The geometric interpretation of the Semimicroscopic Algebraic Cluster Model and the role of the Pauli principle

H. Yépez-Martínez (UACM) , G. E. Morales-Hernández , P. O. Hess , P. Fraser (ICN-UNAM)

Content

- Some trivialities: resuming the SACM
- Structure of the Hamiltonian
- Geometrical mapping (coherent state and semiclassical potential).
- Study of phase transitions
- Conclusions

SACM: Some basic definitions

Basic degrees of freedom:

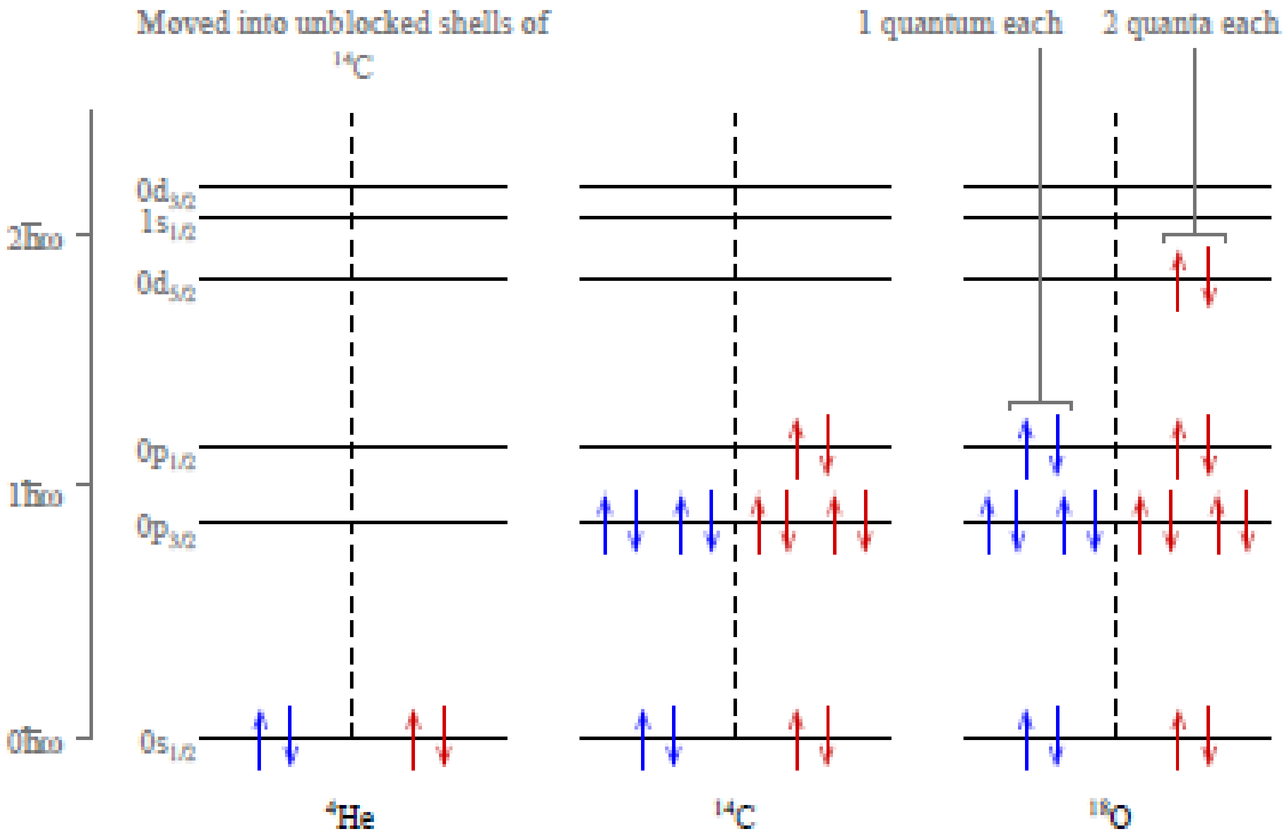
$$\pi_m^\dagger \ , \ \pi_m \quad , \quad m = 0, \pm 1 \qquad \pi^m = (-1)^{1-m} \pi_{-m}$$

U(4) group structure for the relative motion:

$$\pi_m^\dagger \pi^{m'} \quad , \quad \pi_m^\dagger \sigma \quad , \quad \sigma^\dagger \pi^m \quad , \quad \sigma^\dagger \sigma$$

Introducing a cutoff!: $N = n_\pi + n_\sigma$

Wildermuth condition: minimal numer of Π-bosons n_0



n_π=0 n_π=10 n_π=16

→ n_0=6

Some important dynamical symmetries

SU(3)-limit:

$$\begin{array}{ccccccc} \mathrm{U}_R(4) & \supset & \mathrm{SU}_R(3) & \supset & \mathrm{SO}_R(3) & \supset & \mathrm{SO}_R(2) \\ [N,0,0,0] & & (n_\pi,0) & & L_R & & M_R, \end{array}$$

SO(4)-limit:

$$\begin{array}{ccccccc} \mathrm{U}_R(4) & \supset & \mathrm{SO}_R(4) & \supset & \mathrm{SO}_R(3) & \supset & \mathrm{SO}_R(2) \\ [N,0,0,0] & & (\omega,0) & & L_R & & M_R, \end{array}$$

Basis used

$$\begin{array}{ccccc} SU_{C_1}(3) \otimes SU_{C_2}(3) \otimes SU_R(3) & \supset & SU_C(3) \otimes SU_R(3) & \supset \\ (\lambda_1,\mu_1) \quad (\lambda_2,\mu_2) \quad (n_\pi,0) & & (\lambda_C,\mu_C) & \\ SU(3) \supset SO(3) \supset SO(2) \\ (\lambda,\mu) \quad \kappa L \quad M, \end{array} \qquad (8)$$

Additional limit, weak interaction limit:

$$\begin{array}{l} SU_C(3) \otimes U_R(4) \supset SO_C(3) \otimes SO_R(3) \supset SO(3) \supset SO(2) \\ (\lambda_C.\mu_C)\,[N,0,0,0] \quad L_C \quad L_R \quad L \quad M. \end{array} \qquad (10)$$

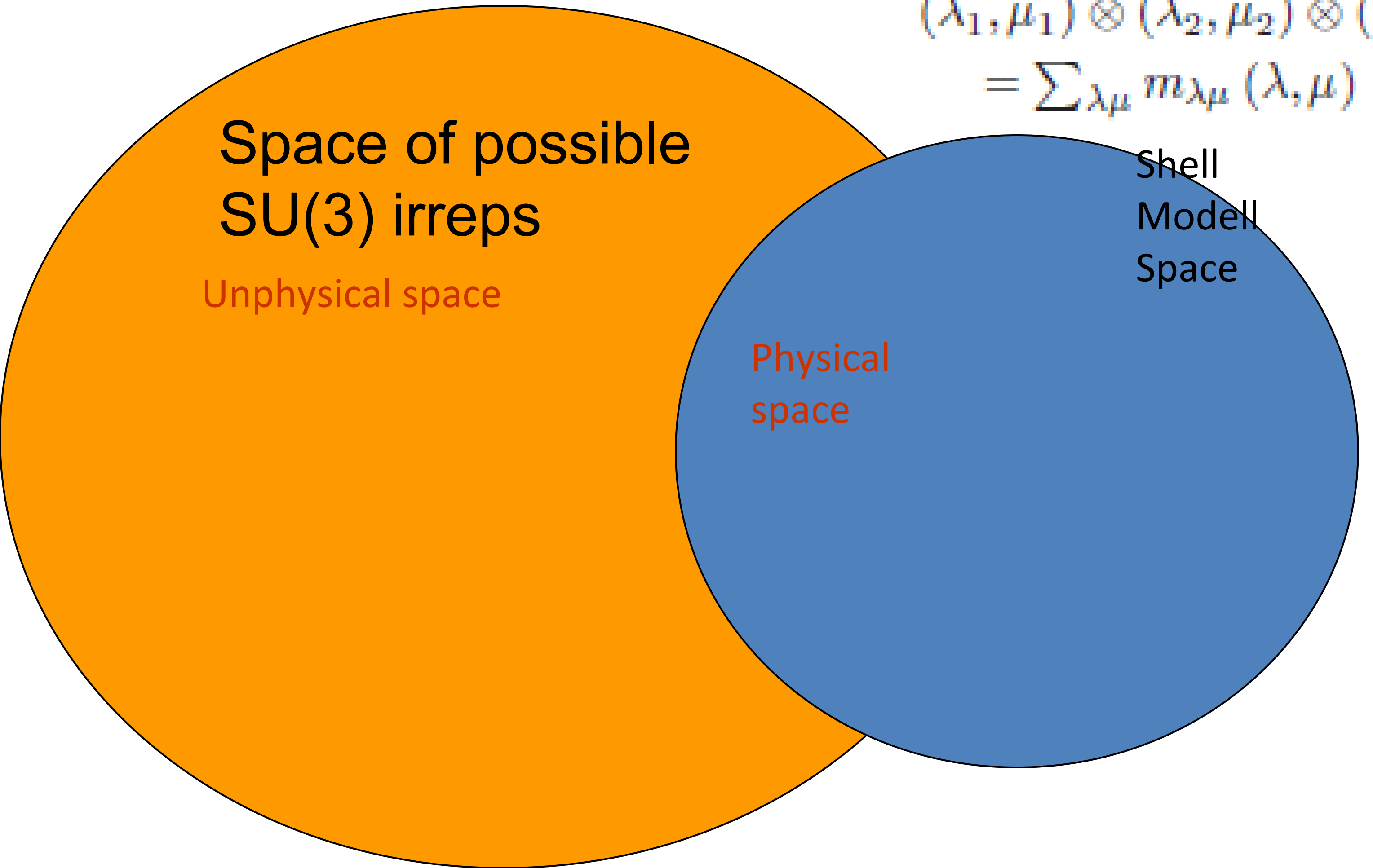
(does not satisfy de “definition” of dynamical symmetry)

→weak interaction limit

Satisfying the Pauli-Exclusion Principle:

Wildermuth condition: **minimal number of relative oscillation quanta +**

$$(\lambda_1,\mu_1)\otimes(\lambda_2,\mu_2)\otimes(n_\pi,0) = \sum_{\lambda\mu} m_{\lambda\mu}(\lambda,\mu) \quad ,$$



Structure of the Hamiltonian

$$\Delta n_\pi = n_\pi - n_0$$

$$\boldsymbol{H} = xy\boldsymbol{H}_{SU(3)} + y(1-x)\boldsymbol{H}_{SO(4)} + (1-y)\boldsymbol{H}_{SO(3)} \quad (11)$$

with

$$\begin{aligned}
\boldsymbol{H}_{SU(3)} &= \hbar\omega\boldsymbol{n}_\pi + a_{Clus}C_2(\lambda_C,\mu_C) \\
&\quad +(a - b\Delta\boldsymbol{n}_\pi)C_2(\lambda,\mu) + (\bar{a} - \bar{b}\Delta\boldsymbol{n}_\pi)C_2(\boldsymbol{n}_\pi,0) \\
&\quad +\gamma\boldsymbol{L}^2 + t\boldsymbol{K}^2 \\
\boldsymbol{H}_{SO(4)} &= a_C\boldsymbol{L_C}^2 + a_R^{(1)}\boldsymbol{L_R}^2 \\
&\quad +\gamma\boldsymbol{L}^2 + \frac{c}{4}\left[(\boldsymbol{\pi}^\dagger\cdot\boldsymbol{\pi}^\dagger) - (\sigma^\dagger)^2\right]\left[(\boldsymbol{\pi}\cdot\boldsymbol{\pi}) - (\sigma)^2\right] \\
\boldsymbol{H}_{SO(3)} &= \hbar\omega\boldsymbol{n}_\pi + a_{Clus}C_2(\lambda_C,\mu_C) \\
&\quad +a_C\boldsymbol{L_C}^2 + a_R^{(1)}\boldsymbol{L}_R^2 + \gamma\boldsymbol{L}^2 \quad , \qquad (12)
\end{aligned}$$

The geometrical mapping

Coherent trial state:

$$|\alpha\rangle = \mathcal{N}_{N,n_0}(\boldsymbol{\alpha}\cdot\boldsymbol{\pi}^\dagger)^{n_0}\left[\boldsymbol{\sigma}^\dagger + (\boldsymbol{\alpha}\cdot\boldsymbol{\pi}^\dagger)\right]^N |0\rangle$$
$$= \mathcal{N}_{N,n_0}\frac{N!}{(N+n_0)!}\frac{\mathrm{d}^{n_0}}{\mathrm{d}\gamma_1^{n_0}}\left[\boldsymbol{\sigma}^\dagger + \gamma_1(\boldsymbol{\alpha}\cdot\boldsymbol{\pi}^\dagger)\right]^{N+n_0}|0\rangle$$

$$\alpha_m^* = (-1)^{1-m}\alpha_{-m} \qquad (\boldsymbol{\alpha}\cdot\boldsymbol{\alpha}) = \sum_m (-1)^{1-m}\alpha_m\alpha_{-m} = \alpha^2 \quad ,$$

Semi-classical potential:

$$V(\boldsymbol{\alpha}) = \langle\alpha|\, \boldsymbol{H}\, |\alpha\rangle$$

Minimal distance: $\langle r_0 \rangle \propto \sqrt{n_0}$ → SU(3) NOT vibrational limit

$$\langle \boldsymbol{H} \rangle = \mathcal{C}(x,y) - (b+\bar{b})xy \left(A(x,y)\alpha^2 \frac{F_{11}(\alpha^2)}{F_{00}(\alpha^2)} \right.$$

$$\rightarrow \quad -B(x,y)\alpha^4 \frac{F_{22}(\alpha^2)}{F_{00}(\alpha^2)} + \alpha^6 \frac{F_{33}(\alpha^2)}{F_{00}(\alpha^2)}$$

$$\left. -C(x,y)\alpha^2 \frac{F_{20}^{N-2}(\alpha^2)}{F_{00}(\alpha^2)} \right)$$

← Only three effective parameters!!!

$$\mathcal{C}(x,y) =$$
$$\langle (a_{Clus} + a + bn_0) C_2(\lambda_C, \mu_C) + \gamma \boldsymbol{L_C}^2 + (1-xy) a_C \boldsymbol{L_C}^2 \rangle$$
$$+ xyt\langle \boldsymbol{K}^2 \rangle + \frac{c}{4}(N+n_0)(N+n_0-1)y(1-x) \qquad (18)$$

$$\begin{aligned}
A(x,y) &= -\frac{1}{(b+\bar{b})xy} \Big(\hbar\omega [yx+1-y] \\
&+2\left[\gamma + (1-yx)a_R^{(1)}\right] \\
&+xy[a-b][4+\Gamma_1+\Gamma_2] \\
&+4xy[\bar{a}-\bar{b}] + xybn_0[4+\Gamma_1+\Gamma_2] + 4xyn_0\bar{b} \\
&-bxyC_2(\lambda_C,\mu_C) - \frac{c}{2}y(1-x)(N+n_0-1) \Big) \\
B(x,y) &= \frac{1}{(b+\bar{b})xy} \Big(xy[a+\bar{a}-6b-6\bar{b} \\
&-b\{\Gamma_1+\Gamma_2\} + n_0(b+\bar{b})] + \frac{c}{2}y(1-x) \Big) \\
C(x,y) &= -\frac{\frac{c}{2}y(1-x)}{(b+\bar{b})xy} , \qquad (19)
\end{aligned}$$

$$F_{pq}(\alpha^2) = \frac{N!^2}{(N+n_0)!} \frac{(N+n_0)!}{(N+n_0-\max(p,q))!}$$
$$\times \sum_{k=\max(n_0-p,n_0-q,0)}^{N+n_0-\max(p,q)} \binom{N+n_0-\max(p,q)}{k}$$
$$\times \left[\frac{(k+p)!}{(k+p-n_0)!}\right] \left[\frac{(k+q)!}{(k+q-n_0)!}\right] \alpha^{2k} .$$

$$\begin{aligned}
\Gamma_k &= \langle (\lambda_k,\mu_k) | \boldsymbol{Q}_m^{Cluster(k)} | (\lambda_k,\mu_k) \rangle \\
&= \sqrt{\frac{5}{\pi}} \left[n_k + \frac{3}{2}(A_k - 1) \right] \alpha_{2m}(k) \\
&= \sqrt{\frac{5}{\pi}} N_{0,k}\beta_k .
\end{aligned}$$

Some limits

i) Limit $\alpha \to \infty$:

$$\alpha^2 \frac{F_{11}}{F_{00}} \to (N + n_0)$$
$$\alpha^4 \frac{F_{22}}{F_{00}} \to (N + n_0)(N + n_0 - 1)$$
$$\alpha^6 \frac{F_{33}}{F_{00}} \to (N + n_0)(N + n_0 - 1)(N + n_0 - 2)$$
$$\alpha^2 \frac{F_{20}^{N-2}}{F_{00}} \to N(N-1)\frac{1}{\alpha^2} \to 0 \quad . \qquad ($$

ii) Limit $\alpha \to 0$:

$$\alpha^2 \frac{F_{11}}{F_{00}} \to n_0$$
$$\alpha^4 \frac{F_{22}}{F_{00}} \to n_0(n_0 - 1)$$
$$\alpha^6 \frac{F_{33}}{F_{00}} \to n_0(n_0 - 1)(n_0 - 2)$$
$$\alpha^2 \frac{F_{20}^{N-2}}{F_{00}} \to N(N-1)(n_0+1)(n_0+2)\frac{\alpha^2}{2} \to 0$$

Conditions of phase transition

Structure of the potential: $\tilde{V} = \sum_k p_k \alpha^{m_k} f_k(\alpha)$

Minima: $\bar{\alpha}_\tau$, $\tau = 1,2$

$\tau = 1$ "Spherical" → α_1=0 $\frac{d^n \tilde{V}(\alpha_1 = 0)}{dp_k^n} = 0, \quad n = 1,2,\ldots$

$\tau = 2$ "deformed" → α_2>0

→ $$\frac{d\tilde{V}}{dp_k} = \frac{\partial \tilde{V}}{\partial p_k} + \frac{\partial \tilde{V}}{\partial \bar{\alpha}_i}\frac{\partial \bar{\alpha}_i}{\partial p_k} = \frac{\partial \tilde{V}}{\partial p_k},$$ → $$\frac{d\tilde{V}}{dp_k} = \bar{\alpha}_2^{m_k} f_k(\bar{\alpha}_2)$$

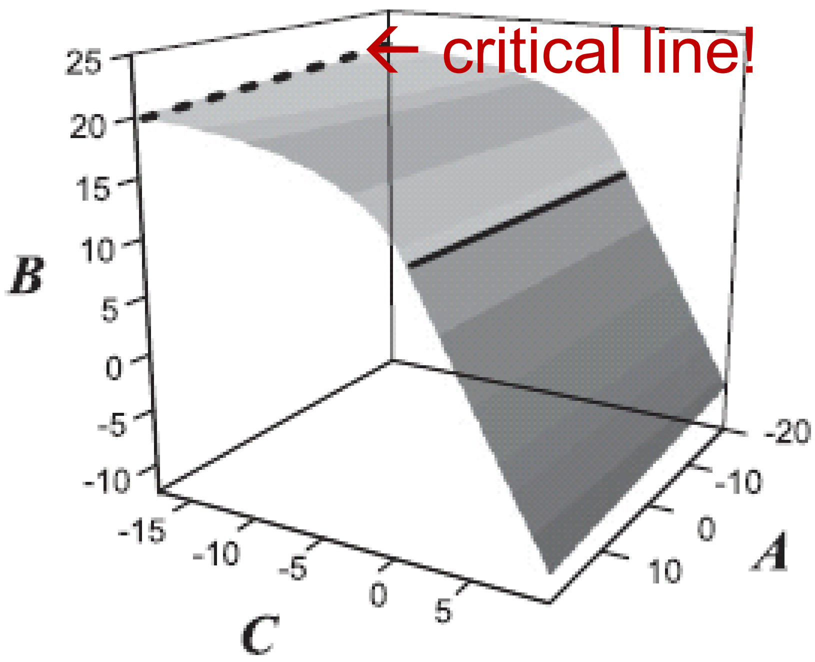
α _2>0 → 1. order

α _2=0 → 2. order

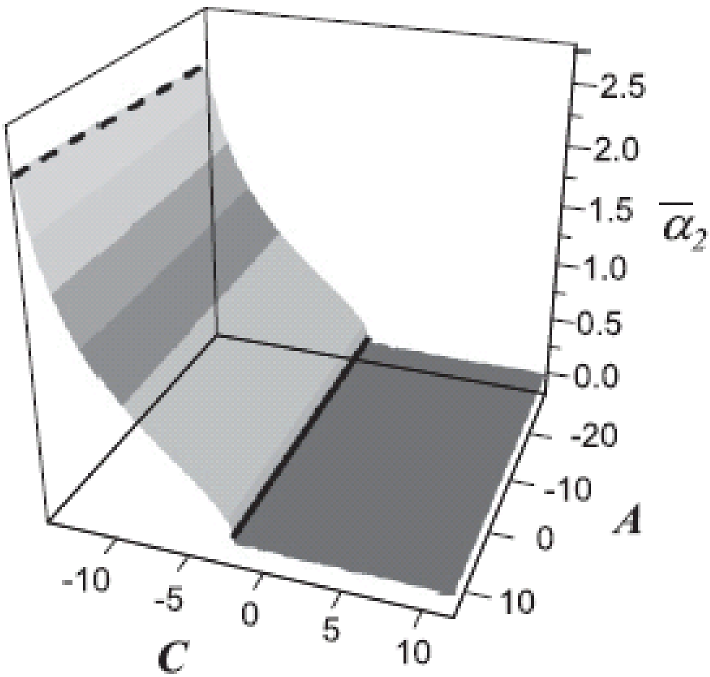
$$\frac{d^2\tilde{V}}{dp_k^2} = \frac{d}{dp_k}\left(\frac{\partial \tilde{V}}{\partial p_k}\right) = \frac{\partial \bar{\alpha}_2^{m_k}}{\partial p_k} f_k(\bar{\alpha}_2) + \bar{\alpha}_2^{m_k} \frac{\partial f_k(\bar{\alpha}_2)}{\bar{\alpha}_2}\frac{\partial \bar{\alpha}_2}{\partial p_k}$$

General structure of phase transitions:

Surface of phase transition:

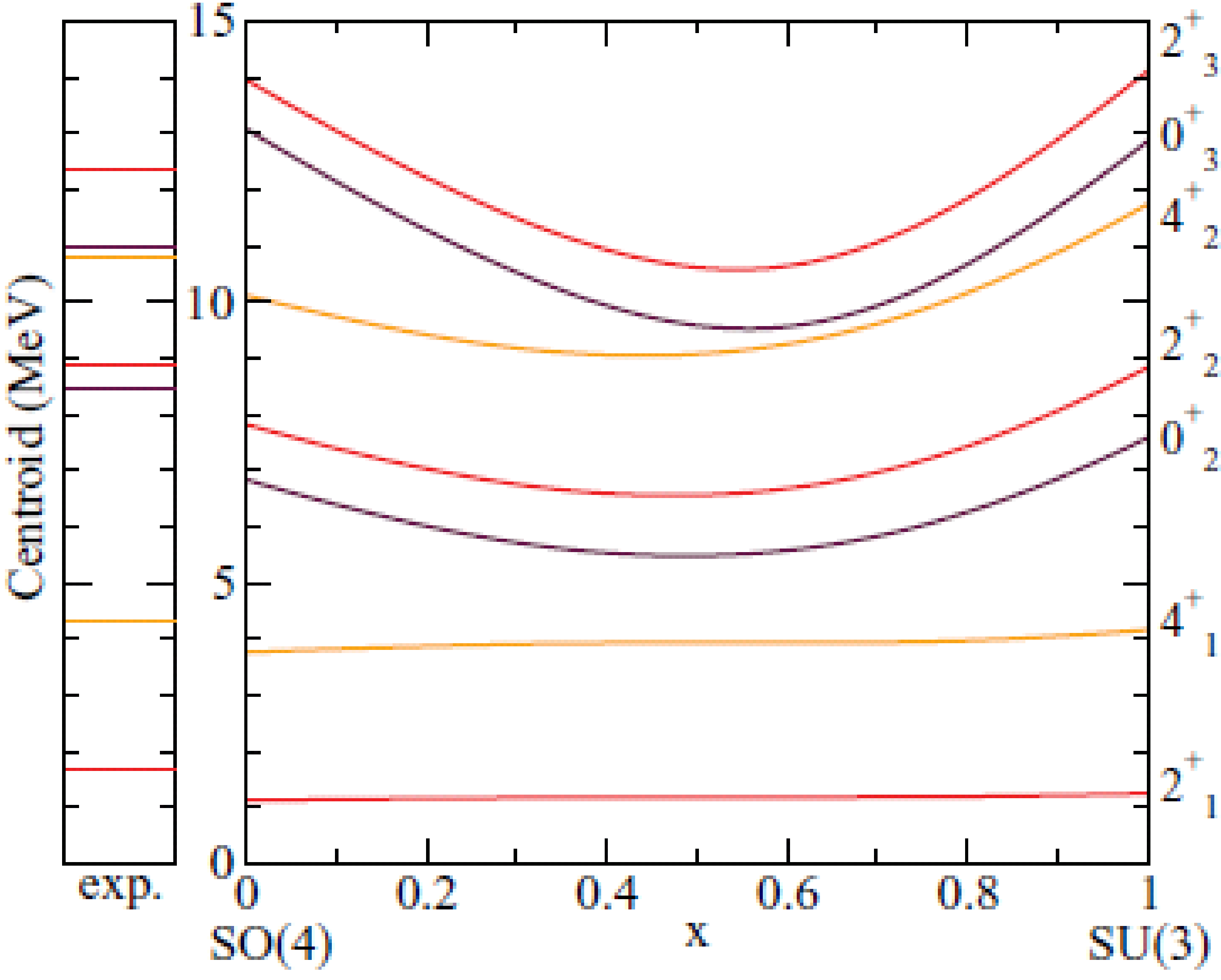


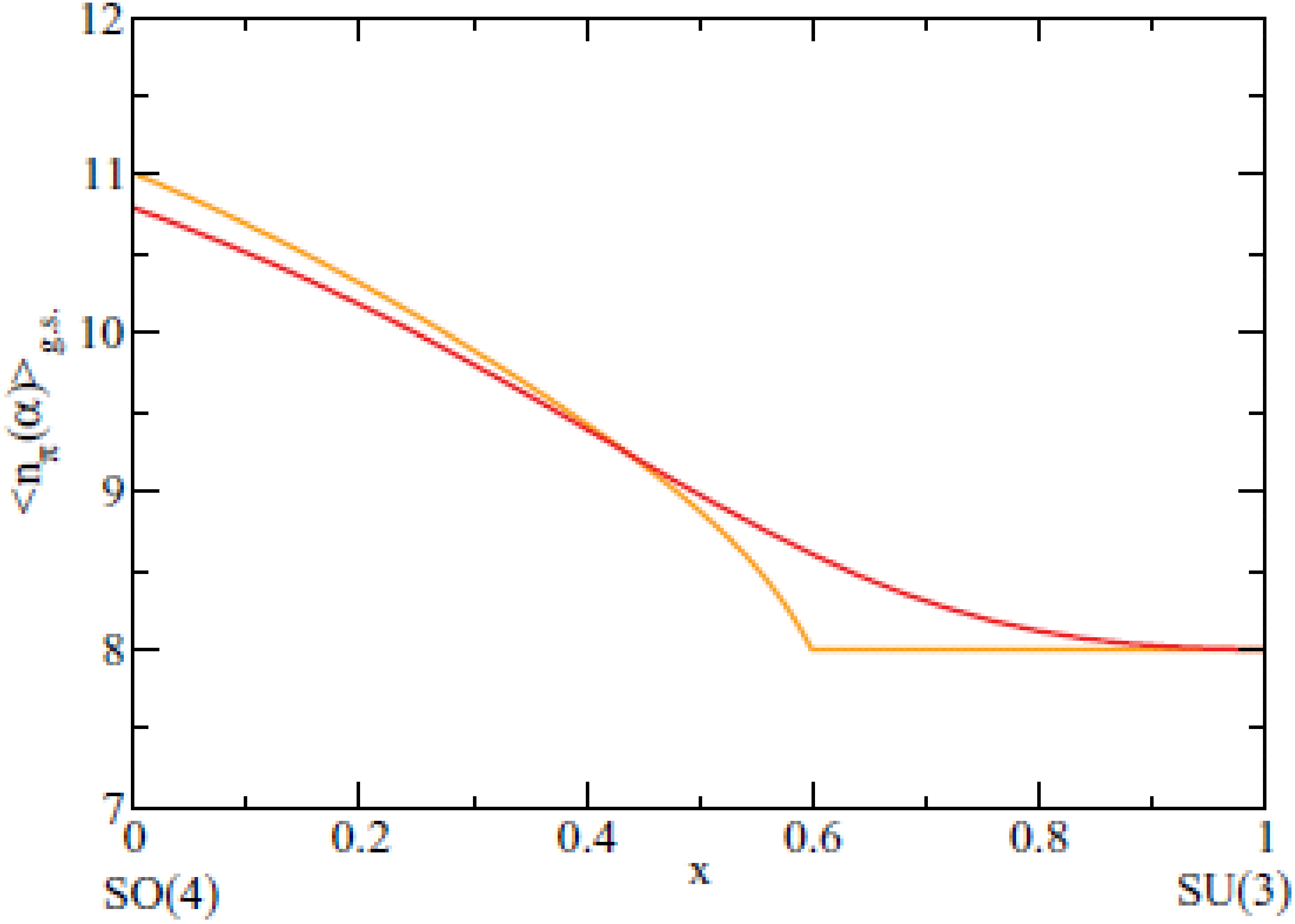
Value of α AT the point of phase transition:



- α=0 → second order
- α>0 → first order

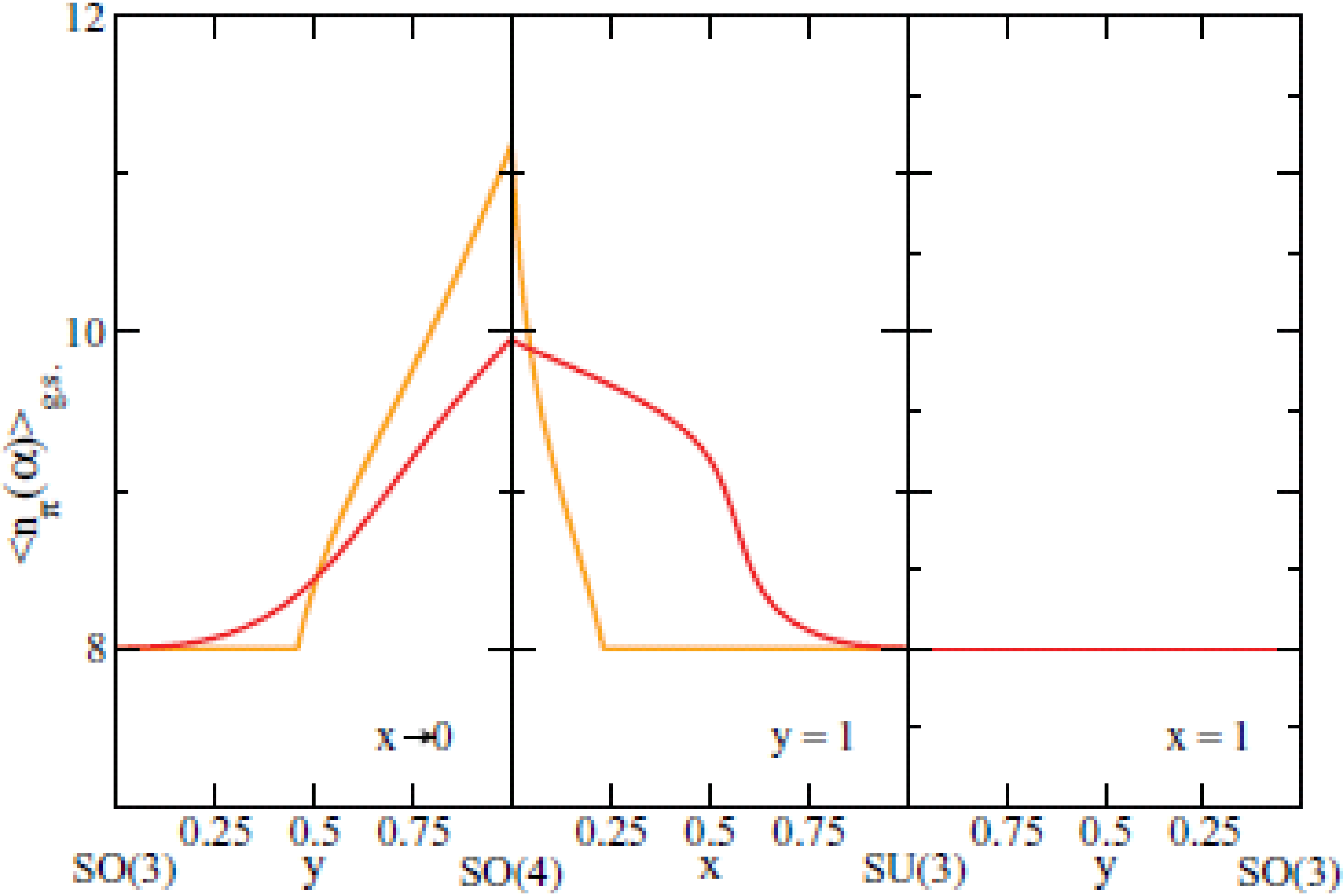
16O+alpha → 20Ne:

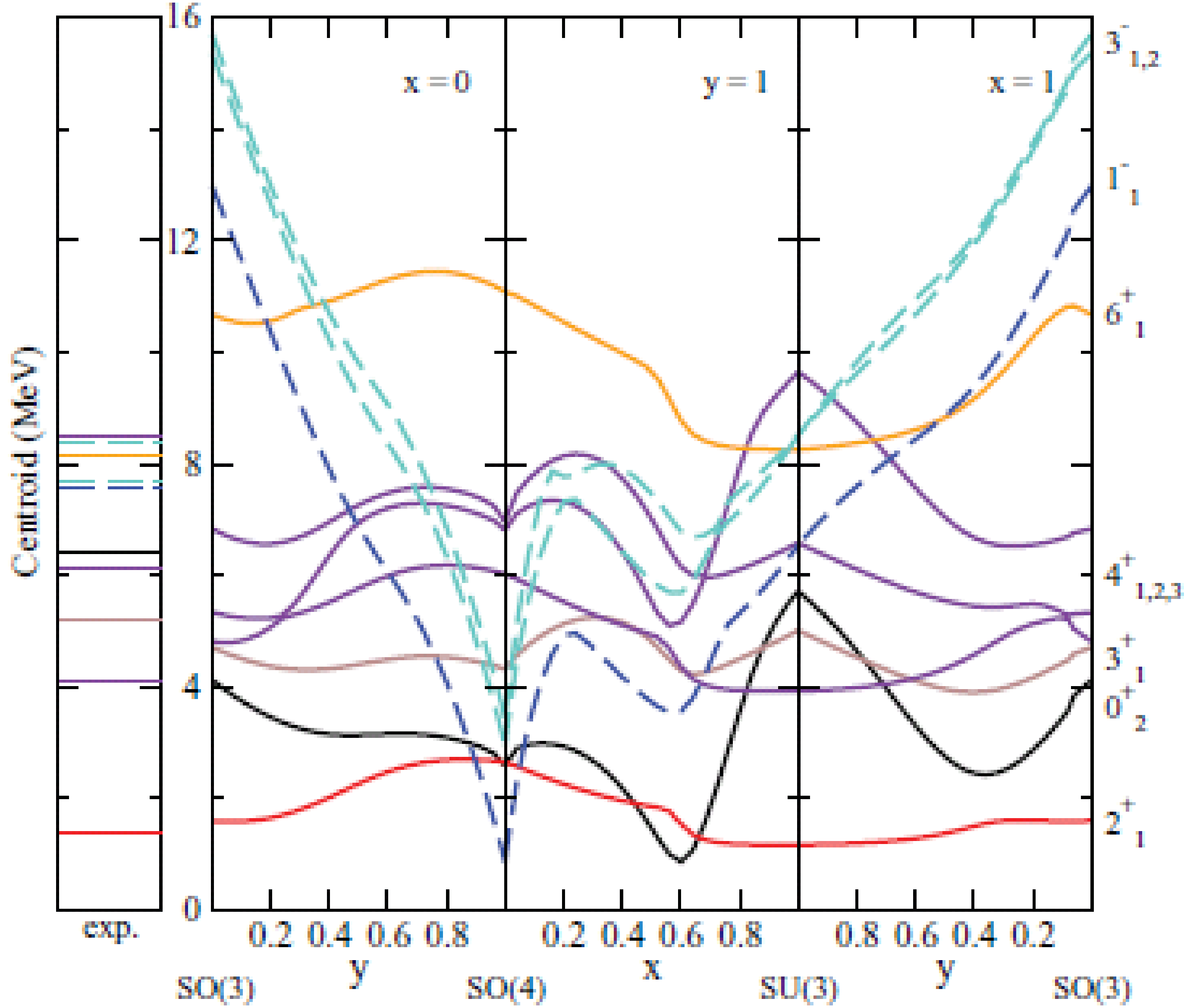




12
11
10
9
8
7
0
0.2
0.4
0.6
0.8
1
x
SO(4)
SU(3)
$\langle n_g(\alpha)\rangle_{g.s.}$

20Ne+alpha → 24Mg:





Centroid (MeV)
16
12
8
4
0
exp.
x = 0
y = 1
x = 1
0.2 0.4 0.6 0.8
0.2 0.4 0.6 0.8
0.8 0.6 0.4 0.2
y
x
y
SO(3)
SO(4)
SU(3)
SO(3)
$3^-_{1,2}$
1^-_1
6^+_1
$4^+_{1,2,3}$
3^+_1
0^+_2
2^+_1

The PACM

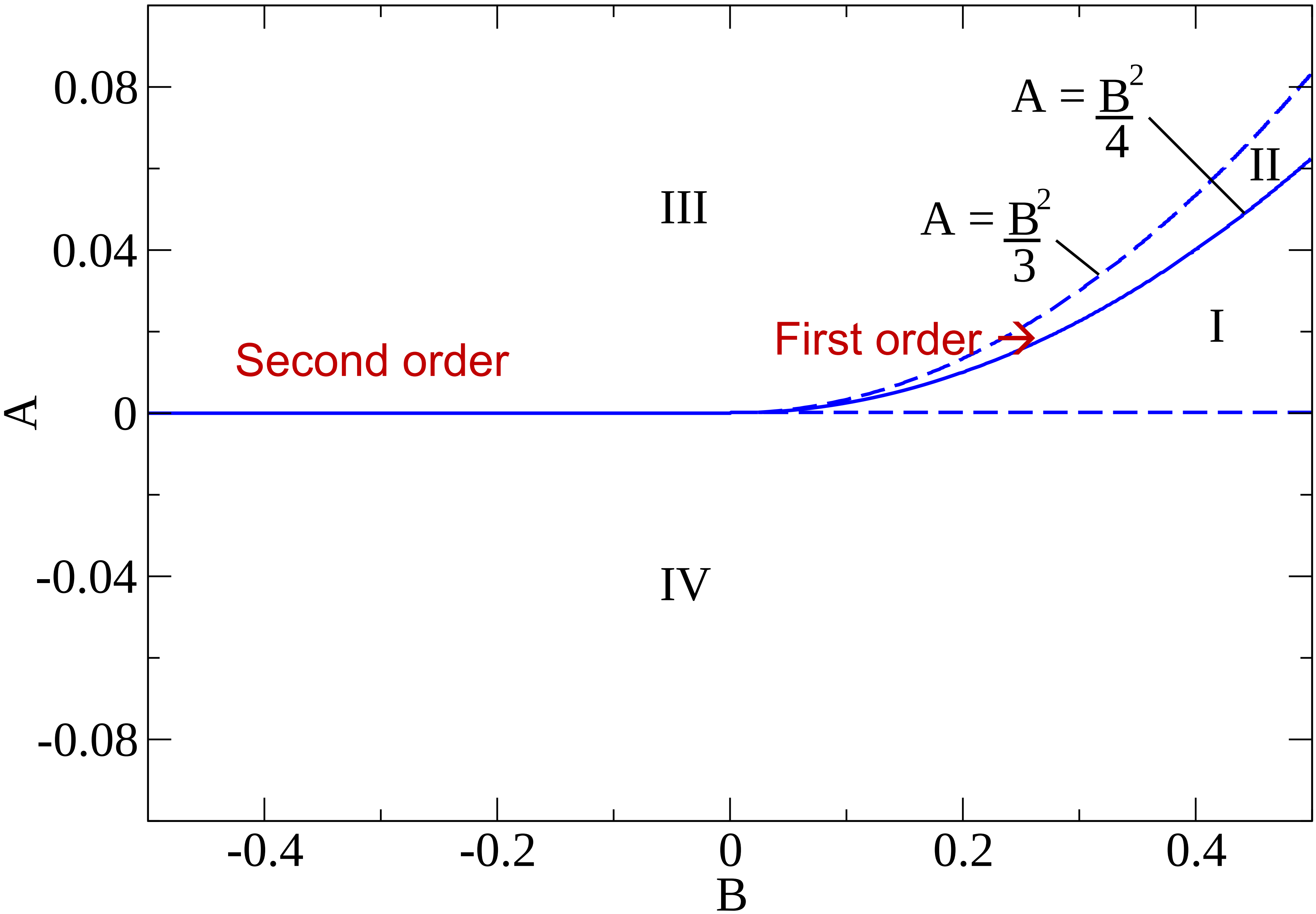
USE: $\beta^2 = \frac{\alpha^2}{1+\alpha^2}$

$$
\begin{aligned}
V &= N(N-1)(N-2)(-(b+\bar{b})xy)\left\{A\beta^2 - B\beta^4 + \beta^6\right\} \\
&\quad +\mathcal{C} \quad . \qquad (32)
\end{aligned}
$$

$$
\tilde{V} = \left\{A\beta^2 - B\beta^4 + \beta^6\right\}
$$

$$
\begin{aligned}
A &= -\left[(b+\bar{b}\,)\,xy(N-1)(N-2)\right]^{-1} \\
&\quad \times \Big[\hbar\omega\,(yx+1-y) + 2\left(\gamma + (1-yx)a_R^{(1)}\right) \\
&\qquad + 4xy\left(\overline{a} - \overline{b}\right) + xy\,(a-b)\,(4+\Gamma_1+\Gamma_2) \\
&\qquad - bxyC_2\,(\lambda_C,\mu_C) - y\,(1-x)\,c\,(N-1)\Big] \\
B &= \frac{xy\left(a+\overline{a} - 6\left(b+\overline{b}\right) - b\,(\Gamma_1+\Gamma_2)\right) + cy\,(1-x)}{(N-2)\left(b+\overline{b}\,\right)xy} \\
\mathcal{C} &= \left\langle (a_{Clus}+a)\,C_2\,(\lambda_C,\mu_C) + \gamma \boldsymbol{L_C}^2 + (1-xy)\,\boldsymbol{L_C}^2 \right\rangle \\
&\quad + xyt\langle \boldsymbol{K}^2\rangle + \frac{c}{4}N\,(N-1)\,y\,(1-x) \quad , \qquad (31)
\end{aligned}
$$

General structure of phase space transitions:



Conclusions

- Structure of phase space transitions in the SACM were investigated.
- Third order interaction already allow phase transitions of first order.
- Present in the SACM: First and second order AND the presence of a critical line.
- Without the Pauli exclusion principle, same results can be obtained but with a quite complicated Hamiltonian.