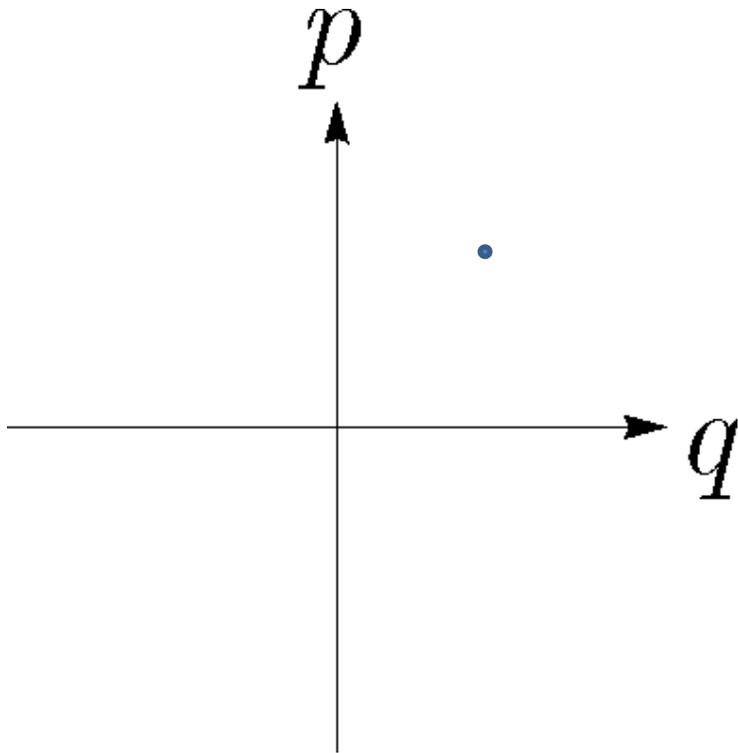
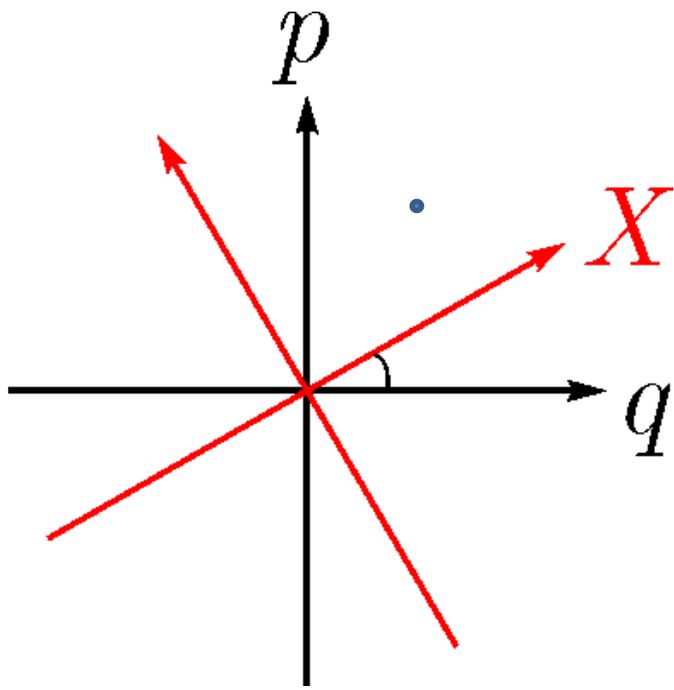


Classical Picture



$$\int f(q, p) dq dp = 1$$



$$X = q \cos \theta + p \sin \theta$$

$$\begin{aligned} w(X, \theta) &= \langle \delta (X - q \cos \theta - p \sin \theta) \rangle \\ &= \int f(q, p) \delta (X - q \cos \theta - p \sin \theta) dq dp \end{aligned}$$

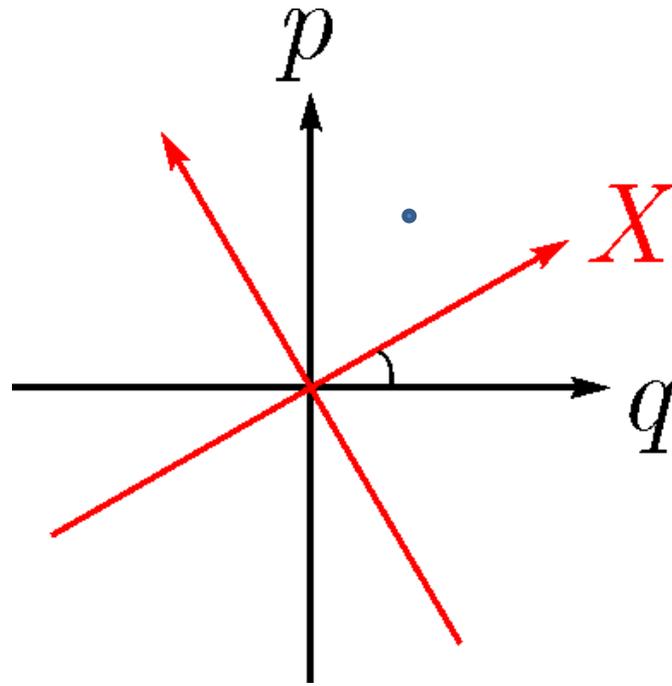
$$\begin{aligned} M(X, \mu, \nu) &= \langle \delta (X - \mu q - \nu p) \rangle \\ &= \int f(q, p) \delta (X - \mu q - \nu p) dq dp \end{aligned}$$

$$\begin{aligned} \mu &= s \cos \theta \\ \nu &= s^{-1} \sin \theta \end{aligned}$$

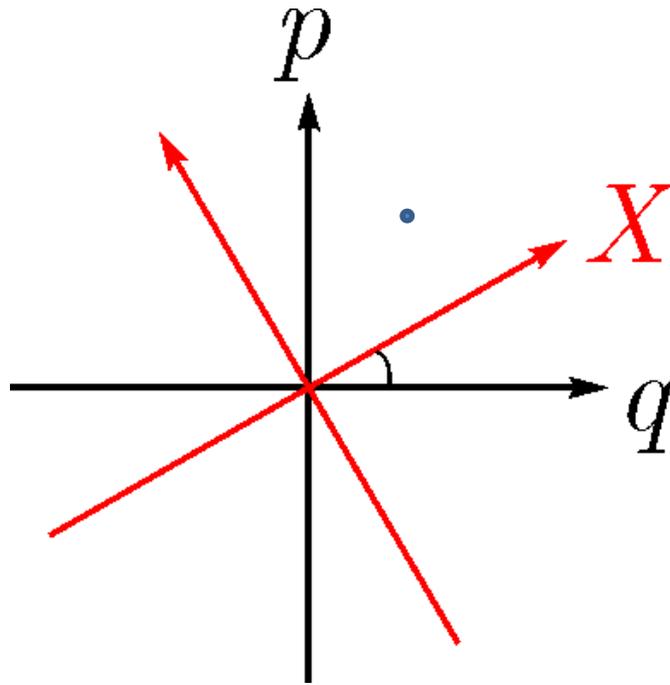
Man'ko, O. V., Man'ko, V. I., J. Russ. Laser Res. 18, 407--444 (1997)

$$w(X, \theta) = M(X, \cos \theta, \sin \theta)$$

$$M(X, \mu, \nu) = \frac{1}{\sqrt{\mu^2 + \nu^2}} w \left(\frac{X}{\sqrt{\mu^2 + \nu^2}}, \tan^{-1} \frac{\nu}{\mu} \right)$$

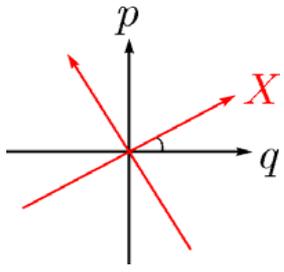


$$f(q, p) = \frac{1}{4\pi^2} \int M(X, \mu, \nu) e^{i(X - \mu q - \nu p)} dX d\mu d\nu \geq 0$$



$$\langle q^n \rangle = \int M(X, 1, 0) X^n dX,$$

$$\langle p^n \rangle = \int M(X, 0, 1) X^n dX$$



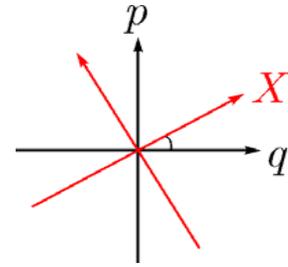
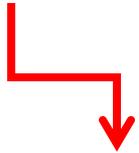
Quantum Picture

$$w(X, \theta) = \langle \delta (X - \hat{q} \cos \theta - \hat{p} \sin \theta) \rangle$$

$$M(X, \mu, \nu) = \langle \delta (X - \mu \hat{q} - \nu \hat{p}) \rangle$$

$$\int M(X, \mu, \nu) dX = 1$$

Mancini, S., Man'ko, V. I., Tombesi, P, Found. Phys. 27, 801-824 (1997).

 $\psi(y)$ 

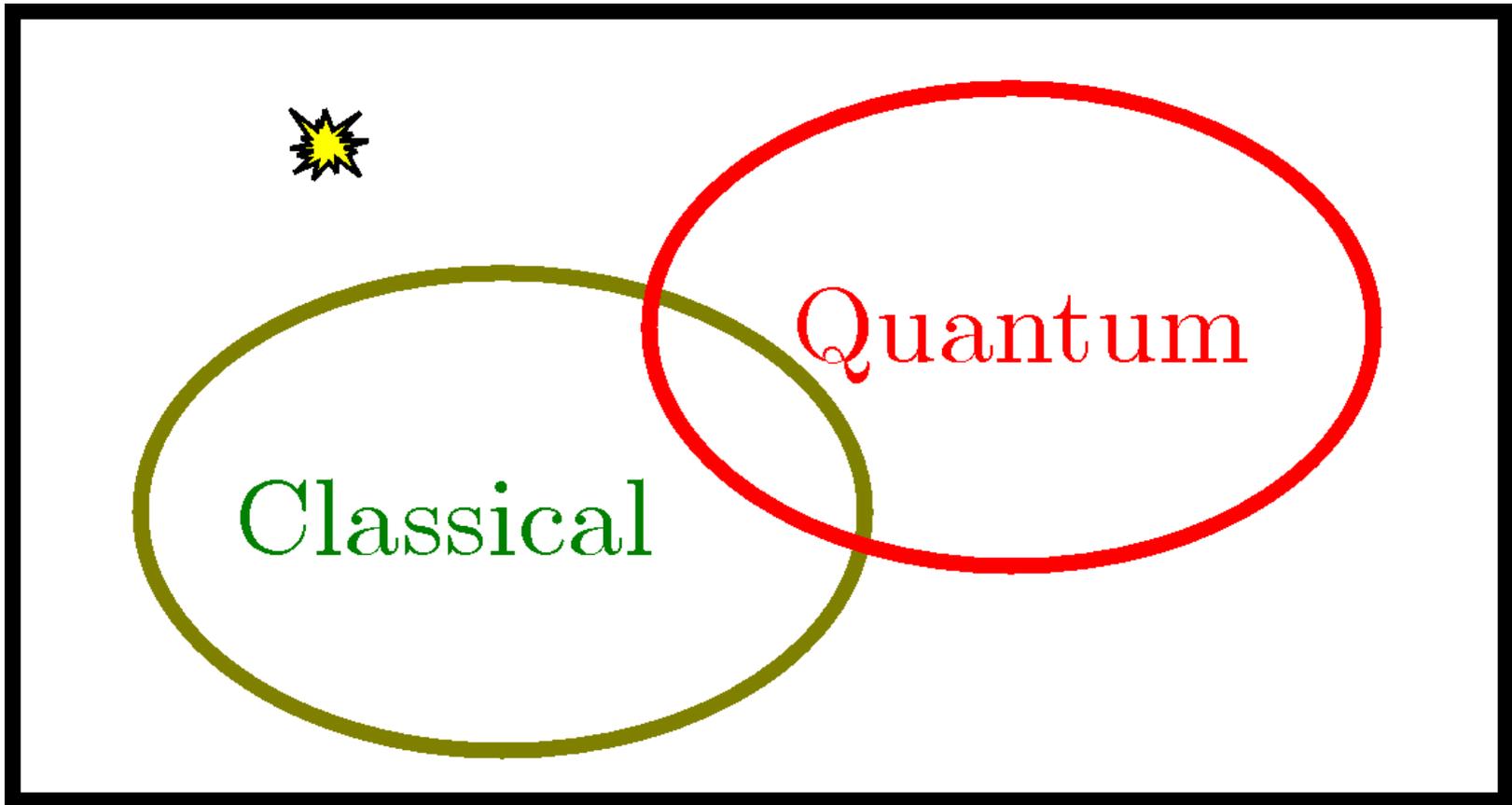
$$M(X, \mu, \nu) = \frac{1}{2\pi|\nu|} \left| \int \psi(y) \exp \left[i \left(\frac{\mu}{2\nu} y^2 - \frac{Xy}{\nu} \right) \right] dy \right|^2$$

Man'ko V. I., Mendes, R. V, Phys. Lett. A 263, 53--56 (1999) [ArXiv Physica/9712022]

$$\hat{\rho} = \frac{1}{2\pi} \int M(X, \mu, \nu) e^{i(X - \mu\hat{q} - \nu\hat{p})} dX d\mu d\nu$$

See also review:

Ibort, A., Man'ko, V. I., Marmo, G., Simoni, A., Ventriglia F., Phys. Scr. 79, 065013 (2009)



Review “An introduction to the tomographic picture of quantum mechanics”
Ibort, A., Man'ko, V. I., Marmo, G., Simoni, A., Ventriglia F., Phys. Scr. 79, 065013
(2009)

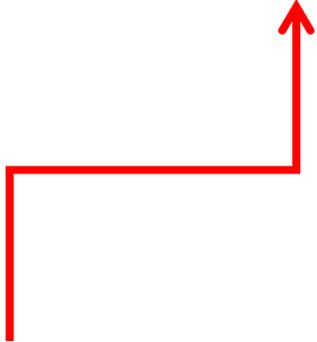
$$\langle X^n \rangle (\mu, \nu) = \int X^n M(X, \mu, \nu) dX, \quad n = 1, 2, \dots$$

$$\begin{aligned} \sigma_{PP}\sigma_{QQ} &= \left(\int X^2 M(X, 0, 1) dX - \left[\int X M(X, 0, 1) dX \right]^2 \right) \\ &\times \left(\int X^2 M(X, 1, 0) dX - \left[\int X M(X, 1, 0) dX \right]^2 \right) \geq \frac{1}{4}. \end{aligned}$$

$$\sigma_{QQ}\sigma_{PP} - \sigma_{QP}^2 \geq \frac{1}{4}$$

$$\sigma_{XX}(\mu, \nu) = \mu^2 \sigma_{QQ} + \nu^2 \sigma_{PP} + 2\mu\nu \sigma_{QP}$$

$$\sigma_{QP} = \sigma_{XX} \left(\theta = \frac{\pi}{4} \right) - \frac{1}{2}(\sigma_{QQ} + \sigma_{PP})$$

$$\sigma_{XX} \left(\theta = \frac{\pi}{4} \right) = \langle X^2 \rangle \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) - \left[\langle X \rangle \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) \right]^2$$


$$\begin{aligned}
F(\theta) = & \left(\int X^2 w(X, \theta) dX - \left[\int X w(X, \theta) dX \right]^2 \right) \\
& \times \left(\int X^2 w \left(X, \theta + \frac{\pi}{2} \right) dX - \left[\int X w \left(X, \theta + \frac{\pi}{2} \right) dX \right]^2 \right) \\
& - \left\{ \int X^2 w \left(X, \theta + \frac{\pi}{4} \right) dX - \left[\int X w \left(X, \theta + \frac{\pi}{4} \right) dX \right]^2 \right. \\
& - \frac{1}{2} \left[\int X^2 w(X, \theta) dX - \left[\int X w(X, \theta) dX \right]^2 \right. \\
& \left. \left. + \int X^2 w \left(X, \theta + \frac{\pi}{2} \right) dX - \left[\int X w \left(X, \theta + \frac{\pi}{2} \right) dX \right]^2 \right] \right\}^2 - \frac{1}{4}
\end{aligned}$$

$$F(\theta) \geq 0$$

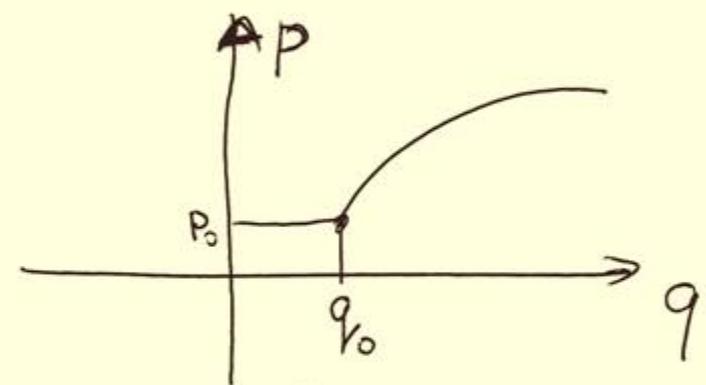
Man'ko, V. I., Marmo, G., Simoni, A., Ventriglia F., Adv. Sci. Lett. 2, 517-520 (2009)

Conclusions

- We reviewed the probability representation of quantum mechanics, where the quantum states are described by probability distributions as an alternative to density operators.
- The probability representation is discussed for continuous variables.
- The essence of this work is the consideration of experiments to check quantum mechanics, or better to say, to study the accuracy with which one can check experimentally the uncertainty relations for photon quadratures.
- It is worth pointing out that in the classical domain one does not have these bounds on $F(\theta)$.

q

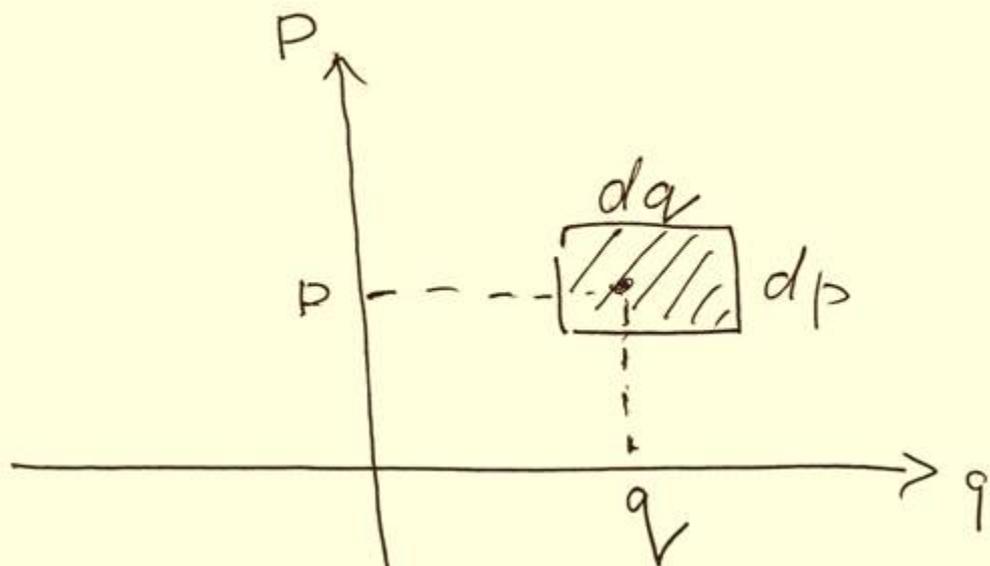
$$p = \dot{q}$$



$q(t)$ $p(t)$

$$\dot{H} = \frac{p^2}{2} + U(q)$$

$$\dot{p} = - \frac{\partial U(q)}{\partial q}$$



$$f(q, p) \geq 0 ; \quad \int f(q, p) dq dp = 1$$

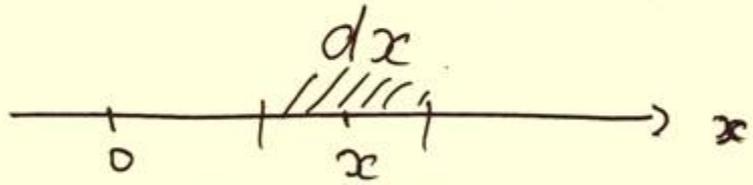
$$\int f(q, p) dp = P(q);$$

$$\int f(q, p) dq = \tilde{\Pi}(p).$$

$$\frac{\partial f(q, p, t)}{\partial t} + p \frac{\partial f(q, p, t)}{\partial q} - \frac{\partial U(q)}{\partial q} \frac{\partial f(q, p, t)}{\partial p} = 0$$

(1926)

$$\Psi(x) = |\Psi(x)| \exp i \Phi(x) = \langle x | \Psi \rangle$$



$$|\Psi(x)|^2 = P(x)$$

$$\hat{H} = -\frac{1}{2} \frac{\partial^2}{\partial x^2} + U(x)$$

$$i \frac{\partial \Psi(x,t)}{\partial t} = -\frac{1}{2} \frac{\partial^2}{\partial x^2} \Psi(x,t) + U(x) \Psi(x,t)$$

$$i \frac{\partial}{\partial t} |\Psi, t\rangle = \hat{H} |\Psi, t\rangle$$

(1927)

-5-

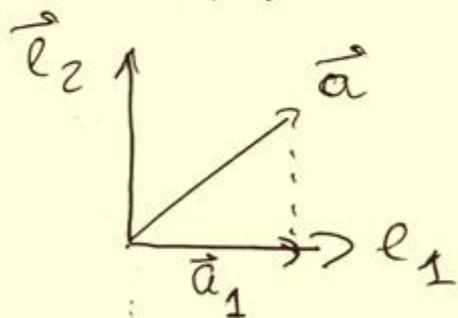
$$\hat{\rho} \longrightarrow \rho(x, x') = \langle x | \hat{\rho} | x' \rangle$$

$$\hat{\rho}_\psi = |\psi\rangle\langle\psi|$$

$$\hat{\rho} = \sum_n P_n |\psi_n\rangle\langle\psi_n|, \quad P_n \geq 0, \quad \sum_n P_n = 1$$

$$|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \langle\psi| = (1 \ 0)$$

$$|\psi\rangle\langle\psi| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1 \ 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \equiv \hat{P}_1$$



$$\hat{P}_1 \vec{a} = \vec{a}_1$$

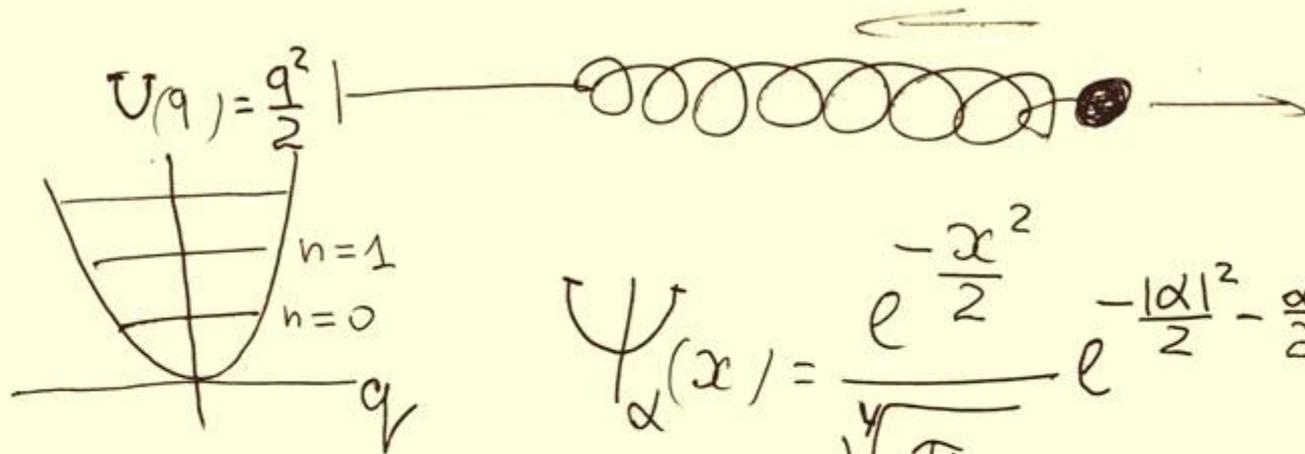
$$\psi_1(x), \quad \psi_2(x)$$

$$\Psi(x) = c_1 \psi_1(x) + c_2 \psi_2(x)$$

$$|\Psi\rangle = \sqrt{P_1} |\psi_1\rangle + \sqrt{P_2} e^{i\varphi} |\psi_2\rangle$$

$$\hat{P}_\Psi = P_1 \hat{P}_1 + P_2 \hat{P}_2 +$$

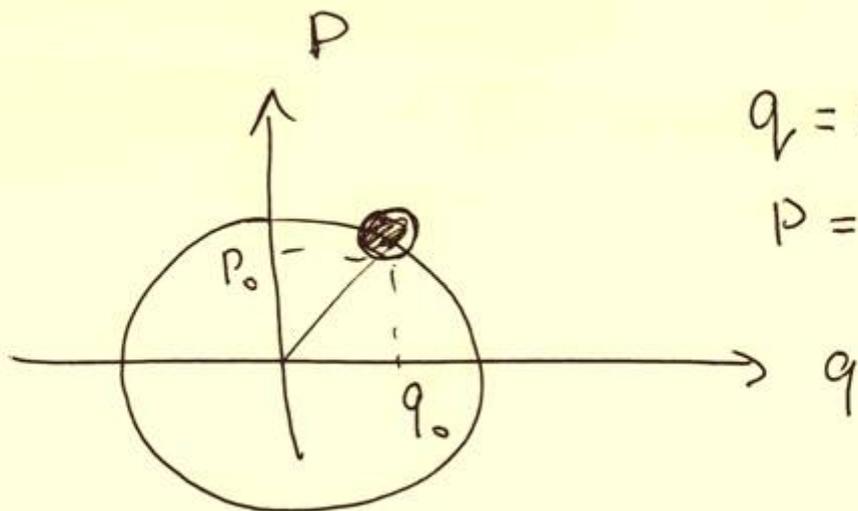
$$+ \sqrt{P_1 P_2} \frac{\hat{P}_1 \hat{P}_0 \hat{S}_2 + \hat{P}_2 \hat{P}_0 \hat{P}_1}{\sqrt{\text{Tr}[\hat{P}_1 \hat{P}_0 \hat{P}_2 \hat{P}_0]}}$$



$$|\Psi_{\alpha}(x)|^2 = \frac{1}{\sqrt{2\pi}\sigma_x^2} \exp - \frac{(x - \bar{x})^2}{2\sigma_x^2}$$

$$\sigma_x^2 = \frac{1}{2}, \quad \bar{x} = \sqrt{2} \operatorname{Re} \alpha, \quad \bar{p} = \sqrt{2} \operatorname{Im} \alpha$$

$$\sigma_p^2 = \frac{1}{2}; \quad \sigma_x^2 \sigma_p^2 = \frac{1}{4}$$



$$q = q_0 \cos t + P_0 \sin t$$

$$P = P_0 \cos t - q_0 \sin t$$

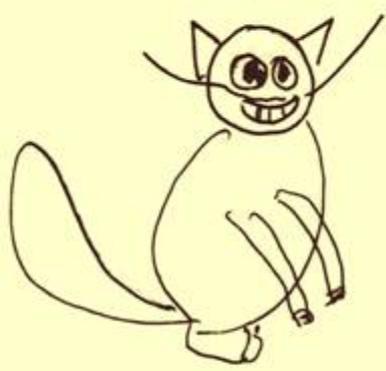
-8-

$$\hat{P}_0(t) = \hat{p} \cos t + \hat{q} \sin t \quad ; \quad \hat{q} = x$$

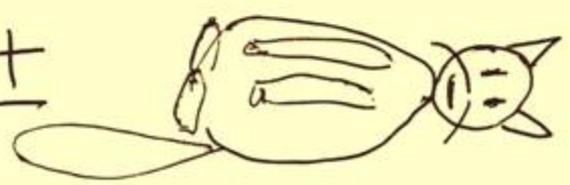
$$(\hat{q}_0(t)) = +\hat{q} \cos t - \hat{p} \sin t$$

$$\hat{p} = -i \frac{\partial}{\partial x}$$

$$\frac{d}{dt} \hat{P}_0(t) = 0, \quad \frac{d}{dt} \hat{q}_0(t) = 0.$$



\pm



(1935)

$$\Psi_{\alpha+}(x) = N_+ (\Psi_{\alpha}(x) + \Psi_{-\alpha}(x))$$

(1974)

$$\Psi_{\alpha-}(x) = N_- (\Psi_{\alpha}(x) - \Psi_{-\alpha}(x))$$

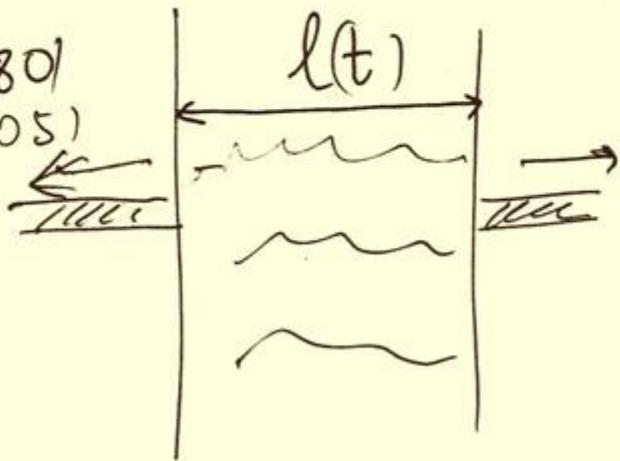
$$\hat{H} = \frac{\hat{p}^2}{2} + \frac{\omega^2(t)\hat{q}^2}{2}; \quad \omega(0)=1$$

(1969)
$$\hat{A}(t) = \frac{i}{\sqrt{2}} (\epsilon(t)\hat{p} - \dot{\epsilon}(t)\hat{q})$$

$$\frac{d\hat{A}(t)}{dt} = 0; \quad \ddot{\epsilon} + \omega^2(t)\epsilon = 0, \quad \epsilon(0)=1, \dot{\epsilon}(0)=i$$

$$\Psi_0(x, t) = \frac{1}{\sqrt{\pi} \sqrt{\epsilon(t)}} \exp\left[\frac{i \dot{\epsilon}(t) x^2}{2 \epsilon(t)}\right]$$

(1980)
(2005)



$$E_n \approx \sum_k \frac{\hbar \omega_k(l)}{2}$$

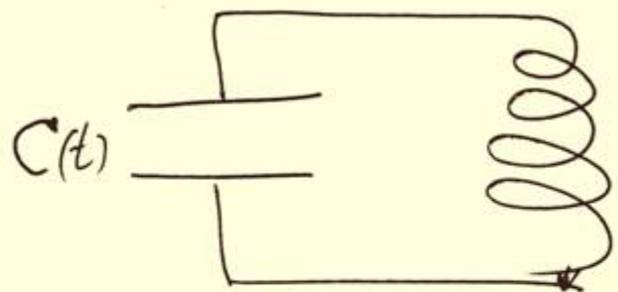
$$F_c \sim \frac{\partial E_n}{\partial l}$$

$$\sigma_x^2 \sigma_p^2 \geq \frac{\hbar^2}{4} \frac{1}{1-r^2}$$

$$\sigma_x^2 = \frac{|E(t)|^2}{2} \leftarrow (\hbar=1)$$

$$\sigma_p^2 = \frac{|\dot{E}(t)|^2}{2} \leftarrow (\hbar=1)$$

$$\sigma_x^2 \sigma_p^2 = \frac{1}{4} \frac{1}{1-r^2}$$



$L(t)$

$$\omega \sim \frac{1}{\sqrt{LC}}$$

$$\hat{q} \sim \hat{V}$$

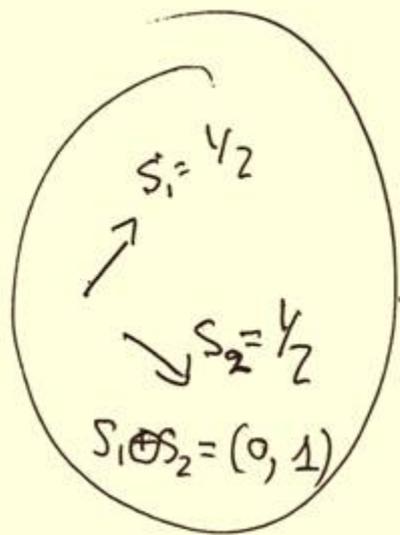
$$\hat{p} \sim \hat{I}$$

$$\frac{\hbar\omega}{T} \gg 1$$

(1980)
|
(2008)

$$\Psi_S(x_1, x_2) = \Phi(x_1) \chi(x_2)$$

$$\Psi_e(x_1, x_2) = c_1 \Phi_1(x_1) \chi_1(x_2) + c_2 \Phi_2(x_1) \chi_2(x_2)$$



$$\rightarrow |S=1, S_z=0\rangle = \frac{1}{\sqrt{2}} \{ |\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2 \}$$

$$\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\rho = \rho^\dagger, \text{Tr} \rho = 1$$

$$\rho \geq 0$$

$$\hat{J}_{SS}(1, 2) = \hat{J}_1(1) \otimes \hat{J}_2(2)$$

$$\hat{J}_S(1, 2) = \sum_n p_n \hat{J}_1^{(n)}(1) \otimes \hat{J}_2^{(n)}(2)$$

$p_n \geq 0, \sum_n p_n = 1$

$$\hat{J}_e(1, 2) \neq \sum_n p_n \hat{J}_1^{(n)}(1) \otimes \hat{J}_2^{(n)}(2)$$

$$t \rightarrow -t, \quad \psi \rightarrow \psi^*$$

$$\psi(x) \psi^*(x') \rightarrow \psi^*(x) \psi(x')$$

$$\rho_\psi \rightarrow \rho_\psi^{tr}, \quad \text{Tr} \rho^{tr} = \text{Tr} \rho = 1, \quad \rho^{tr} \geq 0$$

$$\hat{\rho}_S^{ppt}(1,2) = \sum_n p_n \hat{\rho}_1^{(n)} \otimes \hat{\rho}_2^{(n)tr}$$

$$\left(\hat{\rho}_S^{ppt}(1,2) \right)^+ = \hat{\rho}_S^{ppt}(1,2) \geq 0$$

$$\left[\begin{array}{cc} (a & b) \\ (c & d) \end{array} \otimes \begin{array}{cc} (A & B) \\ (C & D) \end{array} \right] \stackrel{\text{ppt}}{=} \begin{array}{cc} a \begin{array}{cc} (A & B) \\ \swarrow \quad \searrow \\ (C & D) \end{array} & b \begin{array}{cc} (A & B) \\ \swarrow \quad \searrow \\ (C & D) \end{array} \\ c \begin{array}{cc} (A & B) \\ \swarrow \quad \searrow \\ (C & D) \end{array} & d \begin{array}{cc} (A & B) \\ \swarrow \quad \searrow \\ (C & D) \end{array} \end{array}$$

$$P(1,2) \stackrel{PPT}{=} \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix}$$

$$\text{Nil-} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \stackrel{PPT}{=} \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

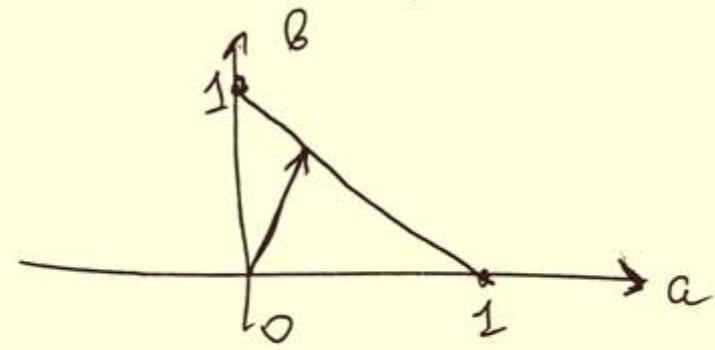
$$\lambda_1 = \frac{1}{2}1, \lambda_2 = \frac{1}{2}1, \lambda_3 = \frac{1}{2}1, \lambda_4 = -\frac{1}{2}$$

!!
 $\lambda_4 = -\frac{1}{2}$

$$1 \geq x, y, z, t \geq 0$$

-18-

$$B_1 = \begin{pmatrix} x & y \\ 1-x & 1-y \end{pmatrix}, \quad B_2 = \begin{pmatrix} z & t \\ 1-z & 1-t \end{pmatrix}$$



$$m_{12} = \pm 1$$

$$\langle m_x \rangle = 2x - 1, \quad \langle m_y \rangle = 2y - 1, \quad \langle m_z \rangle = 2z - 1, \quad \langle m_t \rangle = 2t - 1$$

$$B_{SS} = B_1 \otimes B_2$$

$$I = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix}$$

$$B_{SS} = \begin{pmatrix} xz & xt & yz & yt \\ x(1-z) & x(1-t) & y(1-z) & y(1-t) \\ (1-x)z & (1-x)t & (1-y)z & (1-y)t \\ (1-x)(1-z) & (1-x)(1-t) & (1-y)(1-z) & (1-y)(1-t) \end{pmatrix}$$

$$\langle m_1 m_2 \rangle = 4xz - 2x - 2z + 1$$

$$\text{Tr} \left(\frac{1}{3} B \right) = \langle m_x m_z \rangle + \langle m_x m_t \rangle + \langle m_y m_z \rangle - \langle m_y m_t \rangle = \hat{B}(x, y, z, t)$$

$$= 4(\cancel{z}x + xt + yz - yt) - 4x - 4z + 2$$

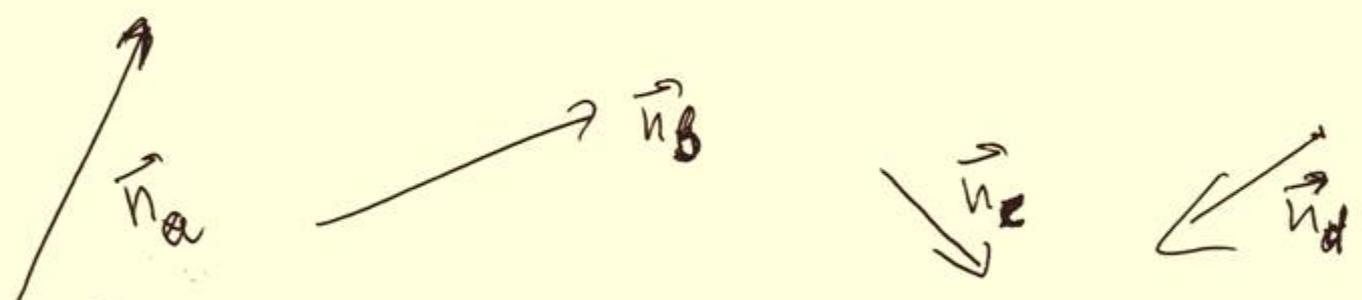
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial t^2} \right) \hat{B}(x, y, z, t) = 0$$

$$\max |B| = 2$$

$$\boxed{|\hat{B}| \leq 2}$$

$$B_s = \sum_n P_n B_1^{(n)} \otimes B_2^{(n)}$$

$$|\hat{B}| = |\text{Tr} \mathbf{I} B_s| \leq 2$$



$$\langle \vec{\sigma}_1 \cdot \vec{n}_a \vec{\sigma}_2 \cdot \vec{n}_b \rangle; \langle \vec{\sigma}_1 \cdot \vec{n}_a \vec{\sigma}_2 \cdot \vec{n}_c \rangle; \langle \vec{\sigma}_1 \cdot \vec{n}_d \vec{\sigma}_2 \cdot \vec{n}_b \rangle; \langle \vec{\sigma}_1 \cdot \vec{n}_d \vec{\sigma}_2 \cdot \vec{n}_c \rangle$$

$$\max |\hat{B}| = 2\sqrt{2} > 2; \quad \langle \cdot \rangle = \text{Tr}(\rho_{(1,2)} \cdot)$$

1932

$$W(q, p) = \int \Psi\left(q + \frac{u}{2}\right) \Psi^*\left(q - \frac{u}{2}\right) e^{-ip u} du$$

$$W^* = W \quad ; \quad \int W(q, p) \frac{dq dp}{2\pi} = 1$$

$$\int W(q, p) \frac{dp}{2\pi} = |\Psi(q)|^2 = \langle q | \Psi \rangle^2$$

$$\int W(q, p) \frac{dq}{2\pi} = |\hat{\Psi}(p)|^2 = \langle p | \Psi \rangle^2$$

$$\frac{W(q, p)}{2\pi} \quad \xrightarrow{\quad ? \quad} \quad f(q, p)$$

$$\langle \hat{q}^n \rangle = \text{Tr} \hat{\rho} \hat{q}^n = \int q^n \frac{W(q, p)}{2\pi} dq dp$$

$$\langle \hat{p}^n \rangle = \text{Tr} \hat{\rho} \hat{p}^n = \int p^n \frac{W(q, p)}{2\pi} dq dp$$

$$W(q, p) \geq 0$$

$$P(x, x') = \frac{1}{2\pi} \int W\left(\frac{x+x'}{2}, p\right) e^{ip(x-x')} dp$$

$$\hat{P} \longleftrightarrow W$$

1932

$$W(q, p) = \int \Psi\left(q + \frac{u}{2}\right) \Psi^*\left(q - \frac{u}{2}\right) e^{-ip u} du$$

$$W^* = W \quad ; \quad \int W(q, p) \frac{dq dp}{2\pi} = 1$$

$$\int W(q, p) \frac{dp}{2\pi} = |\Psi(q)|^2 = \langle q | \Psi \rangle^2$$

$$\int W(q, p) \frac{dq}{2\pi} = |\hat{\Psi}(p)|^2 = \langle p | \Psi \rangle^2$$

$$\frac{W(q, p)}{2\pi} \quad \xrightarrow{\quad ? \quad} \quad f(q, p)$$

$$\langle \hat{q}^n \rangle = \text{Tr} \hat{\rho} \hat{q}^n = \int q^n \frac{W(q, p)}{2\pi} dq dp$$

$$\langle \hat{p}^n \rangle = \text{Tr} \hat{\rho} \hat{p}^n = \int p^n \frac{W(q, p)}{2\pi} dq dp$$

$$W(q, p) \geq 0$$

$$P(x, x') = \frac{1}{2\pi} \int W\left(\frac{x+x'}{2}, p\right) e^{ip(x-x')} dp$$

$$\hat{P} \longleftrightarrow W$$

$$W(X, \theta) = \int f(q, p) \delta(x - q \cos \theta - p \sin \theta) dq dp$$

$$\mu = s \cos \theta, \quad \nu = s^{-1} \sin \theta$$

$$W(\bar{X}, \mu, \nu) = \int f(q, p) \delta(\bar{X} - \mu q - \nu p) dq dp \geq 0$$

$$W(X, \theta) = W(X, \cos \theta, \sin \theta)$$

$$W(\bar{X}, \mu, \nu) = \frac{1}{\sqrt{\mu^2 + \nu^2}} W\left(\frac{\bar{X}}{\sqrt{\mu^2 + \nu^2}}, \arctan \frac{\nu}{\mu}\right)$$

$$\int \psi(X, \mu, \nu) dX = 1$$

$$f(q, p) = \frac{1}{4\pi^2} \int \psi(X, \mu, \nu) e^{i(X - \mu q - \nu p)} dX d\mu d\nu \geq 0$$

$$f(q, p) \longleftrightarrow \psi(X, \mu, \nu)$$

$$\frac{\partial \psi(X, \mu, \nu, t)}{\partial t} - \mu \frac{\partial}{\partial \nu} \psi(X, \mu, \nu, t) - U' \left(-\frac{\partial}{\partial \mu} \left(\frac{\partial}{\partial X} \right)^{-1} \right) \nu \frac{\partial}{\partial X} \psi(X, \mu, \nu, t) = 0$$

$$\hat{\rho} \longleftrightarrow W(X, \mu, \nu)$$

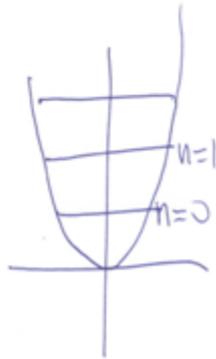
$$\langle \hat{A} \rangle = \text{Tr}(\hat{\rho} \hat{A})$$

$$W(X, \mu, \nu) = \langle \delta(X - \mu q - \nu p) \rangle$$

$$W(X, \mu, \nu) = \langle \delta(X - \mu \hat{q} - \nu \hat{p}) \rangle$$

$$\hat{\rho} = \frac{1}{2\pi} \int W(X, \mu, \nu) e^{i(X - \mu \hat{q} - \nu \hat{p})} dX d\mu d\nu \geq 0$$

$$|0\rangle\langle 0| \longleftrightarrow \omega_0(X, \mu, \nu)$$



$$\omega_0(X, \mu, \nu) = \frac{1}{\sqrt{\pi(\mu^2 + \nu^2)}} e^{-\frac{X^2}{\mu^2 + \nu^2}}$$

$$\omega_n(X, \mu, \nu) = \omega_0(X, \mu, \nu) \frac{1}{2^n n!} H_n^2\left(\frac{X}{\sqrt{\mu^2 + \nu^2}}\right)$$

$$|0\rangle\langle 0| = \frac{1}{2\pi} \int \frac{1}{\sqrt{\pi(\mu^2 + \nu^2)}} e^{-\frac{X^2}{\mu^2 + \nu^2} + i(X - \mu\hat{q} - \nu\hat{p})} dX d\mu d\nu$$

$$\Psi(x) \longleftrightarrow W_{\Psi}(X, \mu, \nu)$$

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$$W_{\Psi}(X, \mu, \nu) = \frac{1}{2\pi|\nu|} \left| \int \Psi(y) e^{\frac{i\mu}{2\nu}y^2 - \frac{iXy}{\nu}} dy \right|^2$$

$$1. i \frac{\partial}{\partial t} \psi(x,t) = \left(-\frac{1}{2} \frac{\partial^2}{\partial x^2} + U(x) \right) \psi(x,t)$$

$$2. i \frac{\partial}{\partial t} (P(x, x', t)) = \left[-\frac{1}{2} \frac{\partial^2}{\partial x^2} + \frac{1}{2} \frac{\partial^2}{\partial x'^2} + U(x) - U(x') \right] P(x, x', t)$$

$$3. \left(\frac{\partial}{\partial t} + p \frac{\partial}{\partial q} \right) W(q, p, t) = -i \left[U \left(q + \frac{i}{2} \frac{\partial}{\partial p} \right) - \text{c.c.} \right] W(q, p, t)$$

$$4. \left(\frac{\partial}{\partial t} - \mu \frac{\partial}{\partial \nu} \right) W(\bar{X}, \mu, \nu, t) = -i \left[U \left(-\frac{\partial}{\partial \mu} \frac{\partial}{\partial \bar{X}} \right)^{-1} + \frac{i}{2} \nu \frac{\partial}{\partial \bar{X}} \right] - \text{c.c.} \right] W(\bar{X}, \mu, \nu, t)$$

$$\left[-\frac{1}{2} \frac{\partial^2}{\partial x^2} + U(x) \right] \psi(x) = E \psi(x) \quad - 8' -$$

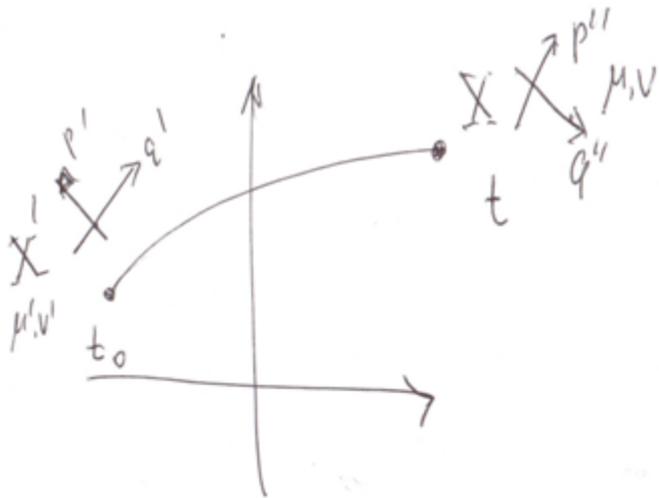
$$1. \frac{1}{2} \left[-\frac{1}{2} \frac{\partial^2}{\partial x^2} - \frac{1}{2} \frac{\partial^2}{\partial x'^2} + U(x) + U(x') \right] \rho(x, x') = E \rho(x, x')$$

$$2. -\frac{1}{4} \left[\left(\frac{1}{2} \frac{\partial}{\partial q} + ip \right) + c.c. \right] W(q, p) + \frac{1}{2} \left[U \left(q + \frac{i}{2} \frac{\partial}{\partial p} \right) + c.c. \right] W(q, p) = E W(q, p)$$

$$3. -\frac{1}{4} \left[\left(\frac{1}{2} \mu \frac{\partial}{\partial X} - i \frac{\partial}{\partial v} \left(\frac{\partial}{\partial X} \right)^{-1} \right) + c.c. \right] W(X, \mu, v) + \frac{1}{2} \left[U \left(-\frac{\partial}{\partial \mu} \left(\frac{\partial}{\partial X} \right)^{-1} + \frac{i}{2} v \frac{\partial}{\partial X} \right) + c.c. \right] W(X, \mu, v) = E W(\bar{X}, \mu, v)$$

$$W(\bar{X}, \mu, \nu, t) = \int \prod (\bar{X}, \mu, \nu, \bar{X}', \mu', \nu', t, t_0) W(\bar{X}', \mu', \nu', t_0) d\bar{X}' d\mu' d\nu'$$

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$$\Psi(x, t) = \int G(x, y, t) \Psi(y) dy$$

$$\Gamma(X, \mu, v, X', \mu', v', t) = \frac{1}{4\pi^2} \int k^2 \delta(y - z - kv') dk dy dz da.$$

$$G\left(a + \frac{kv}{2}, y, t\right) G^*\left(a - \frac{kv}{2}, z, t\right).$$

$$\exp\left[ik\left(X' - X + \mu a - \mu' \frac{y+z}{2}\right)\right].$$

$$G(x, y, t) = \int d[x(t)] e^{iS[x(t)]}$$

$$\hat{U}(\vec{x}), \hat{D}(\vec{x})$$

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$$\text{Tr } \hat{U}(\vec{x}) \hat{D}(\vec{x}') = \delta(\vec{x} - \vec{x}')$$

$$\hat{A} \rightarrow f_A(\vec{x}) = \text{Tr } \hat{A} \hat{U}(\vec{x})$$

$$f_A(\vec{x}) \rightarrow \hat{A} = \int f_A(\vec{x}) \hat{D}(\vec{x}) d\vec{x}$$

$$f_A(\vec{x}) * f_B(\vec{x}) := f_{AB}(\vec{x}) = \int K(\vec{x}_1, \vec{x}_2, \vec{x}) f_A(\vec{x}_1) f_B(\vec{x}_2) d\vec{x}_1 d\vec{x}_2$$

$$K(\vec{x}_1, \vec{x}_2, \vec{x}) = \text{Tr} (\hat{D}(\vec{x}_1) \hat{D}(\vec{x}_2) \hat{U}(\vec{x}))$$

$$\vec{x} = (X, \mu, \nu)$$

$$\hat{U}(\vec{x}) = \delta(X - \mu \hat{q} - \nu \hat{p})$$

$$\hat{D}(\vec{x}) = \frac{1}{2\pi i} \exp i(X - \mu \hat{q} - \nu \hat{p})$$

$$K(X_1, \mu_1, \nu_1, X_2, \mu_2, \nu_2, X, \mu, \nu) = \frac{1}{4\pi^2} \delta[\mu(\nu_1 + \nu_2) - \nu(\mu_1 + \mu_2)] \cdot$$

$$\cdot \exp \left\{ \frac{i}{2} \left[\nu_1 \mu_2 - \nu_2 \mu_1 + 2X_1 + 2X_2 - \left(\frac{\nu_1 + \nu_2}{\nu} + \frac{\mu_1 + \mu_2}{\mu} \right) X \right] \right\}$$

$$\langle \hat{A} \rangle = \text{Tr} \hat{S} \hat{A} = \int w(x, \mu, \nu) f_A^{(d)}(x, \mu, \nu) dx d\mu d\nu$$

$$f_A^{(d)}(x, \mu, \nu) = \text{Tr} \hat{A} \hat{D}(x, \mu, \nu)$$

$$\left\{ \int \underline{X}^2 w(\underline{X}, \Theta) d\underline{X} - \left[\int \underline{X} w(\underline{X}, \Theta) d\underline{X} \right]^2 \right\}.$$

$$\circ \left\{ \int \underline{X}^2 w(\underline{X}, \Theta + \frac{\pi}{2}) d\underline{X} - \left[\int \underline{X} w(\underline{X}, \Theta + \frac{\pi}{2}) d\underline{X} \right]^2 \right\} - \frac{1}{4} \geq 0$$

$$\psi(x) \rightarrow S_x = - \int |\psi(x)|^2 \ln |\psi(x)|^2 dx$$

$$\tilde{\psi}(p) \rightarrow S_p = - \int |\tilde{\psi}(p)|^2 \ln |\tilde{\psi}(p)|^2 dp$$

$$S_x + S_p \geq \ln(\pi e)$$

$$- \int w(x, \theta) \ln w(x, \theta) dx - \int w(x, \theta + \frac{\pi}{2}) \ln w(x, \theta + \frac{\pi}{2}) dx \geq \ln \pi e$$

$$w(x, \mu, \nu) = \int W(q, p) \delta(x - \mu q - \nu p) \frac{dq dp}{2\pi}$$

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$$W(q, p) = \frac{1}{2\pi} \int w(x, \mu, \nu) e^{i(x - \mu q - \nu p)} dx d\mu d\nu$$

$$W(q, p) = \text{Tr} \hat{\rho} \hat{U}(q, p)$$

$$\hat{U}(q, p) = 2 \exp(2\alpha \hat{a}^\dagger - 2\alpha^* \hat{a}) \hat{I}, \quad \alpha = \frac{q + ip}{\sqrt{2}}, \hat{a} = \frac{\hat{q} + i\hat{p}}{\sqrt{2}}$$

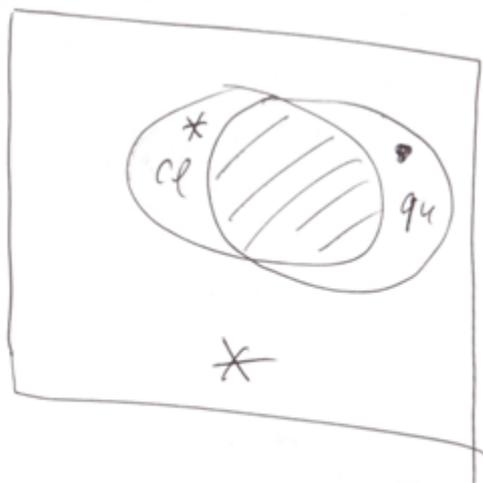
$$\hat{D}(q, p) = \frac{1}{2\pi} \hat{U}(q, p)$$

$$\hat{I} \psi(x) = \psi(x)$$

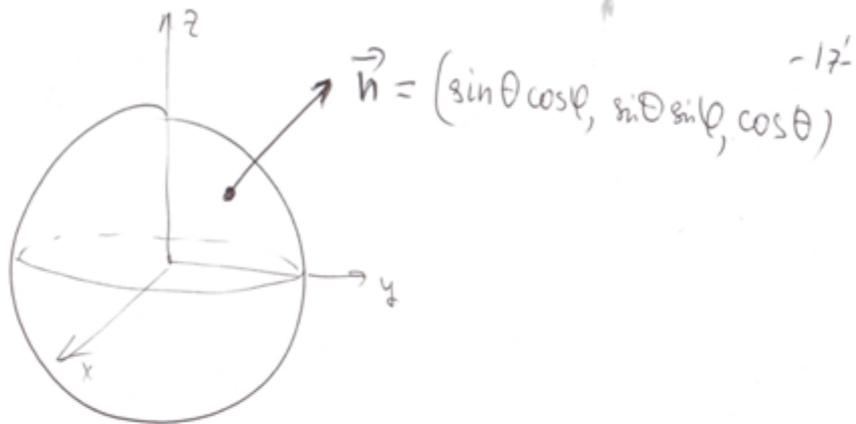
$$1. \int w(x, \mu, \nu) e^{i(x - \mu q - \nu p)} dx d\mu d\nu \geq 0$$

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$$2. \int w(x, \mu, \nu) e^{i(x - \mu \hat{q} - \nu \hat{p})} dx d\mu d\nu \geq 0$$



$$S = \frac{1}{2}$$



$$W(m, u) = \langle m | u^\dagger \rho u | m \rangle \quad m = \pm \frac{1}{2}$$

$$u = \begin{pmatrix} \cos \frac{\theta}{2} e^{i(\varphi+\psi)/2} & \sin \frac{\theta}{2} e^{i(\varphi-\psi)/2} \\ -\sin \frac{\theta}{2} e^{-i(\varphi-\psi)/2} & \cos \frac{\theta}{2} e^{-i(\varphi+\psi)/2} \end{pmatrix}$$

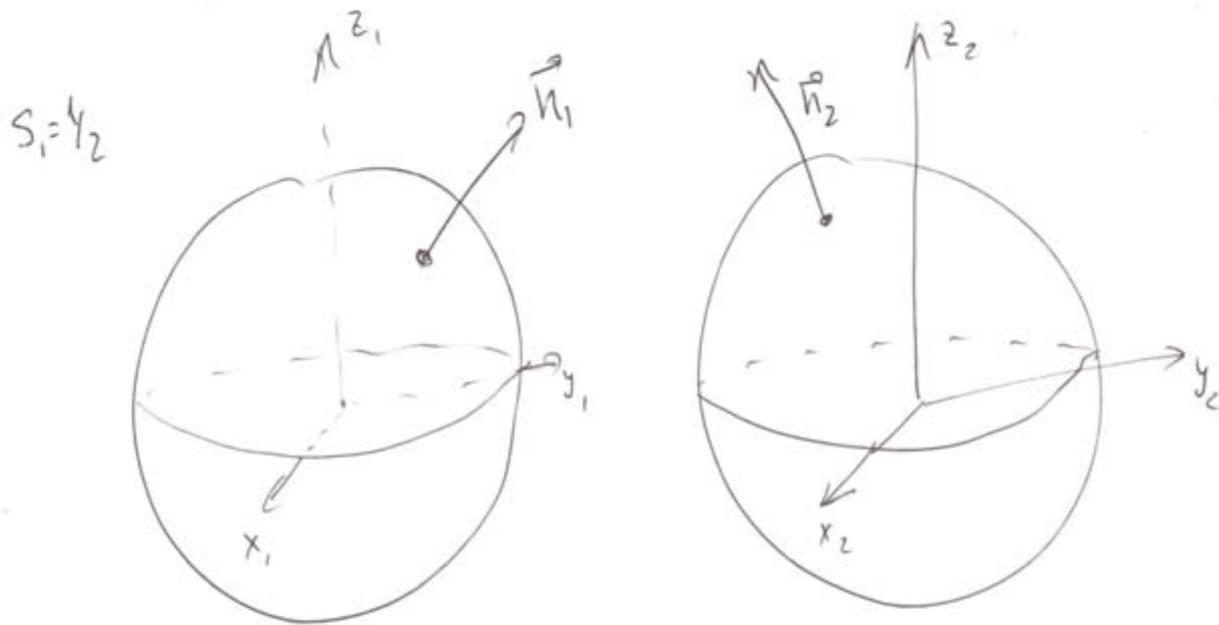
$$W(m, \vec{n}) = \text{Tr} \hat{\rho} \hat{U}(m, \vec{n})$$

$$\hat{U}(m, \vec{n}) = \frac{1}{2\pi} \int_0^{2\pi} e^{im\varphi} e^{-i(\vec{n} \cdot \vec{J})\varphi} d\varphi$$

$$\hat{\rho} = \sum_{m=-j}^j \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^\pi \sin\theta d\theta W(m, \vec{n}) \hat{D}(m, \vec{n})$$

$$\hat{D}(m, \vec{n}) = \frac{2j+1}{\pi} \int_0^{2\pi} \left(\sin^2 \frac{\varphi}{2}\right) e^{im\varphi} e^{-i(\vec{n} \cdot \vec{J})\varphi} d\varphi$$

$$\hat{\rho} \longleftrightarrow W(m, \vec{n})$$



$$W(m_1, \vec{n}_1, m_2, \vec{n}_2) = \langle m_1, m_2 | U_4^\dagger \rho U_4 | m_1, m_2 \rangle$$

$$U_4 = U(\vec{n}_1) \otimes U(\vec{n}_2)$$

$$\rho_{(1,2)} = \sum_n P_n \rho_{(1)}^{(n)} \otimes \rho_{(2)}^{(n)}$$

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$$W(m_1 \vec{n}_1, m_2 \vec{n}_2) = \sum_n P_n W_1^{(n)}(m_1 \vec{n}_1) W_2^{(n)}(m_2 \vec{n}_2)$$

$$M = \begin{pmatrix} w(+ + \vec{a} \vec{b}) & w(+ + \vec{a} \vec{c}) & w(+ + \vec{d} \vec{b}) & w(+ + \vec{d} \vec{c}) \\ w(+ - \vec{a} \vec{b}) & w(+ - \vec{a} \vec{c}) & w(+ - \vec{d} \vec{b}) & w(+ - \vec{d} \vec{c}) \\ w(- + \vec{a} \vec{b}) & w(- + \vec{a} \vec{c}) & w(- + \vec{d} \vec{b}) & w(- + \vec{d} \vec{c}) \\ w(- - \vec{a} \vec{b}) & w(- - \vec{a} \vec{c}) & w(- - \vec{d} \vec{b}) & w(- - \vec{d} \vec{c}) \end{pmatrix} \quad -21-$$

$$I = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix}$$

$$|\text{Tr}(M I)| = |B(\vec{a}, \vec{b}, \vec{c}, \vec{d})| \begin{cases} \leq 2 \\ > 2 \end{cases}$$

