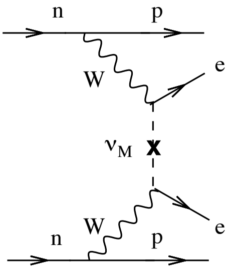
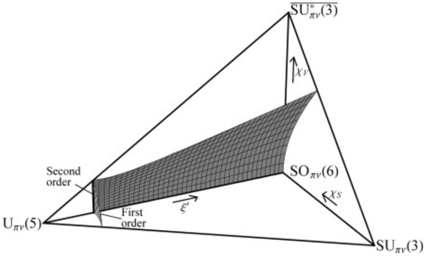
$\beta\beta$ Decay and Algebraic Models

J. Engel

May 24, 2012





Beauty in Physics

FrancoFest '12

Point of This Talk

Using Fermion algebras to test many body approximations in calculation of double-beta-decay matrix elements

Algebras and Methods

- Multi-level SO(5) for testing truncations of shell-model spaces
- SO(8) for testing mean-field based methods: HFB, QRPA, large-amplitude approximations

Neutrinoless Double-Beta Decay

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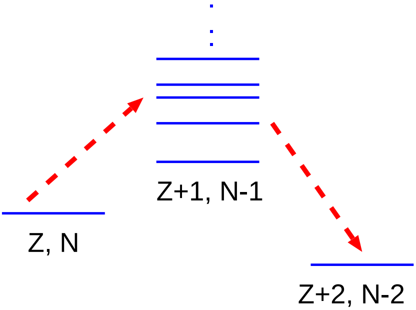

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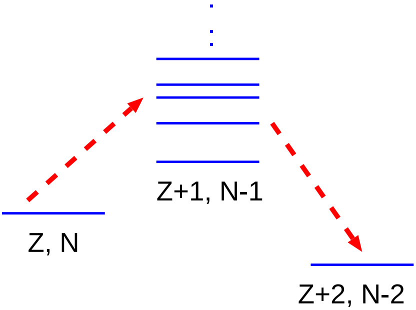

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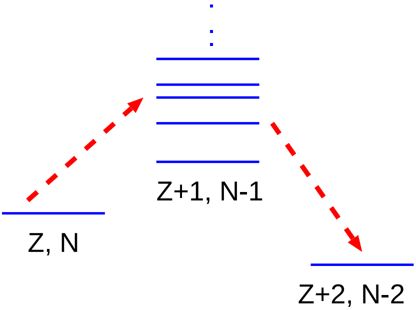

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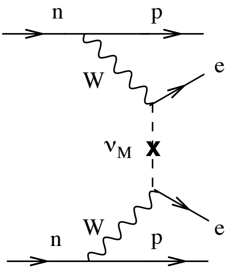

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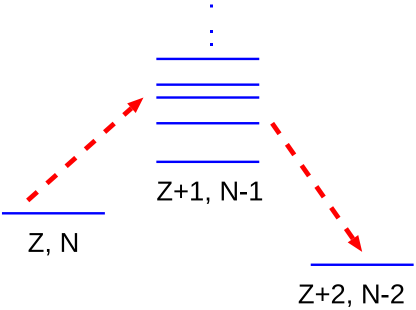

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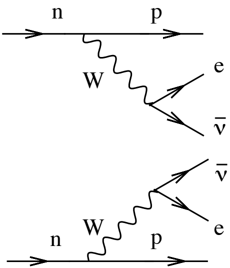

Different from already observed 2ν process.

Usefulness of Neutrinoless Double-Beta Decay

If it's observed, neutrinos are Majorana particles

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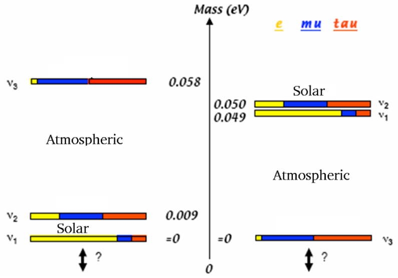
Usefulness of Neutrinoless Double-Beta Decay



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Light-ν-exchange amplitude proportional to "effective mass"

$$m_{ee} \equiv \sum_i m_i U_{ei}^2$$

If lightest neutrino is light:

- $m_{ee} \approx \sqrt{\Delta m^2_{sol}} \sin^2 \theta_{sol}$ (normal)
- $m_{ee} \approx \sqrt{\Delta m^2_{atm}} \cos 2\theta_{sol}$ (inverted)

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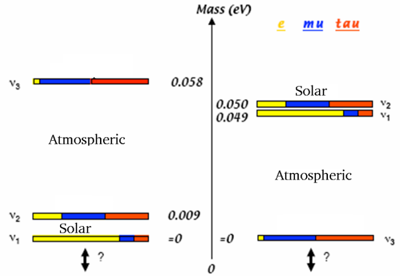

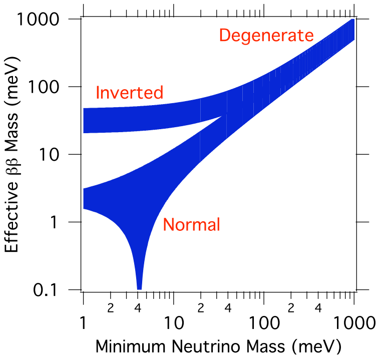

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But rate is also proportional to nuclear matrix element...

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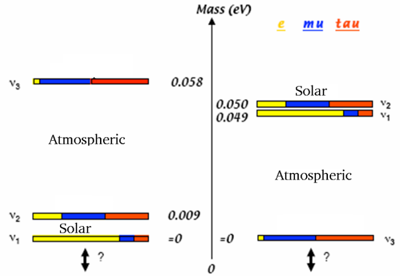

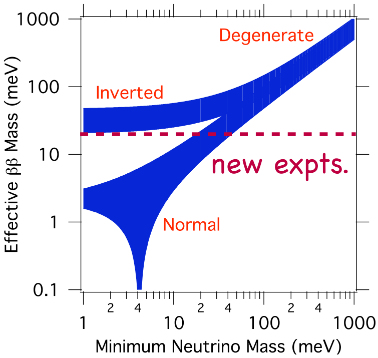

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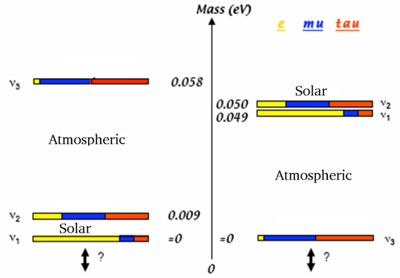

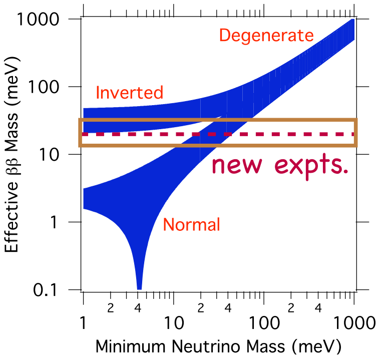

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Nuclear Matrix Element

$$M_{0\nu} = M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^{F} + \ldots$$

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$$M_{0\nu}^{F} = \langle f| \sum_{a,b} H(r_{ab}) \tau_a^+ \tau_b^+ |i\rangle + \ldots$$

$$H(r) \approx \frac{R}{r}$$

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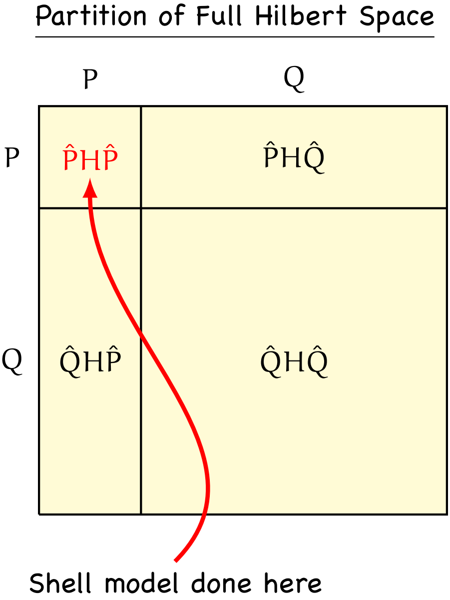
$$H(r) \approx \frac{R}{r}$$

Operators for 2ν decay (in closure approx.) are similar but don't contain $H(r)$.

"Exact" Shell-Model Calculations



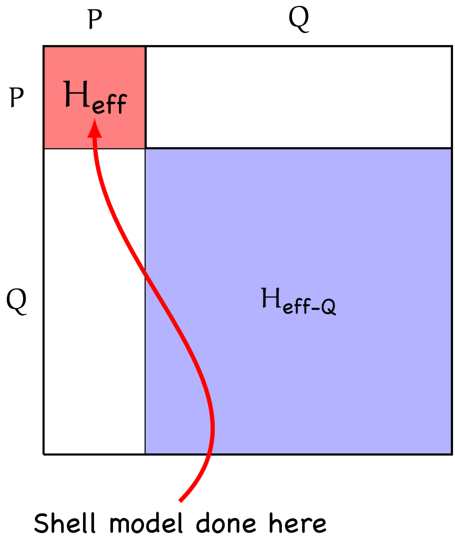
P = valence space (dimension d)
Q = the rest

$$\hat{P} = \sum_{i=1}^{d} |i\rangle\langle i| \qquad \hat{Q} = \sum_{i=d+1}^{\infty} |i\rangle\langle i|$$

Task: Find unitary transformation to make H block-diagonal in P and Q, with H_{eff} in P reproducing d most important eigenvalues.

"Exact" Shell-Model Calculations

Partition of Full Hilbert Space



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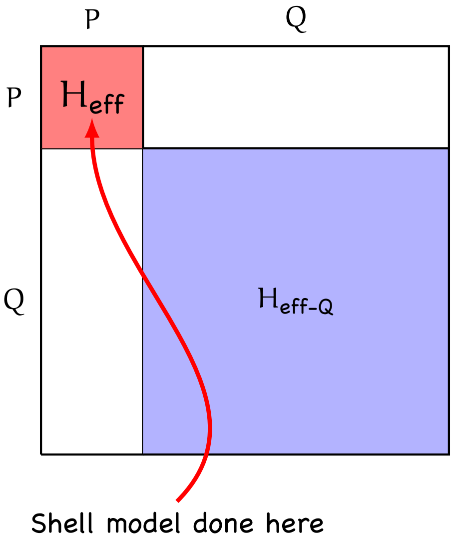

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For transition operator $\mathcal{M}$, must apply same transformation to get $\mathcal{M}_{eff}$.

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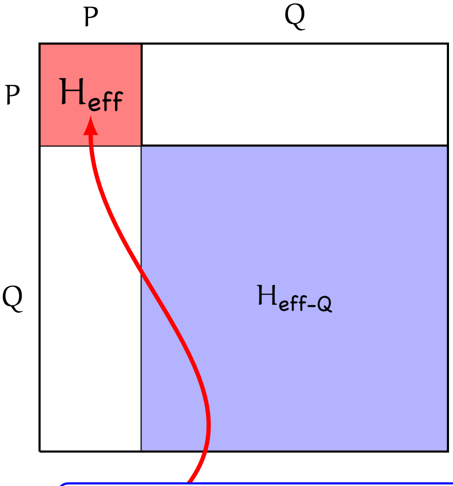

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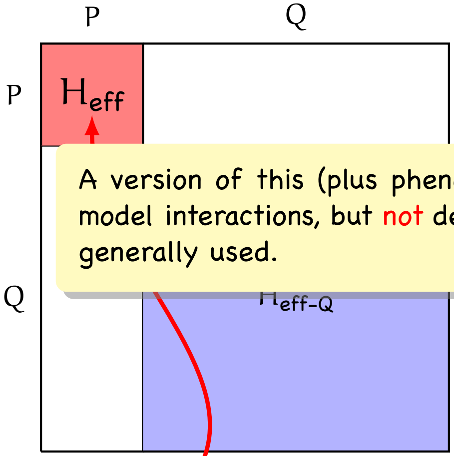

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A version of this (plus phenomenology) used to get shell-model interactions, but not decay operators. Bare operators generally used.

...tion

P

...ucing

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Formulation

Q: Take d full eigenstates $|k\rangle$ of your choice. How do you map these onto normalized P-space states $|\widetilde{k}\rangle$ in a way that maximizes $\sum_{k=1}^{d} \langle k|\widetilde{k}\rangle$?

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A: Lee-Suzuki mapping:

Entirely in P space

$$|\widetilde{k}\rangle \equiv \frac{1}{\sqrt{1+\omega^\dagger \omega}} (P + \omega^\dagger)\, |k\rangle$$

$\omega^\dagger$ takes Q to P with

$$\omega^\dagger_{p,q} = \sum_{k=1,d} \langle p|\underline{k}\rangle \langle k|q\rangle \;, \quad \{\langle p|\underline{k}\rangle\} = \text{inverse}\{\langle k|p\rangle\}$$

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Mapping of operators follows:

$$\langle\tilde{k}|\,O_{\text{eff}}\,|\tilde{k}'\rangle = \langle k|\,O\,|k'\rangle$$

whether O is an interaction H or decay operator $\mathcal{M}$.

Application to Two-Shell SO(5)

Generators

Pair creation operators for each shell:

$$S^{\dagger i}_{\mathsf{pp}} = \sum_{\alpha \in i} p^{\dagger}_{\alpha} p^{\dagger}_{\bar{\alpha}} \qquad S^{\dagger i}_{\mathsf{nn}} = \sum_{\alpha \in i} n^{\dagger}_{\alpha} n^{\dagger}_{\bar{\alpha}} \qquad S^{\dagger i}_{\mathsf{pn}} = \sum_{\alpha \in i} n^{\dagger}_{\alpha} p^{\dagger}_{\bar{\alpha}}$$

where α runs over all levels in shell i.

Other generators:

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Other generators:

$$S^i_{pp} \qquad S^i_{nn} \qquad S^i_{pn} \qquad \vec{\mathcal{J}}_i$$

Hamiltonian

$$H = \epsilon \hat{N}_2 - G \sum_{i,j=1}^{2} \left(S^{\dagger i}_{pp} S^j_{pp} + S^{\dagger i}_{nn} S^j_{nn} + g_{pp} S^{\dagger i}_{pn} S^j_{pn} + g_{ph} \vec{\mathcal{J}}_i \cdot \vec{\mathcal{J}}_j \right)$$

g_{pp} controls strength of np pairing, which is isovector here but plays same role here as isoscalar pairing in real life.

$\beta\beta$ (Closure) Matrix Element in SO(5)

Simplified Fermi transition operators

$$\mathcal{M}^F_{2\nu}(\mathrm{cl.}) = \sum_{i,j} \tau_i^+ \tau_j^+ \propto \mathcal{T}_+ \mathcal{T}_+ \qquad \mathcal{M}^F_{0\nu} = \sum_{i,j} \frac{\tau_i^+ \tau_j^+}{|\vec{r}_i - \vec{r}_j|} \ ,$$

$M^F_{2\nu}(\mathrm{cl.})$ is product of generators, but $M^F_{0\nu}$ contains radial dependence that has nothing to do with SO(5).

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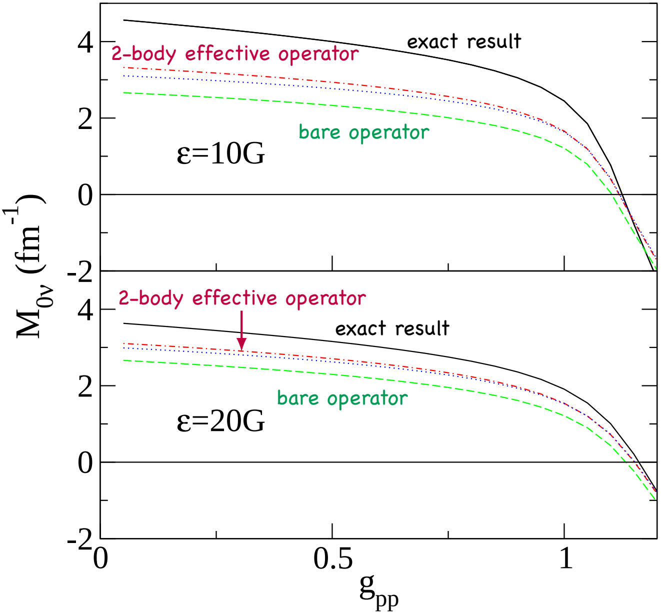
But it can be decomposed into SO(5) tensors and evaluated with help of generalized Wigner-Eckardt Theorem:

$$\langle \Omega_i, \mathcal{N}_i, T_i, M_i | \, \mathcal{M}^{(\omega_1,\omega_2)}_{\mathcal{N}_0, T_0, M_0} \, | \Omega_i, \mathcal{N}'_i, T'_i, M'_i \rangle = \langle (\Omega_i, 0) \| \mathcal{M}^{(\omega_1,\omega_2)} \| (\Omega_i, 0) \rangle \times \langle T'_i M'_i; T_0 M_0 \, | \, T_i M_i \rangle \, \langle (\Omega_i, 0) \mathcal{N}'_i T'_i; (\omega_1, \omega_2) \mathcal{N}_0 T_0 \, \| \, (\Omega_i, 0) \mathcal{N}_i T_i \rangle$$

How Well Does Mapping Work at 2-Body Level?



Answer: Leaves room for imporovement

Phenomenological QRPA

Start with Wood-Saxon potential, G-matrix interaction, usual BCS procedure.

Ansatz for intermediate states:

$$|\nu\rangle = Q^\dagger_\nu |0\rangle \quad \text{where} \quad Q^\dagger_\nu = \sum_{pn} X^\nu_{pn} \alpha^\dagger_p \alpha^\dagger_n - Y_{pn} \alpha_p \alpha_n$$

yields matrix equations:

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \Omega^\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

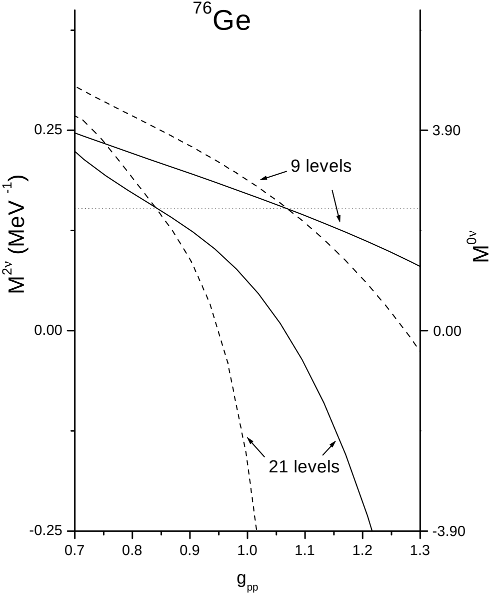
with

$$\begin{aligned} A_{pn,p'n'} = E_{\text{single quasipart.}} &+ V^{ph}_{pn,p'n'} (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}) \\ &+ V^{pp}_{pn,p'n'} (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) \end{aligned}$$

$$B_{pn,p'n'} = (\text{similar expression})$$

$V^{pp}_{pn,p',n'}$ usually contains the adjustable multiplier g_{pp}.

Fiddling with the QRPA



From Simkovic et al.

Generalized HFB

Generalized BCS mixes proton and neutron quasiparticles:

$$\begin{aligned}
\alpha_1^\dagger &= u_{i,p}^{(1)} p_i^\dagger + v_{i,p}^{(1)} p_{\bar{i}} + u_{i,n}^{(1)} n_i^\dagger + v_{i,n}^{(1)} n_{\bar{i}} \\
\alpha_2^\dagger &= u_{i,p}^{(2)} p_i^\dagger + v_{i,p}^{(2)} p_{\bar{i}} + u_{i,n}^{(2)} n_i^\dagger + v_{i,n}^{(2)} n_{\bar{i}} \\
\alpha_1 &= \ldots \\
\alpha_2 &= \ldots
\end{aligned}$$

Generalized HFB combines this with (generalized) Hartree Fock in usual way.

Not much point in generalized QRPA (will see why shortly).

Application to SO(8)

Generators

Pairing operators:

SO(5) pairing operators

isoscalar pairing operators

$$S^\dagger_\nu = \left[a^\dagger \tilde{a}\right]^{S=0,T=1}_{M_T=\nu} \qquad P^\dagger_\mu = \left[a^\dagger \tilde{a}\right]^{S=1,T=1}_{M_S=\mu}$$

$$S_\nu \qquad P_\mu$$

Particle-hole operators:

Gamow-Teller operators

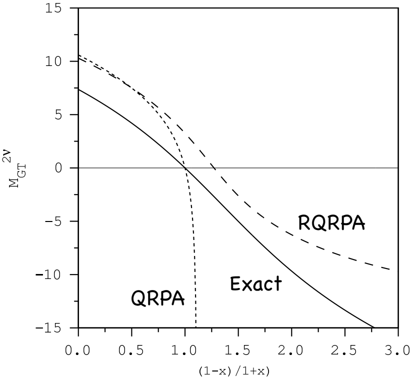
$$\vec{\mathcal{S}} \qquad \vec{\mathcal{T}} \qquad \mathcal{F}^\mu_\nu \equiv \sum_i \sigma(i)_\mu \tau(i)_\nu$$

Hamiltonian

$$H = -\frac{g(1+x)}{2}\sum_\nu S^\dagger_\nu S_\nu - \frac{g(1-x)}{2}\sum_\mu P^\dagger_\mu P_\mu + g_{ph}\mathcal{F}^{\mu\dagger}_\nu\ \mathcal{F}^\mu_\nu$$

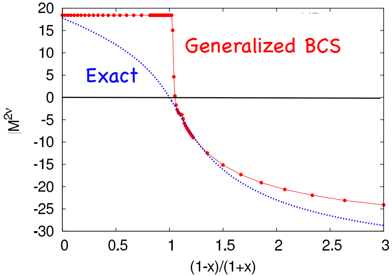
$(1-x)/(1+x)$ (ratio of ioscalar/isovector pairing) is g_{pp}

2νββ (Closure) Matrix Element in SO(8)



Ordinary BCS + QRPA or RQRPA

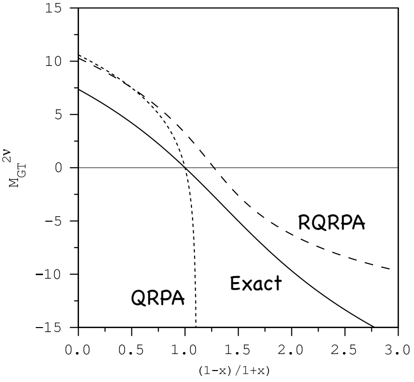
(with ph interaction)



Generalized BCS

(no ph interaction)

2νββ (Closure) Matrix Element in SO(8)



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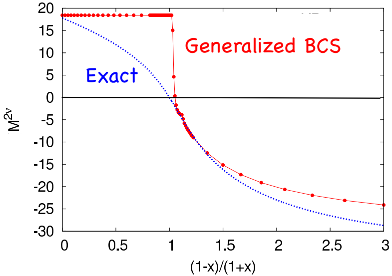


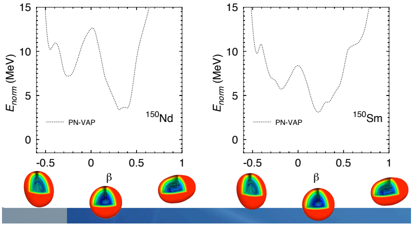
Generalized BCS

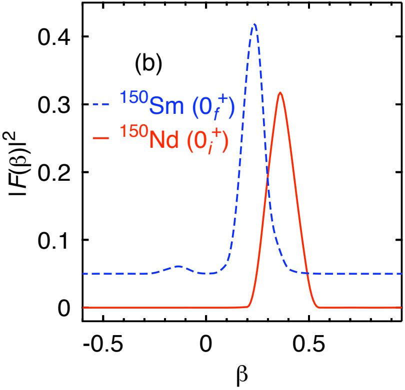
(no ph interaction)

- RQRPA doesn't work in "isoscalar-pairing" phase (right of each fig.)
- Generalized QRPA pointless in "isovector-pairing" phase (left of each fig.)

Beyond QRPA: GCM and Large-Amplitude Motion

For 0ν decay, only need initial and final ground states. Rodgriguez and Martinez-Pinedo have done sophisticated Gogny generator-coordinate calculaton; mix mean fields with different shapes, pairing fields:





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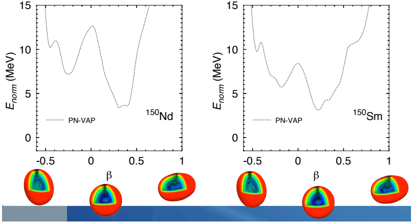
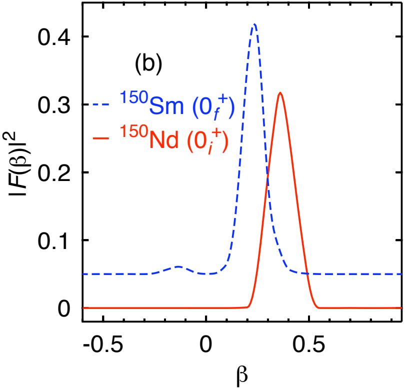
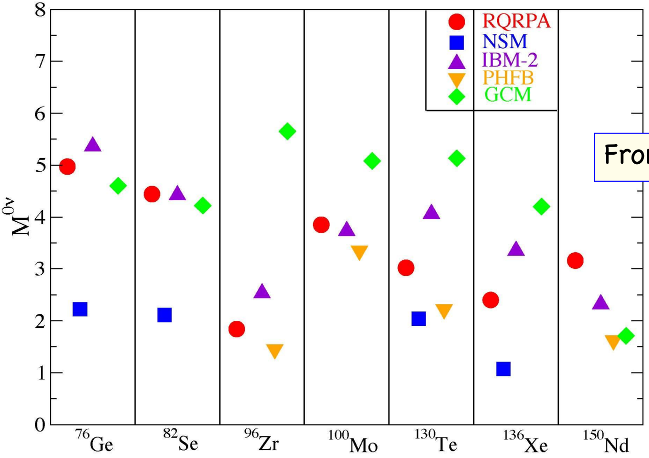




But no explicit np pairing/spin-isospin correlations here. SO(8) says they should be important.

Matrix elements in different schemes

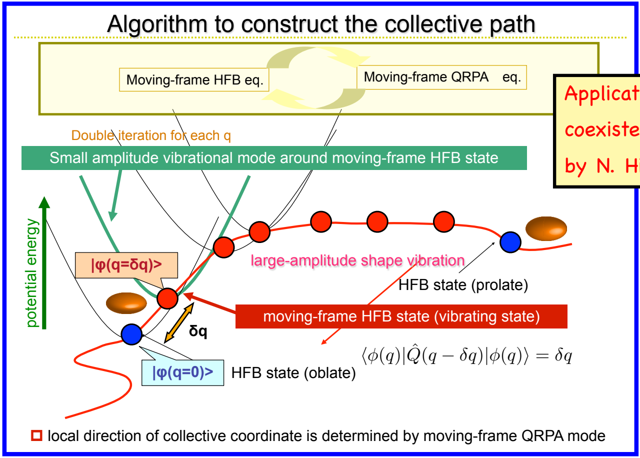


From P. Vogel

GCM numbers tend to be on the high side.

Curent Work: Alternative Large-Amplitude Approx.

Until Now: Induce deformation with constraint operatator Q. Calculate deformed kinetic, potential energies, inertial parameters. Determine most collective Q. Determine equivalent Bohr Hamiltonian.



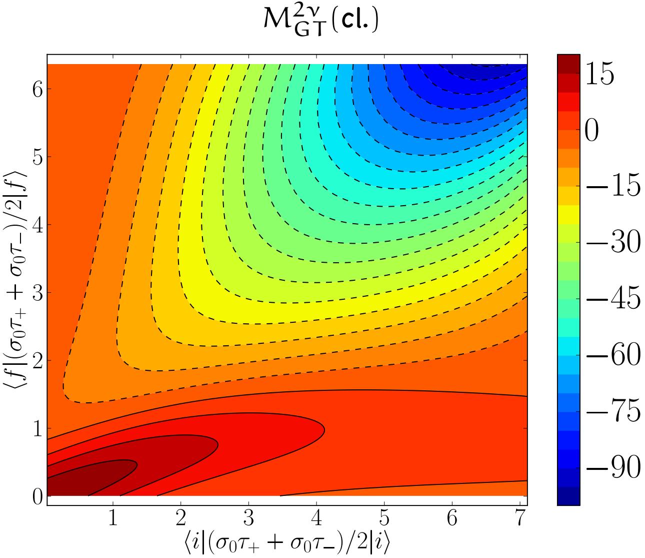
Application to shape coexistence from talk by N. Hinohara

Procedure maps maps adiabatic TDHFB dynamics in collective subspace to 5-d Bohr Hamiltonian.

Now: Inclusion of Collective np Pairing

- Start with generalized moving HFB that includes np mixing.
- Add collective np pair-creation operator to set of constrained operators.
- Map to generalized 6-d Bohr Model (usual 5 plus new collective mode)
- In SO(8) 6-d reduces to 1-d.

Results So Far



Wave functions (not yet calculated) will sample this entire space for all g_{pp}.

Final Question:

IBM also being used to calculate double beta decay. Can we use a fermion algebra to test it?

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Many thanks to Franco for starting me off in this beautiful field.