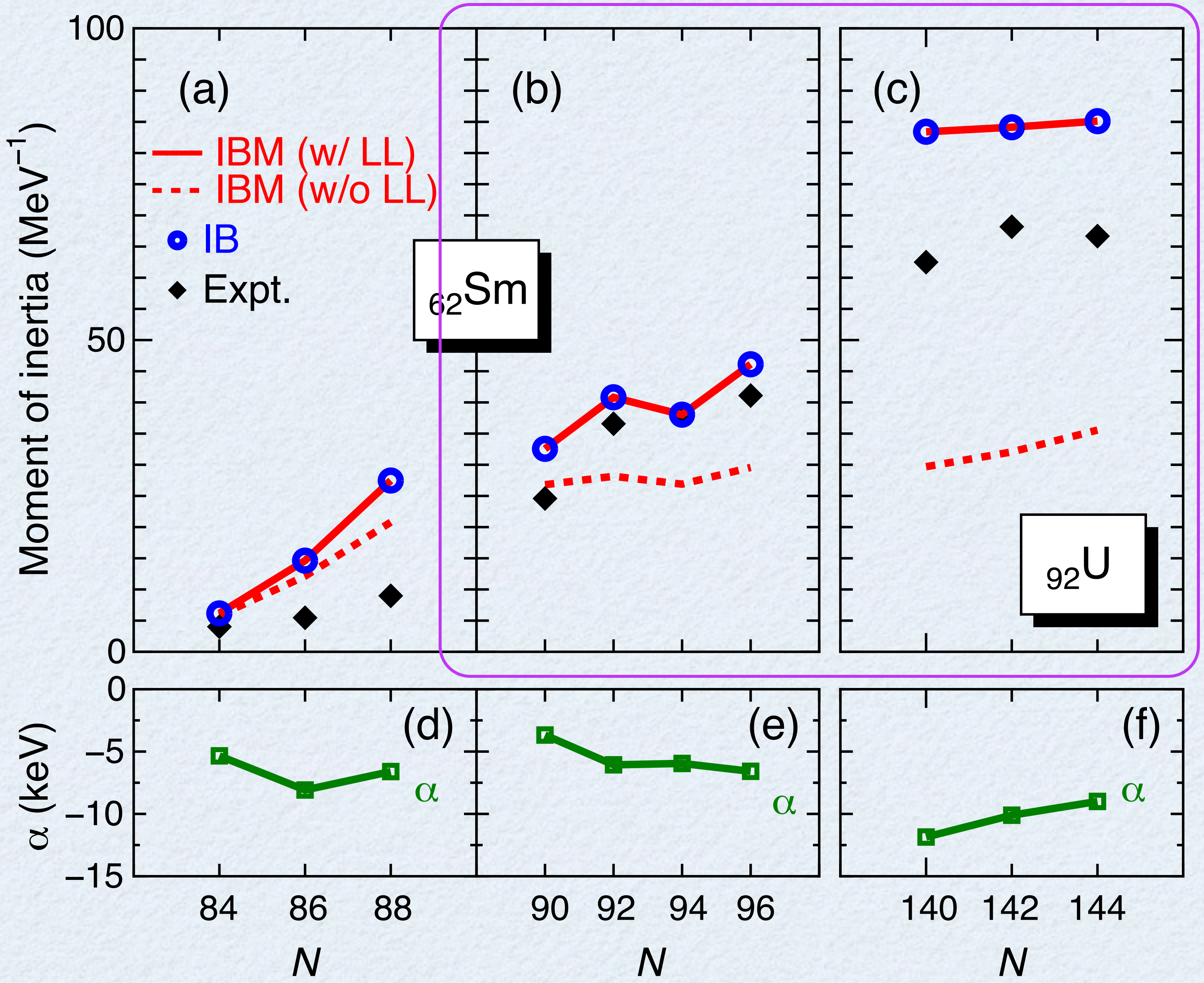
- Debates over the validity of SD-pair truncation:

  Renormalization of J=4 (G) pair, sdg-IBM ..., though still not conclusive.

  - Nilsson+BCS model (T. Otsuka et al., 1982; D. R. Bes et al., 1982)
  - ZB-type boson mapping (M. R. Zirnbauer, 1984)
  - J-projection on intrinsic state (N. Yoshinaga et al., 1984)
  - sdg-IBM and/or sd-IBM with G-pair renormalized (T. Otsuka & J. N. Ginocchio, 1985; T. Otsuka & M. Sugita, 1988)

Cranking mom. of inertia

Large difference between fermion and boson systems



Inglis-Belyaev (IB) formula

$$\mathscr{I}_F = 2 \cdot \sum_{i,j>0} \frac{|\langle i|L_k|j\rangle|^2}{E_i + E_j}(u_i v_j - u_j v_i)^2,$$

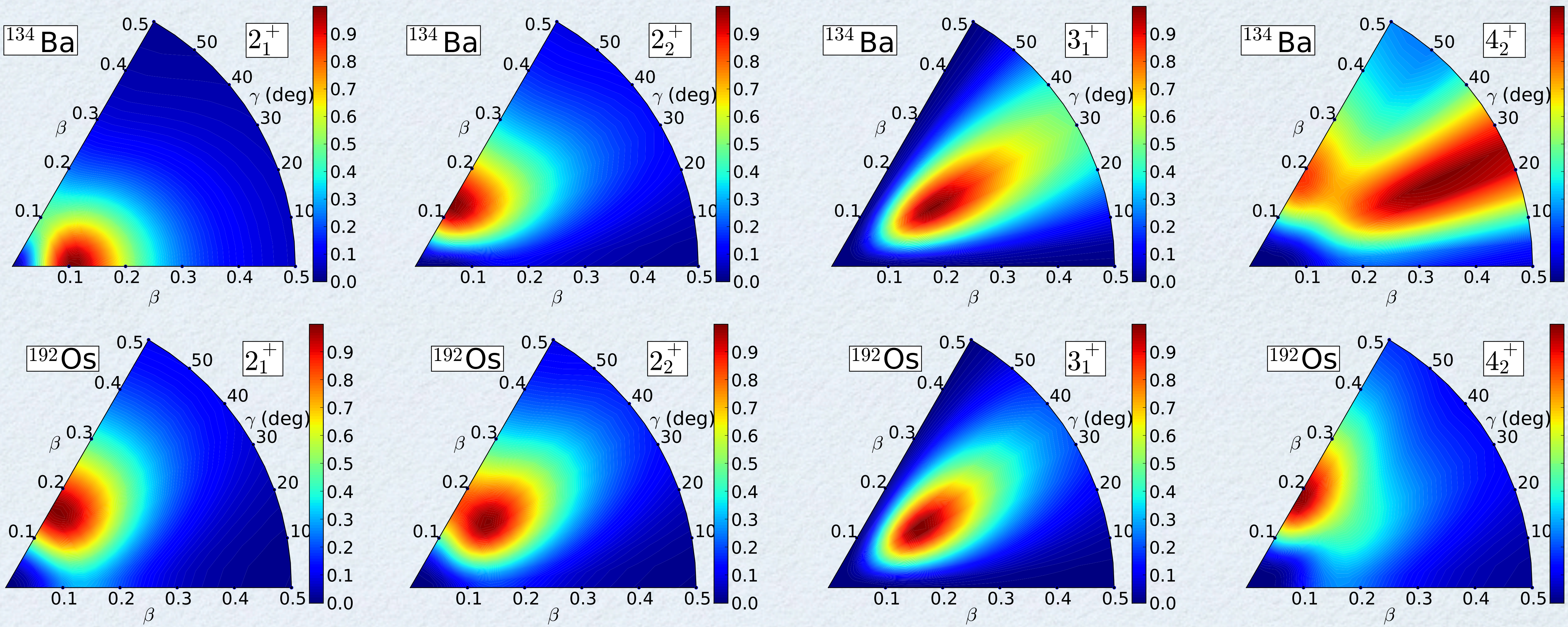
Mom. of inertia in IBM

$$\mathscr{I}_B = \lim_{\omega \to 0} \frac{1}{\omega} \frac{\langle \phi_B|L_k|\phi_B\rangle}{\langle \phi_B|\phi_B\rangle},$$

$\mathscr{I}_B$ is adjusted to $\mathscr{I}_F$

$\Rightarrow$ α value

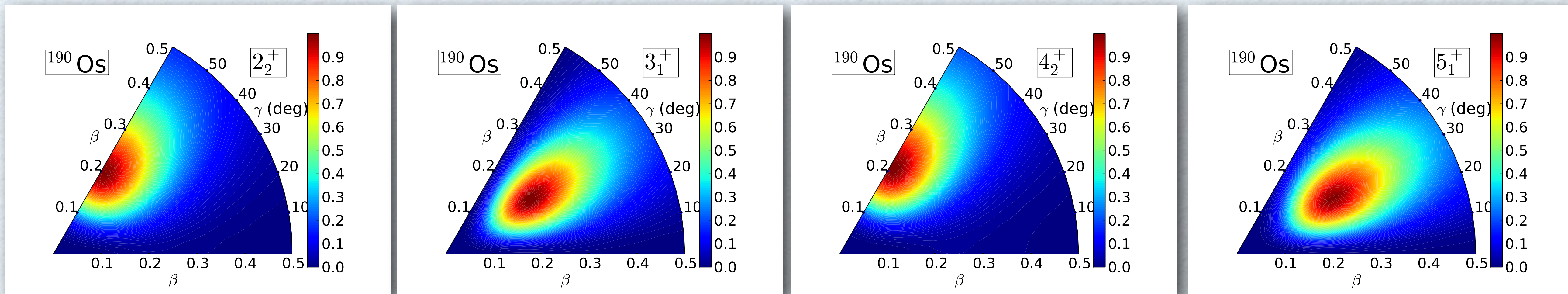
Distributions of wave functions of γ-band states



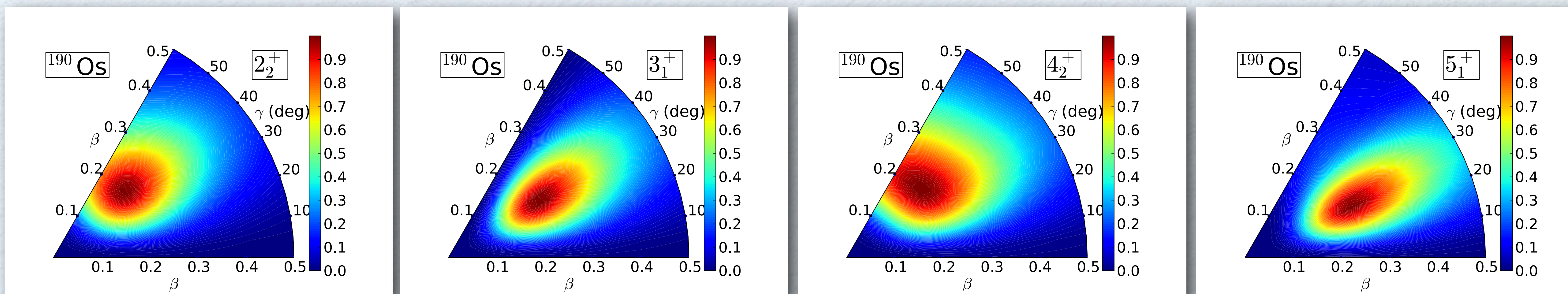
- Configuration mixing is quite strong. e.g., 4_γ state of ^{134}Ba
- 2_γ and 3_γ states are similar ⇒ strong 3_γ ->2_γ E2 transition ?

Wave functions in $\beta\ \gamma$-plains

without 3B



with 3B



- Configuration mixing is strong for 4_γ state
- 2_γ and 3_γ states are similar ⇒ strong 3_γ ->2_γ E2 transition ?

Correlation energy

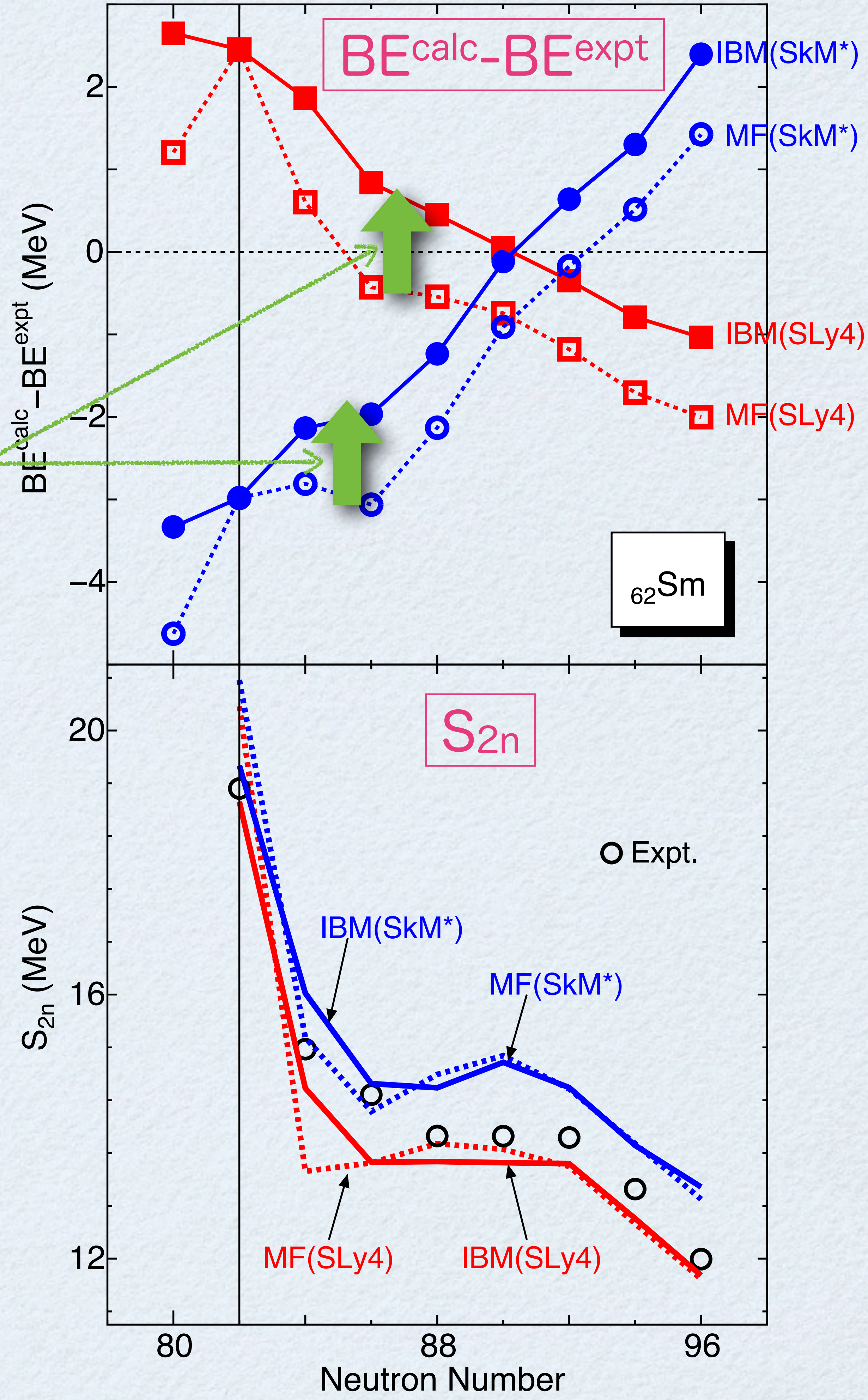
K.N. et al. PRC81 (2010)

Correlation energy included by the IBM hamiltonian

- BE^{IBM}: eigenenergy of H_{IBM}
- BE^{MF}: mean-field solution

Similar arguments

- Skyrme+GCM: Bender et al. 2006
- Gogny+5DCH: Delaroche et al. 2010



Systematics of correlation energy

Maximal in the transitional regions, e.g., Sm and Pt isotopes