Relativistic symmetries in Hadrons and Nuclei

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Beauty in Physics: Theory and Experiment

Hotel Hacienda Cocoyoc, Mexico, May 14-18, 2012

Physics Reports 414 (2005) 165–261

Pseudospin Symmetry in Nuclei

More than thirty years ago a quasi-degeneracy was observed in single-nucleon doublets in nuclei with quantum numbers

$$(n, \ell, j), (n-1, \ell+2, \ j)$$

$$j = \tilde{\ell} \pm \tilde{s}, \quad \tilde{s} = 1/2$$

$\tilde{\ell}$ pseudo-orbital angular momentum, $\tilde{s}$ pseudospin

$2s_{1/2}$, $1d_{3/2}$ — $(\tilde{1})_{3/2, 1/2}$ $2p_{3/2}$, $1f_{5/2}$ — $(\tilde{2})_{5/2, 3/2}$

$1d_{3/2}$, $0g_{5/2}$ — $(\tilde{3})_{7/2, 5/2}$ $1f_{7/2}$, $0h_{9/2}$ — $(\tilde{4})_{9/2, 7/2}$

Hence a quasi-degeneracy in pseudospin

A. Arima, M. Harvey, K. Shimizu, Phys. Lett. B 30 (1969) 517

K.T. Hecht, A. Adler, Nucl. Phys. A 137 (1969) 129.

Rotational bands

built on different

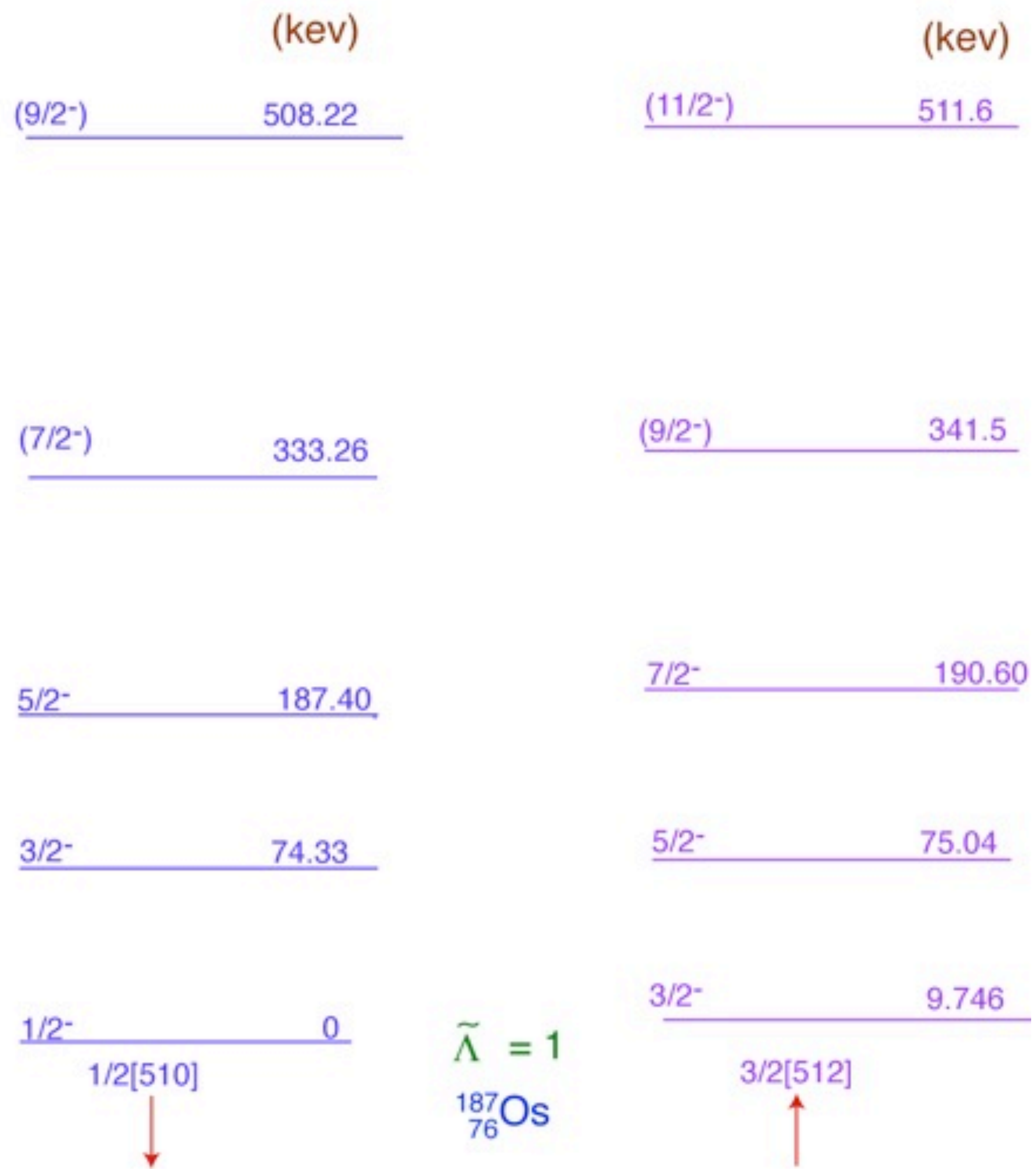
alignments of

pseudospin

along the body

fixed axis.

$\Omega[N\, n_3\, \Lambda]$



A. Bohr, I. Hamamoto, B.R. Mottelson, Phys. Scr. 26 (1982) 267.

The Dirac Hamiltonian

$$H = [\,\vec{\alpha}\cdot\vec{p} + \beta(m + V_S(\vec{r})) + V_V(\vec{r})]$$

α, β are the usual Dirac matrices

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

σ_i = Pauli matrices.

Nucleons move in a scalar, $V_S(\vec{r})$, and vector, $V_V(\vec{r})$, mean fields.

The Dirac Hamiltonian has an invariant SU(2) symmetry for two limits:

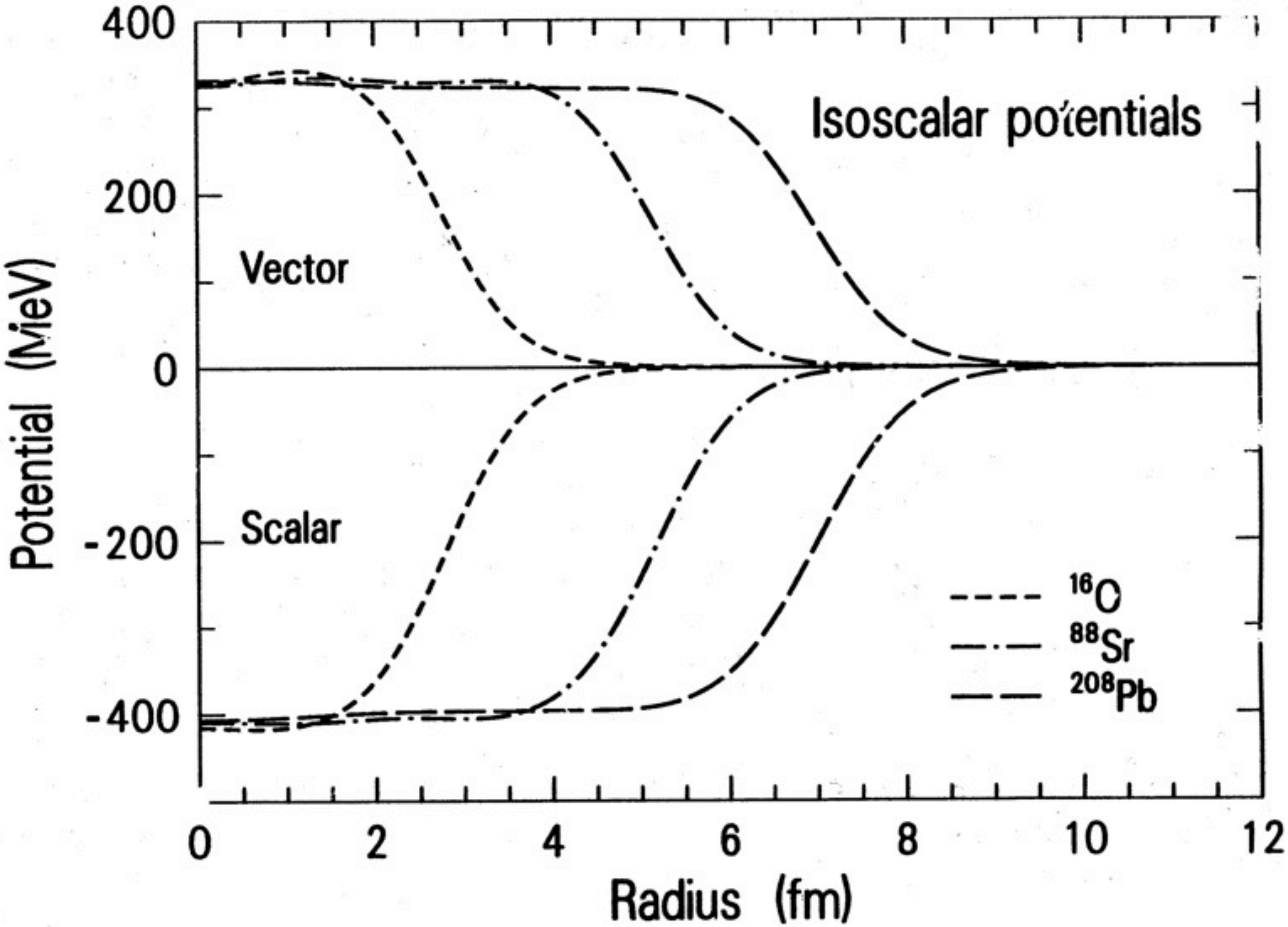
$V_s - V_v = C_s$ Spin Symmetry

$V_s + V_v = C_{ps}$ P-Spin Symmetry

Spin Symmetry occurs in the spectrum of a:

1) meson with one heavy quark (PRL 86, 204 (2001))
2) anti-nucleon bound in a nucleus (Phys. Rep. 315, 231 (1999))

Pseudospin Symmetry occurs in the spectrum of nuclei
PRL 78, 436 (1997)



Isoscalar potentials
Vector
Scalar
Potential (MeV)
Radius (fm)
400
200
0
-200
-400
0
2
4
6
8
10
12
16O
88Sr
208Pb

Figure 9.

QCD SUM RULES

$$\frac{V_S}{V_V} \approx -\frac{\sigma_N}{8m_q}$$

σ_N is the chiral symmetry breaking nucleon sigma term

m_q is the average quark mass

$$\sigma_N \approx 45\ MeV, m_q \approx 5\ MeV$$

$$\frac{V_S}{V_V} \approx\ -1.1$$

Uncannily close to the ratio of central values of mean field potentials

T.D. Cohen, R.J. Furnstahl, D.K. Griegel, X. Jin, Prog. Part. Nucl. Phys. 35 (1995) 221.

Pseudospin Generators

$$\vec{\tilde{S}} = \begin{pmatrix} \vec{\tilde{s}} & 0 \\ 0 & \vec{s} \end{pmatrix}$$

$$\vec{s} = \vec{\sigma}/2 \qquad \vec{\tilde{s}} = U_p \vec{s} U_p \qquad U_p = \vec{\sigma} \cdot \hat{p}$$

These pseudospin generators commute with the Dirac Hamiltonian in the pseudospin limit independent of the form of the potentials, spherical, deformed, triaxial:

$$V_S(\vec{r}) = -V_V(\vec{r}) + C_{ps}$$

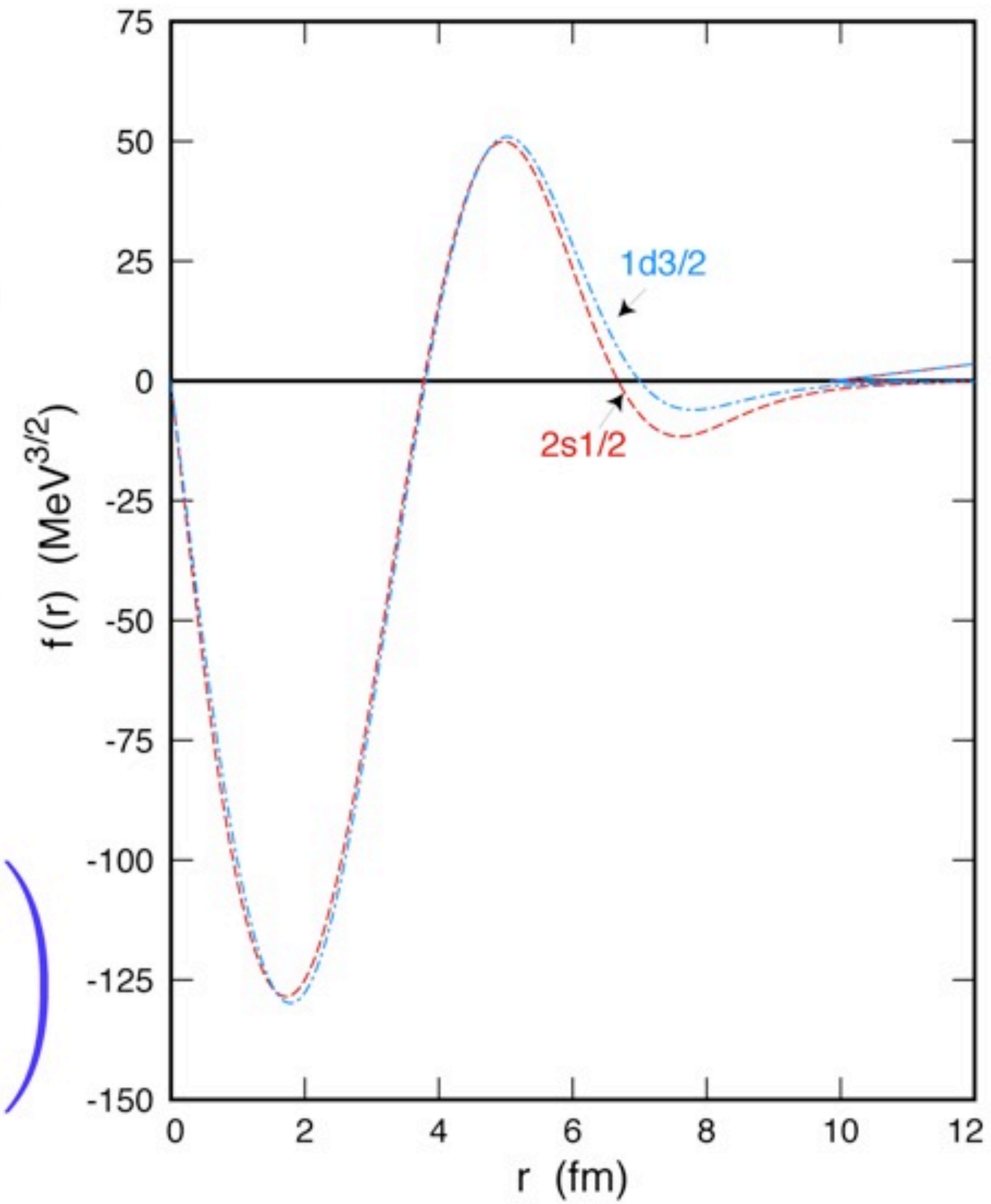
and have spin-like commutation relations

$$[H_{ps}, \vec{\tilde{S}}_i] = 0 \qquad [\vec{\tilde{S}}_i, \vec{\tilde{S}}_j] = i\varepsilon_{ijk} \vec{\tilde{S}}_k$$

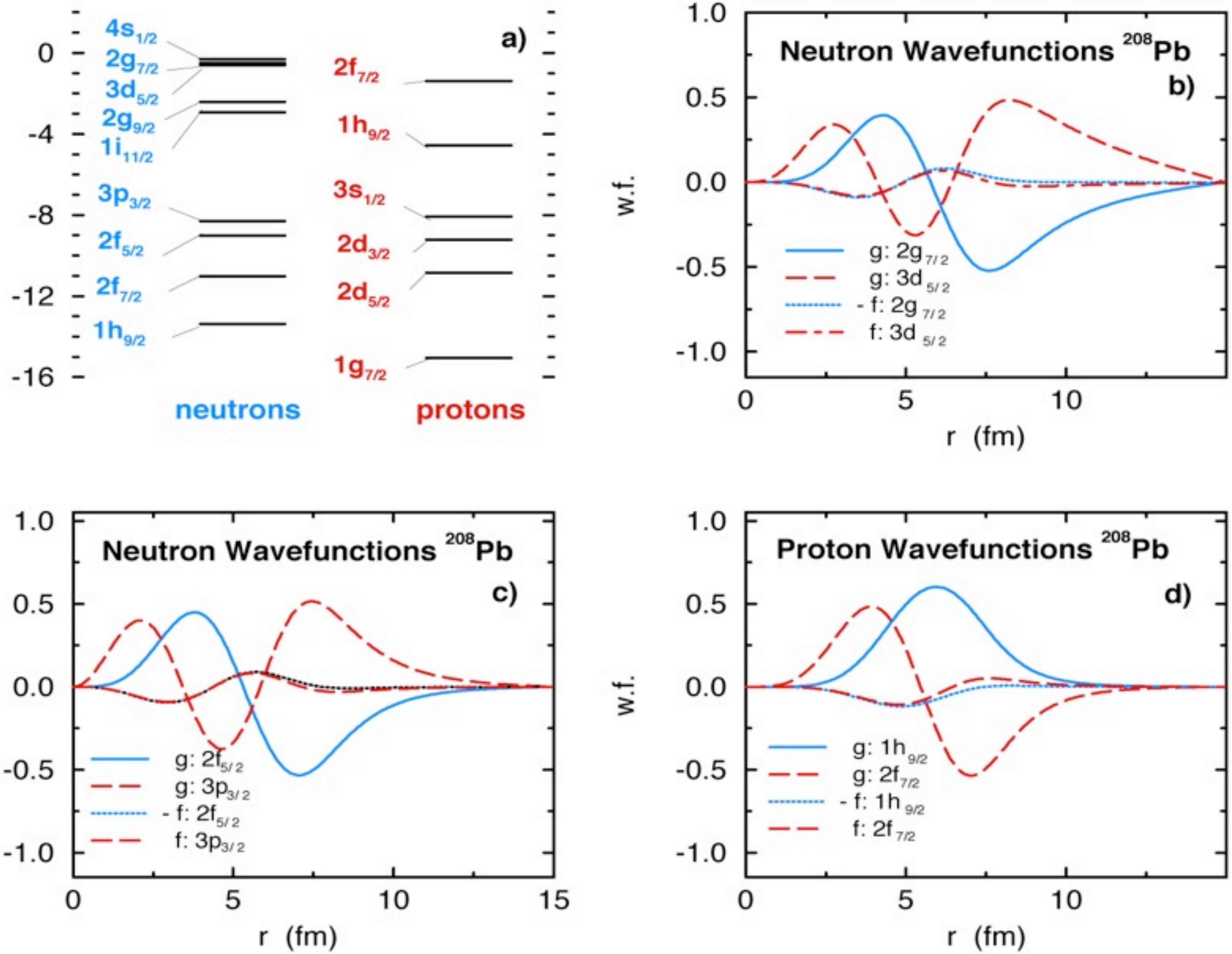
A. L. Blokhin, C. Bahri, and J. P. Draayer, Phys. Rev. Lett. 74, 4149 (1995).

Pseudospin Symmetry predicts that the lower spatial amplitudes of the two eigenstates in the doublets are equal

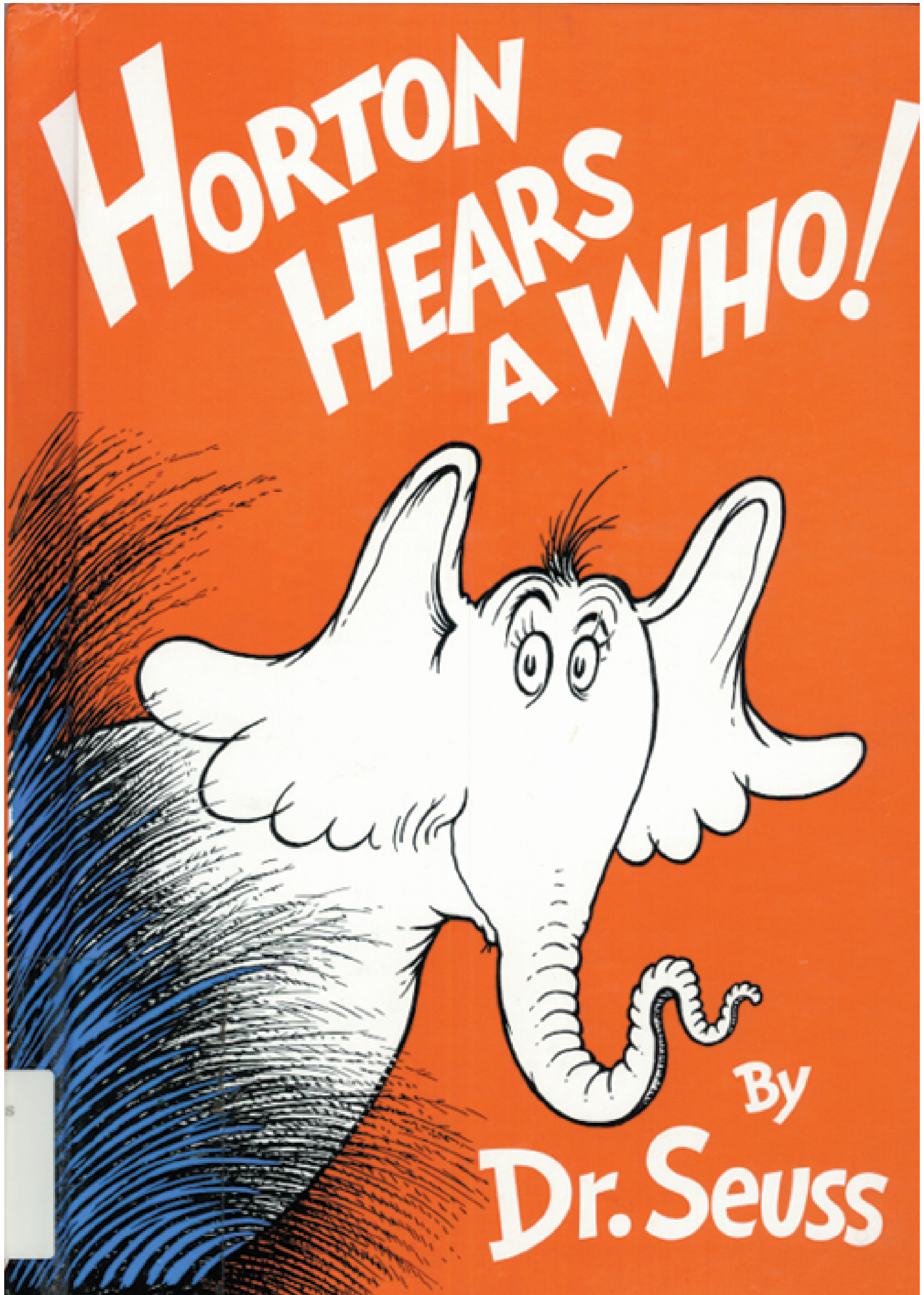
$$\hat{S}_i = \begin{pmatrix} \hat{\tilde{s}}_i & 0 \\ 0 & \hat{s}_i \end{pmatrix}$$



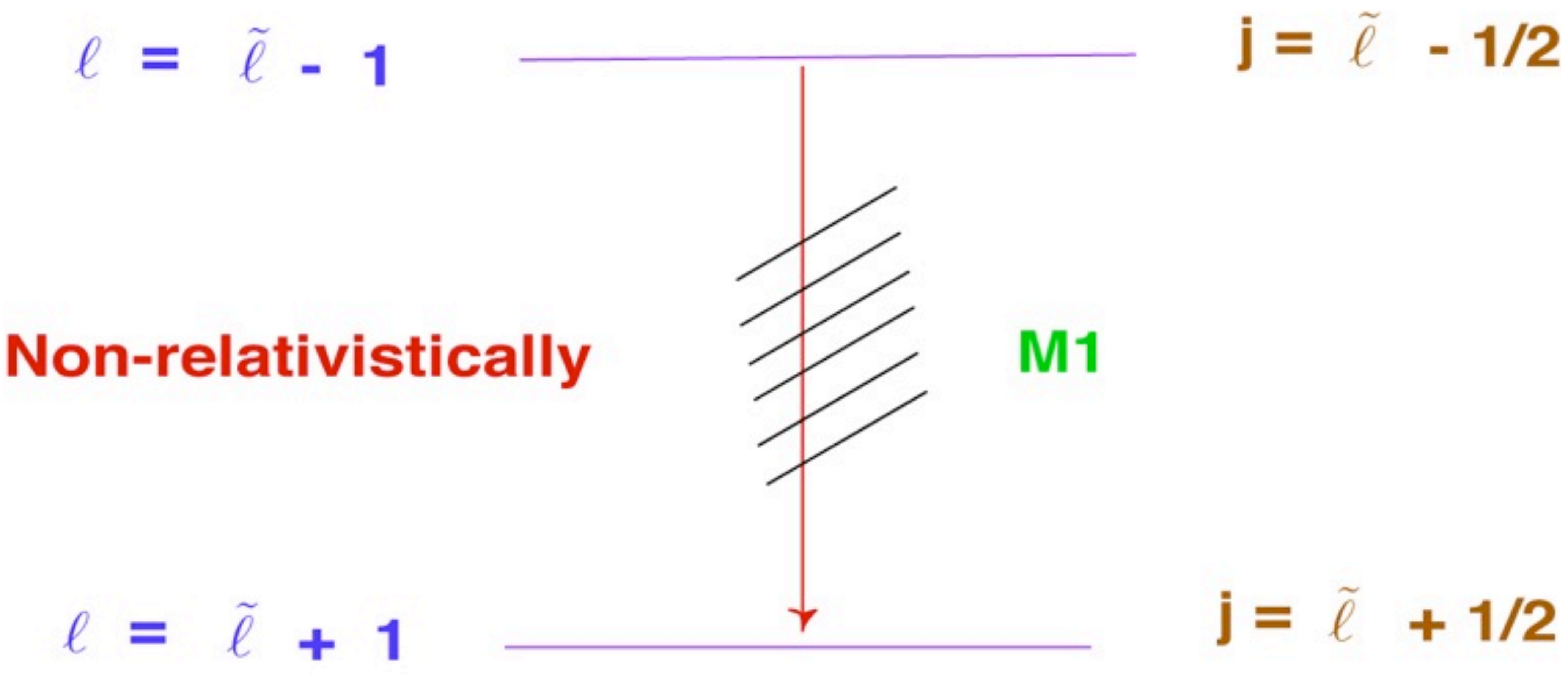
Upper (g) and Lower (f) Radial Wavefunctions



J. Meng, K. Sugawara-Tanabe, S. Yamaji, P. Ring, A. Arima, Phys. Rev. 58 (1998) R628.


HORTON HEARS A WHO!
By
Dr. Seuss


Magnetic Transitions between Pseudospin doublets



Magnetic Transitions for Pseudospin Symmetry

$$B(M1 : j' \rightarrow j\,)_\nu = \frac{j+1}{2j+1}\left[\mu_{j,\nu} - \mu_{A,\nu}\right]^2 \qquad j' = \tilde{\ell} + 1/2$$

$$B(M1 : j' \rightarrow j\,)_\nu = \frac{2j+1}{j+1}\left[\frac{j+2}{2j+3}\left(\mu_{j',\nu} + \frac{j+1}{j+2}\mu_{A,\nu}\right)\right]^2 \qquad j = \tilde{\ell} - 1/2$$

$$\frac{B(M1; j' \rightarrow j)_\nu}{S_j S_{j'}} = \frac{j+1}{2j+1}\left(\frac{\mu_{j,\nu}}{S} - \mu_{A,\nu}\right)^2 \qquad j' = \tilde{\ell} + 1/2$$

$$\frac{B(M1; j' \rightarrow j)_\nu}{S_j\, S_{j'}} = \frac{2j+1}{j+1}\left(\frac{j+2}{2j+3}\right)^2\left(\frac{\mu_{j'}}{S_{j'}} + \frac{j+1}{j+2}\,\mu_{A,\nu}\right)^2 \qquad j = \tilde{\ell} - 1/2$$

Magnetic Transitions for Pseudospin Symmetry

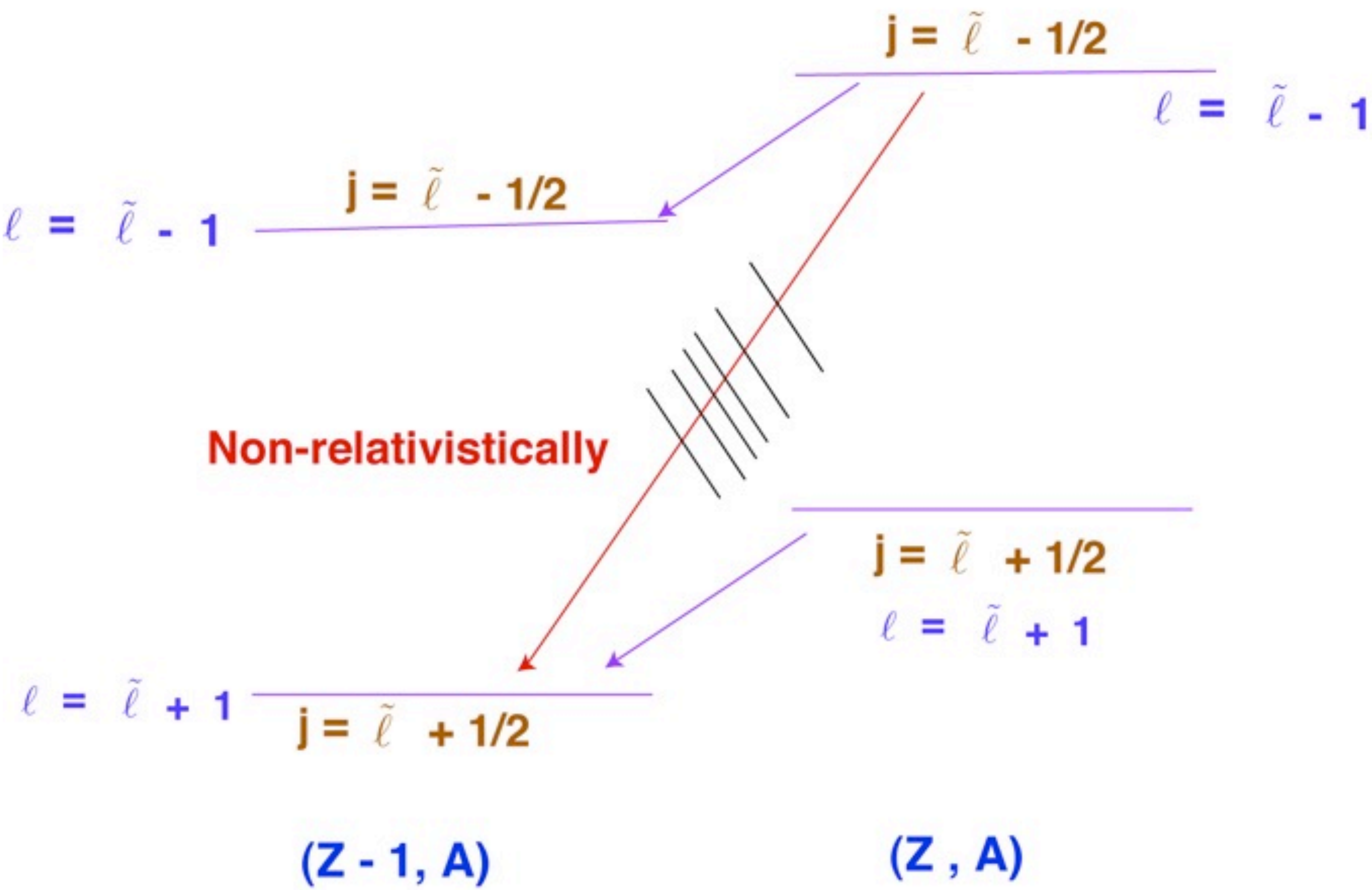
$$B(M1: j' \to j\ )_\nu = \frac{j+1}{2j+1}\left[\mu_{j,\nu} - \mu_{A,\nu}\right]^2 \qquad j' = \tilde{\ell} + 1/2$$

$$B(M1: j' \to j\ )_\nu = \frac{2j+1}{j+1}\left[\frac{j+2}{2j+3}\left(\mu_{j',\nu} + \frac{j+1}{j+2}\mu_{A,\nu}\right)\right]^2 \qquad j = \tilde{\ell} - 1/2$$

Predicted "ℓ forbidden" magnetic dipole transition in ^{39}Ca.

	$B(M1: j, \nu \to j', \nu)$
Predicted Equation	0.0166
Predicted Equation	0.0121
EXP	0.0121 (14)

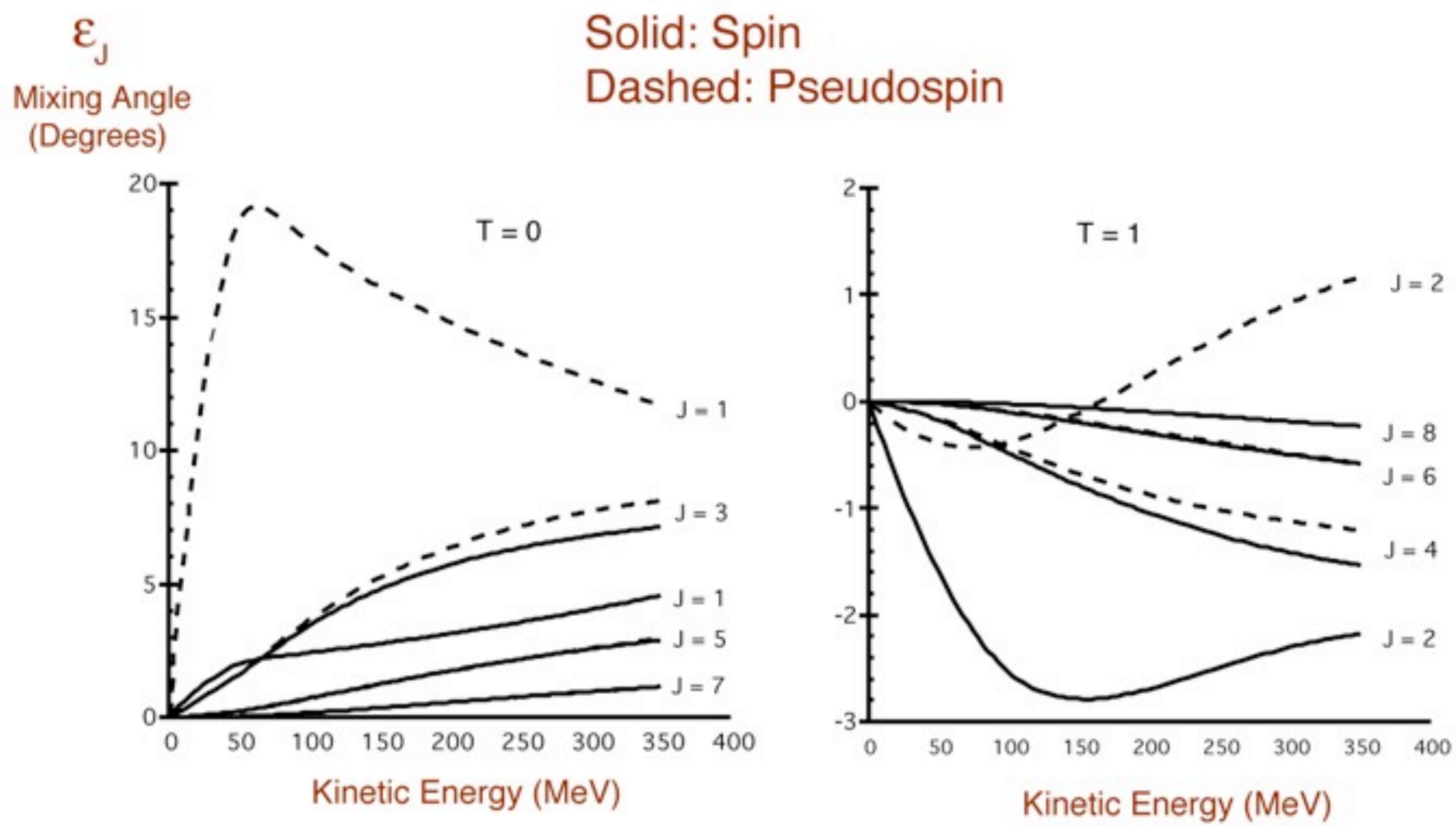
Gamow-Teller Transitions between Pseudospin doublets



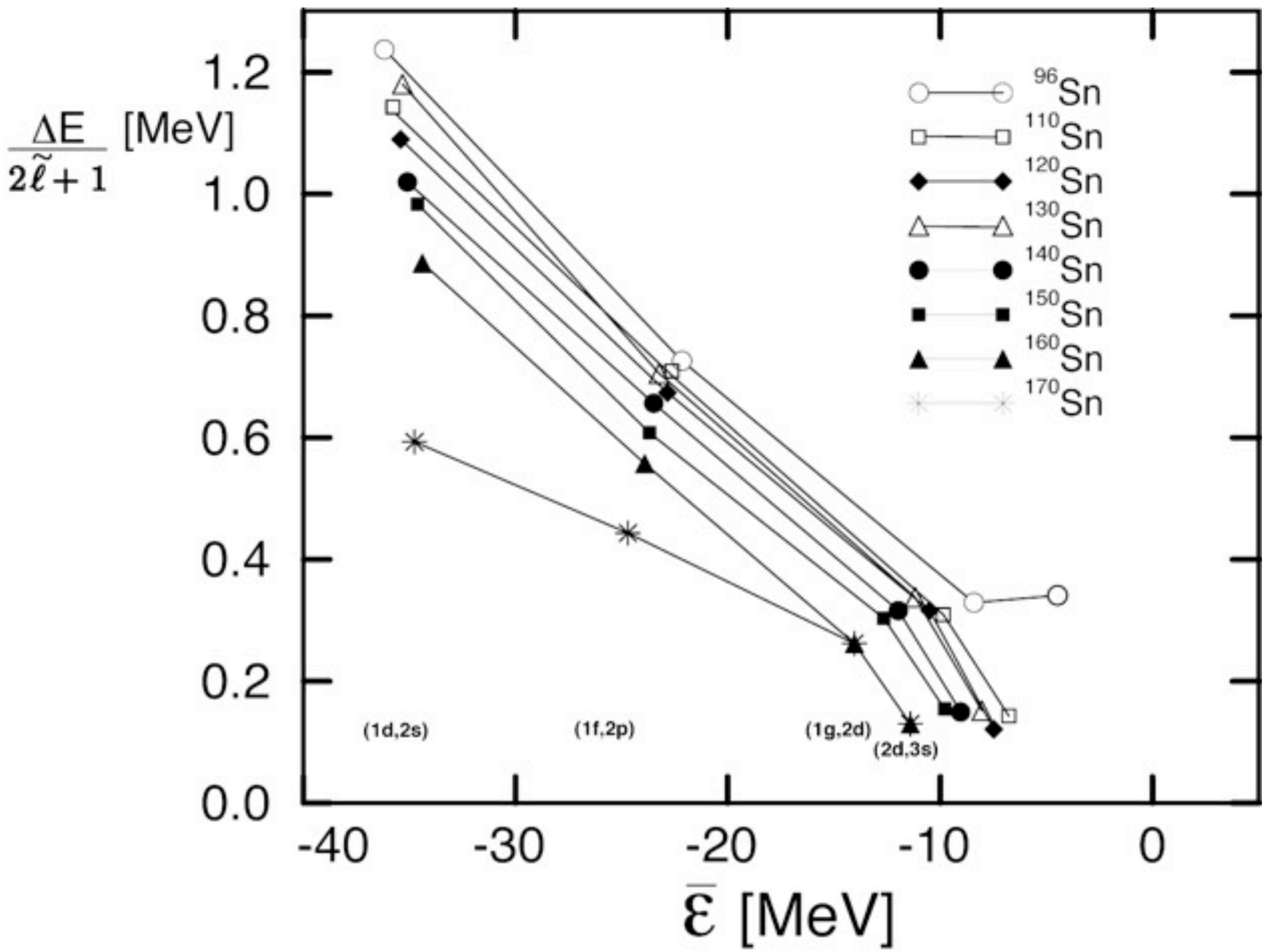
Isovector Effects

Nucleon-Nucleon Scattering

Mixing Angles



Hence we would expect pseudospin symmetry to improve with neutron or proton excess. This isospin dependence can be tested in RIA experiments



The pseudospin energy splitting as a function of the average energy of the doublets. Symmetry improves closer to the Fermi sea and with neutron excess.

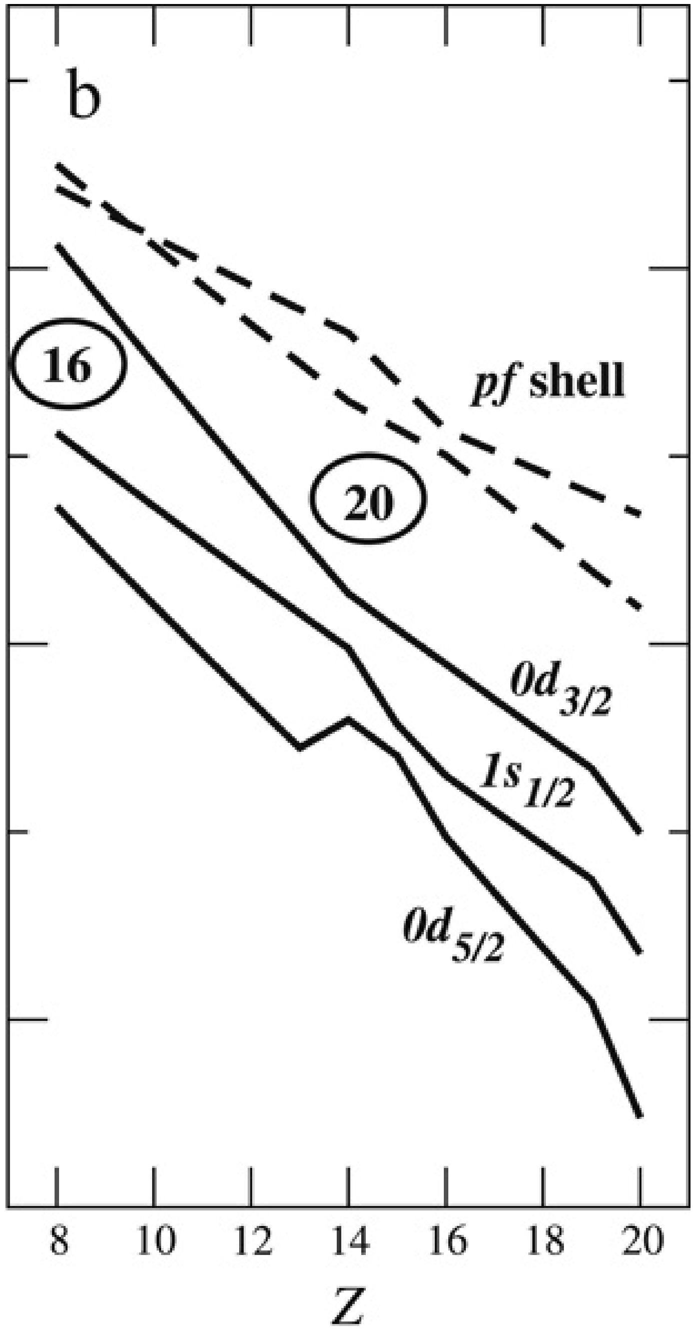
J. Meng, K. Sugawara-Tanabe, S. Yamaji, A. Arima, Phys. Rev. C 59 (1999) 154.

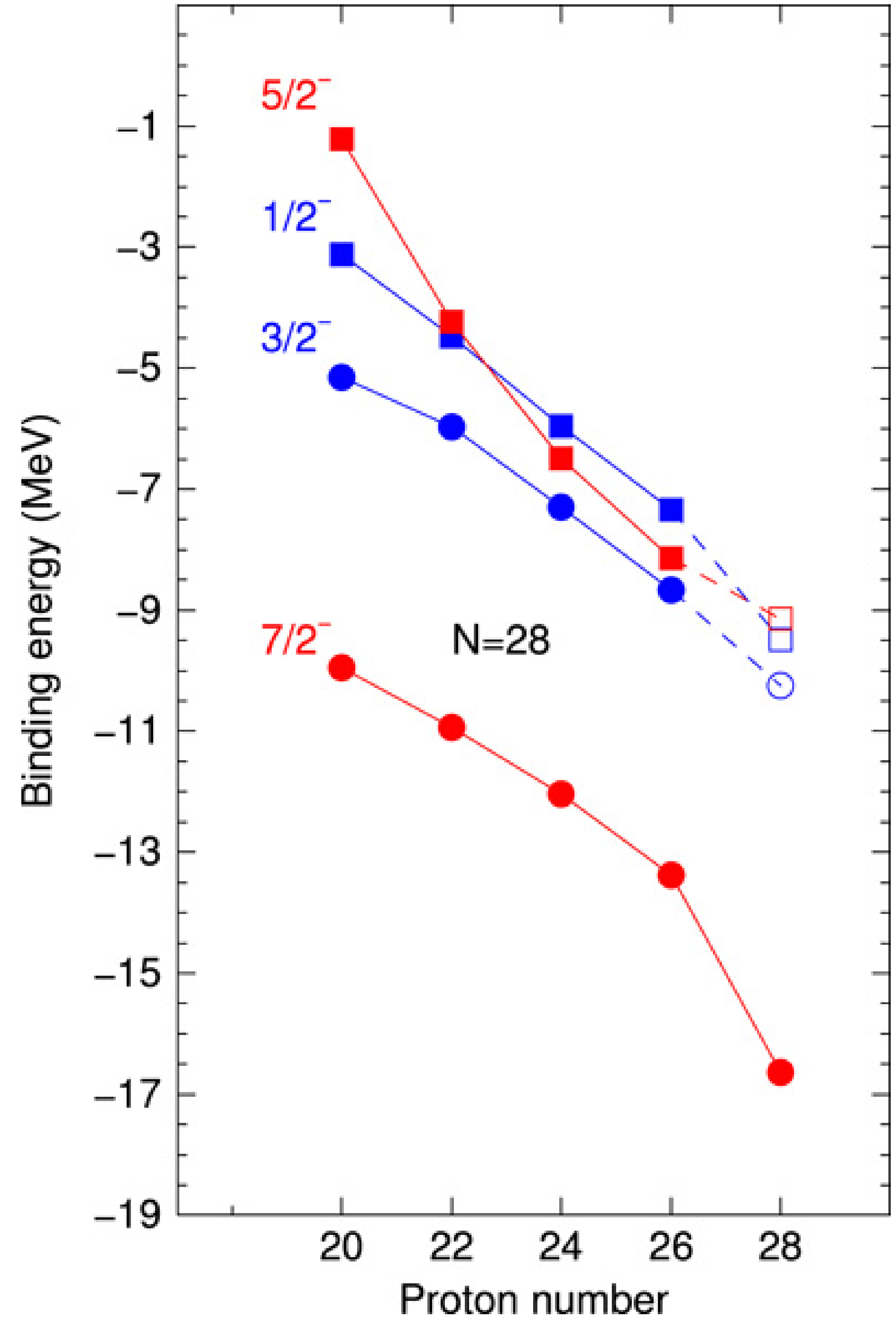
PHYSICAL REVIEW C **81**, 031302(R) (2010)

Nuclear halo structure and pseudospin symmetry

Wen Hui Long , Peter Ring, Jie Meng , Nguyen Van Giai, and Carlos A. Bertulani

Nuclear halo structure and conservation of relativistic symmetry are studied within the framework of the relativistic Hartree-Fock-Bogoliubov theory. Giant halos as well as ordinary ones are found in cerium isotopes close to the neutron drip line. The conservation of pseudospin symmetry on the proton side plays an essential role in stabilizing the neutron halo structures.



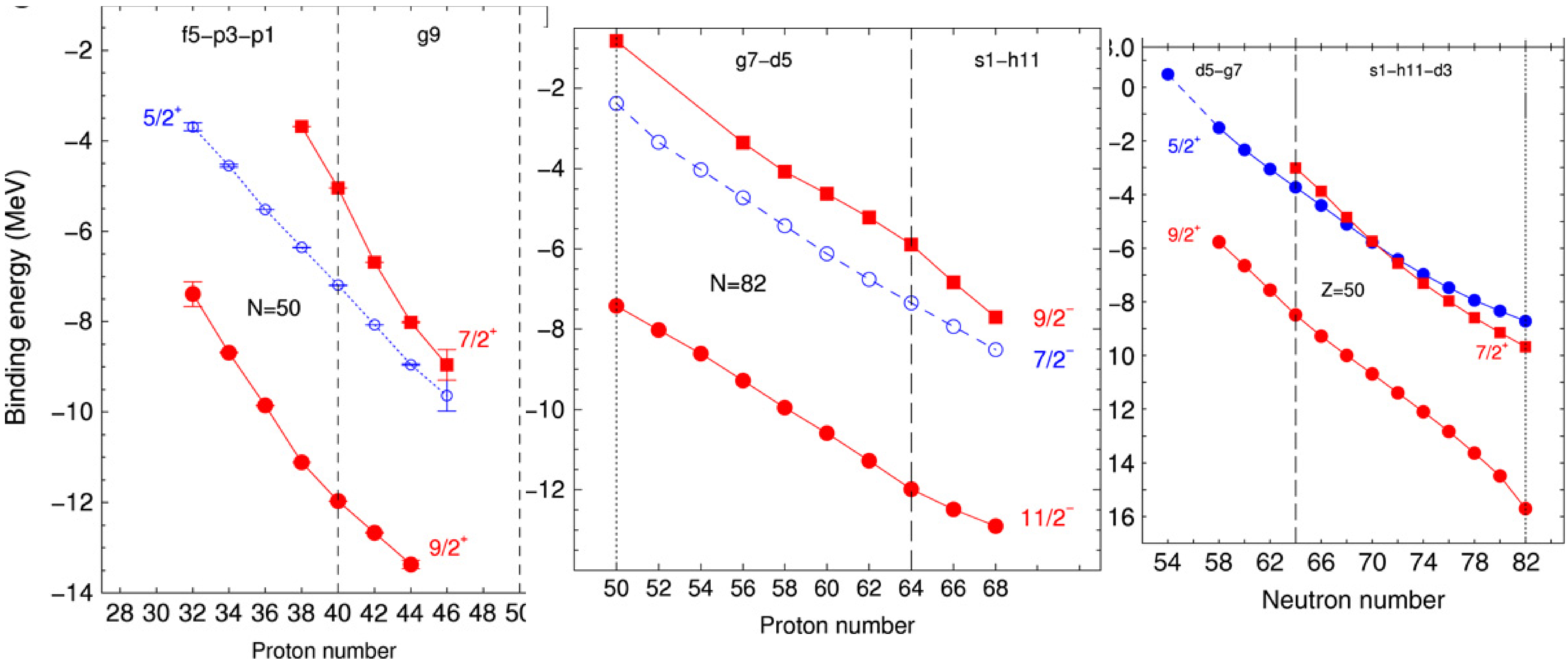


$N = 20$ isotones with $8 < Z < 20$

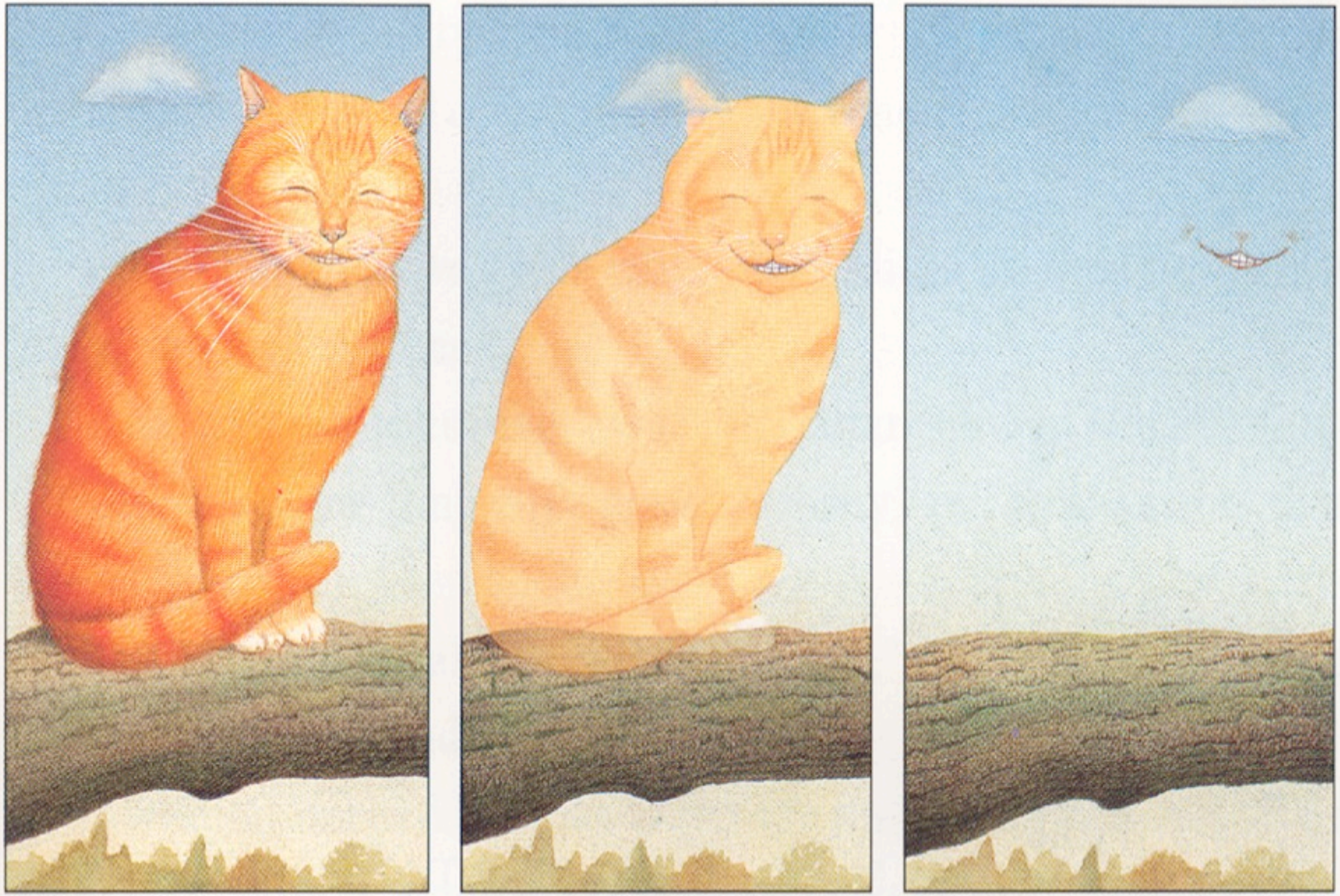
N = 28 isotones

Experimental Data

Progress in Particle and Nuclear Physics 61 (2008) 602–673



Progress in Particle and Nuclear Physics 61 (2008) 602–673



"Well! I've often seen a cat without a grin," thought Alice, "but a grin without a cat! It's the most curious thing I ever saw in all my life!"
Lewis Carroll, *Alice's Adventures in Wonderland*

Antinucleon Spectrum

Charge Conjugation

$$\bar{V}_S(\vec{r}) = C^{\dagger} V_S(\vec{r}) C = V_S(\vec{r})$$

$$\bar{V}_V(\vec{r}) = C^{\dagger} V_V(\vec{r}) C = -V_V(\vec{r})$$

$$\bar{V}_S(\vec{r}) \approx \bar{V}_V(\vec{r})$$

Anti-nucleon potential will have absorption potential as well. However, the vector and scalar absorption potentials must be equal so that under charge conjugation the sum of the them will be zero for nucleons. Therefore, the absorption potential will conserve spin symmetry as well.

Spin polarization in antiproton scattering from Carbon is almost zero supporting this prediction, but data set is limited (NPA 487, 563 (1988)).

Perhaps additional antiproton scattering will be forthcoming at GSI

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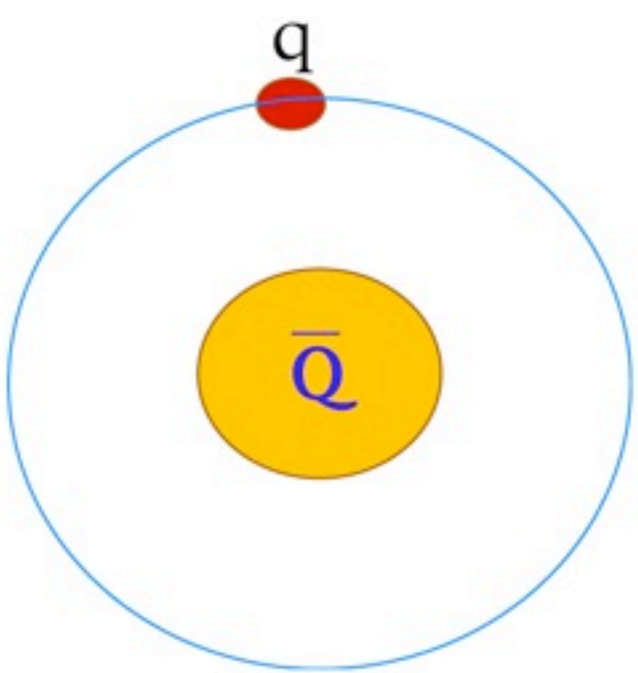
Dirac Hamiltonian has a spin symmetry if

$$V_S(x) = V_V(x) + C_S$$

Explains spin degeneracies of Light Quark - Heavy Quark Mesons



Phys. Rev. L ett. 86, 204 (2001); hep-ph/0002094

Relativistic Harmonic Oscillator

$$V_S(\vec{r}) = \frac{M}{2}\sum_{i=1}^{3} \omega_{S,i}^2 x_i^2$$

$$V_V(\vec{r}) = \frac{M}{2}\sum_{i=1}^{3} \omega_{V,i}^2 \, x_i^2$$

In the symmetry limits the eigenfunctions and eigenenergies can be solved analytically. The eigenfunctions are similar to the non relativistic limit. That is, the upper and lower components can be written in terms of Gaussians and Laguerre polynomials. This true for spherical and non-spherical harmonic oscillator.

JNG, PRC 69, 034318 (2004)

Energy Eigenvalues in the Spin Symmetry and Spherical Symmetry Limit

$$\omega_S = \omega_V = \omega$$

$$E_N = M\left[B(A_N) + \frac{1}{3} + \frac{4}{9\,B(A_N)}\right]$$

$$B(A_N) = \left[\frac{A_N + \sqrt{A_N^2 - \frac{32}{27}}}{2}\right]^{\frac{2}{3}}$$

$$A_N = C\,(N + \frac{3}{2}), C = \frac{\sqrt{2}\,\omega}{M},$$

$$N = 2n + \ell = 0, 1, \ldots.$$

Energy Eigenvalues in the Spin and Spherical Symmetry Limit

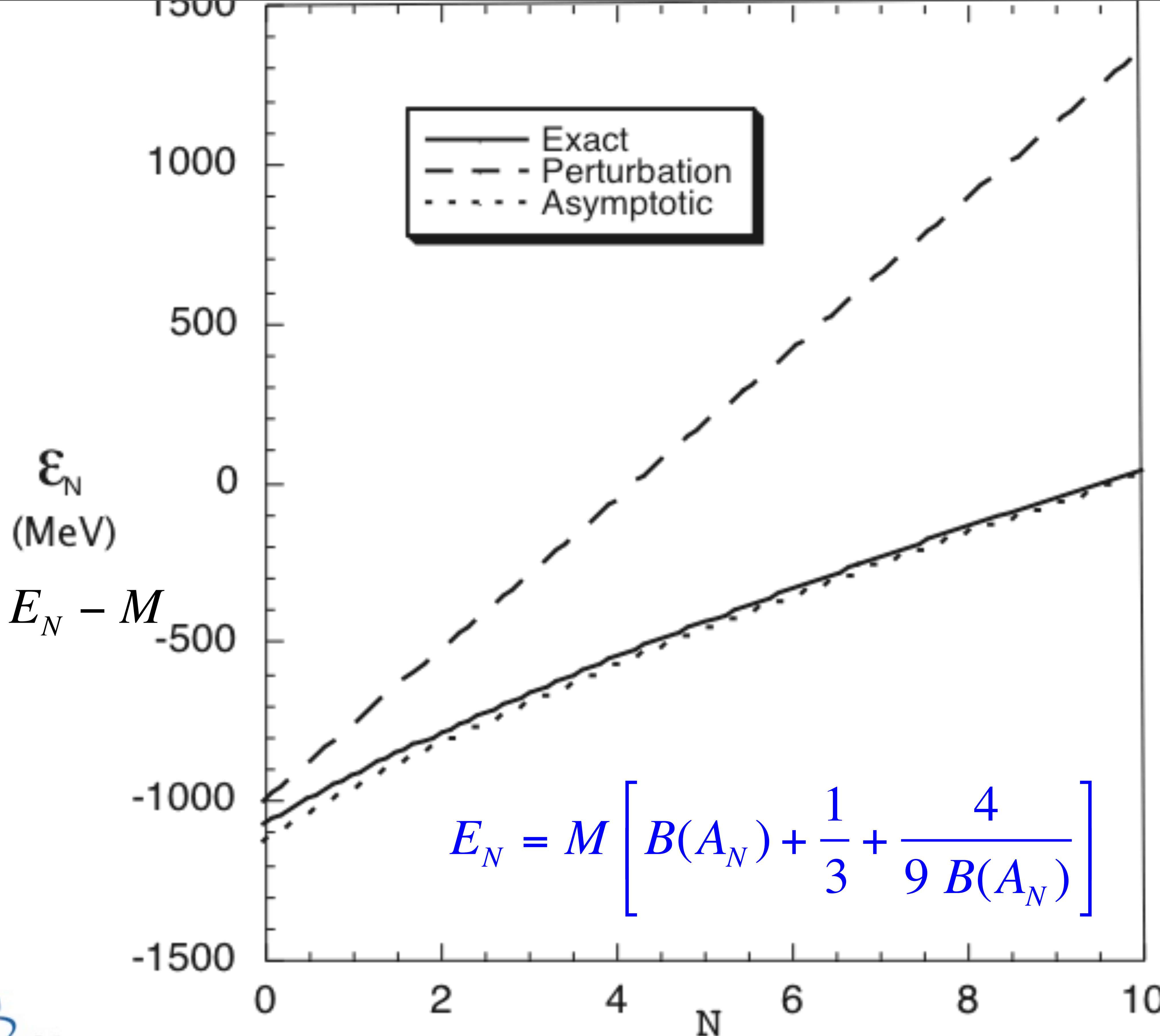
For the mass large compared to the potential, the eigenenergies go approximately like

$$E_N \approx M\,(1 + \frac{C\,(N + \frac{3}{2})}{\sqrt{2}} + \cdots)$$

that is linearly with N like the non-relativistic spectrum. For the mass small the spectrum goes like

$$E_N \approx M\,[C\,(N + \frac{3}{2})]^{\frac{2}{3}} + \frac{1}{3} + \cdots)$$

or approximately like N to the 2/3 power. Therefore the harmonic oscillator in the relativistic limit is not harmonic!



$$= E_N - M$$

$$E_N = M\left[B(A_N) + \frac{1}{3} + \frac{4}{9\,B(A_N)}\right]$$

Energy Eigenvalues in the Pseudospin Symmetry and Spherical Symmetry Limit

$$\omega_V = -\omega_S = \tilde{\omega}$$

$$E_{\tilde{N}} = M\left[B(A_{\tilde{N}}) + \frac{1}{3} + \frac{4}{9\,B(A_{\tilde{N}})}\right]$$

$$B(A_{\tilde{N}}) = \left[\frac{A_{\tilde{N}} + \sqrt{A_{\tilde{N}}^2 - \frac{32}{27}}}{2}\right]^{\frac{2}{3}}$$

$$A_{\tilde{N}} = C\,(\tilde{N} + \frac{3}{2}), C = \frac{\sqrt{2}\,\tilde{\omega}}{M},$$

$$\tilde{N} = 2\tilde{n} + \tilde{\ell} = 0,1,\ldots.$$

Spherical Relativistic Harmonic Oscillator
U(3) and p-U(3)

In these symmetry limits the energy depends only on N or $\tilde{N}$, as in the non-relativistic harmonic oscillator. In the non-relativistic case this is because the Hamiltonian has an U(3) symmetry.

Although the energy spectrum dependence on N is different than in the non-relativistic case, the Dirac Hamiltonian for the spherical harmonic oscillator has been shown to have an U(3) symmetry for the spin symmetry limit and a pseudo-U(3) in the pseudosin limit.

JNG, PRL 95, 252501 (2005)

Generators of Spin and U(3) Symmetry

$$\vec{S} = \begin{pmatrix} \vec{s} & 0 \\ 0 & U_p \vec{s}\, U_p \end{pmatrix}, \vec{L} = \begin{pmatrix} \vec{\ell} & 0 \\ 0 & U_p \vec{\ell}\, U_p \end{pmatrix}$$

$$\vec{s} = \vec{\sigma}/2, \ \vec{\ell} = \frac{(\vec{r} \times \vec{p})}{\hbar}, \ U_p = \frac{\vec{\sigma} \cdot \vec{p}}{p}$$

Quadrupole Generators

Non-relativistic quadrupole generator:

$$q_m = \frac{1}{\hbar M\omega}\sqrt{\frac{3}{2}}\left(2M^2\omega^2[rr]_m^{(2)} + [pp]_m^{(2)}\right)$$

From the examples of the spin and orbital angular mometum the following ansatz seems plausible but, in fact, does not work:

$$Q_m = \begin{pmatrix} q_m & 0 \\ 0 & U_p\, q_m\, U_p \end{pmatrix}$$

Quadrupole Generators

$$Q_m = \begin{pmatrix} (Q_m)_{11} & (Q_m)_{12}\,\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p}\,(Q_m)_{21} & \vec{\sigma}\cdot\vec{p}\,(Q_m)_{22}\,\vec{\sigma}\cdot\vec{p} \end{pmatrix},$$

Conditions for Generators to Commute with the Hamiltonian

$$[Q_m, H] = 0$$

Conditions for Generators to Commute with the Hamiltonian

$$[Q_m, H] = 0$$

implies that

$$(Q_m)_{12} = (Q_m)_{21},$$

$$2[(Q_m)_{11}, V] + [(Q_m)_{12}, p^2] = 0,$$

$$2[(Q_m)_{12}, V] + [(Q_m)_{22}, p^2] = 0,$$

$$(Q_m)_{11} = (Q_m)_{12}\, 2(V + M) + (Q_m)_{22}\, p^2.$$

A solution is:

$$Q_m = \lambda_2 \begin{pmatrix} M\omega^2 (M\omega^2 r^2 + 2M)[rr]_m^{(2)} + [pp]_m^{(2)} & M\omega^2 [rr]_m^{(2)} \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p}\, M\omega^2 [rr]_m^{(2)} & [pp]_m^{(2)} \end{pmatrix}$$

A solution is:

$$Q_m = \lambda_2 \begin{pmatrix} M\omega^2 (M\omega^2 r^2 + 2M)[rr]_m^{(2)} + [pp]_m^{(2)} & M\omega^2 [rr]_m^{(2)} \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} M\omega^2 [rr]_m^{(2)} & [pp]_m^{(2)} \end{pmatrix}$$

where the norm is determined by the by the commutation relations

$$\lambda_2 = \sqrt{\frac{3}{M\omega^2\hbar^2(H+M)}}$$

JNG, PRL 95, 252501 (2005)

Number of Quanta Operator

$$\hat{N} = \lambda_0 \begin{pmatrix} M\omega^2 (M\omega^2 r^2 + 2M)r^2 + p^2 & M\omega^2 r^2 \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p}\, M\omega^2 r^2 & p^2 \end{pmatrix} - \frac{3}{2}$$

$$\lambda_0 = \frac{1}{2\hbar\sqrt{(H + M)M\omega^2}}$$

U(3) Commutation Relations

$$[\hat{N},\vec{L}] = [\hat{N},Q_m] = 0,$$

$$[\vec{L},\vec{L}]^{(t)} = -\sqrt{2}\ \vec{L}\ \delta_{t,1},$$

$$[\vec{L},Q]^{(t)} = -\sqrt{6}\ Q\ \delta_{t,2},$$

$$[Q,Q]^{(t)} = 3\sqrt{10}\ \vec{L}\ \delta_{t,1}.$$

This can be simplified to

$$\hat{N} = \frac{\sqrt{H+M}\,(H-M)}{\hbar\sqrt{2M\omega^2}} - \frac{3}{2}$$

Pseudo U(3) Generators

$$\vec{\tilde{S}} = \begin{pmatrix} U_p \, \vec{s} \, U_p & 0 \\ 0 & \vec{s} \end{pmatrix}, \quad \vec{\tilde{L}} = \begin{pmatrix} U_p \, \vec{\ell} \, U_p & 0 \\ 0 & \vec{\ell} \end{pmatrix},$$

$$\tilde{Q}_m = \sqrt{\frac{3}{M\omega^2\hbar^2(\tilde{H} - M)}} \begin{pmatrix} [pp]^{(2)}_m & \vec{\sigma}\cdot\vec{p} \, M\omega^2 \, [rr]^{(2)}_m \\ M\omega^2 \, [rr]^{(2)}_m \, \vec{\sigma}\cdot\vec{p} & M\omega^2 \, (M\omega^2 \, r^2 - 2M)[rr]^{(2)}_m + [pp]^{(2)}_m \end{pmatrix}$$

$$\widetilde{N} = \frac{\sqrt{H - M}\,(H + M)}{2\hbar\sqrt{M\omega^2}} - \frac{3}{2}.$$

Relativistic Harmonic Oscillator with no symmetries: $V_S(\vec{r}) \neq \pm V_V(\vec{r})$

The pseudospin limit is approximately valid for nuclei. However in this limit there are no bound Dirac valence states, only bound Dirac hole states. Therefore we would like to solve analytically the Dirac Hamiltonian for general scalar and vector harmonic oscillator potentials to obtain eigenfunctions with a realistic spectrum relevant for nuclei.

Furthermore, this is an interesting problem because there exists a symmetry limit for which perturbation theory is not valid even though

$$V_S(\vec{r}) \approx -V_V(\vec{r})$$

as is the case for nuclei.

Summary

- Dirac Hamiltonian has an SU(2) symmetry for

$$V_S\left(\vec{r}\right) - V_V\left(\vec{r}\right) = C_s \quad spin \;\; symmetry$$

$$V_S\left(\vec{r}\right) + V_V\left(\vec{r}\right) = C_{ps} \quad pspin \;\; symmetry$$

- Nuclei exhibit pseudospin symmetry which may improve as isospin increases
- Predicts that anti-nucleons in a nuclear environment exhibit spin symmetry
- Mesons and Baryons seem to exhibit spin symmetry

Summary

- The relativistic harmonic oscillator has a spin and U(3) symmetry when the scalar and vector potentials are equal. This limit is relevant for hadrons and anti-nucleons in a nuclear environment and perturbation theory is possible.
- The relativistic harmonic oscillator has a pseudospin and a pseudo U(3) symmetry when the scalar and vector potentials are equal in magnitude and opposite in sign. This limit is relevant for nuclei, but, in the exact limit, there are no bound Dirac valence states so perturbation is not an option.
- Therefore we are attempting to solve analytically the relativistic harmonic oscillator with arbitrary vector and scalar potentials.

Future

More fundamental rationale for pseudospin symmetry

1) What is the connection between QCD and pseudospin symmetry suggested by QCD sum rules?

2) Why do hadrons have spin symmetry whereas nuclei have pseudospin symmetry?

HAPPY BIRTHDAY FRANCO

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- The relativistic harmonic oscillator has a pseudospin and a pseudo U(3) symmetry when the scalar and vector potentials are equal in magnitude and opposite in sign. This limit is relevant for nuclei, but, in the exact limit, there are no bound Dirac valence states so perturbation is not an option.
- Therefore we are attempting to solve analytically the relativistic harmonic oscillator with arbitrary vector and scalar potentials.

Spherical Relativistic Harmonic Oscillator

κ **is the conserved quantum number for the spherical harmonic oscillator**

$$\kappa = -(j + \frac{1}{2}) \quad \text{for aligned} \quad j = \ell + \frac{1}{2}$$

$$\kappa = (j + \frac{1}{2}) \quad \text{for unaligned} \quad j = \ell - \frac{1}{2}$$

The Dirac Hamiltonian leads to two coupled linear equations for the upper and lower components. We do the usual transformation to a second order differential equation for the upper component

$$g_\kappa(r) = r^{|\kappa|-1} G_\kappa(x), x = \lambda r^2$$

Differential Equation

$$G''(x) + \left(\frac{2|\kappa|+1}{2x} - \frac{a_1}{A(x)} \right) G'(x) + \frac{A(x)B(x)}{x} G(x) = 0$$

$$A(x) = a_0 + a_1 x, \qquad B(x) = b_0 + b_1 x$$

$$a_0 = E_\kappa + M, \qquad a_1 = \frac{M}{2\lambda}(\omega_S^2 - \omega_V^2)x$$

$$b_0 = E_\kappa - M, \qquad b_1 = \frac{M}{2\lambda}(\omega_S^2 + \omega_V^2)x$$

Future

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1) What is the connection between QCD and pseudospin symmetry suggested by QCD sum rules?

2) Why do hadrons have spin symmetry whereas nuclei have pseudospin symmetry?

SUMMARY

1) Pseudospin symmetry is a relativistic symmetry of the Dirac Hamiltonian for

$$[\, p_i \,, V_S(\vec{r}) + V_V(\vec{r}) \,] = 0$$

2) Pseudo-spin symmetry is approximately conserved for realistic mean fields,

$$V_S(\vec{r}) \approx - V_V(\vec{r})$$

3) QCD sum rules suggest a connection with spontaneously broken chiral symmetry

4) Relativistic spherical and deformed mean field eigenfunctions satisfy approximately the conditions for pseudospin symmetry as do realistic non-relativistic eigenfunctions.

5) Pseudospin symmetry M1 predictions approximately valid.

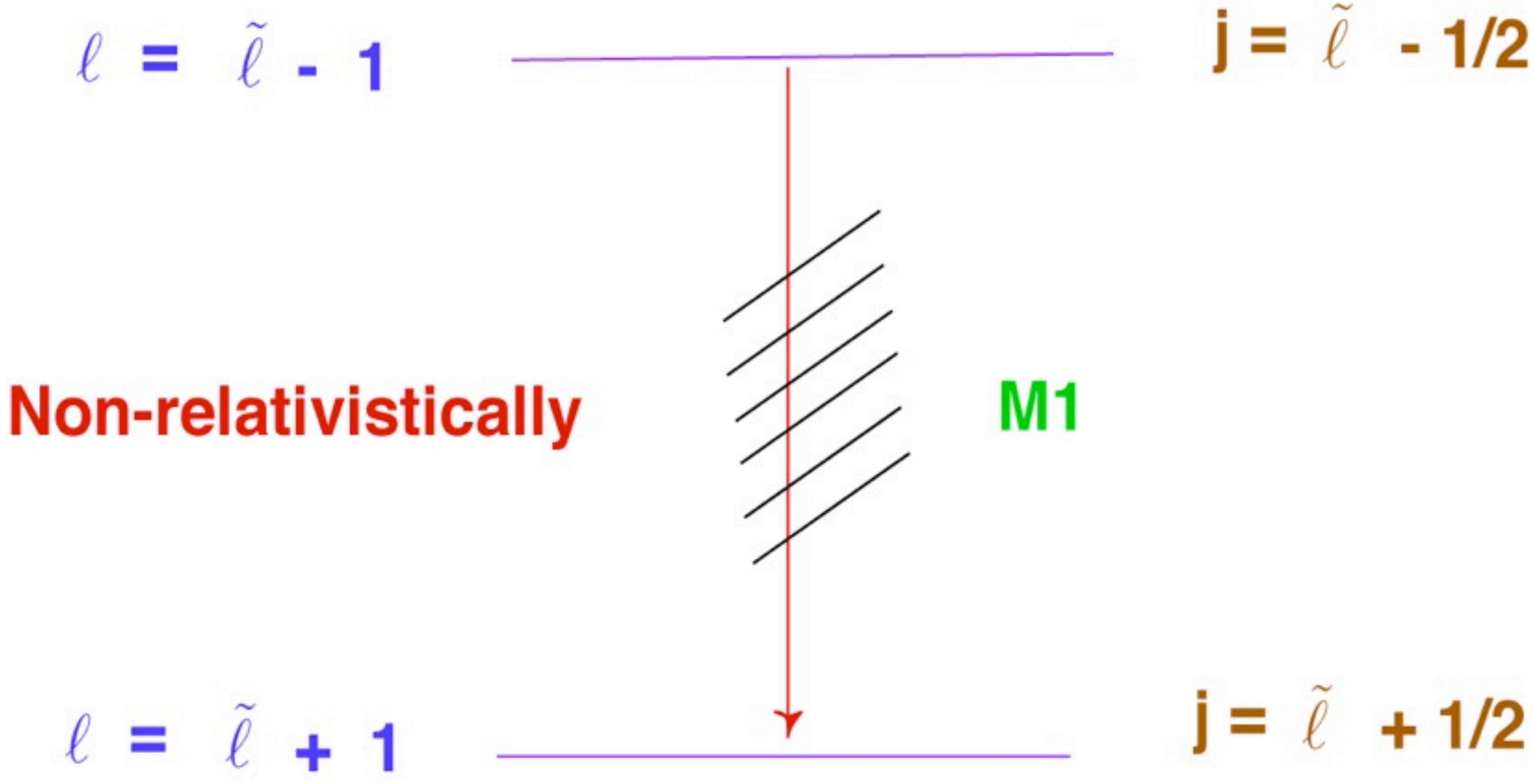
6) Pseudospin symmetry approximately conserved in medium energy nucleon-nucleus scattering.

7) Pseudospin symmetry may improve for neutron rich nuclei.

8) Charge conjugation predicts that an antinucleon imbedded in a nucleus will have spin symmety.

Happy Birthday Akito

Magnetic Transitions between Pseudospin doublets



Magnetic Transitions for Pseudospin Symmetry

$$B(M1: j' \to j)_\nu = \frac{j+1}{2j+1} [\mu_{j,\nu} - \mu_{A,\nu}]^2 \qquad j' = \tilde{\ell} + 1/2$$

$$B(M1: j' \to j)_\nu = \frac{2j+1}{j+1} \left[\frac{j+2}{2j+3} \left(\mu_{j',\nu} + \frac{j+1}{j+2} \mu_{A,\nu} \right) \right]^2 \qquad j = \tilde{\ell} - 1/2$$

Magnetic Transitions for Pseudospin Symmetry

$$B(M1:j' \rightarrow j\,)_\nu = \frac{j+1}{2j+1}\left[\mu_{j,\nu} - \mu_{A,\nu}\right]^2 \qquad j' = \tilde{\ell} + 1/2$$

$$B(M1:j' \rightarrow j\,)_\nu = \frac{2j+1}{j+1}\left[\frac{j+2}{2j+3}\,(\mu_{j',\nu} + \frac{j+1}{j+2}\mu_{A,\nu})\right]^2 \qquad j = \tilde{\ell} - 1/2$$

$$\frac{B(M1;j' \rightarrow j)_\nu}{S_j S_{j'}} = \frac{j+1}{2j+1}\left(\frac{\mu_{j,\nu}}{S} - \mu_{A,\nu}\right)^2 \qquad j' = \tilde{\ell} + 1/2$$

$$\frac{B(M1;j' \rightarrow j)_\nu}{S_j S_{j'}} = \frac{2j+1}{j+1}\left(\frac{j+2}{2j+3}\right)^2\left(\frac{\mu_{j'}}{S_{j'}} + \frac{j+1}{j+2}\,\mu_{A,\nu}\right)^2 \qquad j = \tilde{\ell} - 1/2$$

$S_j \; S_{j'}$ **spectroscopic factors**

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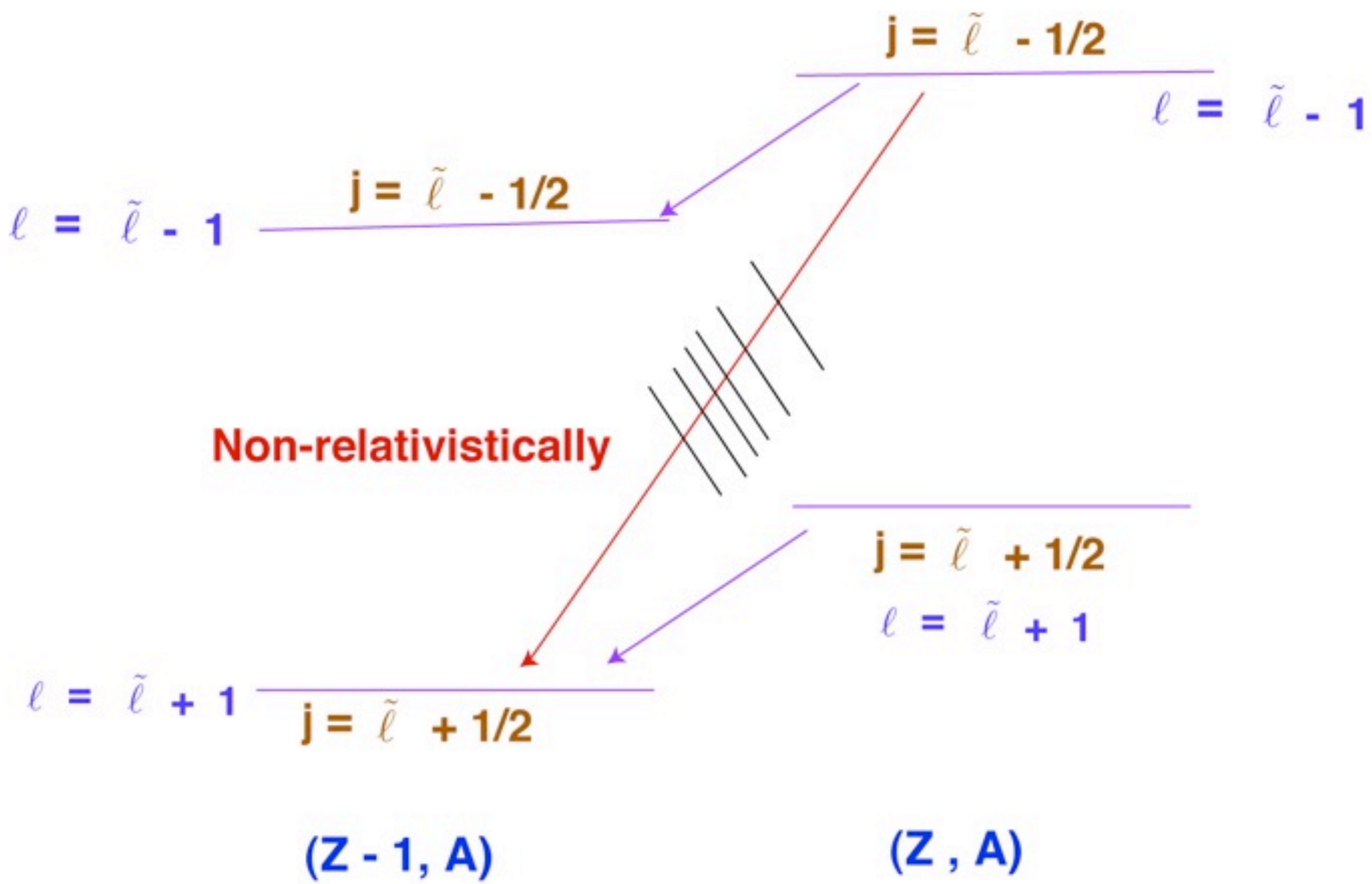
Predicted "ℓ forbidden" magnetic dipole transition in ^{39}Ca.

	$B(M1: j,\nu \rightarrow j',\nu)$
Predicted Equation	0.0166
Predicted Equation	0.0121
EXP	0.0121 (14)

Gamow-Teller Transitions between Pseudospin doublets



Physics Reports 414 (2005) 165–261

Pseudospin Symmetry

$$V_S(\vec{r}) + V_V(\vec{r}) = C_{ps}, \text{ nuclear matter}$$

$$V_S(\vec{r}) = - \ V_V(\vec{r}), \text{ finite nulei}$$

$$[\tilde{S}_i, H_{ps}] = 0,$$

$$[\tilde{S}_i, \tilde{S}_j] = i\epsilon_{ijk} \ \tilde{S}_k,$$

$$\tilde{S}_i = \frac{\vec{\alpha} \cdot \vec{p} \ \hat{s}_i \ \vec{\alpha} \cdot \vec{p}}{p^2} \frac{(1-\beta)}{2} + \hat{s}_i \frac{(1+\beta)}{2},$$

$\hat{s}_i = \sigma_i/2$, σ_i the Pauli matrices.

Simplifies to:

$$\hat{\tilde{S}}_i = \begin{pmatrix} \hat{\tilde{s}}_i & 0 \\ 0 & \hat{s}_i \end{pmatrix}$$

$$\hat{\tilde{s}}_i = U_p \; \hat{s}_i \; U_p = \frac{2\hat{s} \cdot p}{p^2} \, p_i - \hat{s}_i .$$

$$U_p = \frac{\sigma \cdot p}{p}$$

U_p is the momentum - helicity unitary operator introduced in

A. L. Blokhin, C. Bahri, and J. P. Draayer, Physical Review Lett. **74**, 4149 (1995).

Isovector Effects

σ, ω isoscalar mesons

ρ isovector meson

$$V_V^{\pi} = V_{\omega} - (N - Z)\, V_{\rho}\,,$$

$$V_V^{\nu} = V_{\omega} + (N - Z)\, V_{\rho}$$

As neutron excess increases

$V_V^{\pi} + V_S^{\pi}$ **increases**

$V_V^{\nu} + V_S^{\nu}$ **decreases**

Pseudospin symmetry may improve for neutrons for neutron rich nuclei measured in RIA experiments!

Relativistic Harmonic Oscillator

$$V_S(\vec{r}) = \frac{\tilde{M}}{2}\sum_{i=1}^{3} \omega_{S,i}^2 x_i^2$$

$$V_V(\vec{r}) = \frac{\tilde{M}}{2}\sum_{i=1}^{3} \omega_{V,i}^2 \, x_i^2$$

In the symmetry limits the eigenfunctions and eigenenergies can be solved analytically. The eigenfunctions are similar to the non relativistic limit. That is, the upper and lower components can be written in terms of Gaussians and Laguerre polynomials. This true for spherical and non-spherical harmonic oscillator. **JNG, PRC 69, 034318 (2004)**

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Summary

- There are shell model Hamiltonians which have pseudospin and pseudo-orbital angular momentum as dynamical symmetries.

- These Hamiltonians have spin-spin interactions, orbital angular momentum-orbital angular momentum interactions, and spin-orbit interactions.

- In addition these Hamiltonians have tensor, dipole, and momentum dependent spin-orbit interactions which do not conserve spin and orbital angular momentum.

- Calculations with these Hamiltonians are in progress.

Bound States in the Pseudospin Limit

In the pseudospin limit there are no bound valence Dirac states. The bound states are Dirac hole states. Therefore, even though nuclei are near this limit, it is impossible to do perturbation theory around the exact limit, since in the exact pseudospin symmetry limit there are no bound states. In fact, that is what makes this limit fascinating. This also is a motivation to solve analytically the harmonic oscillator with general vector and scalar potentials so that the system can be studied analytically as the pseudospin limit is approached.

4) Relativistic spherical and deformed mean field eigenfunctions satisfy approximately the conditions for pseudospin symmetry as do realistic non-relativistic eigenfunctions.

5) Pseudospin symmetry M1 predictions approximately valid.

6) Pseudospin symmetry approximately conserved in medium energy nucleon-nucleus scattering.

7) Pseudospin symmetry may improve for neutron rich nuclei.

8) Charge conjugation predicts that an antinucleon imbedded in a nucleus will have spin symmety.

SUMMARY

1) Pseudospin symmetry is a relativistic symmetry of the Dirac Hamiltonian for

$$[\, p_i \,, V_S(\vec{r}) + V_V(\vec{r}) \,] = 0$$

2) Pseudo-spin symmetry is approximately conserved for realistic mean fields,

$$V_S(\vec{r}) \approx - V_V(\vec{r})$$

3) QCD sum rules suggest a connection with spontaneously broken chiral symmetry