Resumen

- simetrías más generales QFT
  Super Poincaré , internal.

- esquemáticamente:

$$[M,M] = M$$
$$[M,P] = P$$
$$[T,T] = T$$

$Q_\alpha \quad \bar{Q}_{\dot\alpha}$

$$\{Q_\cdot, \bar{Q}_\cdot\} = P$$

- $SO(3,1) \simeq SU(2) \otimes SU(2) \Rightarrow$ rep of $SO(3,1)$ identificada con 2 etiquetas $SU(2) \times SU(2)$

realmente $\quad \mathcal{L}_c\,(SO(3,1)) \simeq \underbrace{\mathcal{L}_c\,(SU(2) \times SU(2))}_{SL(2,\mathbb{C})}$

$(\frac{1}{2}, \frac{1}{2})$, $(0, \frac{1}{2})$ etc fundamental rep of $SL(2,\mathbb{C})$: $N_\alpha^{\ \beta}$ acts on L h. Weyl ψ_β

anti fundamental " " " $(N^*)_{\dot\alpha}^{\ \dot\beta}$ acts on R h Weyl $\bar\chi_{\dot\beta}$

- ψ_α , $\bar\chi_{\dot\alpha}$ $\alpha = 1, 2$

indices espinoriales se suben y bajan con $\epsilon_{\alpha\beta}$

" espaciales " " " " " $\eta_{\mu\nu}$.

- $(\sigma^{\mu\nu})_{\alpha}^{\ \beta}$

SUSY : bosones $\leftrightarrow$ fermiones

SUSY : traslaciones en superespacio

$$Z^M = (x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$$

$d=4$
si $\mu = 1, 2, 3, 4$
superespacio
es 8 dimensional

$\alpha = 1, 2$

θ^α y $\bar{\theta}_{\dot{\alpha}}$ variables de Grassman.
es decir
anticonmutan

Cálculo en Superespacio

$\theta_\alpha \qquad \bar{\theta}_{\dot{\alpha}}$ espinores de Weyl $\alpha = 1, 2$

θ_1 , θ_2 son Grassmann.

$\{ \theta_\alpha , \theta_\beta \} = 0 \qquad \theta_1 \theta_2 + \theta_2 \theta_1 = 0$

$\theta_1 \theta_2 = -\theta_2 \theta_1$

$\Rightarrow \theta_\alpha^2 = 0 \qquad \theta_1 \theta_1 = -\theta_1 \theta_1 \Rightarrow \theta_1 \theta_1 = 0$

(no suma) $\theta_\alpha \theta_\alpha = 0$

$\theta\theta \equiv \theta^\alpha \theta_\alpha \neq 0$

$$\theta^2 = \theta^1\theta_1 + \theta^2\theta_2$$

$$= \epsilon^{1\beta}\theta_\beta\theta_1 + \epsilon^{2\beta}\theta_\beta\theta_2$$

$$= \epsilon^{12}\theta_2\theta_1 + \epsilon^{21}\theta_1\theta_2 = -\epsilon^{12}\theta_1\theta_2 + \epsilon^{21}\theta_1\theta_2$$

$$\epsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = -\epsilon_{\alpha\beta}$$

$$= -2\theta_1\theta_2$$

$$\boxed{\theta^2 = -2\theta_1\theta_2}$$

$$\boxed{\bar{\theta}^2 = 2\bar{\theta}^{\dot{1}}\bar{\theta}^{\dot{2}}}$$

Si Tenemos una funcion de θ_α y queremos expandir en potencias de θ_α, lo $\boxed{\text{mas general}}$ posible es (con un solo θ_α)

$$f(\theta_\alpha) = f_0 + f_1 \theta_\alpha \qquad \alpha = 1 \text{ ó } 2$$

Diferenciación

$$\partial_\alpha \theta^\beta = \frac{\partial \theta^\beta}{\partial \theta^\alpha} = \delta^\beta_\alpha$$

regla de producto

$$\partial_\alpha (\theta^{\beta_1}\theta^{\beta_2}\cdots\theta^{\beta_n}) = (\partial_\alpha \theta^{\beta_1})\theta^{\beta_2}\cdots\theta^{\beta_n} - \theta^{\beta_1}(\partial_\alpha \theta^{\beta_2})\theta^{\beta_3}\cdots\theta^{\beta_n}$$

$$+ \theta^{\beta_1}\theta^{\beta_2}\partial(\theta^{\beta_3})\ldots\theta^{\beta_n} + \cdots\cdots$$

$$+ (-1)^{n-1}\theta^{\beta_1}\theta^{\beta_2}\ldots\left(\partial_\alpha \theta^{\beta_n}\right).$$

<u>Integración</u>

definimos $\boxed{\int d\theta_\alpha \equiv 0}$ $\boxed{\int d\theta_\alpha\, \theta_\alpha \equiv 1}$

y pedimos linearidad

$$\int d\theta_\alpha \left(c_1 f(\theta_\alpha) + c_2 g(\theta_\alpha)\right)$$

$$\equiv c_1 \int d\theta_\alpha f(\theta_\alpha) + c_2 \int d\theta_\alpha\, g(\theta_\alpha)$$

Notar que para $f(\theta) = f_0 + \theta f_1$

$$\int d\theta\, f(\theta) = f_1 \quad \rightarrow \quad \int = \partial$$

$$\int d\theta\, \delta(\theta)\, f(\theta) = f_0 \qquad \theta = \delta(\theta)$$

generalizando a mas de un θ, θ_1, θ_2, $\bar\theta_1, \bar\theta_2$

$$f(\theta,\bar\theta) = f_0 + \theta f_1 + \bar\theta \tilde f_1 + \theta\theta f_2 + \bar\theta\bar\theta \tilde f_2 + \cancel{\theta}\theta\bar\theta\bar\theta f_4$$
$$+ \bar\theta\bar\theta\theta\theta \tilde f_4$$

no hay terminos cúbicos $\theta\theta\theta$, $\theta\theta\bar\theta$, ...

porque?

ni $\theta\bar\theta$

ni terminos quinticos $\theta\theta\theta\theta\theta$

definimos la medida de integración

$d^4\theta = d^2\theta\, d^2\bar{\theta}$ $\quad d^2\theta = \frac{1}{4} d\theta^\alpha d\theta_\alpha$ $\quad d^2\bar{\theta} = \frac{1}{4} d\bar{\theta}_{\dot\alpha} d\bar{\theta}^{\dot\alpha}$

$\Rightarrow$ se puede probar que

$$\int d^2\theta\, \theta\theta = \int d^2\bar{\theta}\, \bar{\theta}\bar{\theta} = 1$$

ejercicio

$$\int d^2\theta\, d^2\bar{\theta}\, \theta\theta\bar{\theta}\bar{\theta} = 1$$

$$\Rightarrow \int d^2\theta\, f(\theta,\bar{\theta}) = \int d^2\theta \left(f_0 + f_1\theta + \tilde{f}_1\bar{\theta} + f_2\theta\theta + \dots \right)$$

$$= f_2$$

$$\int d^2\bar{\theta}\, F(\theta,\bar{\theta}) = f_2$$

$$\int d^2\theta\, d^2\bar{\theta}\, f(\theta,\bar{\theta}) = f_4$$

$$\int d^2\theta\, d^2\bar{\theta}\, f(\theta,\bar{\theta}) \sim f(\theta,\bar{\theta})\Big|_{\theta^2\bar{\theta}^2}$$

$$\int d^2\theta\, f(\theta,\bar{\theta}) \sim f(\theta,\bar{\theta})\Big|_{\theta^2}$$

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interludio físico: unidades!!

θ^α tiene unidades o es adimensional?

- $\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$

- un elemento del grupo de super Poincaré

$$g \in SP$$

$$g = \exp\left[-i X^\mu P_\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot\alpha}\bar{Q}^{\dot\alpha} + \frac{i}{2}\omega_{\rho\mu} M^{\rho\mu}\right]$$

$$[P_\mu] = M \Rightarrow [Q^2] = M \Rightarrow [Q] = M^{1/2}$$

$$[\theta Q] = 0 \Rightarrow \boxed{[\theta] = M^{-1/2}}$$

Derivadas Covariantes

derivada invariante bajo SUSY

i.e. q' conmute con los SUSY generators

$$P_\mu = i\,\partial_\mu \qquad \checkmark$$

se puede construir algo mas general

$$D_\alpha = \partial_\alpha + i(\sigma^\mu \bar{\theta})_\alpha \partial_\mu$$

$$\bar{D}_{\dot\alpha} = -\partial_\alpha - i(\theta \sigma^\mu)_{\dot\alpha} \partial_\mu$$

satisfacen

$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = 0$$

$$\{D_\alpha, \bar{D}_{\dot\beta}\} = -2\,\sigma^\mu{}_{\alpha\dot\beta}\,P_\mu$$

$$\Rightarrow \quad D^\alpha \bar{D}^2 D_\alpha = \bar{D}_{\dot\alpha} D^2 \bar{D}^{\dot\alpha}$$

Pero tiene demasiados grados de libertad para ser una rep irreducible de SUSY con spin ≤ 1

$\Rightarrow$ obtener campos irreducibles (smallest building block) imponiendo restricciones en el superfield.

Vamos a ver 2 tipos de restricciones

Left handed chiral $\bar{D}_{\dot{\alpha}} \Phi = 0$

right handed chiral $D_{\alpha} \Phi = 0$

chiral superfield

$\Phi = \Phi^{\dagger}$

vector (real) superfield

CHIRAL SUPERFIELDS

dado queremos resolver - encontrar que condiciones deben satisfacer los componentes:

$$\bar{D}_{\dot{\alpha}} \Phi = 0$$

recordar que: $\bar{D}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} - i(\theta \sigma^{\mu})_{\dot{\alpha}\mu}$

..... no es fácil ....

viejo truco de cambio de coordenadas

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

$$\bar{y}^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$$

es fácil ver que (ejercicio)

$$\bar{D}_{\dot\alpha} y^\mu = D_\alpha \bar{y} = 0$$

$$\bar{D}_{\dot\alpha}\theta_\beta = D_\alpha\bar{\theta}_{\dot\beta} = 0$$

$\Rightarrow$ en coordenadas y

$$\bar{D}_{\dot\alpha}\Phi(y,\theta,\bar\theta) = \bar{D}_{\dot\alpha}\Big(f(y) + \theta(\,) + \cancel{\bar\theta(\,)} + \theta\theta(\,) + \cancel{\bar\theta\bar\theta}(\,)$$

$$+ \theta\sigma^\mu V_\mu + \cancel{\theta\bar\theta\bar\theta}(\,) + \cancel{\bar\theta\bar\theta\theta}(\,) + \cancel{\theta\theta\bar\theta\bar\theta}(\,)\Big) = 0$$

$\Rightarrow$ para tener $\bar{D}_{\dot\alpha}\Phi = 0$ imponemos

$$\Phi = \Phi(x,\theta)$$

$\Rightarrow$ chiral superfield in "y" coordinate

$$\boxed{\Phi(y,\theta) = \underbrace{A(y)}_{2\,\text{dof}} + \sqrt{2}\,\theta\underbrace{\psi(y)}_{4\,\text{dof}} + \theta\theta \underbrace{F(y)}_{2\,\text{dof}}}$$

4f dof
4b dof

antichiral

$$\boxed{\Phi^{\dagger}(y,\theta) = A^{*}(y) + \sqrt{2}\,\bar\theta\,\bar\psi(y) + \bar\theta\bar\theta\, F^{*}(y)}$$

$\bar\theta$ esta escondida en y.

Si regresamos a la variable x se obtiene (ejercicio)

$$\Phi(x,\theta,\bar\theta) = \phi(x) + \sqrt{2}\,\theta\psi(x) + i\theta\sigma^{\mu}\bar\theta\,\partial_{\mu}\phi(x)$$

$$- \theta\theta F(x) - \frac{i}{\sqrt{2}}\theta\theta\,\partial_{\mu}\psi(x)\sigma^{\mu}\bar\theta - \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\Box\phi(x)$$

Esto tiene menos grados de libertad q' el superfield general

superfield chiral. $\left(\phi(x) , \psi(x) , F(x) \right)$ F(x) no es físico, es auxiliar: puede ser eliminado x q' sus eom no tienen derivadas

$\downarrow$ 2dof $\downarrow$ 4 dof $\downarrow$ 2dof

" componentes "

fermiones $\longleftrightarrow$ bosones

como cambian campos bajo SUSY?

$$\tilde{S}(x^\mu; \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) = e^{i(\epsilon Q + \bar{\epsilon}\bar{Q})} S(x^\mu; \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$$

$$\Rightarrow \quad \delta S = i(\epsilon Q + \bar{\epsilon}\bar{Q}) S(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$$

Los generadores Q y $\bar{Q}$ se pueden expresar como

$$Q_\alpha = -i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu)_{\alpha\dot\beta}\,\bar\theta^{\dot\beta}\frac{\partial}{\partial x^\mu}$$

$$\bar{Q}_{\dot\alpha} = i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \theta^\beta(\sigma^\mu)_{\beta\dot\alpha}\frac{\partial}{\partial x^\mu}$$

$\Rightarrow$ mostrar que (ejercicio)

$$\delta\phi = \sqrt{2}\,\epsilon\psi$$

$$\delta\psi_\alpha = \sqrt{2}\,i(\sigma^\mu\bar\epsilon)_\alpha\,\partial_\mu\phi - \sqrt{2}\,\epsilon_\alpha F$$

$$\delta F = i\sqrt{2}\,\partial_\mu(\psi\sigma^\mu\bar\epsilon)$$

$$\frac{\partial\theta^\alpha}{\partial\theta^\beta} = \delta^\alpha_\beta \qquad \frac{\partial\bar\theta^{\dot\alpha}}{\partial\bar\theta^{\dot\beta}} = \delta^{\dot\alpha}_{\dot\beta}$$

$$\frac{\partial\theta_\alpha}{\partial\theta_\beta} = -\delta^\alpha_\beta \qquad \frac{\partial\bar\theta_{\dot\alpha}}{\partial\bar\theta_{\dot\beta}} = -\delta^{\dot\alpha}_{\dot\beta}$$

$$\delta\phi = \sqrt{2}\,\epsilon\psi$$

$$S(y,\theta,\bar\theta) = A(y) + \sqrt{2}\,\theta\psi(y) + \theta\theta F(y)$$

$$\delta S = i(\epsilon Q + \bar\epsilon\bar Q)S$$

donde

$$Q_\alpha = -i\frac{\partial}{\partial\theta^\alpha}$$

$$\bar Q_{\dot\alpha} = i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + 2\theta^\beta(\sigma^\mu)_{\beta\dot\alpha}\frac{\partial}{\partial x^\mu}$$

para encontrar $\delta\phi(x)$ identificar componentes que luego de aplicar $i(\epsilon Q + \bar\epsilon\bar Q)S$ no tienen ni θ ni $\bar\theta$

$$\delta S = \left(\epsilon\frac{\partial}{\partial\theta^\alpha} - \bar\epsilon\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\right)S + \left(i\bar\epsilon 2\theta\sigma^\mu\frac{\partial}{\partial x^\mu}\right)S$$

$$= \epsilon\sqrt{2}\psi(y) + \epsilon\theta F(y) + 2i\bar\epsilon\theta\sigma^\mu\partial_\mu A + \sqrt{2}\,2i\,\theta\sigma^\mu\theta\partial_\mu\psi + i\bar\epsilon\theta\cancel{\theta}\,\cancel{\theta\sigma^\mu}\partial_\mu F$$

(ejercicio)

Notar q' el productos de supercampos chirales es otro supercampo chiral

$\Phi_i \Phi_j$ chiral

$\Phi_i \Phi_j \Phi_k$ chiral

Pero

$\Phi_i^{+} \Phi_j$ no es chiral

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otra restriccion q' podemos imponer sobre el supercampo mas general es "realidad" $\Rightarrow$ supercampo vectorial (real)

$$V = V^{+}$$

recordar que el campo mas general es

$$\Phi(x,\theta,\bar\theta) = f(x) + \theta\psi(x) + \bar\theta\bar\chi(x) + \theta\theta\, m(x) + \bar\theta\bar\theta\, n(x) + \theta\sigma^\mu\bar\theta\, V_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\rho(x) + \theta\theta\bar\theta\bar\theta\, d(x)$$

$\Rightarrow$ para q' sea real

$$\psi(x) = \chi(x) \qquad f(x) = f^*(x)$$

$$m(x) = n^*(x)$$

$$\rho(x) = \lambda(x)$$

$$d(x) = d^*(x)$$

$$\Rightarrow V(x,\theta,\bar\theta) = f(x) + \theta\chi(x) + \bar\theta\bar\chi + \theta\theta\, m(x) + \bar\theta\bar\theta\, m^*(x) + \theta\sigma^\mu\bar\theta\, V_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\lambda(x) + \theta\theta\bar\theta\bar\theta\, D(x)$$

f, D : real scalars

ψ, λ : Weyl spinors

m : complex scalar

V_μ : real Lorentz 4 vector

Supergauge Transformation
(abelian)

$$V(x,\theta,\bar\theta) \to V'(x,\theta,\bar\theta) = V(x,\theta,\bar\theta) + \Phi(x,\theta,\bar\theta) + \Phi^{+}(x,\theta,\bar\theta)$$

$$= V(x,\theta,\bar\theta) + i\,(\Lambda(x,\theta,\bar\theta) - \Lambda^{+}(x,\theta,\bar\theta))$$

se puede mostrar que $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ invariante

Wess-Zumino (WZ) gauge

Transformacion de gauge por un vector superfield.

$$\Phi : (A(x), \psi(x), F(x))$$

donde

$$F(x) = -M(x)$$

$$A(x) + A^{*}(x) = -f(x)$$

$$\psi(x) = -\frac{1}{\sqrt{2}}\chi(x)$$

$$\Rightarrow \quad V_{WZ}(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta \overbrace{\left(v_\mu(x) + i\,\partial_\mu\left(A(x) - A^*(x)\right)\right)}^{V_\mu}$$

$$+ \; \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\,\theta\lambda(x) + \theta\theta\bar\theta\bar\theta\, D(x)$$

$$\boxed{V(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta\, V_\mu + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\lambda(x) + \theta\theta\bar\theta\bar\theta D(x)}$$

gauge bosons

gaugino

campo escalar auxiliar D-term.

INTERACCIONES