From Exact to Partial Dynamical Symmetries: Lessons From the Interacting Boson Model

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A.L., Prog. Part. Nucl. Phys. **66** (2011) 93-143

International Conference "Beauty in Physics: Theory and Experiment",
In honor of Francesco Iachello on the occasion of his 70th birthday
Cocoyoc, Mexico, May 14-18, 2012

Dynamical Symmetry

$$\begin{array}{ccccc} G_{\rm dyn} & \supset & G & \supset \cdots \supset & G_{\rm sym} \\ \downarrow & & \downarrow & & \downarrow \\ [N] & & \langle \Sigma \rangle & & \Lambda \end{array}$$

$$\hat{H} = \sum_{G} a_G \hat{C}_G$$

- *Solvability of the* ***complete*** *spectrum*
- *Quantum numbers for* ***all*** *eigenstates*

eigenvalues $E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$

eigenstates $|[N]\langle\Sigma\rangle\Lambda\rangle$

operators $\hat{T}_{[n]\langle\sigma\rangle\lambda}$

- IBM: s (L=0) , d (L=2) bosons, N conserved *(Arima, Iachello 75)*

$U(6) \supset U(5) \supset O(5) \supset O(3)$	$\vert[N]\ n_d\ \tau\ n_\Delta\ L\rangle$	Spherical vibrator
$U(6) \supset SU(3) \supset O(3)$	$\vert[N]\ (\lambda\ ,\mu\ )\ K\ L\ \rangle$	Axial rotor
$U(6) \supset O(6) \supset O(5) \supset O(3)$	$\vert[N]\ \sigma\ \tau\ n_\Delta\ L\ \rangle$	γ-unstable rotor

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- K-band degeneracies $E_\beta(L) = E_\gamma(L)$

SU(3)

- rotational splitting $CL(L+1)$

  deviations: odd-even staggering in the γ-band

- band anharmonicities

$$R = \frac{E(v=2)}{E(v=1)} - 2$$

$$R_{O(6)} = -\frac{2}{N+1}$$

- QPTs: incompatible symmetries

$\Rightarrow$ need to break the exact dynamical symmetry (how?)

higher order terms (1+2+3 body terms 2+7+17 parameters)

Dynamical Symmetry

$$\begin{array}{ccccc} G_{\rm dyn} & \supset & G & \supset \cdots \supset & G_{\rm sym} \\ \downarrow & & \downarrow & & \downarrow \\ [N] & & \langle\Sigma\rangle & & \Lambda \end{array}$$

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$$E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$$

eigenstates $|[N]\langle\Sigma\rangle\Lambda\rangle$

operators $\hat{T}_{[n]\langle\sigma\rangle\lambda}$

Partial Dynamical Symmetry

- Only **part** of these properties are obeyed

PDS

$$G_{\text{dyn}} \supset G \supset \cdots \supset G_{\text{sym}}$$

$[N]$ $\langle\Sigma\rangle$ Λ

n-particle annihilation operator

$$\hat{T}_{[n]\langle\sigma\rangle\lambda} \,|\, [N]\, \langle\Sigma_0\rangle\, \Lambda\rangle = 0$$

for **all** possible Λ contained in the irrep $\langle\Sigma_0\rangle$ of G

Equivalently:

$$\hat{T}_{[n]\langle\sigma\rangle\lambda} \,|\, [N]\, \langle\Sigma_0\rangle\, \rangle = 0$$

| Lowest weight state ⟩

- Condition is satisfied if $\langle\sigma\rangle \otimes \langle\Sigma_0\rangle \not\in [N-n]$

n-body $\hat{H}' = \sum_{\alpha,\beta} A_{\alpha\beta}\, \hat{T}^{\dagger}_{\alpha} \hat{T}_{\beta}$

$$\hat{H}_{PDS} = \hat{H}_{DS} + \hat{H}'$$

DS is **broken** but **solvability** of states with $\langle\Sigma\rangle = \langle\Sigma_0\rangle$ Is **preserved**

SU(3) PDS

U(6) ⊃ SU(3) ⊃ SO(3)

[N] **(λ,μ)** **K** **L**

$$\hat{B}^{\dagger}_{[n](\lambda,\mu)\ell m} \qquad \left.\begin{array}{l} P^{\dagger}_{0} = d^{\dagger}\cdot d^{\dagger} - 2\,(s^{\dagger})^{2} \\ P^{\dagger}_{2,\mu} = 2\,s^{\dagger}d^{\dagger}_{\mu} + \sqrt{7}(d^{\dagger}d^{\dagger})^{(2)}_{\mu} \end{array}\right\}$$

(λ,μ) = (0,2)

$$P_{\ell,\mu}|[N](2N,0)L\rangle \;=\; 0 \qquad |N;\beta=\sqrt{2}\,\rangle \;=\; (N!)^{-1/2}(b^{\dagger}_{c})^{N}|0\rangle$$

(λ,μ) = (2N,0)

$$P_{\ell,\mu}|N;\beta=\sqrt{2}\,\rangle \;=\; 0 \qquad b^{\dagger}_{c} \;=\; (\sqrt{2}\,d^{\dagger}_{0} + s^{\dagger})/\sqrt{3}$$

SU(3) PDS

$$H \;=\; h_0 P^{\dagger}_0 P_0 + h_2 P^{\dagger}_2 \cdot \tilde{P}_2$$

(λ,μ) = (0,0)⊕(2,2)

SU(3) DS

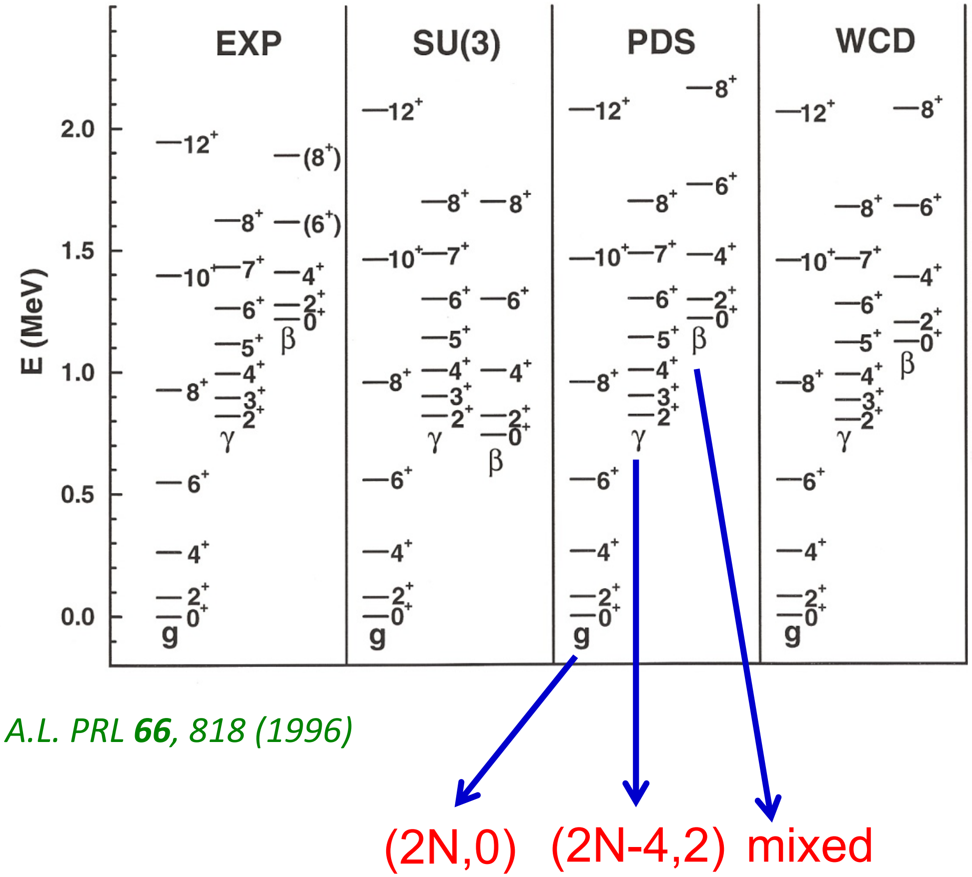
$$H(h_0 = h_2) = \left[-\hat{C}_{SU(3)} + 2\hat{N}(2\hat{N}+3)\right]$$

(λ,μ) = (0,0)

$$P_0|[N](2N-4k,2k),K=2k,L\rangle = 0 \qquad k=1,2,\ldots$$

- **Solvable** bands: g(**K**=0) , γ^{k}(**K**=2k) **good SU(3) symmetry (2N-4k,2k)** $E_k = 6h_2(2N+1-2k)$
- Other bands: **mixed**

^{168}Er



*A.L. PRL **66**, 818 (1996)*

SU(3) PDS

B(E2) branching ratios from states in the γ band

J_i^π	J_f^π	EXP	PDS	WCD	J_i^π	J_f^π	EXP	PDS	WCD
2_γ^+	0_g^+	54.0	64.27	66.0	6_γ^+	4_g^+	0.44	0.89	0.97
	2_g^+	100.0	100.0	100.0		6_g^+	3.8	4.38	4.3
	4_g^+	6.8	6.26	6.0		8_g^+	1.4	0.79	0.73
3_γ^+	2_g^+	2.6	2.70	2.7		4_γ^+	100.0	100.0	100.0
	4_g^+	1.7	1.33	1.3		5_γ^+	69.0	58.61	59.0
	2_γ^+	100.0	100.0	100.0	7_γ^+	6_g^+	0.74	2.62	2.7
4_γ^+	2_g^+	1.6	2.39	2.5		5_γ^+	100.0	100.0	100.0
	4_g^+	8.1	8.52	8.3		6_γ^+	59.0	39.22	39.0
	6_g^+	1.1	1.07	1.0	8_γ^+	6_g^+	1.8	0.59	0.67
	2_γ^+	100.0	100.0	100.0		8_g^+	5.1	3.57	3.5
5_γ^+	4_g^+	2.91	4.15	4.3		6_γ^+	100.0	100.0	100.0
	6_g^+	3.6	3.31	3.1		7_γ^+	135.0	28.64	29.0
	3_γ^+	100.0	100.0	100.0					
	4_γ^+	122.0	98.22	98.5					

$$T(E2) = \alpha\,\hat{Q} + \theta\,(d^\dagger s + \tilde{d} s)$$

□□g ratios: **parameter-free** predictions

SU(3) PDS and higher-order terms

U(6) ⊃ SU(3) ⊃ SO(3)
[N] (λ,μ) K L

$\hat{B}^{\dagger}_{[n](\lambda,\mu)K;\ell m}$ $\hat{H}_{PDS} = \sum_{\alpha\beta} u_{\alpha\beta}\hat{B}^{\dagger}_{\alpha}\hat{B}_{\beta}$

$\hat{B}_{\alpha}\,|\,$**[N] (2N,0)K=0,L**$\rangle = 0$

for **all** **L** contained in the SU(3) irrep **(λ,μ)** = **(2N,0)**

n = 2 **(λ,μ)** = (0,2)

$$\hat{\theta}_2 \equiv P_0^{\dagger}P_0 + P_2^{\dagger}\cdot\tilde{P}_2 = \left[-\hat{C}_2 + 2\hat{N}(2\hat{N}+3)\right]$$

$$P_0^{\dagger} = d^{\dagger}\cdot d^{\dagger} - 2(s^{\dagger})^2\ ,\ \ P_{2,\mu}^{\dagger} = 2s^{\dagger}d_{\mu}^{\dagger} + \sqrt{7}(d^{\dagger}d^{\dagger})^{(2)}_{\mu}$$

$$\hat{C}_2 = 2\hat{Q}\cdot\hat{Q} + \frac{3}{4}\hat{L}\cdot\hat{L}$$

n = 3 **(λ,μ)** = (2,2)

$$W_0^{\dagger} = 5P_0^{\dagger}s^{\dagger} - P_2^{\dagger}\cdot d^{\dagger}\quad ,\quad W_{2,\mu}^{\dagger} = P_0^{\dagger}d_{2,\mu}^{\dagger} + 2P_{2,\mu}^{\dagger}s^{\dagger}$$

$$V_{2,\mu}^{\dagger} = 6P_0^{\dagger}d_{2,\mu}^{\dagger} - P_{2,\mu}^{\dagger}s^{\dagger}\ \ ,\ \ W_{\ell,\mu}^{\dagger} = (P_2^{\dagger}d^{\dagger})^{(\ell)}_{\mu}\quad \ell = 3,4$$

n = 3 **(λ,μ)** = (0,0)

$$2\Lambda^{\dagger}\Lambda = \hat{C}_3 - 3(2\hat{N}+3)\hat{C}_2 + 4\hat{N}(2\hat{N}+3)(\hat{N}+3)$$

$$\Lambda^{\dagger} = P_0^{\dagger}s^{\dagger} + P_2^{\dagger}\cdot d^{\dagger}$$

$$\hat{C}_3 = -4\sqrt{7}\,\hat{Q}\cdot(\hat{Q}\times\hat{Q})^{(2)} - \frac{9}{2}\sqrt{3}\,\hat{Q}\cdot(\hat{L}\times\hat{L})^{(2)}$$

$$\hat{H}_{DS} = \xi_1 \hat{\theta}_2 + \xi_2 \Lambda^\dagger \Lambda + \rho \hat{L} \cdot \hat{L}$$

full DS $|[N](\lambda,\mu)K,L\rangle \quad (\lambda,\mu) = (2N-4k-6m, 2k)$ **solvable**

$$\hat{H}_{PDS} = \sum_{\alpha\beta} u_{\alpha\beta} \hat{B}^\dagger_\alpha \hat{B}_\beta$$

PDS $\hat{B}_\alpha |[N](2N,0), K=0, L\rangle = 0$ **g(K=0) solvable, other bands mixed**

$$\hat{H}_{PDS-1} = \hat{H}_{DS} + \xi_3 P_0^\dagger P_0 + \xi_4 P_0^\dagger s^\dagger s P_0 + \xi_5 \left(\Lambda^\dagger s P_0 + P_0^\dagger s^\dagger \Lambda\right)$$

PDS-1
$$\left.\begin{array}{l} P_0 |[N](2N-4k,2k), K=2k, L\rangle = 0 \\ \Lambda |[N](2N-4k,2k), K, L\rangle = 0 \end{array}\right\}$$
g(K=0), γ^k(K=2k) solvable

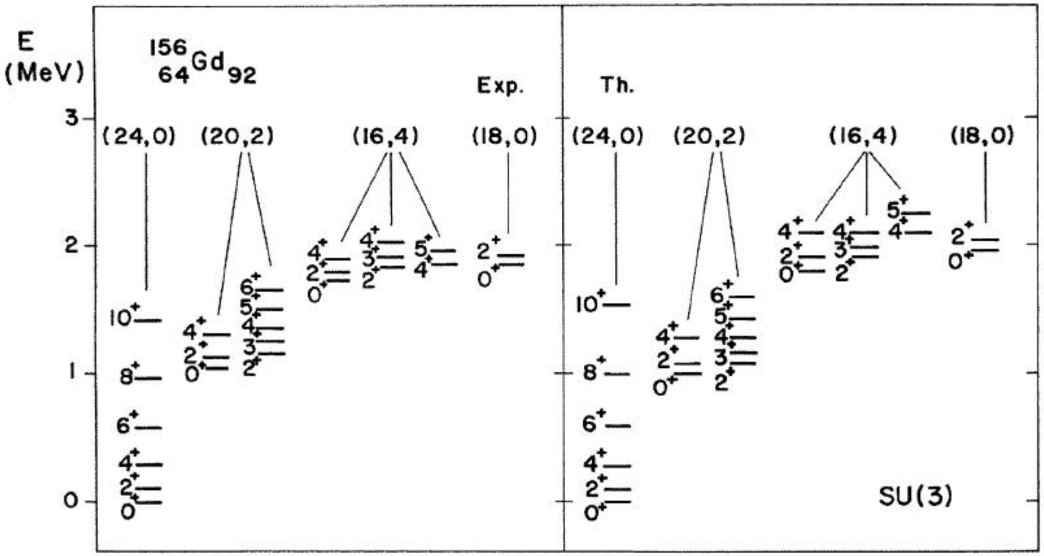
$$\hat{H}_{PDS-2} = \hat{H}_{DS} + \xi_6 W_2^\dagger \cdot \tilde{W}_2 + \xi_7 W_3^\dagger \cdot \tilde{W}_3$$

PDS-2 $W_{\ell,\mu} |[N](2N-4,2), K=0, L\rangle = 0 \quad \ell = 2,3$ **g(K=0), β(K=0) solvable**

$\hat{\Omega} = -4\sqrt{3}\,\hat{Q} \cdot (\hat{L} \times \hat{L})^{(2)}$ **solvable,** diagonal in (λ,μ), lifts K-degeneracy!

can be expressed in terms of $\hat{H}_{PDS-1}$ and $\hat{H}_{PDS-2}$

SU(3) PDS and staggering in the γ-band



$U(6) \supset SU(3) \supset SO(3)$

[N] (λ,μ) K L

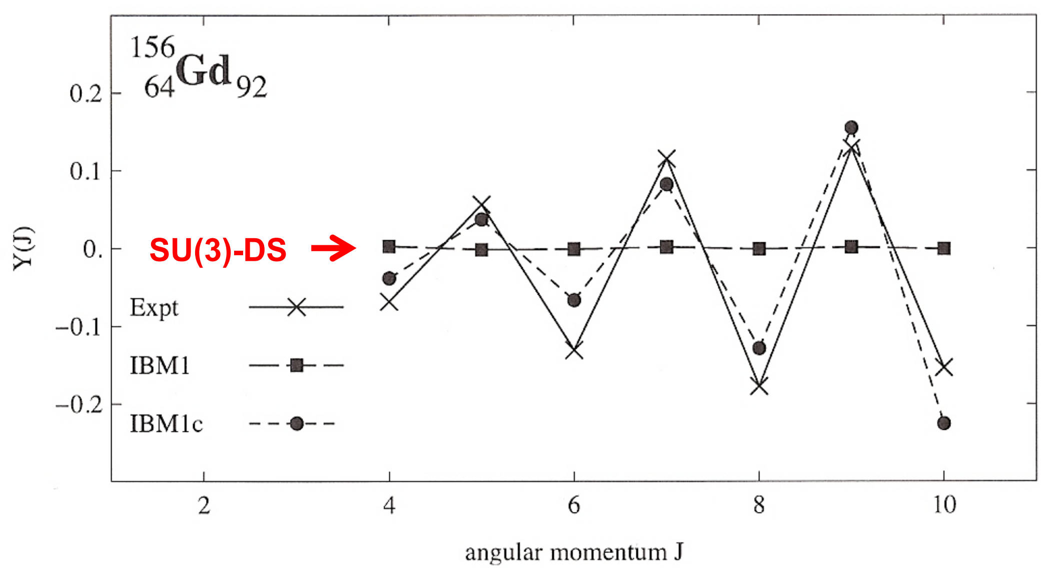
$E_{DS} = A\, f(\lambda,\mu) + B\, L(L+1)$

- ^{156}Gd a good example of SU(3)-DS
- SU(3)-PDS provides a good description of **ground (2N,0)** and β **(2N-4,2)** bands

Staggering index

$$Y(L) = \frac{(2L-1)}{L}\left[\frac{E(L)-E(L-1)}{E(L)-E(L-2)}\right] - 1$$

rigid rotor: Y(L)=0 indep. of L



- **poor** description of odd-even staggering in the γ**-band** at high L

- SU(3)-DS: **good** description of states in the ground **(2N,0)** and beta **(2N-4,2)** bands
  poor description of odd-even staggering in the gamma band

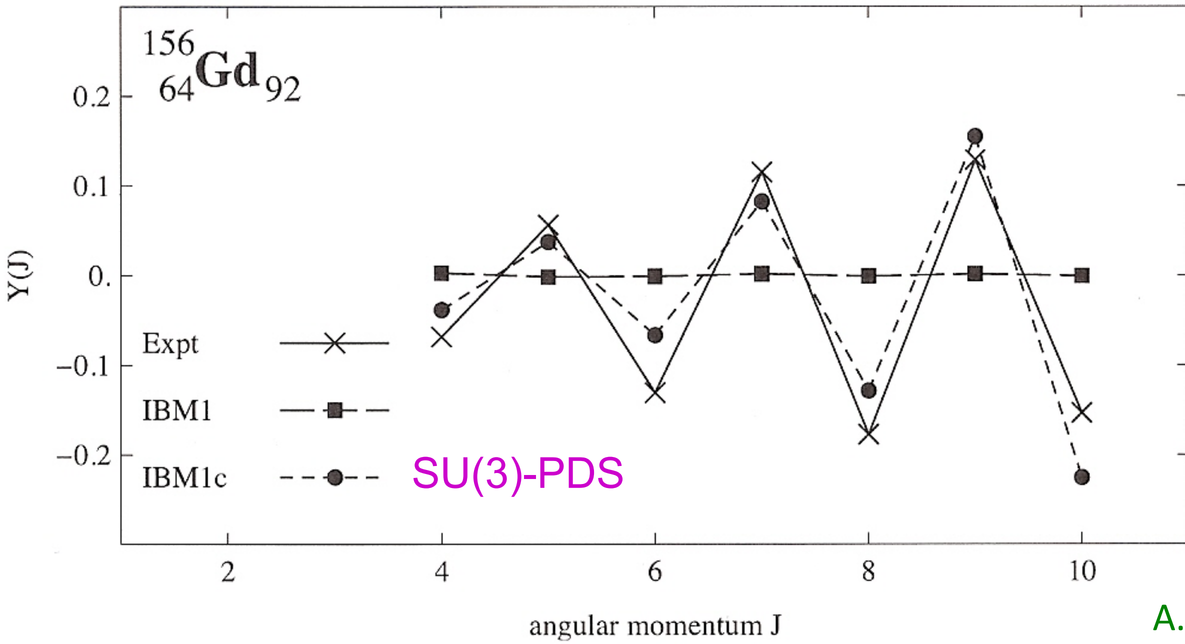
- Need to **break** the SU(3)-DS in the gamma band
  but **preserve** it in the ground and beta bands

$\Rightarrow$ **Partial Dynamical Symmetry (PDS)**

$$\hat{H}_{PDS-2} = \hat{H}_{DS} + \xi_6\, W_2^\dagger \cdot \tilde{W}_2 + \xi_7\, W_3^\dagger \cdot \tilde{W}_3$$

g(2N,0), β(2N-4,2) solvable

γ-band mixed



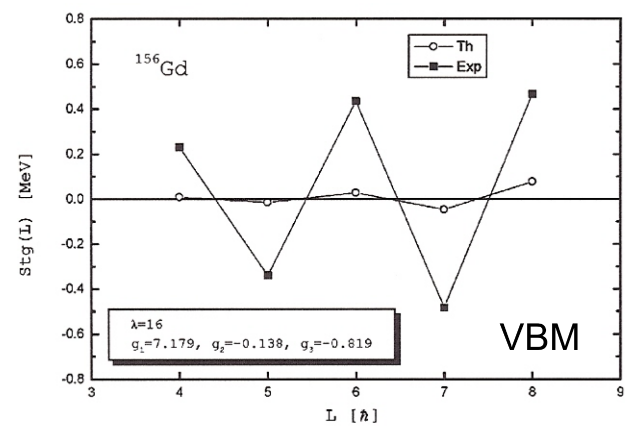
$$W^\dagger_{3,\mu} = \sqrt{7}[(d^\dagger d^\dagger)^{(2)} d^\dagger)]^{(3)}_\mu$$

$$E(\beta,\gamma) \;\propto\; \xi_7\, \beta^6 \sin^2 3\gamma$$

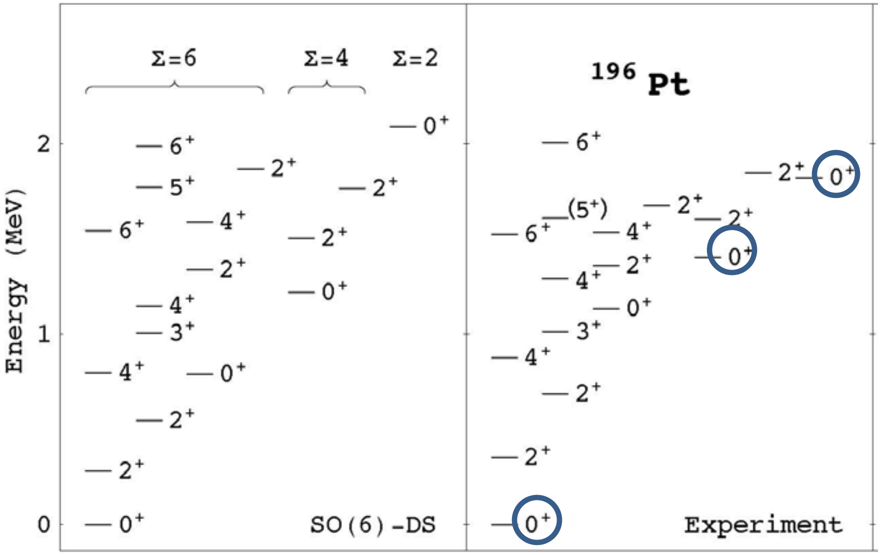
A.L., Garcia-Ramos, Van Isacker, in progress

odd even staggering (OES) in terms of band mixing

- SU(3) conserving: $\hat{H} = \hat{H}_{\rm DS} + \hat{\Omega}$ **β ,γ** ∈ (2N-4,2) (Bonatsos, 1988)
  cannot describe OES in nuclei for which $E_\beta > E_\gamma$ (e.g. ^{156}Gd)
- VBM: **g ,γ** ∈ (λ,2)
  cannot reproduce OES in ^{156}Gd (Minkov et al., 2000)
- SU(3)-PDS: **γ band** mixed with **higher bands**



O(6) PDS and higher-order terms



$$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$$
$$[N] \quad \langle\Sigma\rangle \quad (\tau) \quad L$$

$$E_{DS} = A\,\Sigma(\Sigma+4) + B\tau(\tau+3) + C\,L(L+1)$$

- SO(6)-DS provides a good description for states in the **ground band** ($\Sigma = N$)
- SO(6)-DS is manifested empirically in ^{196}Pt

Transition	Experiment	DS
$2_1^+ \to 0_1^+$	0.274 (1)	0.274
$2_2^+ \to 2_1^+$	0.368 (9)	0.358
$2_2^+ \to 0_1^+$	3.10^{-8}(3)	0.0018
$4_1^+ \to 2_1^+$	0.405 (6)	0.358
$0_2^+ \to 2_2^+$	0.121 (67)	0.365
$0_2^+ \to 2_1^+$	0.019 (10)	0.003
$4_2^+ \to 4_1^+$	0.115 (40)	0.174
$4_2^+ \to 2_2^+$	0.196 (42)	0.191
$4_2^+ \to 2_1^+$	0.004 (1)	0.001
$6_1^+ \to 4_1^+$	0.493 (32)	0.365
$2_3^+ \to 0_2^+$	0.034 (34)	0.119
$2_3^+ \to 4_1^+$	0.0009 (8)	0.0004
$2_3^+ \to 2_2^+$	0.0018 (16)	0.0013
$2_3^+ \to 0_1^+$	0.00002 (2)	0
$6_2^+ \to 6_1^+$	0.108 (34)	0.103
$6_2^+ \to 4_2^+$	0.331 (88)	0.221
$6_2^+ \to 4_1^+$	0.0032 (9)	0.0008

anharmonicity $$R = \frac{E(v=2)}{E(v=1)} - 2$$

EXP $$R = -0.70$$ ^{196}Pt (N=6)

SO(6)-DS $$R = -\frac{2}{N+1} = -0.29$$

- fit to energies of excited bands is **quite poor**

- SO(6)-DS: **good** description of states in the ground band (Σ **=N**)
  poor description of anharmonicity of excited bands

- IBM: **large anharmonicities** can be incorporated only by the inclusion of at least cubic terms Garcia-Ramos, Arias, Van Isacker, PRC **62**, 064309 (2000)

  17 possible three-body interactions

- Need to select suitable higher-order terms that can **break** the SO(6)-DS in excited bands but **preserve** it in the ground band

$\Rightarrow$ **Partial Dynamical Symmetry (PDS)**

O(6) PDS and higher-order terms

U(6) ⊃ SO(6) ⊃ SO(5) ⊃ SO(3)
[N] **⟨Σ⟩** **(τ)** **L**

$$\hat{B}^{\dagger}_{[n]\langle\sigma\rangle(\tau)\ell m} \quad \sigma < n$$

$$\tilde{B}_{[n^5]\langle\sigma\rangle(\tau)\ell m} \,|\, [N]\ \langle\Sigma\rangle = \langle N\rangle (\tau)\ L\rangle = 0$$

for **all** τ and **L** contained in the SO(6) irrep ⟨Σ⟩ = ⟨**N**⟩

$$n = 2 \qquad \hat{B}^{\dagger}_{[2]\langle 0\rangle(0)00} = \hat{P}_{+} \equiv d^{\dagger}\cdot d^{\dagger} - (s^{\dagger})^2$$

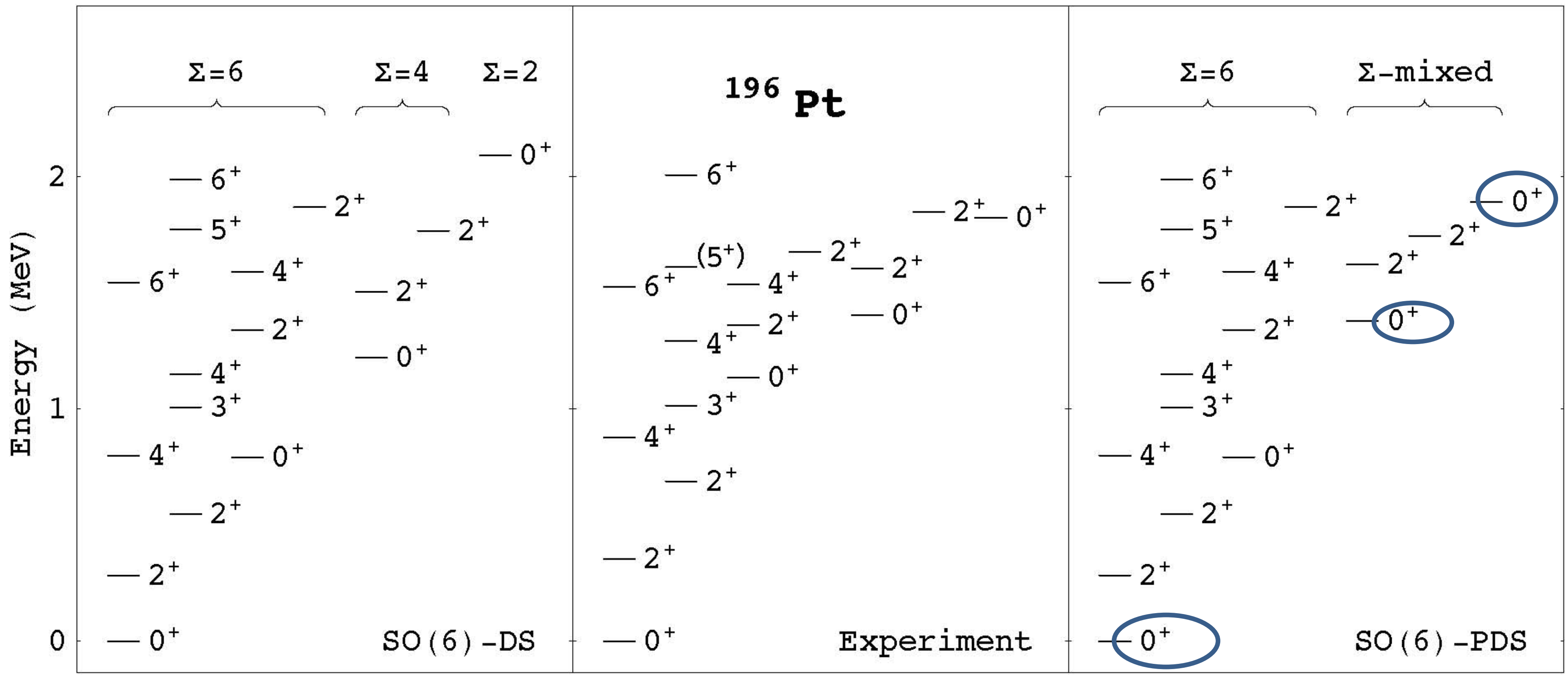
$$\hat{P}_{+}\hat{P}_{-} = \left[-\hat{C}_{SO(6)} - \hat{N}(\hat{N}+4)\right]$$

$$n = 3 \qquad \hat{B}^{\dagger}_{[3]\langle 1\rangle(1)2m} = \hat{P}_{+}\, d^{\dagger}_{m}$$

$$\hat{B}^{\dagger}_{[3]\langle 1\rangle(0)00} = \hat{P}_{+}\, s^{\dagger}$$

$$\hat{P}_{+}\hat{n}_d\hat{P}_{-} \ , \qquad \hat{P}_{+}\hat{n}_s\hat{P}_{-}$$

3-body terms



$$\hat{H}_{\mathrm{PDS}} = \hat{H}_{\mathrm{DS}} + \eta \hat{P}_{+} \hat{n}_s \hat{P}_{-}$$

SO(6) PDS (type I)

- States in the **ground band** — **solvable** with good SO(6) symmetry **⟨Σ⟩ = ⟨N⟩ = 6**
  same energy **$E=E_{DS}$** and **same w.f.** ⇒ **same B(E2)**
- States in **excited bands** — **strongly mixed (25-40 %)**
- **Anharmonicity** — R = -0.70 (EXP)   R= -0.29 (DS)   R= -0.63 (PDS)

Garcia-Ramos, Leviatan, Van Isacker, PRL **102**, 112502 (2009)

Summary

- **PDS**: **role** in nuclear spectroscopy, K-degeneracy, odd-even staggering in the γ-band, band anharmonicity, quantum phase transitions, **mixed** regular and chaotic dynamics

- Systematic procedure for **identifying** and **selecting** interactions of a given **order** with PDS

- Construction of Hamiltonians that **break** the DS but **retain** selected subsets of solvable eigenstates with **good symmetry**

- Interactions with a PDS can be introduced **without destroying** results previously obtained with a DS for a segment of the spectrum

- Quantum many-body Hamiltonians can accommodate simultaneously eigenstates with different symmetry character
  $\Rightarrow$ PDS appear to be **generic**

- PDS: symmetry is preserved in some states
  but is broken in the Hamiltonian
  $\Rightarrow$ **``Simplicity out of complexity''**

- **IBM** provides a rich environment for nurturing new concepts of symmetry!

Thank you

PDS and Quantum Phase Transitions

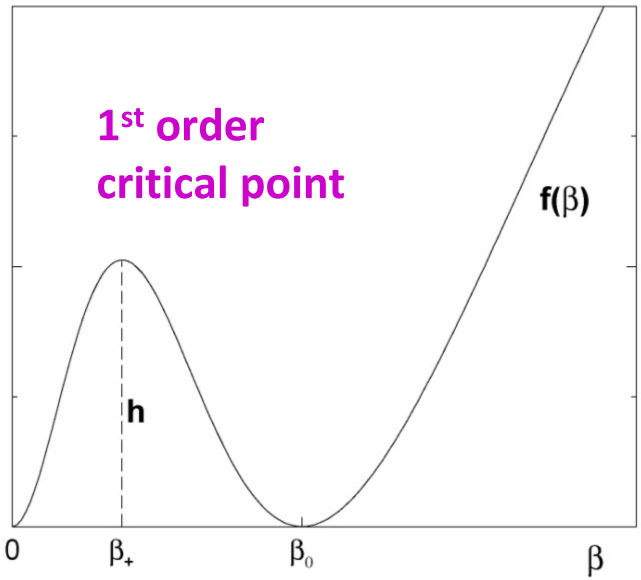
$$H = \alpha H_{G1} + (1-\alpha) H_{G2}$$

G1 and **G2** incompatible symmetries

- Are there any symmetries at the critical point $\alpha = \alpha_c$?

$\Rightarrow$ **Partial Dynamical Symmetry (PDS)**

PDS and Quantum Shape-Phase Transitions



$$H_{cri}(\beta_0) = h_2 P_2^\dagger \cdot \tilde{P}_2$$

$$P^\dagger_{2,\mu} = \beta_0\sqrt{2}\, s^\dagger d^\dagger_\mu + \sqrt{7}(d^\dagger d^\dagger)^{(2)}_\mu$$

$$E_{cri}(\beta,\gamma=0) \;=\; 2h_2 N(N-1) f(\beta)$$

$$f(\beta) \;=\; (1+\beta^2)^{-2}\beta^2(\beta-\beta_0)^2$$

eigenstates : $\beta_0 = \sqrt{2}$

$g(K=0)$ (λ,μ) = (2N,0)

$\gamma^k(K=2k)$ (λ,μ) = (2N-4k,2k)

? **SU(3)**

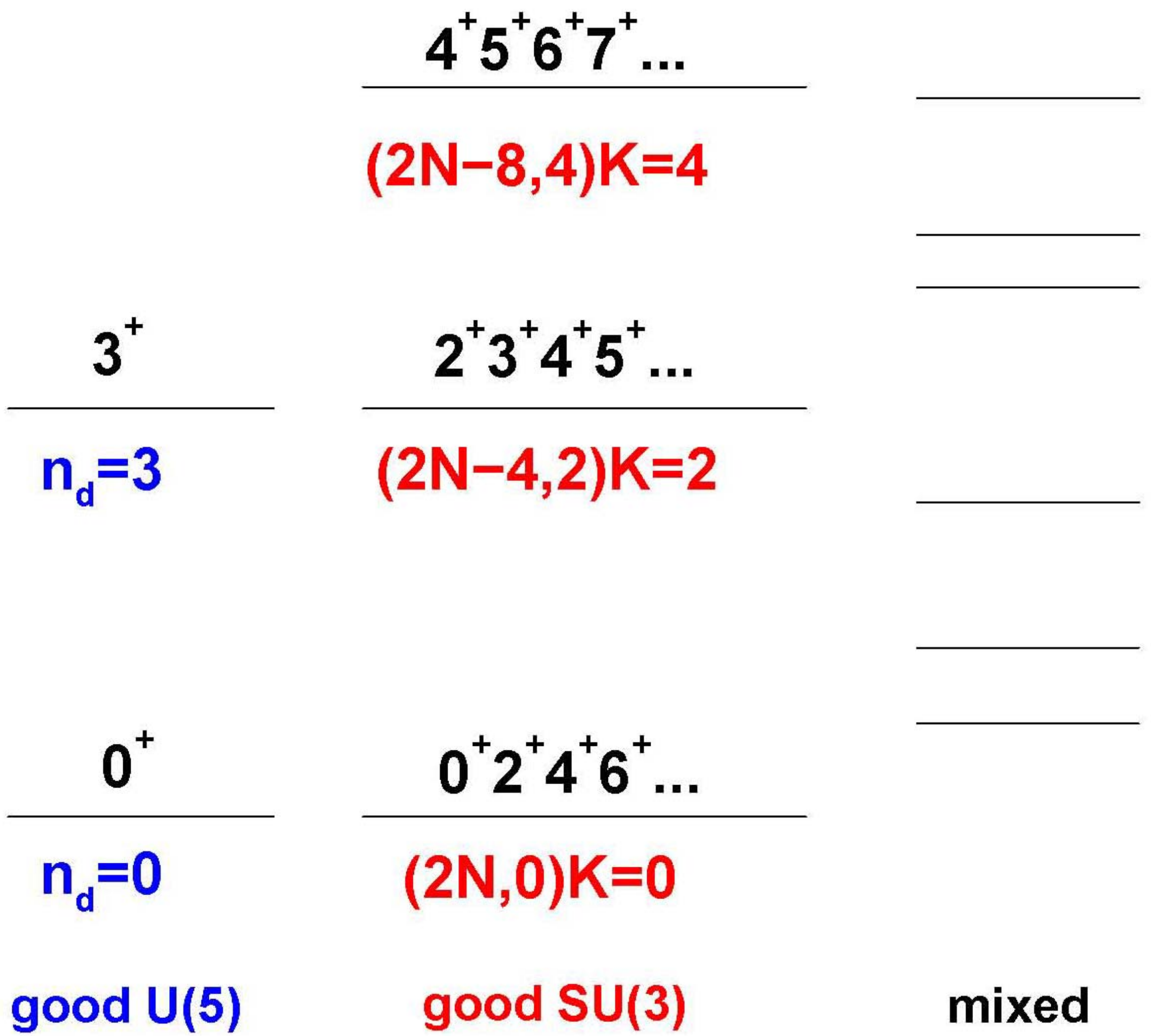
$|s^N\rangle \equiv |N, n_d = \tau = L = 0\rangle$

$|N, n_d = \tau = L = 3\rangle$

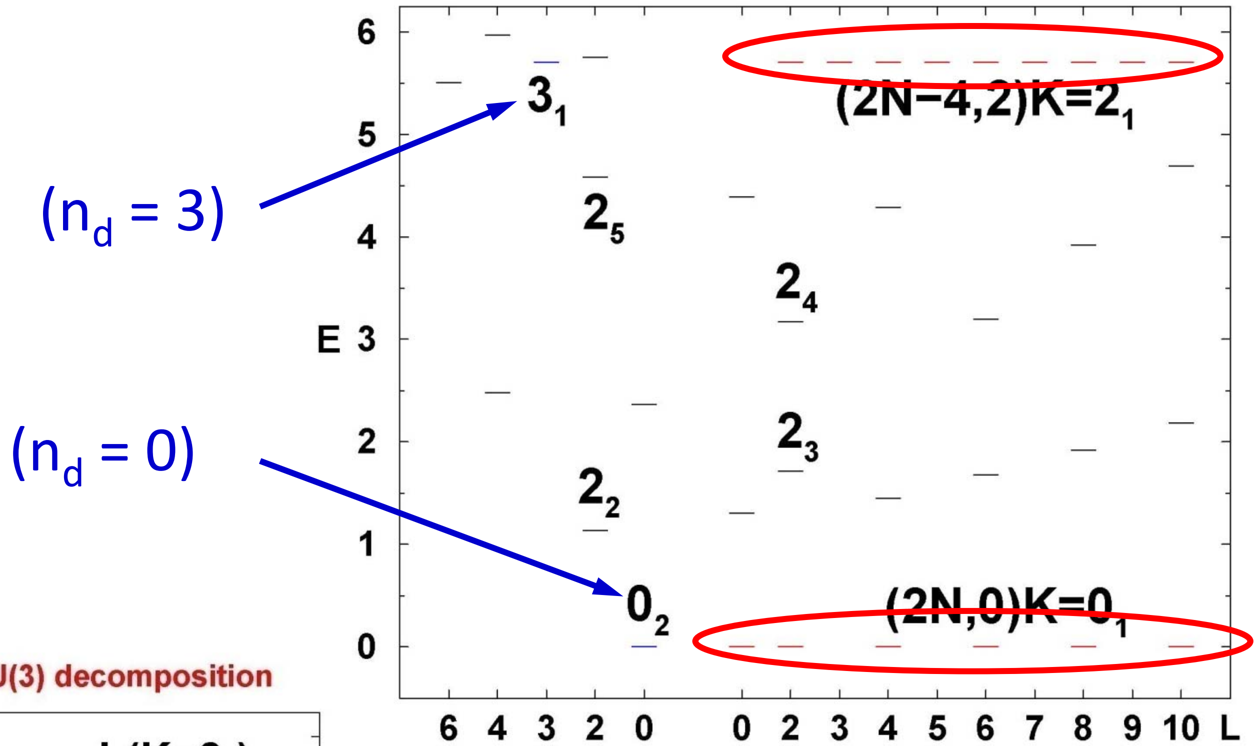
? **U(5)**

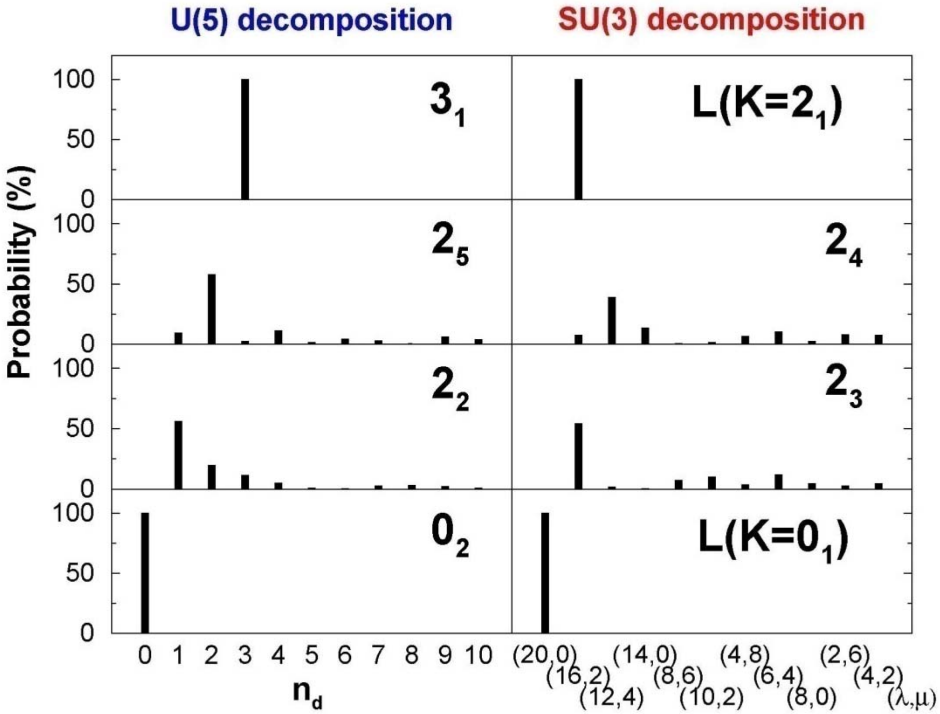
U(5) & **SU(3)** PDS (type I)

*Leviatan, PRL **98**, 242502 (2007)*



U(5) & SU(3) PDS (type I)





U(5) & **SU(3)** PDS (type I)

*Leviatan, PRL **98**, 242502 (2007)*

PDS and mixed regular and chaotic dynamics

SYMMETRY	SYSTEM	DYNAMICS
dynamical	integrable	regular
broken	non-integrable	chaotic
partial	partly-integrable	mixed

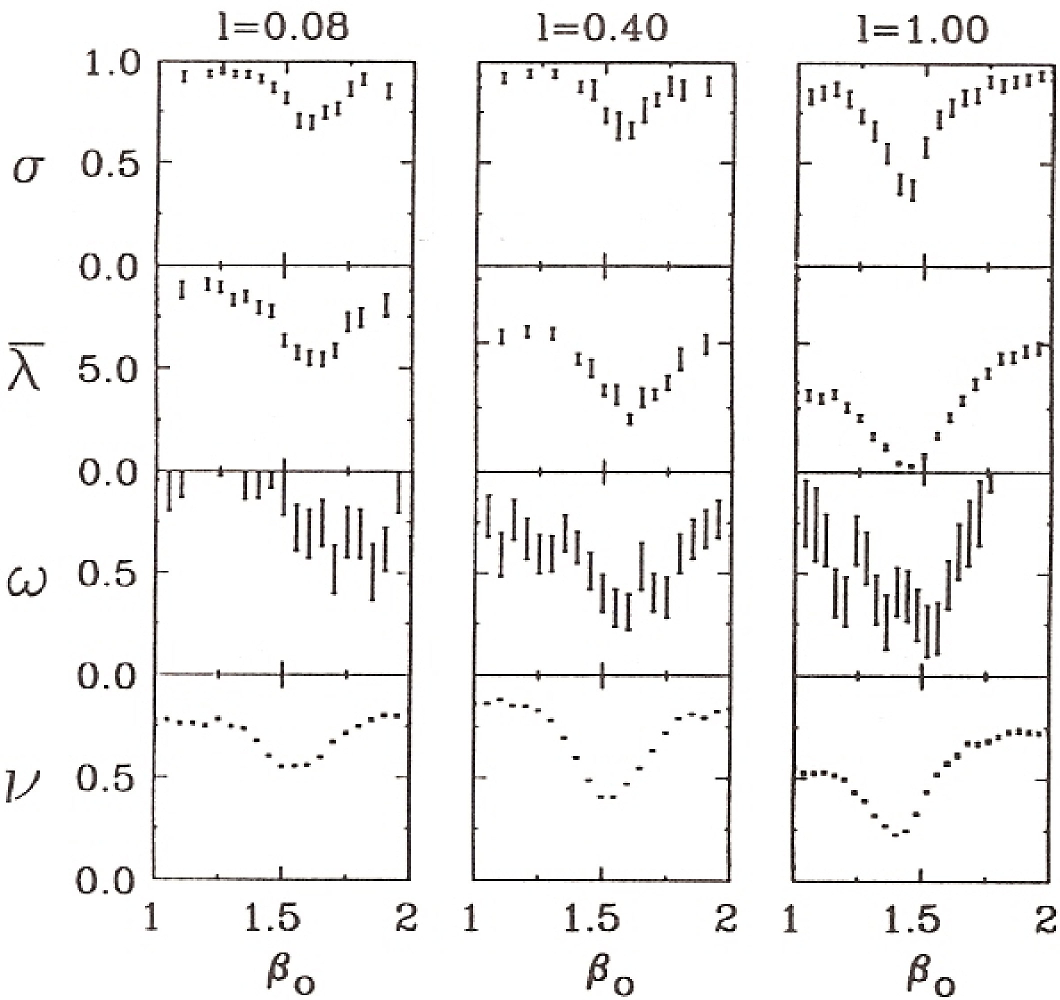
PDS and the Suppression of Chaos

*Whelan, Alhassid, Leviatan, PRL **71**, 2208 (1993)*
*Leviatan, Whelan, PRL **77**, 5202 (1996)*

$$H(\beta_0) \; = \; h_0 P_0^\dagger P_0 + h_2 P_2^\dagger \cdot \tilde{P}_2$$

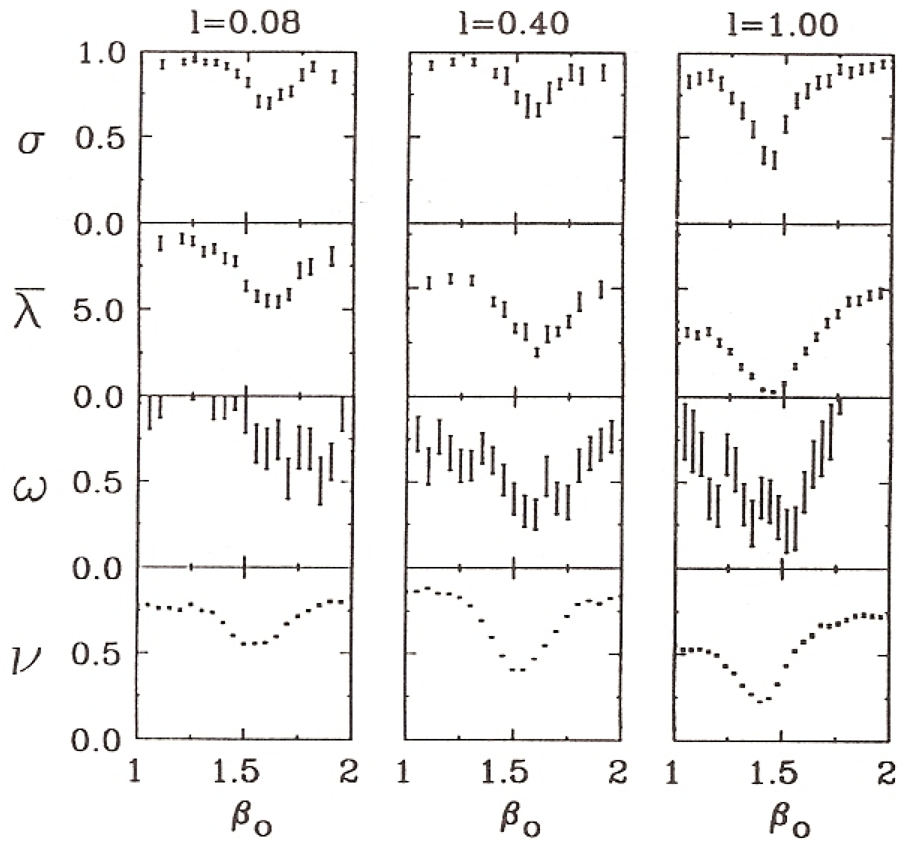
$$P_0^\dagger = d^\dagger \cdot d^\dagger - \beta_0^2 \, (s^\dagger)^2$$

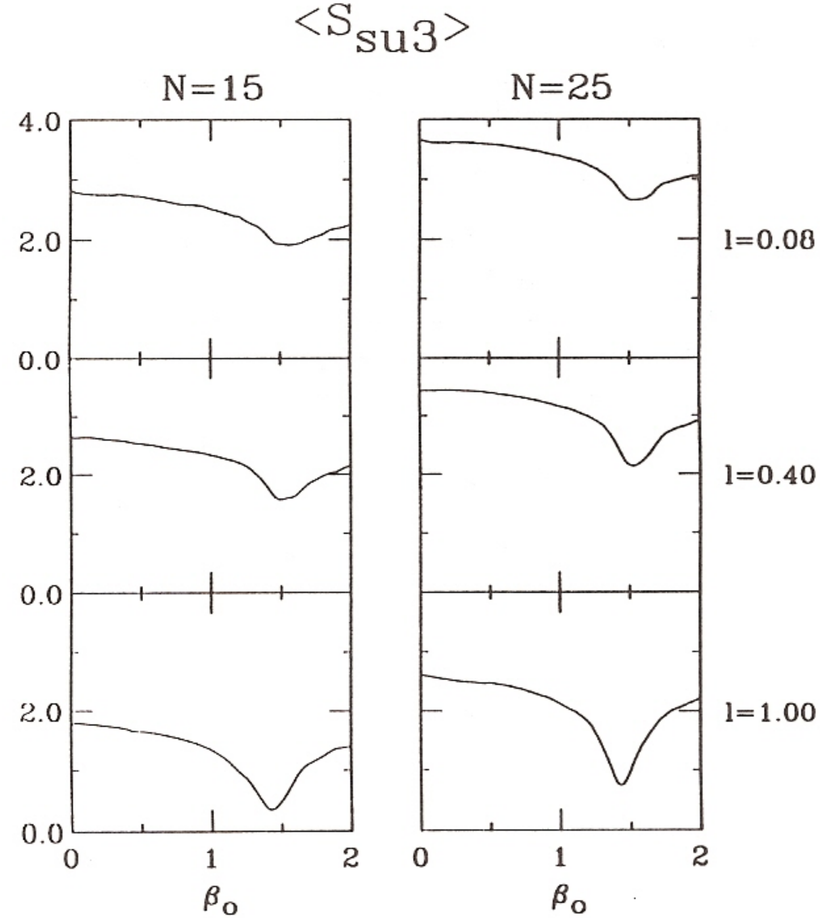
$$P_{2,\mu}^\dagger = \beta_0 \sqrt{2} \, s^\dagger d_\mu^\dagger + \sqrt{7} (d^\dagger d^\dagger)_\mu^{(2)}$$



$\beta_0 = \sqrt{2}$: **SU(3) PDS**

Suppression of chaos





Reduction of chaos observed even though the fraction of solvable states approaches zero in the classical limit

SU(3) entropy

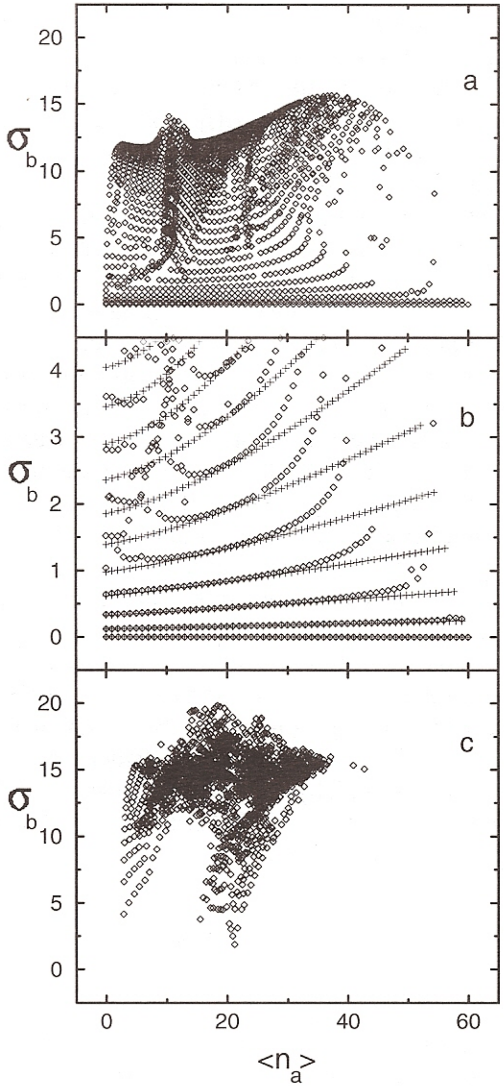
$$S^{(\alpha)}_{SU(3)} = -\sum_{\lambda,\mu} p^{(\alpha)}_{\lambda\mu} \ln p^{(\alpha)}_{\lambda\mu}$$

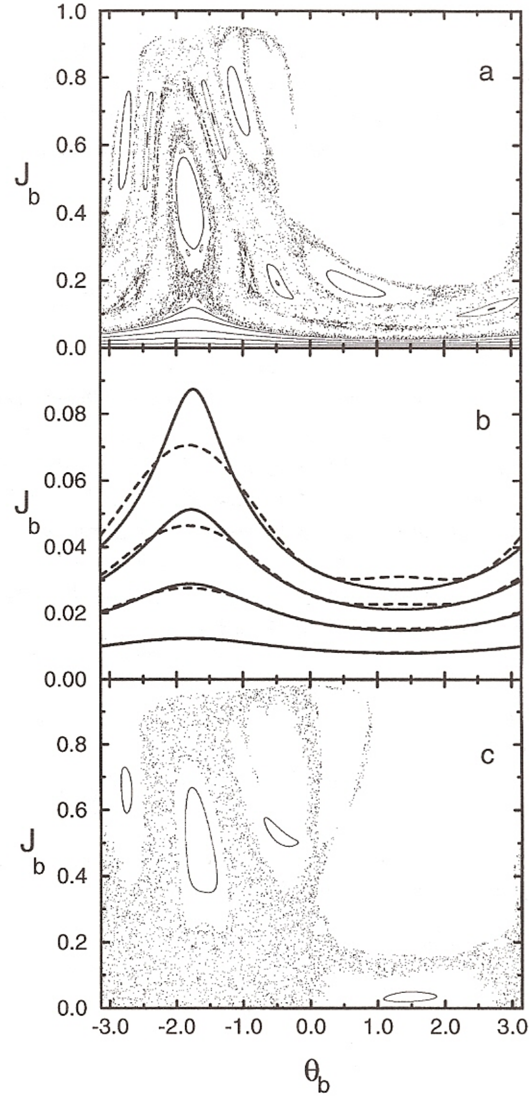
*Whelan, Alhassid, Leviatan, PRL **71**, 2208 (1993)*

PDS

PDS

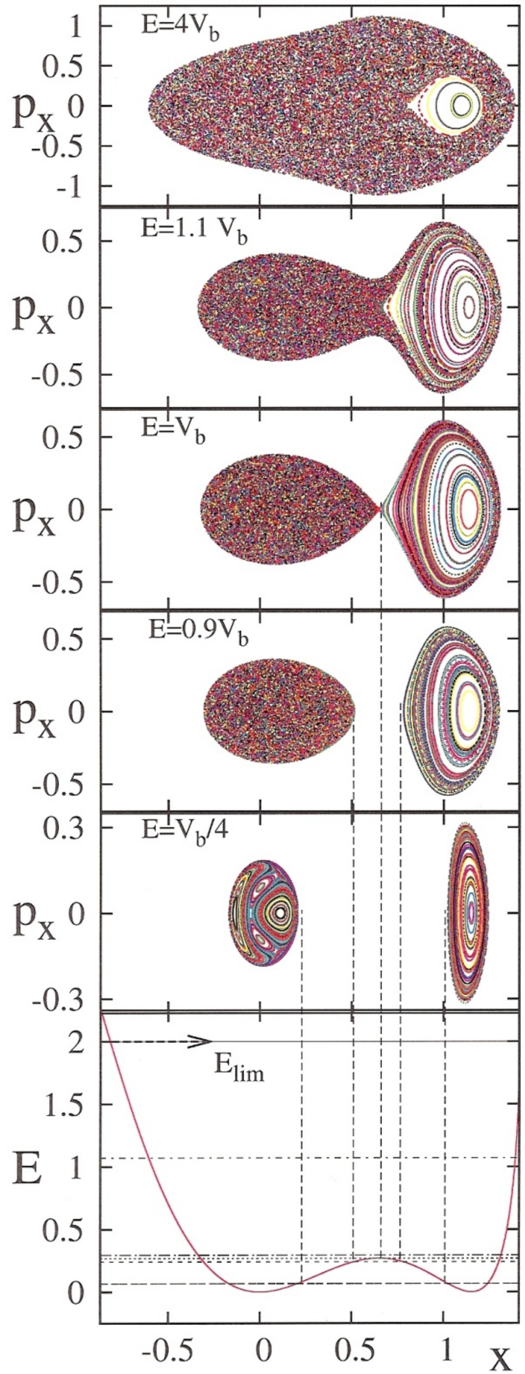
BROKEN
Symmetry

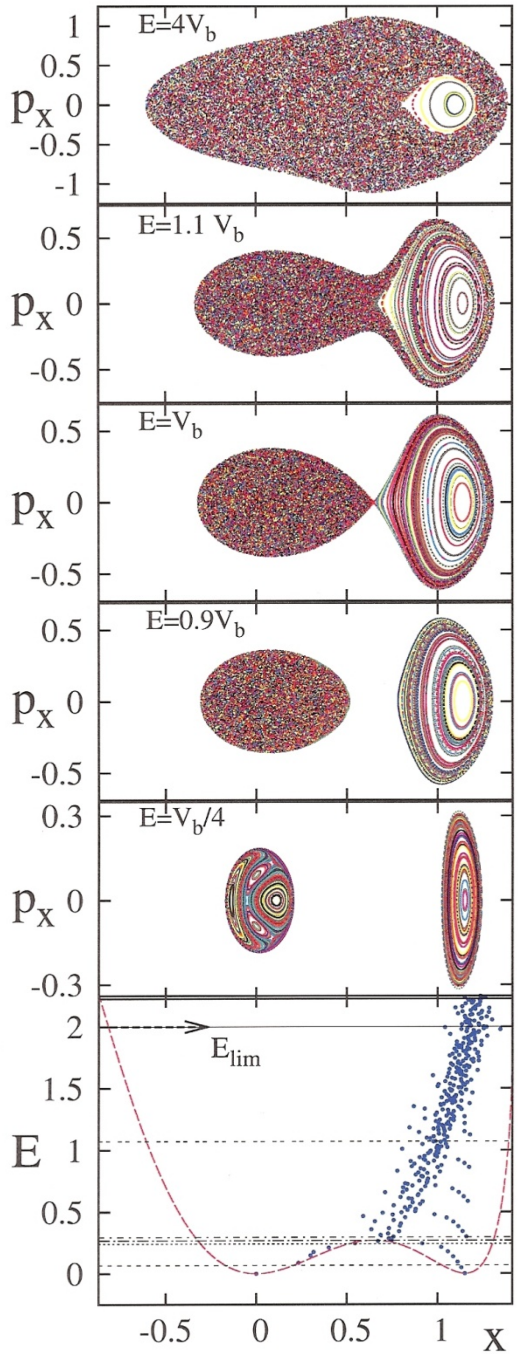




- PDS affects the purity of other states in the system
- Classically, region of phase space near the special torus has also toroidal structure

*Leviatan, Whelan, PRL **77**, 5202 (1996)*


E=4V_b
E=1.1 V_b
E=V_b
E=0.9V_b
E=V_b/4
p_x
E
E_lim
x
1
0.5
0
-0.5
-1
0.3
-0.3
2
1.5



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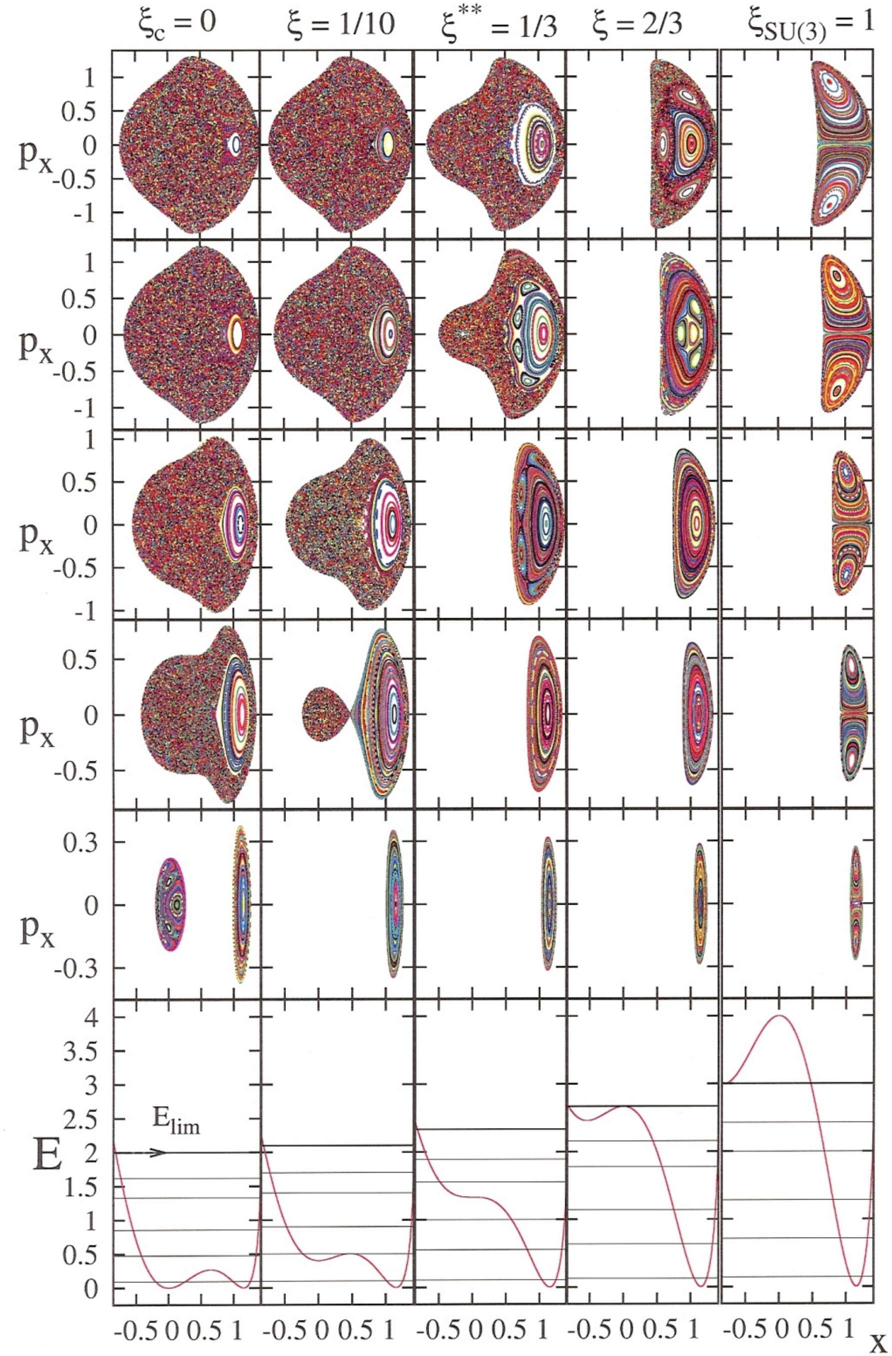
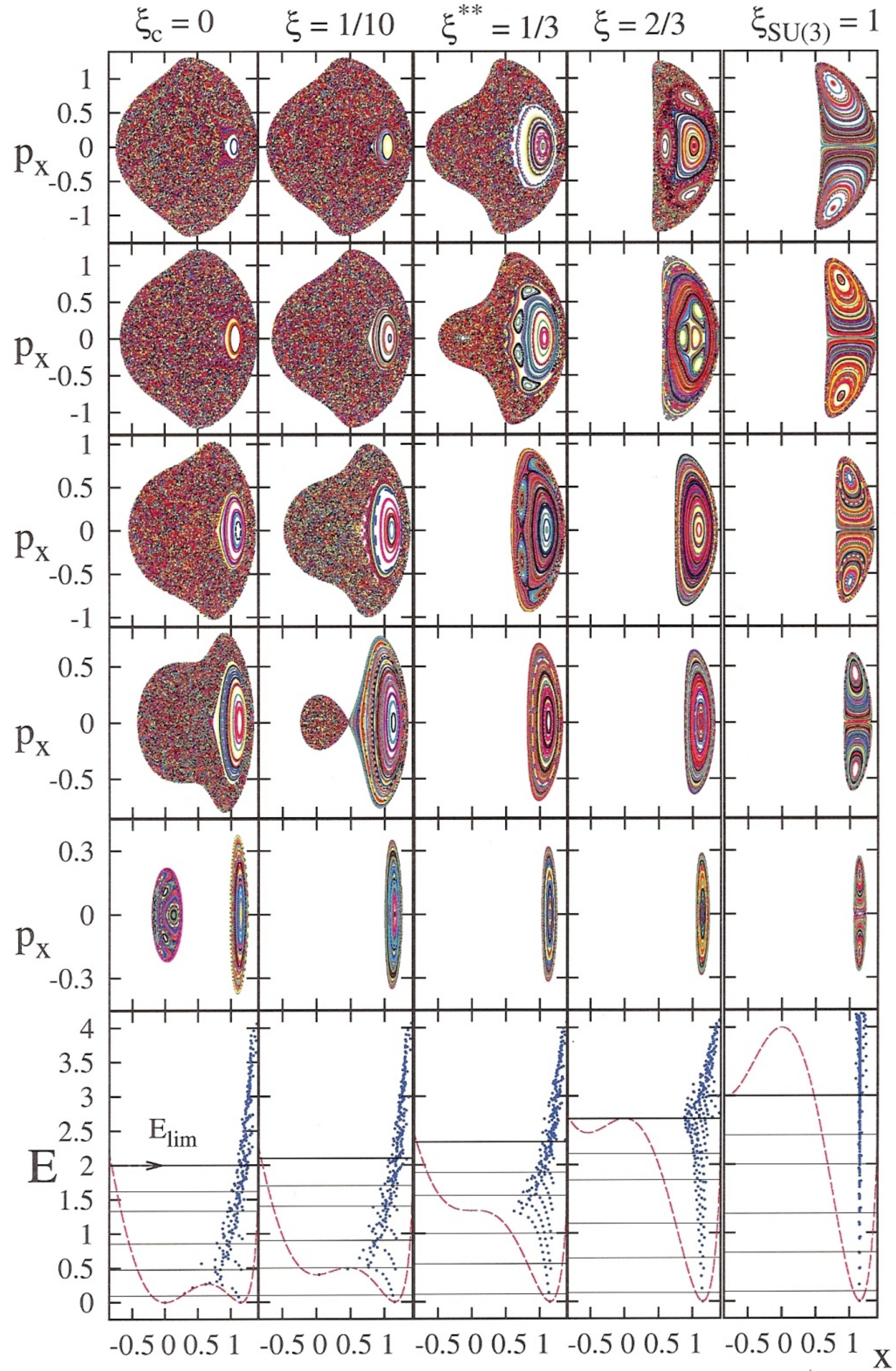


$\xi_c = 0$
$\xi = 1/10$
$\xi^{**} = 1/3$
$\xi = 2/3$
$\xi_{SU(3)} = 1$
p_x
E
E_{lim}
x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$\xi = 2/3$
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p_x
E
E_{lim}
x

exact symmetry **degeneracy**

all states **with** good symmetry

$$[H\,,\,g_i\,] = 0 \qquad g_i \in G$$

dynamical symmetry **spliting no mixing**

all states solvable, **with** good symmetry

$$G_0 \supset G_1 \supset \ldots \supset G_n$$

$$|\alpha_0, \alpha_1, \ldots, \alpha_n\rangle$$

$$[H\,,\,\hat{C}_{G_k}\,] = 0 \qquad H = \sum_k a_k \hat{C}_{G_k}$$

partial symmetry

part of the degenerate states (**solvable**) **with** good symmetry

$$[H\,,\,g_i\,] \neq 0$$

$$[H\,,\,g_i\,]|\Psi\rangle = 0 \qquad g_i \in G$$

partial dynamical symmetry

part of non-degenerate states (**solvable**) **with** good symmetry

$$[H\,,\,\hat{C}_G\,] \neq 0$$

$$[H\,,\,\hat{C}_G\,]|\Psi\rangle = 0$$

broken symmetry

none of the states **solvable, without** good symmetry

$$[H\,,\,g_i\,] \neq 0 \qquad g_i \in G$$